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## The Babelogic of Mathematics

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# The Babelogic of Mathematics

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### Synopsis

How would the Bible written about a Mathematical God start, describing the Creation of Mathematics and Logic? How would Rigveda's "Nasadiya sukta" read if it were describing the Void before mathematics was "born"? Here is an attempt at a partial answer, one which takes the original Genesis chapter and the Nasadiya sukta and makes suitable changes to create a fairly consistent, if somewhat anachronistic narrative (with the slight mixing up of Bertrand Russell and Lobachevsky / Bolyai attributable to "Babelogic"), along with a new ending to the Beginning...

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#### The Hymn of Creation

#### The Babelogic of Mathematics

A Rigvedic-Biblical ReCreation

In which an account is given of the Creation of Logic, Mathematics, and Axiomatics; the loss of Paradise to the Serpent of Paradox and the rise of another Paradise in its place...

#### Pre-Genesis - Beginning-minus<sup>1</sup>

At first was neither Logic nor Illogic. There was not Arithmetic nor Geometry, nor yet Mathematics beyond. What was wrapping? Where? In whose protection? Were the Waters of Thought there, unfathomably deep?

There was no Axiom then, nor yet non-Axioms; of Deductions or Inferences there was not any sign. The Geometrical One breathed without breadth, by its own impulse and angles. Other than that was Nothing at all. Nonunderstanding was there, all wrapped around by unknowing Darkness, and all was Chaos indiscriminate.

Then that which was hidden by the Void, that One, emerging, stirring, through power of Conjecture, came to be.

In the beginning Absolute Omega<sup>2</sup> arose, which was primal germ cell of mind.

<sup>&</sup>lt;sup>1</sup> Compare line by line with "The Nasadiya Sukta, Rigveda"; see, for example, http://mesosyn.com/myth2h-4-2.html, last accessed on January 23, 2023, which showcases the wonderful translation by Prof. Raimundo Pannikar.

 $<sup>^2</sup>$  In the Cantorian set theory of mathematics, "Absolute Omega" / "Absolute Infinity" represents something greater than all transfinite numbers which form the hierarchy of cardinal numbers numbers which quantify the sizes of collections (of numbers, abstract objects, and so on). Indeed, Absolute Omega itself is not a cardinal number at all, as illustrated by something called the Burali-Forti Paradox (see footnote 15). It is "too large" to fit consistently into many of the mathematical theories of transfinite numbers, just like Bertrand Russell's "set of all sets which do not contain themselves" (though there are versions of set theory which get around this issue).

The Seers, searching in their hearts with wisdom, discovered in non-Alpha the Absolute Omega.

A crosswise line cut Consistent from Contradiction. What was described above it, what below? Bearers of axiomatic seed there were and mighty forces, thrust from below and impulsed above.

Who really knows? Who can presume to tell it? Whence was it born? Whence issued this creation? Even the Supreme Logician, the Absolute Omega, came after its emergence. Then who can tell from whence it came to be?

That out of which Creation has arisen, whether it held it firm or it did not, She who surveys it in the highest Ivory Tower, She surely knows or perhaps She knows not...

\_\_\_\_\_o\_\_\_\_\_o\_\_\_\_\_o\_\_\_\_\_

Genesis<sup>3</sup>

#### 1.0 Beginning-plus

**1-1**: In the Beginning the Absolute Omega, the Unreachable, created Itself. And saw that its existence was good.

**1-2**: Now the Infinite Realm beneath was formless and empty, darkness was over the surface of the deep, and the Spirit of Absolute Omega was hovering over the Waters of Nothing.

1-3: And Absolute Omega said, "Let There be Logic!", and Lo!, there was Logic.

**1-4**: Absolute Omega saw that Logic was good, and He separated the Logic from the Illogic.

**1-5**: Absolute Omega called the Logic "*Enlightenment*" and the Illogic He called "*Darkness*". And there was Irrationality, and there was Rationality — the first day.

<sup>&</sup>lt;sup>3</sup> Compare verse by verse with the King James Version of Genesis, Bible. See, for example, https://bibleportal.com/passage?search=Genesis+1&version=KJV, last accessed on January 23, 2023.

**1-6**: And Absolute Omega said, "Let there be an expanse in the midst of the Waters of Logic to separate logic from Logic".

