THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Parametric Estimation Techniques for Space-Time Adaptive Processing

with applications in airborne bistatic radar systems

Jacob Klintberg

CHALMERS
UNIVERSITY OF TECHNOLOGY

Department of Electrical Engineering
Chalmers University of Technology
Gothenburg, Sweden, 2023

Parametric Estimation Techniques for
Space-Time Adaptive Processing
with applications in airborne bistatic radar systems

JACOB KLINTBERG
ISBN 978-91-7905-793-0

Doktorsavhandlingar vid Chalmers tekniska högskola
Ny serie nr 5259
ISSN 0346-718X

Department of Electrical Engineering
Chalmers University of Technology
SE-412 96 Gothenburg, Sweden
Phone: +46 (0)31 772 1000

Printed by Chalmers Digitaltryck
Gothenburg, Sweden, January 2023

To my family.

Parametric Estimation Techniques for Space-Time Adaptive Processing

with applications in airborne bistatic radar systems

Jacob Klintberg
Department of Electrical Engineering
Chalmers University of Technology

Abstract

This thesis considers parametric scenario based methods for Space-Time Adaptive Processing (STAP) in airborne bistatic radar systems. STAP is a multidimensional filtering technique used to mitigate the influence of interference and noise in a target detector. To be able to perform the mitigation, an accurate estimate is required of the associated space-time covariance matrix to the interference and noise distribution. In an airborne bistatic radar system geometry-induced effects due to the bistatic configuration introduces variations in the angle-Doppler domain over the range dimension. As a consequence of this, clutter observations of such systems may not follow the same distribution over the range dimension. This phenomena may affect the estimator of the space-time covariance matrix.

In this thesis, we study a parametric scenario based approach to alleviate the geometry-induced effects. Thus, the considered framework is based on so called radar scenarios. A radar scenario is a description of the current state of the bistatic configuration, and is thus dependent on a few parameters connected to the two radar platforms which comprise the configuration. The scenario description can via a parametric model be used to represent the geometry-induced effects present in the system. In the first topic of this thesis, an investigation is conducted of the effects on scenario parameter residuals on the performance of a detector. Moreover, two methods are presented which estimate unknown scenario parameters from secondary observations. In the first estimation method, a maximum likelihood estimate is calculated for the scenario parameters using the most recent set of secondary data. In the second estimation method, a density is formed by combination of the likelihood associated with the most recent set of radar observations with a prior density obtained by propagation of previously considered scenario parameter estimates through a dynamical model of the scenario platforms motion over time. From the formed density a maximum a posteriori estimate of the scenario parameters can be derived. Thus, in the second estimation method, the radar scenario is tracked over time. Consequently, in the first topic of the thesis, the sensitivity between scenario parameters and detector performance is evaluated in various aspects, and two methods are investigated to estimate unknown scenario parameters from different radar scenarios.

In the second part of the thesis, the scenario description is used to estimate a space-time covariance matrix and to derive a generalized likelihood

ratio test for the airborne bistatic radar configuration. Consequently, for the covariance matrix estimate, the scenario description is used to derive a transformation matrix framework which aims to limit the non-stationary behavior of the secondary data observed by a bistatic radar system. Using the scenario based transformation framework, a set of non-stationary secondary data can be transformed to become more stationarily distributed after the transformation. A transformed set of secondary data can then be used in a conventional estimator to estimate the space-time covariance matrix. Furthermore, as the scenario description provides a representation of the geometry-induced effects in a bistatic configuration, the scenario description can be used to incorporate these effects into the design of a detector. Thus, a generalized likelihood ratio test is derived for an airborne bistatic radar configuration. Moreover, the presented detector is adaptive towards the strength of both the clutter interference and the thermal noise.

Keywords: Airborne Bistatic Radar Systems, Parametric Space-Time Adaptive Processing.

List of Publications

This thesis is based on the following publications:

[A] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, "Mitigation of Ground Clutter in Airborne Bistatic Radar Systems". Published in IEEE Sensor Array and Multichannel Signal Processing Workshop, June 2020.

[B] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, "A Parametric Approach to Space-Time Adaptive Processing in Bistatic Radar Systems". Published in IEEE Transactions on Aerospace and Electronic Systems, April 2022.

[C] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, "Scenario Tracking for Airborne Bistatic Radar Systems". Submitted for publication in IEEE Transactions on Aerospace and Electronic Systems, Jan 2023.

[D] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, "Scenario Based Transformations for Compensation of Non-Stationary Radar Clutter". Submitted for publication in IEEE Transactions on Aerospace and Electronic Systems, Nov 2022.

[E] **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert, "A Parametric Generalized Likelihood Ratio Test for Airborne Bistatic Radar Systems". Published in IEEE Radar Conference. March 2022.

Other publications by the author, not included in this thesis, are:

[F] **J. Klintberg**, T. McKelvey, "An Improved Method for Parametric Spectral Estimation". *Proc. 44th IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, Brighton, UK, May 13-17 2019, pp. 5551-5555.

Acknowledgments

As this thesis marks the end of my time as a PhD Student, it is time to acknowledge some of the people that have offered support on the way.

First, I would like to express my gratitude towards my two supervisors Tomas McKelvey and Patrik Dammert, for your guidance and support throughout my PhD studies. You have always initialized discussions with a curious mind and with many interesting research ideas. Working with you has been a pleasure for the last 4.5 years.

I am also grateful towards my current and former colleagues in the signal processing research group and at the electrical engineering department. In particular, I am grateful for Jakob Lindqvist and Ebrahim Balouji. The two colleagues I shared office with the longest time, and whom I no longer just recognize as colleagues but also as two good friends. You have indeed made my time at Chalmers more enjoyable.

Moreover, I am grateful towards the Swedish Governmental Agency for Innovation Systems (VINNOVA) for funding my work within the nationella flygtekniska forskningsprogrammet (NFFP).

Finally, and most importantly, I would like to express my gratitude towards my friends and family. Ett särskilt tack till mina föräldrar som alltid tror på mig och stöttar mig.

Acronyms

CFAR:	Constant False Alarm Rate
CPI:	Coherent Processing Interval
CUT:	Cell Under Test
GLRT:	Generalized Likelihood Ratio Test
ICM:	Intrinsic Clutter Motion
IID:	Independent and Identically Distributed
LRT:	Likelihood Ratio Test

MAP:	Maximum A Posteriori
MLE:	Maximum Likelihood Estimate
PDF:	Probability Density Function
RADAR:	Radio Detection and Ranging
SBT:	Scenario Based Transformation
SINR:	Signal to Interference and Noise Ratio
STAP:	Space-Time Adaptive Processing
ULA:	Uniform Linear Array

Contents

Part I

Overview

CHAPTER 1

Introduction

1.1 Introduction

In 1904 the German inventor Christian Hülsmeyer patented a device that used radio waves to detect the presence of a distinct metallic object at a distance. From a spark gap, Hülsmeyer generated pulsed signals which radiated from the device. He demonstrated the technique by detecting a ship in dense fog. The invention was based upon experiments performed by Heinrich Hertz in 1886 where he discovered the polarization dependent reflection of an electromagnetic wave [1]. The device build by Hülsmeyer would later be denoted as Radio Detection and Ranging, though it is today most known by its acronym *RADAR*. Although the radar was invented in the early 20th century, the development and the usage of the radar first started to accelerate during the Second World War. In the war, many countries used the radar as a vital tool for detection and tracking of enemy aircrafts [2].

In modern society, the radar is still an important sensor used in military operations, but is further used in a broader spectrum of applications. As the radar is a sensor that uses electromagnetic waves to measure the surrounding environment, it has some advantages compared to other sensors, such as a

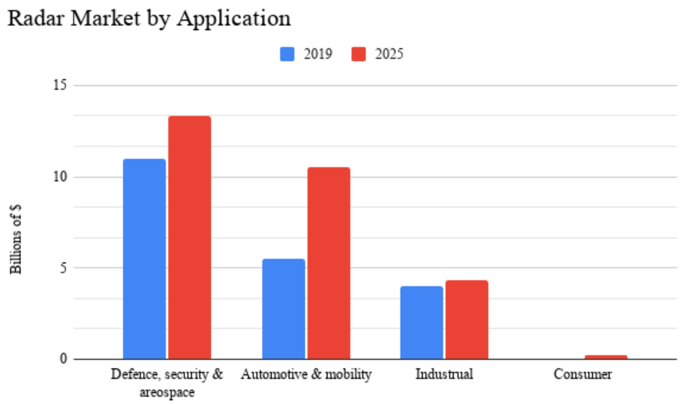

Figure 1.1: Radar market per field of application, and a forecast of the radar market in 2025 [3].

camera or a lidar, used for the same purpose. Thus, the radar can operate in all weather and lighting conditions, and can provide measurements for both long and short ranges. For these reasons, radar systems can be found in the applications of civil air traffic control, weather forecasting and remote sensing. Moreover, the radar system is an important sensor which enables ships to navigate and provides useful information for advanced driver assistance systems and autonomous driving in vehicles. In Fig 1.1, the distribution of the radar market per application is visualized. As seen in the figure, most of the growth in the radar market arises from automotive application, strongly connected to the introduction of more autonomous features in vehicles. Although radar systems can be found in multiple applications, this thesis concern radar systems used in military applications.

One of the main objectives of a radar system is to provide target detections. That is, from the electromagnetic waves reflected by the environment surrounding the radar, determine the presence or the absence of any targets in the environment. To be able to perform the target detection, the radar is equipped with both hardware and software. Depending on the application, various requirements are set on both the hardware and the software for the radar to provide target detections suited for the specific application. Thus, various design choices can be made for both the hardware and the software when constructing a radar system. In terms of hardware, some design choices can involve the usage of a pulsed signal waveform or a continuous signal waveform, as well as the number of array channels comprising the radar antenna. A pulsed signal waveform can in general provide measurements for a longer distance and can be more robust against jamming interference signals compared to a continuous signal waveform. However, a continuous signal waveform is not associated with any minimum distance of where targets can be detected, and may require less power for the wave to be transmitted over time. The number of array channels on an antenna affects the directivity of a transmitter antenna and the spatial resolution of a receiver antenna. The software of a radar system will, for instance, affect the systems ability to mitigate disturbance signals, to resolve targets from each other and the number of targets a system is able to track over time. The accuracy of such operations is affected by numerous parameters, e.g. the quality of the signals observed by the receiver antenna, the amount of computational processing capacity available in the processor and the accuracy of the radar signal processing algorithms. In this thesis, and in the appended papers, methods for radar signal processing is further investigated with the intention to improve their accuracies and decrease their computational complexity.

