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A novel delay time modelling method for incorporating reuse actions in three-state single-component systems

Augusto César de Jesus Santos[1,2], Cristiano Alexandre Virginio Cavalcante[1], Junru Ren[2], Shaomin Wu[2*]
[1]Department of Production Engineering, Federal University of Pernambuco, Recife CEP: 50740-550, Brazil
[2]University of Kent, Kent Business School, UK

Abstract

This paper presents a new delay time modelling method for reusing single-component systems with two defective states and one failure state. It assumes that a component may be reused for the purposes of resource, economic and environmental sustainability. The possibility of reusing industrial components is not generally considered in maintenance models, which represents a knowledge gap in the literature, especially in the delay time related models. To address this gap, this paper proposes a method based on the delay time modelling method to investigate different scenarios of component reusability and uses real-world systems in the mining industry to illustrate its applicability. The paper then derives the expected cost rate, obtains lower and upper bounds of the expected total cost, considers the improving learning rate of correctly classifying defective components and incorporates the environmental impact of disposed components in optimization of the inspection interval. Results discuss when the reuse action may provide economic benefits even when the reused item may have different reliability than new one.

Keywords: reuse of deteriorating components; delay time; component heterogeneity; misclassification problem; cone crusher equipment

1. Introduction

1.1 Background

New regulations, such as the ‘right to repair’, have been extensively discussed in some countries such as the USA [1] and the UK [2]. Encouraged by these innovative rules and motivated by the idea that reusable industrial components should be reused [3-5], this paper analyses the reusability of deteriorating components in a technical system. It aims to reflect the growing awareness of the need to protect the environment and is directly associated with two of the seventeen sustainable development goals of the United Nations (goals 9 and 12) [6]. In fact, the reuse of components is one of the 3R (Reduce-Reuse-Recycle) concept to promote inclusive and sustainable industrialization and ensure sustainable production.

In order to analyse the reusability of a deteriorating component, this paper proposes a method that uses the delay time model for a system with four states, including one perfectly working state, two defective states, and one failed state. The delay time model assumes that an item passes a period in the

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[*] Corresponding author. Email: s.m.wu@kent.ac.uk. Telephone: 0044 1227 827940

defective state prior to the failure state [7,8]. It is widely used to model the deteriorating process of systems in many publications. The reader is referred to [9] for an excellent review paper. In the current paper, a component is assumed reusable if it is correctly classified at the minor defective state, but it cannot be reused anymore if it is classified at the major defective state or the failed state. As a component classified as minor defective necessarily passes by a refurbishment process in order to return to the system later, it is assumed that used components can have same lifetime distributions, but they can be quite different from the lifetime distributions of new components. Concisely, refurbished components can have the same lifetime distributions as the new components. The term "refurbishment" differs from repair. According to British Standard (GB3811, 1993), repair refers to *the maintenance carried out after fault recognition and intended to put an item into a state in which it can perform a required function* whereas refurbishment is an *extensive work intended to bring plant or buildings up to current acceptable functional conditions, often involving modifications and improvements* [41]. In addition, the state of a component may be mistakenly assessed. Consequently, a reusable component may be mistakenly classified non-reusable or vice versa. This causes a problem of misclassification. As such, the paper then also considers component heterogeneity and misclassification problems.

1.2 Motivating examples

Along with the theoretical development, the application of maintenance models in practical contexts should be emphasised on demonstrating their applicability in real-world cases [10]. In this section, we show a physical degradation process of a single-component system that inspired us to develop this paper.

The system is the mantle and the bowl liner of cone crusher equipment. The components in the system operate together as a component and a socket, both of which perform one operational function and can be considered as a single-component system [11,12]. During the operation period, the system suffers a continuous wearing process since it needs to crush hard materials into small fragments [13], as depicted on the left drawing in Figure 1. This wearing process is an intrinsic characteristic of the system and is one of the main causes of failures [13,14]. In addition, as a consequence of different material gradation, the normal wear can turn into a more severe stage of degradation presented by the abnormal wear, as depicted on the right drawing in Figure 1.