1-7: So Absolute Omega made the expanse and separated the Water of the Finite under the expanse from the Water of Infinities above it. And it was so.

**1-8**: Absolute Omega called the expanse "*Mathematiks*". And there was Irrationality, and there was Rationality — the second day.

**1-9**: And Absolute Omega said, "Let the Waters of Logic underlying the Mathematiks be gathered to one place, and let Dry Logic appear." And it was so.

**1-10**: Absolute Omega called the Dry Logic "*Deductions*" and the gathered waters He called "*Axioms*". And Absolute Omega saw that this Axiomatic System was good.

1-11: Then Absolute Omega said "Let the Deductions produce Inferences — Lemmabearing plants and trees of Dry Logic that bear conjectures with other theorems in them according to their various kinds." And it was so.

**1-12**: The Deductions produced Inferences: Lemma-bearing plants and trees bearing conjectures with theorems according to their kinds. And Absolute Omega saw that the Tree of Mathematiks was good.

1-13: And there was Irrationality, and there was Rationality — the third day.

**1-14**: And Absolute Omega said, "Let there be Logics in the expanse of Mathematiks to separate Rationality from Illogic, and let them serve as signs to mark intelligence and cogitation and analysis,

**1-15**: and let them be Logic in the expanse of Mathematiks to give logic on the earth." And it was so.

1-16: Absolute Omega made two great Logics — the Standard Logic to govern real numbers and the Non-Standard Logic to govern the Infinitesimals.<sup>4</sup> And Absolute Omega made the two great lights: the greater source of Algebra to rule the day and the lesser source of Geometry to rule the night and the stars.

<sup>&</sup>lt;sup>4</sup> The traditional methods of calculus utilize the concepts of "infinitely small" or infinitesimal numerical quantities; quantitative variables which vanish "in the limit" of going toward zero size. This calculation approach of calculus is also known as the  $\epsilon$ - $\delta$  method. Informally, the results of calculus can also be obtained by manipulating symbols which represent infinitesimal quantities (e.g.,  $\Delta x$ ) and treating them as actual, "real-like" numbers. To make this rigorous by insisting that such vanishingly small quantities are, in fact, non-zero but smaller than any positive quantity, Abraham Robinson created a very rich, consistent extension of the standard set-theory in the early 1960s, called Non-Standard Analysis. These Robinson infinitesimals (also called nonstandard numbers) enable one to do regular calculus without having to wave hands "in the limit".

**1-17**: Absolute Omega set them in the expanse of Mathematiks to give Logic on the Earth,

**1-18**: to govern the rationality and the irrationality, and to separate Enlightenment from Illogic. And Absolute Omega saw that it was good.

1-19: And there was Irrationality, and there was Rationality — the fourth day.

**1-20**: And Absolute Omega said, "Let the Waters of Logic teem with Surreal numbers, and let the infinities soar above the finites, across the expanse of Mathematiks."

**1-21**: So Absolute Omega created the great Theorems of Mathematiks and every living and moving Arithmetik with which the logos teems, according to their kinds, and every soaring Number according to its kind. And Absolute Omega saw that it was good.

**1-22**: Absolute Omega blessed Number Theory and said, "Be fruitful and increase in Prime Numbers and fill the Arithmetik in the logos, and let the numbers multiply in Prime on the firmaments of Algebra and Calculus"

1-23: And there was Irrationality, and there was Rationality — the fifth day.

**1-24**: And Absolute Omega said, "Let the Axioms be sources of light in the firmament of Mathematiks, to generate surreal numbers<sup>5</sup> from gossamer according to their kinds: complex numbers that move along the Wessel-Argand plains, and wild quaternions and octonions,<sup>6</sup> each according to its kind." And it was so, as the Surreal essence hung in the sky.

**1-25**: Absolute Omega made wild quaternions and forbidding octonions according to their kinds, and all the complex numbers that move along the Wessel-Argand plains according to their kinds. And Absolute Omega saw that it was good.

**1-26**: Then Absolute Omega said, "Let us make Logician in our image, in our likeness, and let them rule over theorems of the Mathematiks and the Logos of the Logics, over the numbers and geometries, over all the topology, and over all the complex numbers that move along the Wessel-Argand plains."