In most radar systems, the processing of the received electromagnetic signals is performed by a chain of various signal processing algorithms. Consequently, the term *radar signal processing* refers to a collection of multiple processing techniques of a radar system. In the chain of processing, the aim of each step is either to transform the signal or extract information from the signal to facilitate for succeeding calculations in the chain. As an example of a processing chain; reflections of unwanted objects are removed from the electromagnetic signals before a detector determine the target state, or that target detections are aided to a tracking algorithm to be tracked over time. Moreover, a radar system usually is required to be able to operate in various environments and under various scenarios. Such requirements implies that the chain of processing must be robust towards varying characteristics of the received signals. To satisfy the criteria of robustness, the signal processing algorithms are commonly designed to be adaptive. Hence, the processing algorithms will adapt towards phenomena present in the current measured signal. Such adaptive behavior of the processing may be obtained by describing the electromagnetic signals using statistical tools. Thus, an algorithm is designed

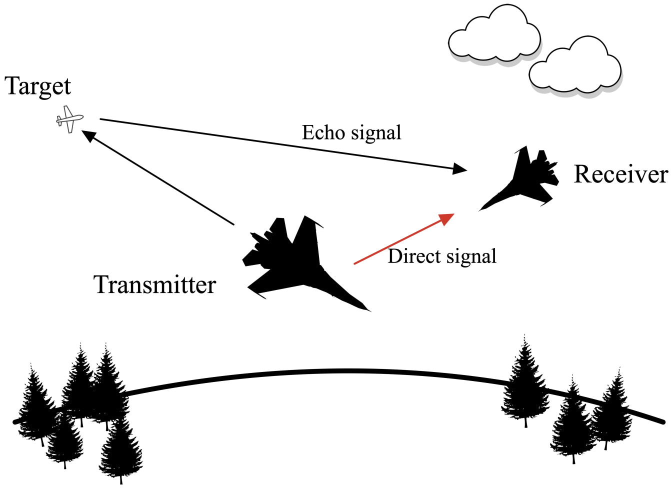

Figure 1.2: Visualization of an airborne bistatic radar configuration.

to manage a distribution of possible signals rather than one deterministic case. Moreover, with a statistical framework, the algorithms may use observations in the adaptation of the processing and hence become data dependent.

Although many radar signal processing techniques are designed to be adaptive, it may be beneficial to incorporate knowledge about the specific radar configuration in the design of the processing techniques as the configuration of the radar system affect the statistical properties of the received signals. Thus, to illustrate the effect of the configuration on the processing, consider the airborne *monostatic* configuration and the airborne *bistatic* configuration. The difference between the two configurations is related to the location of antennas. In a monostatic configuration, the same antenna is used to emit the electromagnetic waves and to receive the radar echo signals. In a bistatic configuration two antennas which are separated at a distance collaborate with each other to create a radar system. Thus, one of the antennas emits an electromagnetic wave and the other antenna receives the radar echoes. A visualization of an airborne bistatic radar system is shown in Fig 1.2. The monostatic configuration is the most common configuration of operational radar systems today, but in recent years an increased research interest has been shown for the bistatic configuration. As an indication of the increased

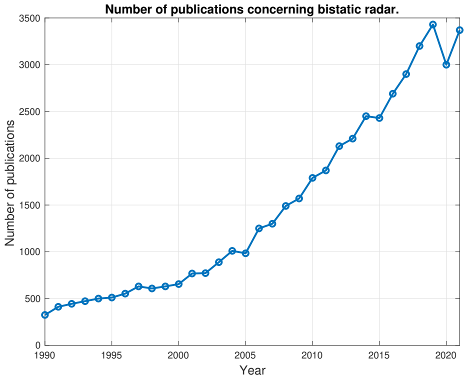

Figure 1.3: Number of publications on Google Scholar involving the search term "Bistatic radar".

research interest, the number of publications on Google Scholar including the search term *bistatic radar* is shown in Fig 1.3. Part of the increased interest arises from the introduction of digitalization in a higher degree in the radar systems, i.e. that each antenna channel is sampled as a separate digital channel [4]. A digital radar has the possibility to fully take advantage of the potential of the bistatic configuration [5]. One of the advantages associated with a bistatic configuration is the silent operation of the receiver platform. Thus, as the receiver antenna receives radar echoes emitted by the transmitter antenna, the receiver platform does not reveal its position by emitting a signal. Moreover, a target using stealth techniques to decrease the targets radar cross section, may be detected with a higher probability in a bistatic configuration than in a monostatic configuration due to the oblique bounces of the electromagnetic signals in a bistatic system. Additionally, in a bistatic configuration, the transmitting antenna has the possibility to emit a signal constantly, and thus emit more electromagnetic energy over time compared to a corresponding antenna operating in a monostatic configuration. This is possible as a transmitter antenna of a bistatic configuration is not required to be silent when receiving radar echoes. In theory, this property can lead to higher signal-to-noise-ratios in a bistatic configuration compared to a monostatic configuration.

To utilize the full potential of a bistatic radar configuration, additional processing functionality is needed compared to what is required in a monostatic radar configuration. As an example, in a bistatic configuration there can be a direct signal between the transmitter platform and the receiver platform. As the direct signal travel one direction, in contrast to radar echoes which travels from the transmitter to an object and to the receiver, the direct signal may have a significantly larger signal power compared to the radar echoes. Thus, if the direct signal is not mitigated, it may disturb the processing significantly. Moreover, to be able to operate a bistatic radar system, the platform used as a transmitter and the platform used as a receiver must be synchronized. In a military application, it may be problematic to establish a communication channel between the two platforms where the synchronization is performed, as it may expose the positions of the platforms. Using the direct signal can be one approach to perform the synchronization between the platforms. Furthermore, in a bistatic system, the statistical properties of the observations depend on the geometry of the radar configuration. Thus, the positions and the velocities of the transmitter platform and the receiver platform affect the characteristics of the measurements. As a consequence of this, a geometry-induced range dependency is present in the angle-Doppler domain of observations in a bistatic radar configuration. In the appended papers of this thesis, a parametric scenario based approach which considers the geometry-induced effects is investigated in various aspects.

1.2 Outline of the thesis

This thesis is divided into two parts. In the first part, the theoretical background of the thesis is presented with the intention of introducing the topic and prepare the reader to the second part of the thesis. In the second part, the contributions of the author in the field of parametric estimation techniques in airborne bistatic detectors and bistatic space-time adaptive processing are presented in terms of five appended publications.

The first part of the thesis is structured as follows; In Chapter 2 the fundamentals and the working principles of a radar system is introduced. In Chapter 3 an introduction to detectors is presented, and a brief introduction to parameter tracking is given in Chapter 4. Space-Time Adaptive Processing is presented in Chapter 5, and a summary of the appended papers of the thesis is presented in Chapter 6. In Chapter 7 the thesis is concluded and some possible future research directions are presented.

CHAPTER 2

Fundamentals of Radar Systems

In this chapter, a short introduction to the radar sensor and to the fundamental theory of a radar measurement is given. A more comprehensive overview is given in e.g. [6].

2.1 Radar signal modelling

In a radar system, an antenna emits electromagnetic energy of some waveform into the environment surrounding the radar. The electromagnetic energy is reflected by the objects in the environment, and echoes from a small fraction of the emitted energy are received by an antenna used by the radar system. The radar system can measure the distance, speed and direction towards the objects which reflected energy is received by the receiving antenna. In this section, a short description of these measurements is given for a monostatic radar. However, the same principles holds also for other radar configurations.

The distance, or the range, to an object is measured by the time between the signal has been emitted, reflected by the object and received by the receiving antenna. The range R to an object is for an monostatic radar system is given by

$$R = \frac{c\Delta t}{2} \tag{2.1}$$

where c is the speed of light and Δt is the duration of time the signal travels from the radar platform to the object and back to the radar platform.

A signal echo reflected by a moving object will, relative to the emitted signal, either be compressed or stretched in time depending on the direction of movement of the object relative to the location of the radar antenna. For a signal of narrow bandwidth, a compression or a stretch of the signal result in a shifted frequency of the signal compared to a signal reflected by a non-moving object. The frequency shift is known as the Doppler effect. The frequency shift is given by

$$f_{\mathrm{d}} = -\frac{2v}{\lambda} \tag{2.2}$$

where λ is the wavelength of the transmitted signal and v is the relative velocity between the radar platform and the object.

The direction to an object can be determined by either considering the look direction of the radar antenna, or by consider some direction-of-arrival (DOA) estimation scheme. For the determination of the direction to an object via the look direction of the radar antenna, it may be beneficial to employ a beam pattern with a narrow main beam. Objects which generate a radar echo is then simply considered to be located in the direction of the transmitted main lobe of the gain pattern. As for a DOA estimation scheme, it can be assumed that echo signals reflected by objects at a large distance encounter the receiving array antenna as a wave of planar wavefront [6]. Under such assumptions, a signal of narrow bandwidth introduce a linear phase shift between adjacent array channels if received by a Uniform Linear Array (ULA) receiver antenna. The phase shift between the array channels is proportional to the sine of the DOA angle of the echo signal. An illustration of this phenomena is shown in Fig 2.1. In the figure, the DOA angle of the signal wave encounter the array is denoted with θ, and the ULA have N array channels with equal distance d between two adjacent array channels.

