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Using Bloom's Taxonomy for Math Outreach Within and Outside the Classroom

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Synopsis

Not everyone is a great artist, but we don't often hear, "I dislike art." Most people are able to appreciate visual arts, music and sports, without necessarily excelling in it themselves. On the other hand, the phrase "I dislike math" is widely prevalent. This is especially ironic in our current society, where mathematics affects our day-to-day activities in essential ways such as e-commerce and e-mail. This paper describes the opportunity to popularize mathematics by focusing on its fun and creative aspects, and illustrates this opportunity through a brief discussion of interdisciplinary topics that expose the beauty, elegance and value of mathematics within and beyond the typical K-16 curriculum. We share practical outreach methods inspired by real mathematics, and our experience with a liberal-arts 'math for poets' course which aims to develop math appreciation without losing its rigor or depth. While many of us are not comfortable doing live performances or making fancy videos, we all can use these methods to encourage a more positive mindset about mathematics in our communities.

Keywords: math appreciation, beauty of mathematics, elegance, creativity, outreach, popularization, mathematics education

1. Outreach Philosophy

Most people are not afraid of hard work when it comes to playing sports or music. Their passions lead them to push their physical and mental boundaries as they strive to excel, be a good team member and get better than their adversary. Learning mathematics is not as difficult as making it to a sports team, and it is established that math skills lead to upward social mobility, but math does not invoke a similar enthusiasm and zeal, and is considered difficult or out of reach. John von Neumann guides us out of this situation:

> "If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is."[1]

In order to motivate our students to learn mathematics, it is important to first understand motivation itself. Motivation is the process by which activities are started, directed, and continued, so that physical or psychological needs or wants are met [11]. Extrinsic motivation (such as prize money, bonus, etc.) usually lead to an outcome that is outside of the self. On the other hand, with intrinsic motivation, a person performs an action because the act itself is fun, rewarding, challenging, or satisfying in some internal manner. Harvard psychologist Teresa Amabile and colleagues [1] found that in an experimental group of children, extrinsic motivation decreased the degree of creativity in their artwork, as compared to creativity levels of children in an intrinsically motivated group. It is therefore important to convince our communities that mathematics can be fun and rewarding, in order to develop intrinsic motivation towards doing mathematics.

Mathematics is about finding patterns, building models, and solving puzzles – it involves critical thinking, analyzing facts or situations, and logical deduction. It involves 'abstraction' of the main point of an issue or an argument while ignoring the 'not so relevant' aspects in order to deduce useful results. While most people like puzzles and games, many are not cognizant of the *essence* of mathematics, and the fun involved in mathematical problem solving. To them, mathematics does not come across as creative or exciting. In his book [5], Edward Frankel states:

> "What if you had to take an art class in which you were only taught how to paint a fence? What if you were never shown the paintings of Van Gogh and Picasso, weren't even told they existed? Alas, this is how math is taught, and so for most of us it becomes the intellectual equivalent of watching paint dry."

[1] For the origin of this quote, see http://homepage.math.uiowa.edu/~jorgen/vonneumannquotesource.html, last accessed on January 31, 2023.

Traits such as curiosity and exploration can stimulate motivation [3]. According to the arousal theory of motivation, people seek to maintain an optimal level of tension, by decreasing or increasing stimulation. A very high level of arousal (such as severe test anxiety) or a very low level (such as boredom) both lead to a lower level of task performance. As mathematics educators, we frequently see both extremes in our students. The relation between a task performance and stimulus intensity is explained by the Yerkes-Dodson law [13], which states that performance is related to arousal; moderate levels of arousal lead to better performance. Further, this effect varies with the difficulty of the task; easy tasks require a high-moderate level of arousal, and more difficult tasks require a low-moderate level [12].The sports psychologists, for example, try to get their athlete in the zone where their arousal is not too high and not too low in order to maximize their athletic skills and performance. It is important that we get our students "in the zone," avoiding both extremes of arousal. This can be done by choosing activities that invoke curiosity and discovery, and involve explorations at their skill level.