<sup>&</sup>lt;sup>5</sup> Surreal numbers, invented by the legendary mathematician John Conway, can be thought of as a "superset" of all types of numbers — Robinson infinitesimals, real numbers, and transfinite numbers, amongst others.

<sup>&</sup>lt;sup>6</sup> Just as complex numbers extend one-dimensional real numbers into two-dimensional quantities by adding one "perpendicular" axis characterized by the basic imaginary number (square root of minus 1), quaternions extend numerical quantities to four-dimensional numbers using three additional, basic imaginary numbers which act as special vector quantities that follow specific rules of vector multiplication. Similarly, octonions extend the concept of 'quantity' to eight-dimensional numbers. Beyond this, one can keep defining higher-dimensional numbers like Sedenions, Trigintaduonions, etc. by what is known as the Cayley-Dickinson Construction, but these extensions do not have "interesting" properties.

**1-27**: So Absolute Omega created Logician in his own image, in the image of Absolute Omega He created him; Logician and Mathematician, He created them.

**1-28**: Absolute Omega blessed them and said to them, "Be fruitful and prove theorems. Fill the Mathematiks and subdue it. Rule over the geometries of the logos and the proofs which corral higher geometries and over every complex number that moves on the Wessel-Argand plains."<sup>7</sup>

**1-29**: Then Absolute Omega said, "I give you every proof-bearing tool on the face of the whole of Mathematiks and every proof-tree that has theorems with lemmas in it. They will be yours for mental food. Go forth and multiply. Add, subtract and divide as well with equal vigor."

**1-30**: "And to all theorems of Mathematiks and all the real numbers of the Arithmetik and all the complex numbers that move on the Wessel-Argand plains — everything that has the breath of logic in it — I give every new, green conjecture as food for thought." And it was so.

**1-31**: Absolute Omega saw all that He had made, and it was very good. And there was Irrationality, and there was Rationality — the sixth day.

#### 2.0 Euclid and Axiomata

2-1: Thus the Mathematiks and the Logics were completed in all their vast array.

**2-2**: By the seventh day Absolute Omega had finished the math He had been doing; so on the seventh day He rested from all His math.

**2-3**: Then Absolute Omega blessed the seventh day as "*Mathematiks Day*" and made it holy, because on it He rested from all the creating of math that He had done.

**2-4**: This is the account of the Mathematiks and the Logics when they were created, when the Lord Absolute Omega made the Arithmetiks, Algebras and Geometries of the heavens.

**2-5**: Now no lemma had yet appeared in the Mathematiks and no theorem had yet sprung up, for the Lord Absolute Omega had not sent axioms for the Mathematiks and there was no one to work the Logics.

<sup>&</sup>lt;sup>7</sup> The complex number plane, where the x-axis displays the real part of a complex number and the y-axis shows its imaginary part, is known as either the Argand plane after the mathematician, Jean-Robert Argand, or as the Wessel plane, after the mathematician, Caspar Wessel, both of whom had done independent work on creating the geometrical interpretation and representation of complex numbers.

**2-6**: But streams of logos came up from the Logics and watered the whole surface of the Mathematiks.

**2-7**: Then the Lord Absolute Omega formed a Logician named *Euclid* from the eternal Fatou Dust<sup>8</sup> of fractal Geometry and breathed into his nostrils the breath of life, and Euclid became a living mathematician.

**2-8**: Now the Lord Absolute Omega had planted a garden in the east, in Platonia;<sup>9</sup> and there He put Euclid He had formed.

**2-9**: The Lord Absolute Omega made all kinds of theorems grow out of the ground of axioms — schemas that were pleasing to the eye and good for mental food. In the middle of Platonia grew the Tree of Deductions and the Tree of the Knowledge of Unprovable and Paradoxical.

**2-10**: A river of logos watering the Coordinated Garden flowed from Platonia; from there it was separated into four quadrants of Algebraic Geometry.

**2-11**: The name of the first quadrant is PosPos; it winds through the entire line of Positivia, where there are real numbers.

**2-12**: (The real numbers of that land are good; aromatic (re)sine and onyxian co-sine are also positive there.)

**2-13**: The name of the second quadrant is NegPos; it winds through the entire line of Negativia.

**2-14**: The name of the third quadrant is the NegNeg; it runs along the west side of Positivia and south side of Negativia. And the fourth quadrant is the PosNeg.