The signal power of the echo signals received by the receiver antenna is

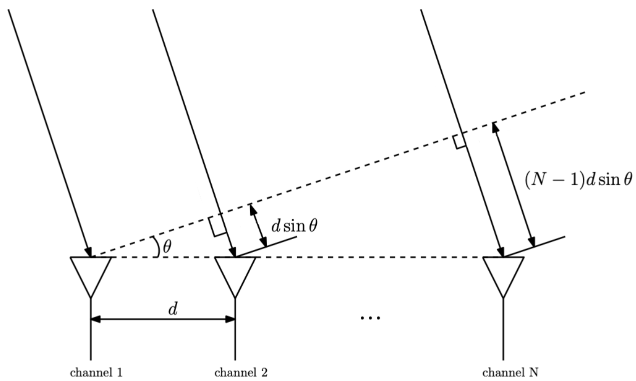

Figure 2.1: Illustration of direction of arrival for an array antenna.

given by the radar range equation

$$P = \frac{\lambda^2 P_{\mathrm{T}} \sigma_{\mathrm{RCS}}}{(4\pi)^3} \frac{G_{\mathrm{T}}}{R_{\mathrm{T}}^2} \frac{G_{\mathrm{R}}}{R_{\mathrm{R}}^2} \tag{2.3}$$

where P_{T} is the power of the transmitted signal, and the gain pattern of the transmitter antenna and the receiver antenna is denoted G_{T} and G_{R}, respectively. The range between the transmitter and the object is R_{T}, and the range between the receiver and the object is R_{R}. The radar cross section of the object is denoted σ_{RCS}.

2.2 Radar Operation Environments

An environment which an airborne radar system operates in is illustrated in Fig 2.2. The environment consists of clutter interference, jamming interference and possible targets which can be both airborne and ground based. Consequently, the observations of a radar system will include clutter interference, jamming interference, possible targets as well as thermal noise generated from the receiver platform. In this section, the components of the radar observations are introduced.

In this work, we consider an antenna of N array channels that receives and processes M pulses of a coherent waveform over $K+1$ range bins. A pulse

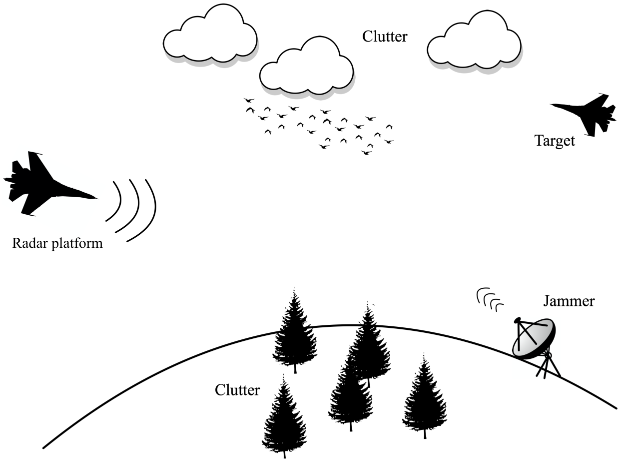

Figure 2.2: Illustration of a radar environment consisting of clutter interference, jamming interference and a target.

compression scheme can be utilized to obtain the observations in the considered framework [7]. By vectorizing the spatial and the temporal response in each range bin, i.e. to vectorize the observations of all pulses and all antenna channels of one range, a space-time snapshot is obtained. The snapshot from an arbitrary range bin k is denoted as $\mathbf{x}_k \in \mathbb{C}^{NM\times 1}$. Assume that the snapshot is additively comprised of clutter interference, $\mathbf{x}_{k,\mathrm{c}} \sim \mathcal{CN}(0, \mathbf{R}_{k,\mathrm{c}}) \in \mathbb{C}^{NM\times 1}$, jamming interference, $\mathbf{x}_{k,\mathrm{j}} \sim \mathcal{CN}(0, \mathbf{R}_{k,\mathrm{j}}) \in \mathbb{C}^{NM\times 1}$, receiver thermal noise, $\mathbf{x}_{k,\mathrm{n}} \sim \mathcal{CN}(0, \mathbf{R}_{k,\mathrm{n}}) \in \mathbb{C}^{NM\times 1}$, and possible targets $\mathbf{x}_{k,\mathrm{s}} = \sigma_\mathrm{s}\mathbf{s}_\mathrm{ts} \in \mathbb{C}^{NM\times 1}$, where σ_s is the intensity of the target and $\mathbf{s}_\mathrm{ts}$ is a space-time steering vector towards the angle-Doppler direction of the target. The notation $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Gamma})$ denotes a complex Gaussian distribution of mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Gamma}$. Consequently, it is assumed that the clutter interference, the jamming interference and the thermal noise are complex Gaussian distributed. This is a common assumption regarding the distribution of the components comprising the radar snapshot [6]. Other formulations regarding the distributions has been presented, such as describing the distribution of the clutter interference using heavy-tailed distributions [8], [9].

In a detector, a test statistics is formed by the multiplication between the radar snapshot of the cell-under test and a weight vector. Thus, for a test in range bin 0 the test statistic becomes $\Lambda(\mathbf{x}_0) = |\mathbf{w}_0^H \mathbf{x}_0|$, where $\mathbf{w}_0$ is the weight vector and superscript 'H' denotes Hermitian transpose. Detectors are further discussed in Chapter 3. However, a useful measure of performance is the signal-to-interference-and-noise-ratio (SINR), which is given by

$$\mathrm{SINR} = \frac{|\mathbf{w}_k^H \mathbf{x}_{k,\mathrm{s}}|^2}{E\{|\mathbf{w}_k^H(\mathbf{x}_{k,\mathrm{j}} + \mathbf{x}_{k,\mathrm{c}}) + \mathbf{w}_k^H \mathbf{x}_{k,\mathrm{n}}|^2\}} \tag{2.4}$$

where $E\{\cdot\}$ denotes expected value operator [10]. The SINR measure have a one-to-one relationship with performance measures of a detector [11]. Hence, maximizing the SINR is equivalent to maximizing the performance of a detector. Thus, in the design of radar signal processing algorithms, and when comparing different radar signal processing algorithms with each other, it is common to consider the SINR measure in such evaluations [12]–[15].

Clutter interference

Unwanted radar echoes originating from reflections of electromagnetic energy generated by the own radar are denoted as clutter interference. In different radar applications, different objects are regarded as wanted or unwanted in the radar echoes. Thus, the terminology clutter interference include echoes from different objects in different applications. In the radar application considered in this work, i.e. target detection of moving objects, echoes from the ground, trees, clouds mountains and buildings are considered as clutter interference [6].

Jamming interference

Signals actively emitted by another system than the own radar, and which interfere with the own radar signals at the used frequency band, are considered to be jamming signals. Jamming can both be unintended, as when two friendly radars are using the same frequency band or when a telecommunication system interfere with the radar, or intended as an electronic warfare technique [6].

A jamming signal is usually observed by the radar receiver antenna via the direct signal and some multipath propagation. The direct signal propagates directly between the emitting platform to the receiver antenna, in contrast to a radar echo which travels two ways; to an object and back to the radar. As a consequence of this, jamming signals may have a significantly larger signal power compared to the power of a radar echo. Thus, jamming signals may cause large problem for the receiver processor. Furthermore, a common jamming signal characteristics is that the signal have a broad Doppler spectrum and originate from a distinct spatial direction [10].

CHAPTER 3

Detection

Target detection serve as one of the primary objectives of a radar system. In this chapter, we introduce the mathematical framework of the binary hypothesis test which a detector performs, and two detectors commonly used in a radar system; the matched filter and the Kelly's detector.

3.1 Binary hypothesis testing

A detection framework can be used in the situation when a measurement can arise from a number of possible hypotheses. The objective of the detection problem is to determine which of the possible hypotheses the measurement arises from. If the measurement is seen as a realization of a stochastic variable, the detection problem may use statistical hypothesis testing to discriminate between the hypotheses. In statistical hypothesis testing, the measurement is used to form a test statistic which is evaluated against some decision boundaries. Depending on the outcome of the test statistic compared to the decision boundaries, the measurement is declared to arise from one of the hypotheses. In the case when the measurement can arise from two possible hypotheses, binary hypothesis testing may be used. In the appended papers of this thesis,

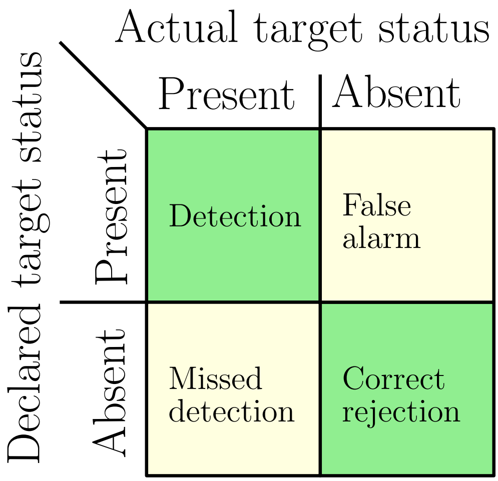

Figure 3.1: The four categories of a binary hypothesis test in terms of actual status and declared status.

the binary hypothesis test has been considered.

In a binary hypothesis test, the measurement is seen as a realization of a stochastic variable. Thus, to determine between the hypotheses a probability density function (pdf) is associated with each of the two hypotheses. For a measurement $\mathbf{x}$, the pdf under the *null-hypothesis*, $\mathcal{H}_0$, is denoted $p_{\mathbf{x}|\mathcal{H}_0}(\mathbf{x})$, while the pdf under the *alternative hypothesis*, $\mathcal{H}_1$, is denoted $p_{\mathbf{x}|\mathcal{H}_1}(\mathbf{x})$.

Given the declaration of a specific measurement, the declaration can be categorized into four different categories depending on the actual hypothesis of the measurement and the assigned hypothesis of the measurement. In radar terminology, the different categories are; detection, missed detection, false alarm and correct rejection. In Fig 3.1, the categories are illustrated together with the actual status and the assigned status.