As mathematicians, we see mathematics as a living and breathing subject, involving the upper layers of Bloom's Taxonomy:[2] create, evaluate, analyze, apply, and understand. We construct examples, formulate theories, and develop conjectures. We create definitions, and use them as handles to create theorems. We evaluate expressions and justify theorems through rigorous mathematical proofs. Mathematical problem solving intrinsically requires finding patterns, building models, and creative thinking. It requires careful organizing, examining and analyzing information, and drawing connections with other fields of study. However, to most people, mathematics just means crunching numbers and remembering and following rules — the lowermost layer of Bloom's taxonomy. When we teach mathematics in the classroom, we slice the Bloom's Taxonomy triangle from Figure 1 horizontally, and share with our students the lowermost layers. If, instead, we slice it at a slant, everyone can leave the classroom with a better understanding of the true essence of mathematics.

[2] Bloom's taxonomy is a framework applied widely throughout K-12 education to describe and hierarchically categorize types of cognitive learning tasks. A good visual representation of the taxonomy is in Figure 1. See [2] for more information on Bloom's taxonomy.

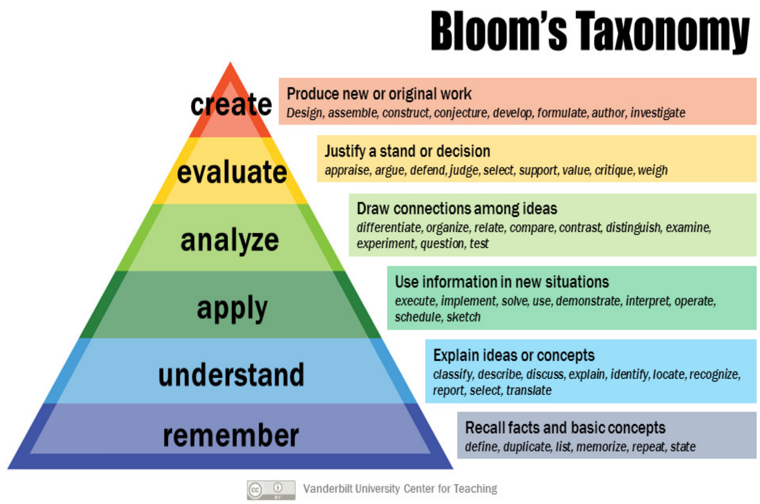

Figure 1: A visual representation of Bloom's taxonomy [2].

In our outreach efforts, we must focus on the upper layers of Bloom's taxonomy and share with our students the 'process' of mathematics, not just the end product, or the 'method'. We must foster inquiry and intellectual curiosity, logical deduction and creative problem solving. With less focus on algebraic manipulations and more on discovery, we must discuss classical problems that have changed the course of history, and the intriguing and ground-breaking mathematical concepts that help solve those problems. Instead of focusing on 'how' to do mathematics, we must focus on the 'why': why does this rule work the way it does, and why should we learn this rule in the first place?

Mathematics is not just the queen of the sciences, it permeates through art, commerce, and in fact, every aspect of modern life. Not only are mathematical concepts deep and powerful, they are also beautiful and elegant. Over the centuries, mathematics has played an important role in almost every aspect of human development, and in many cases has changed the course of history. While everyone understands that mathematics is important, not many understand the full extent of its impact on the modern world. We must reveal the impact and value of mathematics as we seek to popularize it.

2. Cryptology and Other Fun Outreach Topics

Cryptology, the science of coding and decoding secret messages, has played an important role in human lives over the centuries. The Spartans used the *scytale* to send secret messages in 500 BCE and militaries all over the world have used various cryptographic methods over the years. The breaking of secret codes, like the German Enigma and the Japanese military cipher, has often had a decisive effect on the outcomes of major wars. Some authors have argued that as early as 1900 BCE, ancient Egyptians modified standard writing symbols in the writings on the tombs of some of the pharaohs just to intrigue the reader. Deciphering ancient languages has often led to the understanding of those cultures. Thus, cryptography is of interest not only to the military and security systems, but also to the historian, the artist, and the anthropologist. Due to its enigmatical nature, this topic is of natural interest to young and old alike. Even more interesting is its connection with many different fields of study.