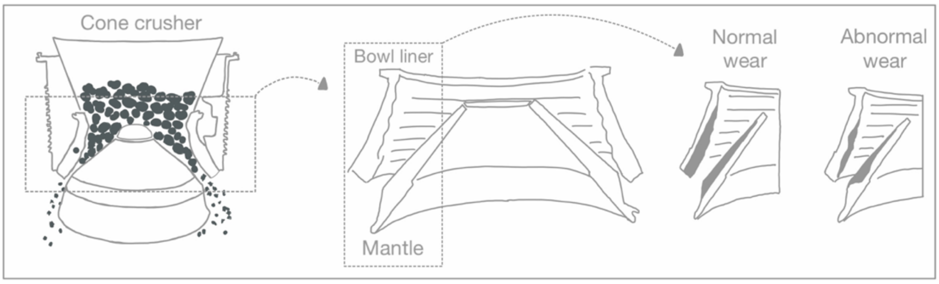


Figure 1. Illustrative example of one possible practical application. On the left, a draft of a cone crusher equipment. In the middle, the bowl liner (external element) and the mantle (internal element). On the right, representations of normal wear (considered as minor defective state) and abnormal wear (considered as major defective state). Source: Adapted from [4], [14] and [42].

As can be seen in Figure 1 and aforementioned, the system has two distinct defective states: the minor defective state and the major defective state, respectively. If the component is found in the major defective state (with abnormal wear), it cannot be refurbished and reused anymore due to the level of its severe degradation. This poses a challenge on when the component should be preventively maintained in order to minimise the relevant cost. Notice that this challenge differs from brand-new components: used items normally may be prone to fail and therefore need more inspections and maintenance whereas brand-new items are more reliable and need few inspections and maintenance. Consequently, the relevant costs incurred are different: despite maintenance on used items may be more frequent than brand new ones, used components have a lower acquisition cost, especially when they can be refurbished in-house. In addition, they are more environmentally friendly and save more resources than brand new ones.

1.3 Literature review and our methods

This brief literature review focuses on what has been studied in the context of reuse. First, it is shown a more general view, since this concept is also studied in other research areas. Then, it is presented how the reuse is generally dealt with in the context of maintenance and reliability, situating the current paper in the literature.

Considering a more general perspective, the most used approaches are based on circular economy [15-17, 20] and reverse logistic [18,19, 21], both with the focus on the product rather than the component. Regarding the circular economy approach, the effect of the original product design on the recovery and reuse of composite products are investigated in [15]. Some practical guidelines for viable recycling business models are proposed in [16]. Possible improvements towards a more circular built environment are discussed in [17]. Wakiru et al. [20] develop an integrated methodology to optimise maintenance, remanufacturing, and multiple spare strategies for the life extension of an ageing multi-component system. In terms of reverse logistic, a two-stage stochastic mixed-integer programming model is developed in [18]. The authors applied this model in a real-world problem to design a reverse logistics network for product reuse, remanufacturing, recycling and refurbishing under uncertainty. Similarly, a redesign of the reverse logistics network is proposed in [19], based on decisions associated with the remanufacturing policies and the location of the collection facility. [21] presents a review of quality, reliability and maintenance issues in closed-loop supply chains with remanufacturing and deals with reverse logistic by using very distinct approaches. The reader is referred to papers [22-24] for other relevant investigations on reverse logistic and closed-loop supply chains.

Concerning maintenance studies, [25] characterises three distinct approaches to maintenance models in terms of sustainability: *(i) "lean maintenance",* which refers to provide maintenance services with the smallest quantity of generated waste [25,26]; *(ii) "green maintenance",* which refers to the management of maintenance operation in respect of the environment [25,27]; *(iii) "sustainable maintenance",* which involves eliminating sources of energy waste [25,28]. Reuse actions of components and equipment may be associated with these approaches that aim to improve sustainability by means of developing better maintenance actions.

[33,34] are examples of interesting investigations that use reuse-related actions in the context of maintenance. [33] investigates a collaborative maintenance service and component sales strategy for original equipment manufacturers challenged by booming used-component sales. The authors consider the possibility of using a preventive replacement policy based on used components in the maintenance service strategy. [34] considers the maintenance policy in which some components are still usable and can be sold as second-hand products. The price of these usable components depends on their original lifetime and the replacement time of the system.

Both approaches in [33,34] consider maintenance polices to deal with the reuse-related actions. This consideration has been extensively investigated in a recent literature review of maintenance models and policies that make use of strategies for reuse and remanufacturing [35]. [35] describes that there is a lack of studies in the area of reuse and remanufacturing, which represents an opportunity for developing maintenance policies that address economic, environmental and social dimensions of sustainability by means of more appropriated maintenance actions. [35] also depicts two main scopes of reuse and remanufacturing in maintenance models and policies. The first and more common scope refers to the reuse or remanufacturing of products to be sold in second-hand markets while the second and less common scope refers to the reuse or remanufacturing of industrial items to be reintroduced in industrial systems.