To determine between the hypotheses, a test statistic, $\Lambda(\mathbf{x})$, is formed. One framework that can be used to select test statistic is the Neyman-Pearson Criteria (NPC) [12], [16]. The objective of the NPC is to maximize the probability of a detection while keeping the probability of a false alarm below some maximum false alarm rate the detector can tolerate. A decision mechanism which obtains the optimal solution to the NPC is the likelihood ratio test (LRT)

[16]. The LRT is given by

$$\Lambda^*(\mathbf{x}) = \frac{p_{\mathbf{x}|\mathcal{H}_1}(\mathbf{x})}{p_{\mathbf{x}|\mathcal{H}_0}(\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{3.1}$$

where the superscript '$*$' denotes optimality. Thus, the LRT forms a test statistic which evaluates the ratio between the pdf of the alternative hypothesis and the pdf of the null-hypothesis with a threshold. If the test statistic exceeds the threshold, the alternative hypothesis is declared to have generated the measurement $\mathbf{x}$. Otherwise, the measurement is declared to be generated from the null-hypothesis.

A binary hypothesis test involve some measures of performance. The NPC considered the probability of detection, P_{D}, and the probability of false alarm, P_{FA}. These probabilities are defined as:

$$P_{\mathrm{D}} = \Pr[\mathcal{H}_1 \text{ declared}|\mathcal{H}_1 \text{ is true}] = \int_{\mathbf{x}\in\Lambda(\mathbf{x})>\gamma} p_{\mathbf{x}|\mathcal{H}_1}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{3.2}$$

$$P_{\mathrm{FA}} = \Pr[\mathcal{H}_1 \text{ declared}|\mathcal{H}_0 \text{ is true}] = \int_{\mathbf{x}\in\Lambda(\mathbf{x})>\gamma} p_{\mathbf{x}|\mathcal{H}_0}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{3.3}$$

where γ is the threshold to determine between the hypotheses. A visualization of the probability density functions of the two hypotheses together with the decision threshold and the corresponding performance measures is shown in Fig 3.2.

To evaluate the LRT, the processor is required to possess full knowledge of both $p_{\mathbf{x}|\mathcal{H}_0}(\mathbf{x})$ and $p_{\mathbf{x}|\mathcal{H}_1}(\mathbf{x})$. In a practical application, it is unlikely that the processor holds that information and rather some parameter that defines the probability density functions is unknown. In such scenarios, the unknown parameter can be replaced with an estimate of the parameter. For a parameter that is replaced by a maximum likelihood estimate, and used in the framework of (3.1), yield a generalized likelihood ratio test (GLRT) [16]. Consequently, the GLRT is given by

$$\Lambda(\mathbf{x}) = \frac{\max_{\theta\in\Omega_1} p_{\mathbf{x}|\mathcal{H}_1}(\mathbf{x}|\theta_1)}{\max_{\theta\in\Omega_0} p_{\mathbf{x}|\mathcal{H}_0}(\mathbf{x}|\theta_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{3.4}$$

where θ_0 and θ_1 are unknown parameter vectors within the sets Ω_0 and Ω_1, respectively.

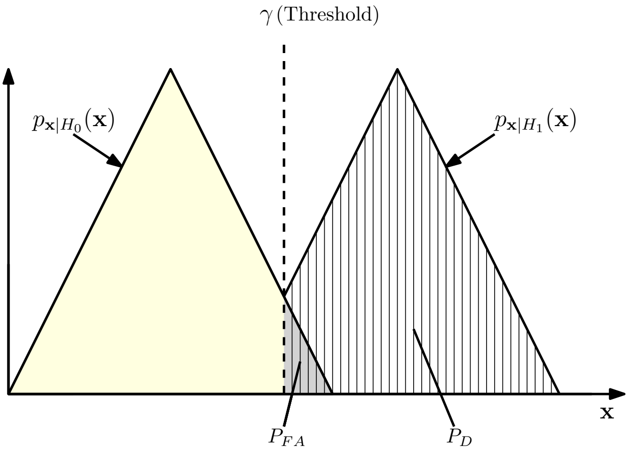

Figure 3.2: An illustration of the probability density functions of the hypotheses involved in a binary hypothesis test.

3.2 Radar detectors

Now we will consider detectors used in radar systems. Consequently, consider a radar system which employs an array antenna of N channels which emits and receives M pulses of coherent waveforms over $K+1$ range bins. Thus, the considered system will observe a measurement $\mathbf{x} \in \mathcal{C}^{NM(K+1)}$. One common discretization of such measurement is to discretize the measurement along the range dimension. Consequently, by vectorizing the observations of all pulses and all array channels in each range bin, the measurement can be reorganized to a set of measurements, $\mathbf{x}_k \in \mathcal{C}^{NM\times 1}\ \forall\ k \in [0, K]$. These manipulations allows for a detection framework to be applied towards each range bin of the observed set of measurement. In this chapter, radar detectors using the discretization over range is investigated.

Recall from Chapter 2 that a radar observation is additively comprised of clutter interference, jamming interference, thermal noise and possible targets. Consider that range bin 0 is associated with the cell-under-test (CUT) which the detector is to declare between target presence or target absence. Thus, formulate two hypotheses

$$\mathcal{H}_0 : \ \mathbf{x}_0 = \mathbf{x}_{0,\mathrm{c}} + \mathbf{x}_{0,\mathrm{j}} + \mathbf{x}_{0,\mathrm{n}} \tag{3.5}$$

$$\mathcal{H}_1 : \ \mathbf{x}_0 = \mathbf{x}_{0,\mathrm{s}} + \mathbf{x}_{0,\mathrm{c}} + \mathbf{x}_{0,\mathrm{j}} + \mathbf{x}_{0,\mathrm{n}} \tag{3.6}$$

Thus, under the null-hypothesis, $\mathcal{H}_0$, a target is absent in range bin 0, and under the alternative hypothesis, $\mathcal{H}_1$, a target is present in range bin 0. Assuming that clutter interference, jamming interference and receiver thermal noise are complex Gaussian distributed, the associated probability density functions are $p_{\mathbf{x}_0|\mathcal{H}_0}(\mathbf{x}) = \mathcal{CN}(0, \mathbf{R}_0)$ and $p_{\mathbf{x}_0|\mathcal{H}_1}(\mathbf{x}) = \mathcal{CN}(\sigma_s \mathbf{s}_{\mathrm{ts}}, \mathbf{R}_0)$ for the null-hypothesis and the alternative hypothesis, respectively. For mutually uncorrelated clutter interference, jamming interference and thermal noise, we have that $\mathbf{R}_0 = \mathbf{R}_{0,\mathrm{c}} + \mathbf{R}_{0,\mathrm{j}} + \mathbf{R}_{0,\mathrm{n}}$.

The test statistic used in the binary hypothesis test of the detector is the scalar output $y_0 = |\mathbf{w}_0^H \mathbf{x}_0|$, where $\mathbf{w}_0$ is a weight vector. The weight vector which yields a maximum SINR is $\mathbf{w}_0 = \mu \mathbf{R}_0^{-1} \mathbf{s}_{\mathrm{ts}}$, where μ is an arbitrary scalar and $\mathbf{s}_{\mathrm{ts}}$ is the space-time steering vector of the currently investigated cell-under-test [11]. This weight vector is selected to mitigate the influence of the interference and the noise in the test, while preserving the signal power of possible targets. The arbitrary constant μ does not affect the SINR of the test. However, some μ can provide additional properties which may be beneficial in subsequent calculations of the test. One common detector have a test statistic of,

$$|\mathbf{w}_0^H \mathbf{x}_0|^2 = \frac{|\mathbf{s}_{\mathrm{ts}}^H \mathbf{R}_0^{-1} \mathbf{x}_0|^2}{\mathbf{s}_{\mathrm{ts}}^H \mathbf{R}_0^{-1} \mathbf{s}_{\mathrm{ts}}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{3.7}$$

where $\mu = 1/\sqrt{\mathbf{s}_{\mathrm{ts}}^H \mathbf{R}_0^{-1} \mathbf{s}_{\mathrm{ts}}}$. This detector is here denoted the *matched filter*, and is associated with two useful properties. First, the matched filter solves the LRT and consequently is an optimal detector. Secondly, the matched filter have the constant-false-alarm-rate (CFAR) property [17]. It implies a normalization factor of the test statistic which compensates for the power of the interference and the thermal noise, to maintain a constant probability of false alarm.

Similarly, a GLRT derived by Kelly,

$$\frac{|\mathbf{s}_{\mathrm{ts}}^H \hat{\mathbf{R}}_0^{-1} \mathbf{x}_0|^2}{\mathbf{s}_{\mathrm{ts}}^H \hat{\mathbf{R}}_0^{-1} \mathbf{s}_{\mathrm{ts}} (1 + \frac{1}{K} \mathbf{x}_0^H \hat{\mathbf{R}}_0^{-1} \mathbf{x}_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{3.8}$$

where $\hat{\mathbf{R}}_0$ is an estimate of $\mathbf{R}_0$, and K is the number of snapshots used in the estimate of the space-time covariance matrix [18]. The snapshots used in the estimate are typically the radar observations that have been observed, but currently not evaluated by the detector. Snapshots available for estimation of unknowns in the range bin associated with the CUT are denoted as secondary data. Here, for a test in range bin 0, the secondary data comprises of the snapshots $\mathbf{x}_k \ \forall \ k \in [1, K]$. Characteristics of the secondary data may affect the space-time covariance matrix estimate, and implicitly the detector (3.8). Further discussions of estimates of space-time covariance matrices is presented in Chapter 5. Moreover, the Kelly's detector can be used when the space-time covariance matrix of the observation is unknown, and it fulfills the CFAR property.

CHAPTER 4

Tracking

In this chapter, we give a brief introduction to the state vector tracking problem from the viewpoint of Bayesian statistics.

4.1 Problem formulation

An unknown state vector of a dynamical system is denoted as $\boldsymbol{\xi}_n \in \mathbf{R}^{d_{\boldsymbol{\xi}}}$, where n is the current discrete time index and $d_{\boldsymbol{\xi}}$ is the dimensionality of the state vector. A state vector of a dynamical system can involve various terms associated with the system. However, in a tracking application, the state parameters usually represent motion characteristics of the system, e.g. positions, velocities and accelerations of components comprising the dynamical system. In most real-world dynamical systems, the state parameters are continuous in time, where the continuous time index t_n corresponds to the discrete time index n. However, measurements from a sensor of the state dynamics are most certainly produced in discrete moments of time. Although the filtering problem can be performed in continuous time, it is most common to be performed in discrete time. The formulation of the filtering presented in this chapter concern the discrete time description.