Until the 1970s, even though many found it entertaining, information secrecy was primarily of interest to the military and the diplomats. With the development of public key cryptography in the 1970s, this 'science of secrecy' does not need to be 'secret' any more. In the modern world, cryptography plays a crucial role in many aspects of life, such as internet banking, e-commerce, email, automatic teller machines, digital signatures, key exchange, privacy, authentication, etc. We make use of cryptography in many of our everyday transactions. Modern cryptographic methods have had tremendous impact on the economic, sociological, and political aspects of our society. Many websites, popular books, movies, and television serials are devoted to this topic, and are easily accessible.

Interestingly, the mathematics behind various cryptosystems is not very complicated, and can be discussed at various ability levels. The problems in this field are easy to understand, although they may be very hard to solve. The study of any cryptosystem immediately raises the questions of how to implement and cryptanalyze the system, leading to discussion of mathematical concepts such as modular arithmetic, exponentials and logarithms, inequalities, matrices, binary and hexadecimal number systems, statistical analysis, complexity, analysis, number theory, computer programming, etc. Instead of the traditional route of theorems followed by their applications, we can introduce the problem, and have the students 'create' solutions to the using mathematics as a tool at an 'as needed' basis. A careful selection of exercises to maximize performance (in accordance to the Yerkes-Dodson law) leads to a much more positive experience with mathematics in general. One of our favorite activities, appropriate for students of all ages, is based on a Sherlock Holmes murder mystery in which the victim and the criminal communicate via a simple substitution cipher involving stick figures [10]. Such topics present mathematics in an exciting way that students appreciate, show them its value though applications, and motivate the study of deeper mathematical concepts.

Apart from Cryptology, there are many other mathematical masterpieces that are relevant at all times. A discussion of infinity and the infinitesimal involves rich and fun ideas such as the Zeno's paradox, Hilbert's infinite hotel, fractals, chaos and dynamical systems. The concept of limits allows us to talk about the instantaneous speed, without needing to divide by zero, even when there is zero time in an instant. Questions such as 'is there more than one type of infinity?' lead to a discussion of one-to-one mappings. Graph theory offers many interesting problems such as the map coloring, the Konigsberg problem, and the travelling salesman problem. The optimization theorems of calculus are easy to visualize when seen through technology [7]. Other topics of interest to a general audience come from number theory, Platonic solids, golden ratio, and conic sections.

3. In the Classroom

While many of us are not comfortable doing live performances or making fancy videos,[3] we can all use the ideas above to encourage a more positive mindset about mathematics in our communities and classrooms. Cryptology and other topics can be used to share the fun, beauty, and true essence of mathematics not just through outreach talks, presentations, math circles etc., but also with undergraduate students through liberal arts courses [4], and upper level courses for math majors and minors [8, 9]. Such endeavors seek to foster inquiry and creative problem solving in students, and focus on the upper levels of Bloom's taxonomy – create, evaluate, analyze, apply – in order to develop positive sentiments towards mathematics.

[3] Alternatively, some hold that math teaching and outreach are both performances [6].

The students in our 'math for poets' course do not need mathematics for their major field of study. Their interests vary from language and literature, communication arts, special education, psychology, philosophy, to sports studies. Almost all of these students have hated mathematics during their K-12 education, so our larger goal is to develop a greater appreciation and an open mindset towards mathematics. We strive to show them the beauty and wonder in mathematics and build on their skills in writing and creative expression. While they may not excel in mathematics, we hope that they do not dislike it either, and can serve as ambassadors of mathematics in the larger communities. Students in this course are required to create a final paper or artifact in which they discuss a mathematics topic of their choice. Students have created interesting work such as poems involving math concepts, graphic art, string art, and more. We conclude this paper with a comment from a student of literature: "Overall this was a great experience. Now I do not cringe when I hear mathematics, I write instead."

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