Within the first mentioned scope, the literature generally deals with warranty policies due to the necessity of determining a type of assurance for second-hand products to be safely used and also to meet customers or dealers' requirements [32, 36-40]. [32] proposes a warranty policy for second-hand products to determine the optimal length of warranty period from the dealer's point of view. [36] investigates an optimal age replacement policy for second-hand products with a second life-cycle in a more severe environment and with an uncertain initial age. [37] develops a stochastic model for obtaining the derivation of the optimal upgrade level for used products sold with warranty and identifies the optimal upgrade action strategy leading to maximisation of the dealer's expected profit. [38] uses a profit model to determine the optimal upgrade level and warranty length so that the expected profit per used item for the producer can be maximised. [40] investigates the worthiness of reliability improvement of repairable second-hand products sold with a two-dimensional warranty from a dealer's viewpoint. The authors propose a new modelling approach that considers the effects of customer usage heterogeneity, PM actions and upgrade on the product reliability. More recently, [39] investigates different PM strategies for second-hand products covered by a two-dimensional warranty from the perspectives of both dealers and customers.

With the second mentioned scope, most papers develop PM models that incorporate the reuse or remanufacturing of items in the industrial system [3-5]. In terms of the number of investigations, this scope has been more neglected in the literature [35], which emphasises the importance of the current paper that deals with the reuse-related actions for industrial items. Some recent contributions that are more specifically related with the scope of this paper are the ones that consider the delay time model for developing preventive maintenance policies that consider reuse-related actions. In [3], the first delay time model for reuse of items is proposed, emphasising how reuse can be incorporated in this type of model. In [5], the effect of different reliability between reused and new components in a maintenance policy subject to human error is investigated. Finally, in [4], a delay time model for a repairable system subject to two defective states prior to the failure is presented. The consideration of two defective states were also interestingly presented before in papers [29,30].

The current paper is an extended version of a conference paper [4] and an extended version of a model proposed in the academic thesis of the first author [42]. It brings an innovative idea related to the practical conditions that determine the possibility to reuse a component. In this paper, different from the previous ones mentioned in the last paragraph, the way to determine if the component is reusable or not, does not require any strong skills regarding the understanding of degradation of the component, neither make this a minor issue that should be faced by the company. Here, we consider that the defective state comprises two steps, the minor defective state, and the major defective state. And the way to define if the component can be reused or not, is based on the stage of defect state after an inspection. This notion not only provides a much more natural understanding for practical use, but it also draws attention to possible errors that may occur when making this judgment. Thus, part of the article is devoted to the analysis of the influence of misclassification on the reusability of a component, since that may be a real issue in the practical application of this model.

1.4 Novelty and contributions

This section emphasises the novelty of this paper and contributions to reliability engineering. First, specific novelty and contributions compared to the existing literature are explained in detail, and then more general contributions are presented. These contributions are also highlighted because the importance of the paper is not restricted to the context of reuse of items but can also be associated with the application of sustainability in reliability engineering.

1.4.1 Specific novelty and contributions compared to the existing literature

Considering the effort to respond to the need for reuse in practical contexts, the specific importance of this paper rests on the following topics. (i) The investigation of the reuse of components rather than only focusing on the reuse of products, noting a product may be composed of more than one component. (ii) The modelling of the related costs associated with the reusing process. In maintenance models that deal with reuse, a special attention needs to be paid into terms of costs. This is due to the conflicting relation between the cheaper cost of acquisition of a reused component and its lower dependability that requires more inspections in the long run. For this reason, the percentage of reused items used in the long run needs to be carefully determined in order to obtain both environmental and economic benefits. In addition, existing research has not considered the uncertainty of the expected cost, as discussed in Section 4 in this paper; (iii) The consideration of the learning rate of correctly classifying the defective items (Section 5) and (iv) The consideration of the environmental impact of disposed items, which has not been tackled in related literature as investigated in Section 6 in this paper.

The novelty of the paper can be mainly associated with the filling of two important gaps in the literature.