Consider that at a certain time index n, we would like to find an estimate of $\boldsymbol{\xi}_n$ with as high accuracy as possible. Thus, we seek $\hat{\boldsymbol{\xi}}_n$ which is close to $\boldsymbol{\xi}_n$ in some measure. In a Bayesian setting [19], the state vector estimate is calculated from the posteriori probability density function (pdf),

$$p(\boldsymbol{\xi}_n|\mathbf{Z}^n) \tag{4.1}$$

where $\mathbf{Z}^n$ denotes a sequence of measurements from time index 1 to n. A measurement at a certain time is not necessary a scalar value, but can be comprised by a vector of measurements. The sequence of measurements is then the ordered set,

$$\mathbf{Z}^n = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_n\} \tag{4.2}$$

where $\mathbf{z}_n$ is the vector of measurements at time n.

Given that the posteriori pdf is known, a state estimate can be derived from it. Various versions of such estimates can be considered. One popular estimate is the minimum mean-square error (MMSE) estimate, given by

$$\hat{\boldsymbol{\xi}}_n^{\text{MMSE}} = E\{\boldsymbol{\xi}_n|\mathbf{Z}^n\} = \int \boldsymbol{\xi}_n p(\boldsymbol{\xi}_n|\mathbf{Z}^n) d\boldsymbol{\xi}_n \tag{4.3}$$

where $E\{\cdot\}$ denotes the expected value operator. Moreover, another popular estimator considers the maximum a posteriori (MAP) estimate. A MAP estimate is the solution to the following optimization problem

$$\hat{\boldsymbol{\xi}}_n^{\text{MAP}} = \arg\max_{\boldsymbol{\xi}_n} p(\boldsymbol{\xi}_n|\mathbf{Z}^n) \tag{4.4}$$

Consequently, the essence of a Bayesian filtering problem, is to calculate the posteriori pdf $p(\boldsymbol{\xi}_n|\mathbf{Z}^n)$, or find an approximation of it, and extract an estimate $\hat{\boldsymbol{\xi}}_n$ from the density.

4.2 Conceptual solution

As discussed in the section above, filtering of a single state vector includes the calculation, or approximation, of the posteriori pdf $p(\boldsymbol{\xi}_n|\mathbf{Z}^n)$. In this section, we present the exact calculation of this density.

Consider that the set of measurements can be split into two parts, $\mathbf{Z}^n = (\mathbf{z}_n, \mathbf{Z}^{n-1})$. The posteriori pdf then becomes

$$p(\boldsymbol{\xi}_n|\mathbf{z}_n, \mathbf{Z}^{n-1}) \tag{4.5}$$

Using Bayes' rule [20], the posteriori pdf can be rewritten as

$$p(\boldsymbol{\xi}_n|\mathbf{Z}^n) = \frac{p(\mathbf{z}_n|\boldsymbol{\xi}_n, \mathbf{Z}^{n-1})p(\boldsymbol{\xi}_n|\mathbf{Z}^{n-1})}{p(\mathbf{z}_n|\mathbf{Z}^{n-1})} \tag{4.6}$$

Given a state vector $\boldsymbol{\xi}_n$, the density of the measurement vector does not depend on previous measurements. Thus, we have,

$$p(\boldsymbol{\xi}_n|\mathbf{Z}^n) = \frac{p(\mathbf{z}_n|\boldsymbol{\xi}_n)p(\boldsymbol{\xi}_n|\mathbf{Z}^{n-1})}{p(\mathbf{z}_n|\mathbf{Z}^{n-1})} \tag{4.7}$$

As seen in (4.7), the posterior density is proportional to the product of the likelihood, $p(\mathbf{z}_n|\boldsymbol{\xi}_n)$, and a prior, $p(\boldsymbol{\xi}_n|\mathbf{Z}^{n-1})$. The likelihood describes how likely a measurement, $\mathbf{z}_n$, is, given the state vector, $\boldsymbol{\xi}_n$. In a tracking application, the likelihood, $p(\mathbf{z}_n|\boldsymbol{\xi}_n)$, is commonly referred to as a *measurement model*. The prior density is found by the marginalization over the previous state

$$\begin{aligned} p(\boldsymbol{\xi}_n|\mathbf{Z}^{n-1}) &= \int p(\boldsymbol{\xi}_n, \boldsymbol{\xi}_{n-1}|\mathbf{Z}^{n-1})d\boldsymbol{\xi}_{n-1} \\ &= \int p(\boldsymbol{\xi}_n|\boldsymbol{\xi}_{n-1}, \mathbf{Z}^{n-1})p(\boldsymbol{\xi}_{n-1}|\mathbf{Z}^{n-1})d\boldsymbol{\xi}_{n-1} \end{aligned} \tag{4.8}$$

Assuming that the dynamical system fulfills the Markov property [21], i.e. that

$$p(\boldsymbol{\xi}_n|\boldsymbol{\xi}_{n-1}, \mathbf{Z}^{n-1}) = p(\boldsymbol{\xi}_n|\boldsymbol{\xi}_{n-1}) \tag{4.9}$$

we get the Chapman-Kolmogorov equation

$$p(\boldsymbol{\xi}_n|\mathbf{Z}^{n-1}) = \int p(\boldsymbol{\xi}_n|\boldsymbol{\xi}_{n-1})p(\boldsymbol{\xi}_{n-1}|\mathbf{Z}^{n-1})d\boldsymbol{\xi}_{n-1} \tag{4.10}$$

Consequently, the integral in (4.10) propagates the posteriori pdf at time $n-1$ though the pdf $p(\boldsymbol{\xi}_n|\boldsymbol{\xi}_{n-1})$. The density $p(\boldsymbol{\xi}_n|\boldsymbol{\xi}_{n-1})$ is often referred to as the *motion model* or the *process model*. The resulting pdf $p(\boldsymbol{\xi}_n|\mathbf{Z}^{n-1})$ is called the predicted density.

Using the Equations (4.7) and (4.10), the posteriori density function at time n, $p(\boldsymbol{\xi}_n|\mathbf{z}_n)$, can be expressed as a function of previous posteriori distribution, $p(\boldsymbol{\xi}_{n-1}|\mathbf{z}_{n-1})$, the motion model $p(\boldsymbol{\xi}_n|\mathbf{z}_{n-1})$ and the measurement model $p(\mathbf{z}_n|\boldsymbol{\xi}_n)$. Consequently, with a filtering process initialized with a prior distribution $p(\boldsymbol{\xi}_0)$ on the initial state $\boldsymbol{\xi}_0$, the equations can be used to calculate $p(\boldsymbol{\xi}_1|\mathbf{z}_1)$, and then $p(\boldsymbol{\xi}_2|\mathbf{z}_2)$, and so on. Thus, (4.7) and (4.10) represents a recursive method to calculate the posteriori probability density function each time a new measurement is available.

The motion model and the measurement model can be described in an alternative way using a system of equations,

$$\boldsymbol{\xi}_n = f_{n-1}(\boldsymbol{\xi}_{n-1}, \mathbf{q}_{n-1}) \tag{4.11}$$

$$\mathbf{z}_n = h_n(\boldsymbol{\xi}_n, \mathbf{w}_n) \tag{4.12}$$

where f_{n-1} and h_n are possible non-linear functions, $\mathbf{q}_{n-1}$ and $\mathbf{w}_n$ are realizations of motion model noise and measurement model noise, respectively. Knowing the motion model and the measurement model is equivalent to knowing the functions f_{n-1} and h_n and the joint density functions of $\mathbf{q}_{n-1}$ and $\mathbf{w}_n$.

4.3 Practical solutions

In the section above, the conceptual solution to the Bayesian filtering problem was presented. Under some circumstances there exists a closed form solution to the filtering problem. One case when a closed form solution exists is when (4.11) and (4.12) have linear dynamics and the noise is additive and Gaussian distributed. Consequently, the system of equations can be written as

$$\boldsymbol{\xi}_n = \mathbf{F}_{n-1}\boldsymbol{\xi}_{n-1} + \mathbf{q}_{n-1} \tag{4.13}$$

$$\mathbf{z}_n = \mathbf{H}_n\boldsymbol{\xi}_n + \mathbf{w}_n \tag{4.14}$$

where $\mathbf{q}_{n-1}$ and $\mathbf{w}_n$ are independently distributed as

$$\mathbf{q}_{n-1} \sim \mathcal{N}(0, \mathbf{Q_{n-1}}) \tag{4.15}$$

$$\mathbf{w}_n \sim \mathcal{N}(0, \mathbf{W_n}) \tag{4.16}$$

in which $\mathbf{Q_{n-1}}$ is the process noise covariance matrix and $\mathbf{W_n}$ is the measurement noise covariance matrix. Further, $\mathbf{F}_{n-1}$ is denoted the system matrix, and $\mathbf{H}_n$ is denoted the measurement matrix.

If the dynamical system and the associated state vector measurements can be written in terms of (4.13) and (4.14), a closed form solution to the filtering problem exists and is given by the *Kalman filter* [22]. The Kalman filter is a recursive processing technique which calculates the first two moments of the posteriori state vector in each time iteration. As the first two moments completely describes a Gaussian pdf, the solution of the Kalman filter gives a complete description of $p(\boldsymbol{\xi}_n|\mathbf{z}_n)$. Moreover, if the mean value of such posterior pdf is used as a state vector estimate, the Kalman filter estimator is the optimal estimator in a mean square error (MSE) sense. This since the a posteriori mean is the minimum mean square error (MMSE) estimate.

The Kalman filter can only operate on linear process models and linear measurement models. To handle non-linear models, extensions of the Kalman filter has been proposed. Two such extensions are the Extended Kalman Filter (EKF) [23] and the Unscented Kalman Filter (UKF) [24]. Other filtering techniques, possible to use for general models, are particle filters and grid-based methods [25]–[27]. With these methods, an approximation of the posterior pdf is made, which is different from the EKF, for example, where the state space models are approximated.