**2-15**: The Lord Absolute Omega took Euclid and put him in the Garden of Platonia to work it and take care of it.

**2-16**: And the Lord Absolute Omega commanded Euclid, "You are free to pluck and prove from any conjecture-tree in the garden;

**2-17**: but you must not pluck from the Tree of the Knowledge of Unprovable and Paradoxical, for when you pluck from it you will certainly die of confusion."

**2-18**: The Lord Absolute Omega said, "It is not good for Euclid to be alone. I will make a helper suitable for him."

<sup>&</sup>lt;sup>8</sup> Fatou dusts are sets of complex numbers which arise in the study of fractal geometries / analysis. These sets are collections of infinitely many disconnected complex points in the Wessel-Argand plane, visually looking like dust patches.

<sup>&</sup>lt;sup>9</sup> The Greek philosopher, Plato, posited that all things extant were imperfect shadows and projections of perfect forms and templates which inhabited an ethereal realm. This Platonic heaven is hereby called Platonia.

**2-19**: Now the Lord Absolute Omega had formed out of the Logics all the wild conjectures and all the geometrics in the sky. He brought them to Euclid to see what he would name them; and whatever Euclid called each point and line and plane, that was its name.

**2-20**: So Euclid gave names to all the concepts in geometry, the parallel lines in the plane and all the wild triangles.

**2-21**: But for Euclid no suitable helper was found. So the Lord Absolute Omega caused Euclid to fall into a deep sleep; and while he was sleeping, He took five of Euclid's ribs and five of his posterior postulates and then closed up the place with linearized planes.<sup>10</sup>

**2-22**: Then the Lord Absolute Omega made a schema from the ribs and postulates He had taken out of Euclid, and He brought the schema to Euclid.

**2-23**: Euclid said,

"This is now the Axiom of my axioms and the Postulate of my postulates; she shall be called "Euclid's Axiomata", for she was taken out of Euclid."

**2-24**: That is why a Logician leaves his Teacher and his Lover and is united only to his Axiomata, and they become one mind.

**2-25**: Euclid and his Axiomata were both uncovered in their axioms, and they felt no shame none the same.

## 3.0 The Fall

**3-1**: Now the Russell of Bertrand was more crafty than any of the unruly logicians the Lord Absolute Omega had made. Russell said to Axiomata, "*Did Absolute Omega really say, 'You must not pluck from any tree of conjectures in the garden'?*"

**3-2**: Axiomata said to Russell, "We may pluck conjectures from the trees in the garden,

**3-3**: but Lord Omega did say, 'You must not pluck conjectures from the tree that is in the middle of the garden, and you must not touch it, or you will die of confusion''.'

 $<sup>^{10}</sup>$  Euclid of Alexandria (3<sup>rd</sup> century BCE), in his collection of thirteen books called the *Elements*, pulled together a diverse set of known results in geometry of and on plane surfaces. He systematized that knowledge in an axiomatic framework, starting with a number of definitions, common notions and unsaid concepts (which were clarified later), along with five postulates — assertions about five truths of geometry which were taken as god-given, self-evident statements from which all other truths / theorems of geometry would follow.

3-4: "You will not certainly die of confusion," Russell of Bertrand said to Axiomata.

**3-5**: "For Absolute Omega knows that when you pluck from the Tree of Paradox your mind will be opened, and you will be like Absolute Omega, knowing Paradoxical and Unparadoxical."

**3-6**: When Axiomata saw that the conjectures of the tree of Paradox were good for the thought and pleasing to the eye of the mind, and also desirable for gaining wisdom, she took the fifth one and puzzled over it. She also gave some to her Euclid, who was with her, and he puzzled over the fifth, too.

**3-7**: Then the eyes of both of them were opened, and they realized their axioms might be redundant or inconsistent; so they had two logicians sew together nine regular, Choice fig leaves of pairing, separation and union, and had the coverings of power-set upon themselves.<sup>11</sup>

**3-8**: Then Euclid and his Axiomata heard the sound of the Lord Absolute Omega as He was walking in Platonia in the cool of the day, and they hid from the Lord among the unproven conjectures and hypotheses of the garden.