- Existing research relating to 3R in the reliability literature generally focuses on the product itself, or a system level, instead of a component level, where the most of maintenance actions is addressed [18,19,31,32].
- Most existing literature refers to reuse as the refurbishment of the component or the system in order to be sold again in second-hand markets [35]. Nevertheless, this paper proposes that reusable components should be repaired in-house in order to be able to make part of the reused spare parts. From this perspective, this paper creates novelty as it considers the sustainability issue in maintenance models to encourage sustainable industrialization. In addition, it is related to a very new tendency in the maintenance environment: that is, the use of 3D printed spare parts. Indeed, few changes in the proposed model may address the possibility of using printed spare parts instead of reused ones.

Additionally, the novelty also includes the following bullets, which were not addressed in maintenance policies where the delay time theory is used:

- The consideration of the learning rate of correctly classifying the defective items; and
- The consideration of the environmental impact of disposed items.

The paper also uses a real-world example as a case to illustrate the applicability of the method proposed in it.

As can be seen, the novelty of this paper is related to new considerations of reuse-related techniques to preventive maintenance, more specifically, to the delay time theory. The contribution of this paper goes beyond the context of reuse of items due to the promotion of sustainability via reliability engineering. As such, Section 1.4.2 shows wider contributions to reliability engineering based on the novelty of this paper and on the contemporaneous context in which sustainability has been advocated as an important issue in reliability engineering.

1.4.2 General contributions to reliability engineering

Research on sustainability has gained a substantial attention in many research areas, including reliability engineering, due to a significant concern with the future of our planet. For example, the number of sustainability papers published in journal *Reliability Engineering and System Safety (RESS)* has considerably increased over the last 20 years, as illustrated in Figure 2, which reflects a growing concern about sustainable issues. As shown in [25], factors such as increasing complexity of industrial process and the search for higher profits require the implementation of sustainable maintenance policies. As a result, adapting maintenance policies that incorporate sustainability is a current challenge for many organisations.

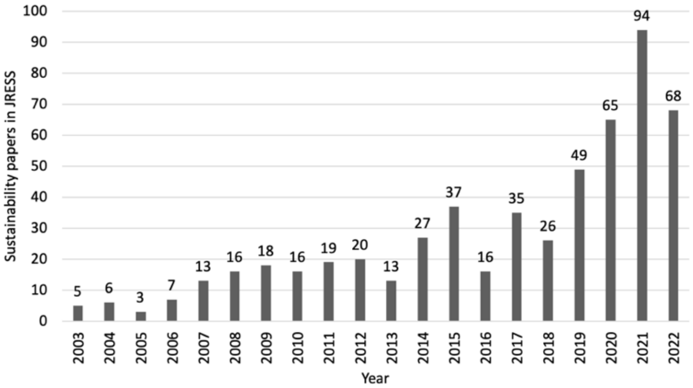


Figure 2. Number of RESS papers that address sustainability over the last 20 years. Source: Elaborated by authors from [43].

The concern of sustainable development motivates us to perform the research of this paper, which addresses goal 9 and goal 12 of the United Nations Sustainable Goals [6]. Goal 9 refers to “build resilient infrastructure, promote sustainable industrialization and foster innovation” and goal 12 refers to “ensure sustainable consumption and production patterns”. Both goals are addressed by the current paper due to the promotion of sustainable industrialization or sustainable consumption patterns, especially by considering the possibility of reusing defective components instead of purchasing new ones. This reinforces the potentiality of adopting reliability techniques to promote a positive impact in terms of sustainability to industries. For this reason, this paper makes an important contribution to reliability engineering, since it clearly illustrates how reliability engineering can be adopted not only in an economic view but also from a sustainable perspective.

The second general contribution to reliability engineering is the consideration of environmental perspective of sustainability in maintenance models, which is a significant gap in the literature. Succinctly, even the maintenance models that adopt reuse or remanufacturing have not been considering this important perspective [35]. In summary, both general impacts of the paper to reliability engineering can be illustrated as follows. Sustainability is an important tendency to be incorporated in the reliability engineering area, and reuse-related actions optimally provided by maintenance policies is one of the interesting ways to promote sustainability via reliability techniques. As such, the paper suggests a model that integrates sustainability to reliability by means of considering the reuse of defective items.

As such, the importance of this paper to reliability engineering, can be assessed in two ways: by providing specific novelties for an important type of preventive maintenance models that are based on the delay time theory, and by promoting the consideration of sustainability in reliability-related area.