In the appended Paper C of this thesis, a Bayesian filtering framework is used to estimate scenario motion parameters of the transmitter platform and the receiver platform comprising a bistatic radar configuration. The state vector estimate used in Paper C is the maximum a posteriori (MAP) state vector. Further, to obtain the second moment of the posteriori state vector, a Taylor series approximation is calculated in the MAP estimate. From the Taylor series expansion, the corresponding state vector covariance matrix is extracted from the second degree Taylor polynomial. In such a manner, the posteriori pdf is approximated by a Gaussian distribution using the MAP estimate as mean value and the corresponding covariance matrix of the Taylor expansion.

CHAPTER 5

Space-Time Adaptive Processing

In this chapter we introduce the space-time adaptive processing (STAP) technique. It can be used to limit the effects of interference and noise in an airborne radar systems.

5.1 Problem formulation

To demonstrate the need of STAP, recall from Chapter 3, that the optimal test statistic in a SINR sense have the form, $\mathbf{w}_0 = |\mu \mathbf{R}_0^{-1} \mathbf{s}_{\mathrm{ts}}|$ [11]. Thus, to obtain a detector of high performance, it is required to know the associated space-time covariance matrix, $\mathbf{R}_0$, of the distribution to the interference and the noise. However, in many radar applications, the covariance matrix is unknown for the processor. In an airborne radar system, space-time adaptive processing is a technique that can be used to estimate the covariance matrix adaptively from radar observations [28].

In an airborne radar system, the motion of the platform makes that non-moving objects have a relative velocity towards the moving platform. Thus, the echoes of the clutter interference will in such systems have energy in the angle-Doppler domain in regions apart from the zero-Doppler dimension. As a consequence of this, the response of the clutter interference and moving targets may coincide in the angle-Doppler domain, making it difficult to distinguish them apart. Consequently, a detector can in such a scenario have a degraded performance. To enhance the performance of the detector in such radar systems, the STAP technique can be used. STAP is a multidimensional filtering technique which combines information from an array antenna with multiple pulses of a coherent waveform. The objective of a STAP scheme is to describe the distribution associated with the interference and the thermal noise [13], [14]. In this work, we have assumed that the clutter interference, the jamming interference and the thermal noise are complex Gaussian distributed. Under those assumptions, it is sufficient for the STAP scheme to describe the distribution of the interference and the noise in terms of the associated space-time covariance matrix. In the following sections, methods are presented where the covariance matrix is adaptively estimated from radar observations.

5.2 Sample Covariance Matrix Estimate

Consider that an array antenna of N channels is receiving and processing M pulses over $K+1$ ranges. Furthermore, the observations of the N channels and the M pulses corresponding to one range bin is vectorized, making the radar system observe a set of measurements of the form $\bar{\mathbf{x}} = \{\mathbf{x}_k\}_{k=0}^{K}$ where $\mathbf{x}_k \in \mathcal{C}^{NM\times 1}\ \forall\ k \in [0, K]$. Thus, the observation in each range bin comprises of temporal and spatial information, and is here called a *snapshot*. Moreover, consider that a detector perform a binary hypothesis test in range bin 0. Snapshots from the range bins $[1, K]$ are consequently currently not involved in the test. Snapshots not involved in the test are denoted as *secondary data*, and can be used to estimate unknowns in the range bin investigated by the detector. Moreover, the secondary data is commonly assumed to be absent of any targets. For the set $\bar{\mathbf{x}}$, we assume that all snapshots of the secondary data are statistically identical and independently distributed (IID), thus the measurements are distributed as $\mathbf{x}_k \sim \mathcal{CN}(0, \mathbf{R})$ for all $k \in [1, K]$. It can be argued that the set $\bar{\mathbf{x}}$ represents observations received by a side-looking monostatic radar [11].

Given the set $\bar{\mathbf{x}}$, a maximum likelihood estimate of the space-time covariance matrix $\mathbf{R}_0$ is the solution to the following optimization problem

$$\hat{\mathbf{R}}_0 = \arg\max_{\mathbf{R}} \mathcal{L}(\bar{\mathbf{x}}|\mathbf{R}) \tag{5.1}$$

where $\mathcal{L}(\cdot)$ denotes the associated likelihood function to the distribution of $\bar{\mathbf{x}}$. The solution to (5.1) is the *sample covariance matrix* (SCM) estimator, given by

$$\hat{\mathbf{R}}_0 = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_k \mathbf{x}_k^H \tag{5.2}$$

A SCM estimator using, at least, $K = 2NM$ snapshots in the estimate, is associated with a -3 dB loss in SINR compared to the SINR of the clairvoyant space-time covariance matrix [13]. The minimum number of snapshots used in the estimate to reach the -3 dB SINR loss is known as the Reed-Mallet-Brennan (RMB) rule.

In some radar systems, it can be assumed that the thermal noise power is known to the processor. In such case, the noise power can be incorporated in the space-time covariance matrix estimate. Two techniques which can be used to incorporate a known thermal noise level are diagonal loading [29] and fast maximum likelihood [30]. In a diagonal loading approach, the covariance matrix estimate becomes

$$\hat{\mathbf{R}}_0^{\mathrm{DL}} = \hat{\mathbf{R}}_0 + \sigma_{\mathrm{n}}^2 \mathbf{I} \tag{5.3}$$

where $\hat{\mathbf{R}}_0$ is a SCM estimate [29]. In the fast maximum likelihood (FML) framework, the eigenvalues of a SCM estimate are set to a minimum threshold value corresponding to the thermal noise power [30]. Consequently, for an eigenvalue decomposition of a SCM estimate $\hat{\mathbf{R}}_0 = \Phi\Sigma_0\Phi^H$, where Φ is a matrix containing the eigenvectors of $\hat{\mathbf{R}}_0$ and Σ_0 is a diagonal matrix with the ordered eigenvalues of $\hat{\mathbf{R}}_0$ on the diagonal. Denote λ_D to be the smallest eigenvalue of $\hat{\mathbf{R}}_0$ which is greater than σ_{n}^2. Set $\Sigma_0^{\mathrm{FML}} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_D, \sigma_{\mathrm{n}}^2, \ldots, \sigma_{\mathrm{n}}^2)$. A space-time covariance matrix estimate with a FML approach then becomes

$$\hat{\mathbf{R}}_0^{\mathrm{FML}} = \Phi\Sigma_0^{\mathrm{FML}}\Phi^H \tag{5.4}$$

5.3 Heterogeneous and non-stationary radar observations

Radar observations from real world applications and for configurations other than the side-looking monostatic configuration are unlikely to be identically distributed over the range dimension. Thus, the assumptions of the set $\bar{\mathbf{x}}$ generally does not hold. There are mainly two factors affecting the response not to be IID; clutter heterogeneity and geometry-induced range dependencies. Variations of the terrain surrounding the radar system may cause a variational response in strength of the clutter interference reflections. Consequently, in the angle-range dimension, the clutter interference have a heterogeneous response. One common approach to manage such heterogeneous clutter is to aid the processor with additional knowledge about the surrounding environment, and incorporate such knowledge in the covariance matrix estimate. Such approaches are typically known as knowledge-aided STAP, and is further discussed in [31], [32]. A geometry-induced range dependency arises from the relative array configuration compared to the heading of the platform. These effects introduce variations of the clutter intensity in the angle-Doppler domain over range. Radar observations affected by geometry-induced effects is here called *non-stationary.* Other monostatic configurations than the side-looking configuration are affected by geometry-induced effects. Consequently, angle-Doppler variations in the range dimension are present for forward-looking arrays [28], [33], circular arrays [34] and conformal arrays [35] Moreover, geometry-induced effects are present in bistatic configurations where the relative orientation and the relative velocity of both the transmitter platform and the receiver platform contribute to the angle-Doppler variations [28], [36], [37].

To demonstrate the impact of a geometry-induced range dependency on a space-time covariance matrix estimate, consider a set of secondary data $\tilde{\mathbf{x}} = \{\mathbf{x}_k\}_{k=1}^{K}$. The distribution of the set is as follows; $\mathbf{x}_k \sim \mathcal{CN}(0, \mathbf{R}_k)$ for all $k \in [1, K]$. Consequently, the set $\tilde{\mathbf{x}}$ is range variant. A SCM estimator applied on the secondary data $\tilde{\mathbf{x}}$ have an expected value of

$$E[\hat{\mathbf{R}}_0] = \frac{1}{K}\sum_{k=1}^{K} E[\mathbf{x}_k \mathbf{x}_k^H] = \frac{1}{K}\sum_{k=1}^{K} \mathbf{R}_k \tag{5.5}$$

Thus, the expected value of the SCM estimator is the average behavior of the range dependent covariance matrices, rather than the covariance matrix for range bin with index zero.

Several techniques have been presented which address the complications of range dependent secondary data. The algorithms can mainly be divided into three different categories based on their processing technique. The first category comprises of techniques which aim to limit the variations within the data itself. This can be accomplished by only considering secondary data in a close vicinity to the CUT, and is often combined with some dimension reduction technique [37], [38]. In the other two categories, the angle-Doppler variations of the secondary data are included in the processing. Thus, in the second category, the processing assumes a model of the variations in the secondary data. This includes a time-varying weight scheme where temporal variations are modeled in the secondary data [37], [39], [40]. In the last category, transformations are applied to the secondary data with the aim of homogenizing the angle-Doppler response over the range dimension. Several transformations have been presented which transforms the secondary data according to various measures [41]–[47]. As an example, in a registration based approach, the associated direction-Doppler curve of the snapshot of each range bin is adjusted via curve fitting to coincide with the direction-Doppler curve of a reference range bin [45]. While in the Adaptive Angle-Doppler Compensation ($\mathrm{A}^2\mathrm{DC}$) method, the eigenvector associated with the dominant eigenvalue of the secondary data in each range bin is rotated to the direction of the corresponding eigenvector of a reference range bin. The argumentation of this approach is to homogenize the dominant subspace of each range of the secondary data [46], [47].