**3-9**: But the Lord Absolute Omega called to Euclid, "Where are you?"

**3-10**: Euclid answered, "I heard you in the garden, and I was afraid that perhaps I was inconsistent and uncovered; so I hid."

**3-11**: And Absolute Omega said, "Who told you that you were inconsistent? Have you pondered from the tree that I commanded you not to pluck from?"

**3-12**: Euclid said, "The Axiomata you put here with me — she gave me the fifth conjecture from the tree, and I pondered over it."<sup>12</sup>

**3-13**: Then the Lord Absolute Omega said to Axiomata, "What is this you have done?" Axiomata said, "The Russell of Bertrand deceived me, and I plucked and pondered."

<sup>&</sup>lt;sup>11</sup> Ernst Zermelo and Abraham Fraenkel were two logicians who put set theory on a firm footing in the early 20<sup>th</sup> century, designed to block certain logical paradoxes created by Bertrand Russell and others. This set theory used eight basic axioms (Axiom of Pairing, Axiom of Separation, etc.), along with a slightly more controversial axiom called the Axiom of Choice. This last axiom enables set theory's logic to pick out arbitrary elements from infinite collections of abstract objects. Together, this axiomatic system is called Zermelo-Fraenkel Set Theory with Choice, shortened as ZFC.

<sup>&</sup>lt;sup>12</sup> Amongst the five postulates of Euclid, the first four were deemed straight-forward. The fifth one asserted that given a straight line L, and a point P outside it, there existed only one unique line which passed through P and stayed parallel to L all the way out to infinity. This gave significant heartburn to onlookers, because the postulate made an assertion about infinitely long objects, and infinity was not be meddled with lightly. The struggle and confusion engendered by this fifth postulate kept geometricians busy for almost two thousand years from the days of Euclid, till they realized that there were equally valid, consistent, "non-Euclidean" geometries in which the fifth postulate failed to hold in an infinite variety of ways.

**3-14**: So the Lord Absolute Omega said to Russell of Bertrand,

"Because you have done this, Cursed are you above all logicians and all wild conjecturists! In your mind you shall crawl on your belly, and eat the dust of logic merged with illogic all the days of your life."

**3-15**: "And I will put enmity between you and Axiomata, and between your offspring and hers; he will crush your logic, and you will strike his confused will."

3-16: To Axiomata He said,

"I will make your pains in the birth of a new set theory very severe; with painful labor you will give birth to a new schema. Your desire will be for Hilbert's Consistency, and his Second Problem will rule over you with an iron fist."

**3-17**: To Euclid He said, "Because you listened to the music of your Axioma and pondered the fifth fruit from the tree about which I commanded you, 'You must not pluck and ponder from it',

Cursed now is the Dry Logic of Deductions because of you; through painful toil you will now prove conjectures from it all the days of your life. And your new, unique name shall be Zermelo, 'the one who waltzes with lost melody'."<sup>13</sup>

**3-18**: "It will produce thorns of paradox and thistles of contradictions for you, and you will ponder your confusion in all your field theories."<sup>14</sup>

**3-19**: "By the sweat of your brow you will prove and prove, until you return to Dry Logic, since from it you were taken; for Fatou dust you are and to Fatou dust you will return."

 $<sup>^{13}</sup>$  Ernst Zermelo was once asked about the origin of his name, and he replied to the effect that it was really supposed to be Walzermelodie — "the melody of a Waltz" — but his parents dropped the first and last syllables to end up with Zermelo.

<sup>&</sup>lt;sup>14</sup> Modern physics, at its deepest levels, is currently formulated using the abstract concept of a field, and the various related physics theories are known as field theories.

**3-20**: Zermelo named his schema ZFC because she would become the mother of all the living Mathematiks.

**3-21**: The Lord Absolute Omega made garments of propositions for Zermelo and his ZFC and clothed them in new logic.

**3-22**: So the Lord Absolute Omega said, "The Logician has now become like one of us, knowing provable and the paradox. He must not be allowed to reach out his mind and pluck also from the Tree of Completeness and Consistency and then conjecture and prove forever."

**3-23**: So the Lord Absolute Omega banished logicians from the Garden of Platonia to work the Fatou Dust from which they had been taken.