1.5 Overview

The remainder of the paper is structured as follows. Section 2 shows the notation and the assumptions. Section 3 presents the method developed based on the delay time model, emphasising its main characteristics. Section 4 derives the lower and upper bounds of the expected total cost of the three cases. Section 5 obtains the expected total cost for the case when the capability of correctly classifying defective items is improving with the number of inspections. Section 6 proposes a new objective function for the case when the environmental impact of disposed items is considered. Section 7 includes a numerical application and provides a discussion on interesting maintenance insights. Section 8 wraps up the paper, discusses its limitations, and proposes future research.

2. Notations and assumptions

2.1 Notations

Prior to the introduction of the proposed method based on the delay time model, we present the notation used in this paper (Table 1) to provide a reference guide to the terminology.

Table 1. Notation.

Decision variable	
T	Interval between inspections
Decision criterion	
$C(T)$	Long run cost per unit of time (cost rate)
Model parameters	
R_{items}	Percentage of reused components. Also, the mixing parameter in Eq. (1)
X, Y, H	Sojourn times in the good state, in the minor defective state and in the major defective state, respectively
β_{x1}, β_{x2}	Shape parameters for Weibull distribution of the arrival of minor defects in reused components and in new components, respectively
β_y, β_h	Shape parameters for Weibull distribution of the arrival of major defects and arrival of failure, respectively
η_{x1}, η_{x2}	Scale parameters for Weibull distribution of the arrival of minor defects in reused components and in new components, respectively
$\eta_y,\ \eta_h$	Scale parameters for Weibull distribution of the arrival of major defects and arrival of failure, respectively
$f_1(x), f_2(x)$	Probability density functions of the arrival of minor defects in reused components and in new components, respectively
$f_x(x)$	Mixture distribution of the arrival of minor defects, based on the R_{items}
$f_y(y), f_h(h)$	Probability density functions of the sojourn time of minor defect and major defect, respectively
p, q	Probability of a minor and a major defective component to be correctly classified, respectively
C_i, C_d	Cost of inspection and disposal cost of a major defective or failed component, respectively
C_{error}, B_r	Penalty cost for not classifying the real state of a major defective component and sent it for repairing, and bonus due to the reuse of the current component classified as minor defective, respectively
C_{ritem}, C_{nitem}	Cost of using a reused component and cost of acquisition of a new component, respectively
C_{r_r}, C_{r_nr} $C_{r\_nr\_e}$	Replacement costs when the current component is in the minor defective state and is correctly classified, when it cannot be reused but is not failed, and when it cannot be reused and is incorrectly classified as reusable
C_{pen}, C_f	Penalty cost due to failure and cost of failure, respectively

2.2 Assumptions

(1) The component has four states: good, minor defective, major defective and failed. The minor defective state does not promote severe damage to the component, whereas the major defective state does. The component can be new or reused. If the component is major defective or fails, it will be disposed of.

(2) The lifetime distributions of used and new components in the good state may be different.

(3) Inspections are performed in order to detect the state of the component and to prevent non-reusing action. Upon inspection, the minor defective component can be correctly classified with probability p, being reused, or mistakenly classified with probability $(1-p)$, being discharged of. Upon inspection, the major defective component can be correctly classified with probability q, being discharged of, or mistakenly classified with probability $(1-q)$, being initially sent to repair but then discharged as well.

(4) The inspections take place every T units of time. Each inspection incurs a cost of C_i.

(5) If the component cannot be reused, there exists an additional cost C_d due to its disposal. If the component can be reused, there exists a discount B_r due to its refurbishment.

(6) If failure occurs, there exists an additional penalty cost C_{pen} due to the negative impacts of a failure. An additional penalty cost due to this judgment error is considered, C_{error}.

(7) The sojourn time in the good state, X, is distributed according to a known mixed distribution based on the level of R_{items}, for which the probability density function is $f_x(x)$.

(8) The sojourn times in the minor defective state, Y, and in the major defective state, H, are distributed according to known Weibull distributions, for which the probability density functions are $f_y(y)$ and $f_h(h)$, respectively.

3. Development of the method

We are trying to determine the optimal inspection interval T in order to minimize the long run cost per unit of time $C(T)$. It is assumed that components come from an inventory composed of reused and brand-new spare parts (Figure 3). The percentage of reused components to be introduced in the system is defined according to the maintenance policy adopted by the company in order to obtain the economic and environmental advantages shown in this paper.