5.4 Reduced dimension techniques

In the sections above, we describe processing techniques applied towards the N spatial channels and the M pulses of coherent waveforms. Such direct formulations are commonly known as the joint-domain STAP. However, from a practical implementation viewpoint the joint-domain STAP is of limited use. Two factors mainly makes the joint-domain STAP intractable in a real world applications: sample support and computational complexity. To exemplify this, consider the radar system used in publicly available data collection program MCARM [48]. In the MCARM program, they have used a radar antenna of $N = 22$ array channels and $M = 128$ pulses of coherent waveform. The Reed-Mallet-Brennan rule requires for these parameters the minimum of $2NM = 5632$ snapshots used in a SCM estimator [13]. This heavily exceeds the available sample support of 630 snapshots in the MCARM dataset. Furthermore, the SCM estimator is associated with the computational complexity of $\mathcal{O}(N^2M^2K)$. Consequently, a large amount of antenna array channels processing multiple pulses may complicate a joint-domain implementation in real world radar systems. As an attempt to limit these implementational complications, sophisticated techniques which reduce the size of the necessary sample support and the associated computational complexity have been presented. Here, we will briefly introduce one of the presented techniques: the reduced-dimension technique. However, another commonly used technique, the reduced-rank technique, which utilizes the low rank nature of clutter interference and the jamming interference is further presented in [28], [49].

In a reduced-dimension technique for STAP, the radar observations are filtered with a data-independent transformation before applying the STAP [11]. The objective of the transformation is to reduce the number of degrees of freedom in the observations, which will reduce the required sample support and the computational complexity of a subsequent STAP technique. Consequently, for a space-time snapshot $\mathbf{x}_k$, a transformed snapshot becomes

$$\breve{\mathbf{x}}_k = \mathbf{T}_k^H \mathbf{x}_k \tag{5.6}$$

where $\mathbf{T}_k \in C^{NM \times J}$ is a transformation matrix. Consequently, the transformed snapshot $\breve{\mathbf{x}}_k$ have dimensions $J \times 1$ where $J < NM$. STAP algorithms, as described in Section 5.2, can then be applied to $\breve{\mathbf{x}}_k$.

Multiple choices of the transformation matrix $\mathbf{T}_k$ are possible. A good selection of $\mathbf{T}_k$ manage the tradeoff between reducing the sample support and reduce the computational complexity, while maintaining the number of degrees of freedom necessary to mitigate the interference and the noise and preserve the power of echo signals of possible targets. Common selections of the transformation matrix involve traditional radar signal processing techniques such as Doppler processing and beamforming [50], [51].

CHAPTER 6

Summary of included papers

This chapter provides a summary of the included papers.

6.1 Paper A

> **Jacob Klintberg**, Tomas McKelvey, Patrik Dammert
> Mitigation of Ground Clutter in Airborne Bistatic Radar Systems
> *Published in IEEE Sensor Array and Multichannel Signal Processing Workshop*,
> pp. 1–5, June. 2020.
> ©2020 IEEE DOI: 10.1109/SAM48682.2020.9104314 .

A space-time adaptive processing algorithm is dependent on an accurate estimate of the space-time covariance matrix to mitigate the effects of interference and noise. In this paper, we investigate the sensitivity of such a covariance matrix estimate calculated from a model describing the current radar scenario. Consequently, a radar scenario is dependent on a set of scenario parameters, connected to the two radar platforms comprising the bistatic configuration, which can be used via a model to calculate a covariance matrix. A scenario parameter describes some characteristic of the radar platform which affects the radar observations of the system. An example of a scenario parameter can be the position or the velocity of a radar platform, and the direction of the transmitter antenna gain pattern. In a real-world application, a set of scenario parameters is typically unknown to the processor in the system. Therefore, in this paper, we investigate the influence of scenario parameter residuals on the covariance matrix estimate. Thus, the evaluation concerns a sensitivity analysis of the covariance matrix on the radar scenario. In numerical simulations, the sensitivity is measured in form of detector performance and the calculated covariance matrix is evaluated against other state-of-the-art methods for estimation of covariance matrices in bistatic radar systems.

6.2 Paper B

Jacob Klintberg, Tomas McKelvey, Patrik Dammert
A Parametric Approach to Space-Time Adaptive Processing in Bistatic Radar Systems
Published in IEEE Transactions on Aerospace and Electronic Systems, vol. 58, no. 2, pp. 1149–1160, April 2022.
©2019 IEEE DOI: 10.1109/TAES.2021.3122520.

This paper considers the space-time covariance matrix estimation problem for airborne bistatic radar systems. In such systems, the distribution associated with the radar observations is range variant due to the geometry of bistatic configuration. In the paper, a covariance matrix estimate calculated from a model describing the radar scenario is considered. The scenario description implies that the geometry-induced effects due to the configuration are incorporated in covariance matrix estimate. Consequently, to obtain the covariance matrix with the considered approach, the set of scenario parameters defining the radar scenario must be known. However, in a radar application, it can be expected that a subset of the scenario parameters is unknown to the processor. Therefore, a maximum likelihood estimate of the scenario parameters is derived using the secondary observations of the system. Moreover, if the scenario approach is used in a detector, it would represent an approximative generalized likelihood ratio test as unknowns are replaced by their maximum likelihood estimates. In numerical simulations, the presented scenario approach is indicated to significantly reduce the associated signal-to-interference-and-noise-ratio loss compared to the other investigated state-of-the-art methods.

6.3 Paper C

Jacob Klintberg, Tomas McKelvey, Patrik Dammert
Scenario Tracking for Airborne Bistatic Radar Systems
Submitted for publication in IEEE Transactions on Aerospace and Electronic Systems, Jan 2023,.

In most implementations of a space-time adaptive processing technique, the required space-time covariance matrix is estimated using secondary observation associated with the most recent coherent processing interval. Consequently, such an estimate is only dependent on the set of measurements currently being observed. In this paper, we derive the covariance matrix using the parametric model of the radar scenario. The model of the radar scenario is dependent on a few parameters which define the state of the scenario. Thus, with this approach, it is sufficient to estimate the scenario parameters to obtain the covariance matrix estimate. Moreover, for scenario parameters which represent motion characteristics of a radar platform, the scenario approach enables that the parameters can be tracked over time by assuming a motion model. Thus, the estimator of the paper derives a scenario parameter estimate using a combination of the likelihood density associated with the most recent set of measurements and a prior density obtained from the propagation of a previous scenario parameter estimate through the assumed dynamical motion model. The derived scenario parameter estimate is a maximum a posteriori estimate. Consequently, the approach of the presented maximum a posteriori estimator implies that information from multiple coherent processing intervals contribute to the considered space-time covariance estimate.

6.4 Paper D

Jacob Klintberg, Tomas McKelvey, Patrik Dammert
Scenario Based Transformations for Compensation of Non-Stationary Radar Clutter

Submitted for publication in IEEE Transactions on Aerospace and Electronic Systems,.

This paper investigates a transformation framework which aim to compensate for angle-Doppler variations in the range dimension of bistatic radar observations. The framework is a combination of an incomplete scenario description and secondary observations, which are used to find a space-time covariance matrix estimate. Thus, the incomplete scenario description is used to find the unitary transformation matrix which, in a Frobenius norm sense, minimizes in each range bin the expected clutter covariance matrix of the scenario description towards the corresponding response in a reference range bin. The unitary condition of the transformation matrix preserves the stationary behavior of the thermal noise under the transformation. With this approach, a set of non-stationary secondary data can be transformed to become more stationary distributed after the transformation. A sample covariance matrix estimator is applied on the transformed set of secondary data to find the space-time covariance matrix estimate. The outlined procedure is denoted as a Scenario Based Transformation (SBT) STAP. The presented method is evaluated and compared with other methods for the considered problem in numerical simulation.

6.5 Paper E

Jacob Klintberg, Tomas McKelvey, Patrik Dammert
A Parametric Generalized Likelihood Ratio Test for Airborne Bistatic Radar Systems
Published in IEEE Radar Conference.,
New York, USA, Mar 21-25 2022, pp. 1-5.

This paper studies the effects of non-stationary secondary data on radar detectors. Thus, in the presence of angle-Doppler variations in the radar observations, a conventional detector approach using a sample covariance matrix covariance matrix estimator together with a detector like the Kelly detector will not represent a generalized likelihood ratio test. In this paper, we derive a generalized likelihood ratio test designed for the airborne bistatic radar system. Thus, in the presented detector, a scenario dependent model is used to represent the geometry-induced range variations of the radar observations. Unknown parameters in the assigned scenario model is estimated by their maximum likelihood estimates. This allows for the derivation of a range and scenario dependent generalized likelihood ratio test of bistatic radar systems. In numerical simulations, the presented generalized likelihood ratio test is compared with other radar detectors used for the considered bistatic radar configuration.

CHAPTER 7

Concluding Remarks and Future Work

In this thesis, we have studied a parametric scenario based approach for airborne bistatic radar systems. The contributions of the publications regard scenario parameter estimation from secondary radar observations, and an investigation of the influence on detector performance of scenario parameter residuals. Moreover, the publications involve methods where the scenario description is used to transform non-stationary secondary data to become more stationary distributed, and to derive a generalized likelihood ratio test for airborne bistatic systems. In this chapter, we provide some comments regarding possible extensions of the results, and directions for future research.

The methods of the appended papers of this thesis have all been evaluated solely on simulated data. Consequently, the proposed methods in this thesis have not been evaluated on observations from a real radar system. Therefore, an evaluation with real radar observations is of interest for future studies.

Further research directions can involve the calculation of a theoretical bound on the accuracy of the scenario parameter estimates for the scenario based framework to the space-time covariance matrix estimation problem. Such a theoretical bound can provide information regarding the identifiability of the scenario parameters given an observed set of measurements, as well as an analysis regarding the sensitivity of the parameter estimates on the number of secondary data in the estimates.

Moreover, a theoretical bound on the scenario parameters enables a cognitive approach to the detection problem in a bistatic radar configuration. Thus, for an area of interest to be evaluated by a detector, the theoretical bound of the scenario approach can propose a radar scenario which yields the optimal detector performance for the considered area of interest. Consequently, the platforms comprising the bistatic configuration can follow the trajectories which would yield an optimal performance of the detector. Such trajectories can be calculated off-line before the bistatic radar system starts to operate.