**3-24**: After He drove the logicians out, He placed on the east side of the Garden of Platonia the Cherubim called *Gödel*, with a flaming sword of Incompleteness flashing back and forth to guard the way to the Tree of Ultimate Knowledge,

**3-25**: keeping in Undecidable Indecision all of Axiomatics; the shadow of Absolute Omega's Spirit hovered over Uncertain Waters, leaving all its Mathematiks Incomplete.

**3-26**: And Woe! Absolute Omega observed His own Ordinal Inconsistency through the eyes of Burali-Forti.<sup>15</sup> And Absolute Omega saw that this Cardinal Sin was not good.

**3-27**: Weeping for a Paradise Lost, Absolute Omega placed on the west side of the Garden of Platonia a Cherubim called  $Woodin^{16}$  and said, "Let there be lower infinities",

3-28: and Lo!, there were inaccessibly many Inaccessible Infinities.

**3-29**: The Spirit of Absolute Omega saw that the Infinities were inaccessibly good, and struck a covenant with Infinity — a compact of Cardinal Virtue;

**3-30**: Absolute Omega separated one infinity from the next, and gave them varying strengths of Consistency, all according to their kinds;<sup>17</sup>

<sup>&</sup>lt;sup>15</sup> Cantor had formulated his transfinite theory using the concept of collections of objects, their various sub-collections, and the relative, comparative sizes of such collections. In turn, this analysis led to a distinction between numbers (called ordinals) which specify the ordering of a collection of numbers, and numbers (called cardinals) which specify the relative sizes of collections of numbers. Cantor himself had understood that thinking about the cardinality of the collection of ALL ordinals leads to a puzzling inconsistency, a fact later discovered by the Italian logician, Cesare Burali-Forti, as well. This puzzle is known as the Burali-Forti Paradox.

<sup>&</sup>lt;sup>16</sup> Hugh Woodin is one of the foremost set theorists currently active.

 $<sup>^{17}</sup>$  In the extremely abstruse, rarefied stratosphere of transfinite number theory lies an area of

**3-31**: and called the largest huge cardinal "Divinely Strongly Compact" and the other largely smallish cardinal "Worldly Weakly Measureable";<sup>18</sup>

**3-32**: and in deference to Yahweh, there was now the Staircase of Transfinities, a well-ordered procession of Bernstein-Schroeder Alephs — the First Transfinite Day in the new paradise.<sup>19</sup>

**3-33**: Then Lord Absolute Omega commanded the Logician Hilbert, "Let no one expel thee from this Paradise which we have created through Cantor for you."<sup>20</sup>

**3-34**: And it was so.

- Amen-

investigation whose theories are grouped under the rubric, "Large Cardinal Theories". This area requires adoption of new axioms, beyond those needed to capture all of "ordinary mathematics". These axioms make assertions about the existence of extremely large transfinite numbers, and are evaluated on various metrics, including a technical concept called consistency strength. Sort of chest-beating done by set theorists as to whose infinity is larger, "more powerful", "more encompassing", etc.

<sup>&</sup>lt;sup>18</sup> While the infinities mentioned in 3-31 are made-up labels, there are actual transfinite cardinal numbers with exotic names like ethereal cardinals, super-almost-huge cardinals, unfoldable cardinals ineffable cardinals, and the like.

<sup>&</sup>lt;sup>19</sup> In Cantor's theory of transfinite numbers, it is not obvious that ALL of his infinitely many infinities (called Alephs) can be compared to each other in terms of relative size and be ordered in increasing largeness, just as we order finite numbers. It is the Cantor-Bernstein-Schroeder Theorem which lets us speak of various infinities as lining up in a linear, growing progression, till we reach the so-called large cardinals (at which point, we reach the limits of the ZFC axioms used to define our regular mathematics)

 $<sup>^{20}</sup>$  Cantor faced fierce resistance from many a set theorist and mathematician when he came up with his seemingly paradoxical notion of infinitely many infinities, even though he had one of the most beautiful demonstrations in all of mathematics to establish its validity — the Diagonal Proof. David Hilbert, the greatest mathematician of his days, was one of Cantor's defenders, having recognized the new vistas opened up by Cantorian set theory. In its defense, Hilbert said, "Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können" — "From the paradise which Cantor has created for us, no one shall drive us out".