Connections between the platforms of a multistatic configuration can be described as multiple bistatic triangles. Consequently, the scenario based framework of this thesis can also be used in a multistatic configuration. In a multistatic configuration the processing for a detection of possible targets can either be performed in each of the receiver platforms comprising the configuration or in a central processing unit. In both of these two processing set-ups, information needs to be communicated between the platforms. In a military application, the use of communication channels between the platforms may be unwanted as it can reveal the position of the platforms. As a consequence of this, it is beneficial to minimize any needed communication between the platforms. The scenario approach provides a compact description, i.e. the scenario parameters, which can be used in the covariance matrix estimation problem, and hence suitable to the used in a multistatic configuration.

Furthermore, with the description of a multistatic configuration is comprised by multiple bistatic connections, it can be expected that some of the bistatic triangles contribute more to the performance of a detector than other bistatic triangles. Moreover, as the processing capabilities of the processor is not unlimited, it may be desirable to perform the processing with a minimum amount of bistatic combinations. Thus, a selection is required to determine the number of bistatic connections, and which bistatic connections, to include in the processing. One further research direction is to consider the scenario based approach in such selection process.

References

[1] H. Hertz, *Electric waves: being researches on the propagation of electric action with finite velocity through space*. Dover Publications, 1893.

[2] J. H. DeWitt, "Technical and tactical features of radar", *Journal of the Franklin Institute*, vol. 241, no. 2, pp. 97–123, 1946.

[3] C. Cédric and A. Bonnabel, "Status of the radar industry: Players, applications and technology trends 2020", Yole Développment, Tech. Rep., 2020.

[4] Y. Wu and J. Li, "The design of digital radar receivers", *IEEE Aerospace and Electronic Systems Magazine*, vol. 13, no. 1, pp. 35–41, 1998.

[5] H. Griffiths, "From a different perspective: Principles, practice and potential of bistatic radar", in *2003 Proceedings of the International Conference on Radar (IEEE Cat. No.03EX695)*, 2003, pp. 1–7.

[6] M. I. Skolnik, "Radar handbook second edition", *McGrawHill*, 1990.

[7] M. N. Cohen, "Pulse compression in radar systems", in *Principles of Modern Radar*, J. L. Eaves and E. K. Reedy, Eds. Boston, MA: Springer US, 1987, pp. 465–501, ISBN: 978-1-4613-1971-9.

[8] Ting Shu, J. He, and Xingzhao Liu, "Robust stap in non-gaussian clutter via infinity-norm snapshot-normalization", in *2008 IEEE Radar Conference*, 2008, pp. 1–6.

[9] K. J. Sangston, F. Gini, and M. S. Greco, "Coherent radar target detection in heavy-tailed compound-gaussian clutter", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 1, pp. 64–77, 2012.

[10] J. R. Guerci, *Space-time adaptive processing for radar.* Artech House, 2014.

[11] W. L. Melvin, "A stap overview", *IEEE Aerospace and Electronic Systems Magazine*, vol. 19, no. 1, pp. 19–35, 2004.

[12] W. Melvin, "Space-time detection theory", *Military Application of Space-Time Adaptive Processing*, 2003.

[13] L. E. Brennan and L. S. Reed, "Theory of adaptive radar", *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-9, no. 2, pp. 237–252, Mar. 1973, ISSN: 2371-9877.

[14] I. S. Reed, J. D. Mallett, and L. E. Brennan, "Rapid convergence rate in adaptive arrays", *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-10, no. 6, pp. 853–863, Nov. 1974, ISSN: 2371-9877.

[15] A. De Maio and S. Greco, *Modern radar detection theory.* The Institution of Engineering and Technology, 2016.

[16] J. A. Rice, *Mathematical statistics and data analysis*, 3rd ed. Cengage Learning, 2006.

[17] W.-S. Chen and I. S. Reed, "A new cfar detection test for radar", *Digital Signal Processing*, vol. 1, no. 4, pp. 198–214, 1991, ISSN: 1051-2004.

[18] E. J. Kelly, "An adaptive detection algorithm", *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-22, no. 2, pp. 115–127, 1986.

[19] C. P. Robert *et al.*, *The Bayesian choice: from decision-theoretic foundations to computational implementation.* Springer, 2007, vol. 2.

[20] A. Stuart and K. Ord, *Kendall's advanced theory of statistics, distribution theory.* John Wiley & Sons, 2010, vol. 1.

[21] G. Grimmett and D. Stirzaker, *Probability and random processes.* Oxford university press, 2020.

[22] R. E. Kalman, "A new approach to linear filtering and prediction problems", *Transactions ASME Journal of Basic Engineering*, vol. 82, pp. 34–45, Mar. 1960.

[23] A. H. Jazwinski, *Stochastic processes and filtering theory.* Courier Corporation, 2007.

[24] S. J. Julier and J. K. Uhlmann, “New extension of the kalman filter to nonlinear systems”, in *Signal processing, sensor fusion, and target recognition VI*, Spie, vol. 3068, 1997, pp. 182–193.

[25] N. J. Gordon, D. J. Salmond, and A. F. Smith, “Novel approach to nonlinear/non-gaussian bayesian state estimation”, in *IEE proceedings F (radar and signal processing)*, IET, vol. 140, 1993, pp. 107–113.

[26] A. Doucet, S. Godsill, and C. Andrieu, “On sequential monte carlo sampling methods for bayesian filtering”, *Statistics and computing*, vol. 10, no. 3, pp. 197–208, 2000.

[27] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman filter: Particle filters for tracking applications.* Artech house, 2003.

[28] R. Klemm, *Principles of Space-Time Adaptive Processing.* IEE Press, 2002.

[29] B. Carlson, “Covariance matrix estimation errors and diagonal loading in adaptive arrays”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 24, no. 4, pp. 397–401, 1988.

[30] M. Steiner and K. Gerlach, “Fast converging adaptive processor or a structured covariance matrix”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, no. 4, pp. 1115–1126, 2000.

[31] W. L. Melvin and G. A. Showman, “An approach to knowledge-aided covariance estimation”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, no. 3, pp. 1021–1042, 2006.

[32] W. L. Melvin and J. R. Guerci, “Knowledge-aided signal processing: A new paradigm for radar and other advanced sensors”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, no. 3, pp. 983–996, 2006.

[33] O. Kreyenkamp and R. Klemm, “Doppler compensation in forward-looking stap radar”, *IEE Proceedings-Radar, Sonar and Navigation*, vol. 148, no. 5, pp. 253–258, 2001.

[34] M. Zatman, “Circular array stap”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, no. 2, pp. 510–517, 2000.

[35] R. Hersey, W. Melvin, and J. McClellan, “Clutter-limited detection performance of multi-channel conformal arrays”, *Signal Processing*, vol. 84, pp. 1481–1500, Sep. 2004.

[36] P. G. Tomlinson, "Modeling and analysis of monostatic/bistatic space-time adaptive processing for airborne and space-based radar", in *Proceedings of the 1999 IEEE Radar Conference. Radar into the Next Millennium (Cat. No.99CH36249)*, 1999, pp. 102–107.

[37] W. L. Melvin, M. J. Callahan, and M. C. Wicks, "Adaptive clutter cancellation in bistatic radar", in *Conference Record of the Thirty-Fourth Asilomar Conference on Signals, Systems and Computers*, vol. 2, Oct. 2000, 1125–1130 vol.2.

[38] B. Himed, J. H. Michels, and Yuhong Zhang, "Bistatic stap performance analysis in radar applications", in *Proceedings of the 2001 IEEE Radar Conference*, May 2001, pp. 198–203.

[39] S. D. Hayward, "Adaptive beamforming for rapidly moving arrays", in *Proceedings of International Radar Conference*, Oct. 1996, pp. 480–483.

[40] S. M. Kogon and M. A. Zatman, "Bistatic stap for airborne radar systems", in *Proc. ASAP*, 2000, pp. 1–6.

[41] W. L. Melvin, M. J. Callahan, and M. C. Wicks, "Bistatic stap: Application to airborne radar", in *Proceedings of the 2002 IEEE Radar Conference*, Apr. 2002, pp. 1–7.

[42] G. K. Borsari, "Mitigating effects on stap processing caused by an inclined array", in *Proceedings of the 1998 IEEE Radar Conference, RADARCON'98. Challenges in Radar Systems and Solutions*, May 1998, pp. 135–140.

[43] F. Pearson and G. Borsari, "Simulation and analysis of adaptive interference suppression for bistatic surveillance radars", Massachusetts Inst of Tech Lexington Lincoln Lab, Tech. Rep., 2001.

[44] B. Himed, Y. Zhang, and A. Hajjari, "Stap with angle-doppler compensation for bistatic airborne radars", in *Proceedings of the 2002 IEEE Radar Conference*, Apr. 2002, pp. 311–317.

[45] F. D. Lapierre, J. G. Verly, and M. Van Droogenbroeck, "New solutions to the problem of range dependence in bistatic stap radars", in *Proceedings of the 2003 IEEE Radar Conference*, May 2003, pp. 452–459.

[46] W. L. Melvin, B. Himed, and M. E. Davis, “Doubly adaptive bistatic clutter filtering”, in *Proceedings of the 2003 IEEE Radar Conference*, May 2003, pp. 171–178.

[47] W. L. Melvin and M. E. Davis, “Adaptive cancellation method for geometry-induced nonstationary bistatic clutter environments”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 43, no. 2, pp. 651–672, Apr. 2007.

[48] B. N. S. Babu, J. A. Torres, and W. L. Melvin, “Processing and evaluation of multichannel airborne radar measurements (mcarm) measured data”, in *Proceedings of International Symposium on Phased Array Systems and Technology*, 1996, pp. 395–399.

[49] J. Ward, “Space-time adaptive processing for airborne radar”, in *1995 International Conference on Acoustics, Speech, and Signal Processing*, vol. 5, 1995, 2809–2812 vol.5.

[50] R. C. DiPietro, “Extended factored space-time processing for airborne radar systems”, in *[1992] Conference Record of the Twenty-Sixth Asilomar Conference on Signals, Systems Computers*, 1992, 425–430 vol.1.

[51] Hong Wang and Lujing Cai, “On adaptive spatial-temporal processing for airborne surveillance radar systems”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 30, no. 3, pp. 660–670, 1994.