Spare parts	
Population of Reused components	Population of new components

Figure 3. Inventory of reused and new components.

Upon inspections, the current defective component is replaced by another one that can be new or reused (a previous component that has been refurbished in order to return back to the system), the same occurs for a failed component. Depending on the state of the component or on the perception of the maintenance personnel about the state, the current component is discharged with a cost C_d or sent to the in-house repair with a bonus B_r due to the possibility of reutilization. The reused component may have the same or a different dependability of a new one due to the different lifetime distribution associated to its good state. This is a practical concern to be considered because a refurbished component is unlikely to have the same characteristics of a new one. In the proposed model, we consider that the sojourn time in the good state (up to the arrival of the minor defect) is influenced according to the status of the component being brand-new or reused. So, the probability density function, $f_1(x)$, of the arrivals of the minor defect in a reused component is different from that of the minor defect in a new component, $f_2(x)$. Also, the probability density function that represents the arrival of the minor defect in a system sometimes composed of a reused component and sometimes composed of a new component is considered as a mixture distribution based on the percentage of reused components R_{items} and new components $(1 - R_{items})$ in the long run (Eq. 1). The sojourn time in the minor defective state and in the major defective state follow the same probability function in new and reused components, respectively $f_y(y)$ for minor defective state and $f_h(h)$ for major defective state.

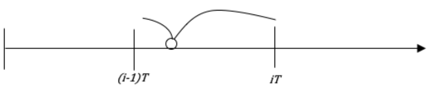
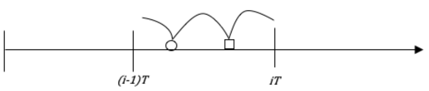
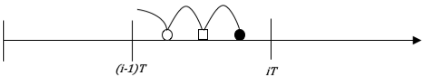
$$f_x(x) = R_{items} f_1(x) + (1 - R_{items}) f_2(x) \tag{1}$$

The analysis of this important characteristic of the model can establish up to which level the dependability of the reused component can be reduced and still be economically viable to be introduced in the system, given a specific R_{items}. The analysis is performed by varying β_{x1}in comparison with β_{x2} and η_{x1}in comparison with η_{x2}. The former is to consider a higher dispersion on the arrival of minor defects in reused components, which is in line with a less standard process of refurbishment when compared to a manufacture process of a brand-new component. The latter is to consider a shorter time to the arrival of the minor defect in reused components, once that even make the best job in refurbishment, it is not possible to make the component be like a new one. Also, it is possible to establish the expected cost rate for varied combinations of new and reused components in order to verify the one that provides the best cost relation.

Another important issue being considered in the model is the mistakes made in classifying the current state of the component being analysed at inspections. We consider that a reusable component can be correctly classified with probability p and mistakenly classified with probability $(1 - p)$. Also, a non-reusable component (currently in the major defective state) can be correctly classified with probability q and mistakenly classified with probability $(1 - q)$. A failed component is always correctly classified due to the interruption of the process. The replacement costs regarding the different possibilities of classification are as follows: (1) when the current component is in the minor defective state and is correctly classified, the cost of replacement is defined as (C_{r_r}), (2) when the current component is in the major defective state (it cannot be reused anymore, but it is not failed yet), the cost of replacement is defined as (C_{r_nr}), and (3) when the current component cannot be reused and is mistakenly classified as reusable, the cost of replacement is defined as ($C_{r\_nr\_e}$).

The decision variable is T, its optimum values are found by the minimization of the objective function, that is, the long run cost per unit of time $C(T)$. All possible disjunct and mutually exclusive renewal events are called by cases. They are represented by Table 2. For each case we develop its respective probability, cost, and cycle length expressions.

Table 2. All possible cases and cost structure. The circumference (empty circle), the square and the circle represent the arrivals of the minor defect, the major defect and the failure, respectively.

<table>
<tr><th colspan="2">The current component is in the minor defective state</th><th>Action and respective cost structure</th></tr>
<tr><td>Case 1</td><td> </td><td>REUSE: when the component is correctly classified with probability p:
$C_{r\_r} = (R_{items})C_{ritem} + (1 - R_{items})C_{nitem} - B_r$
NOT REUSE: when the component is mistakenly classified with probability $(1 - p)$:
$C_{r\_nr} = (R_{items})C_{ritem} + (1 - R_{items})C_{nitem} + C_d$</td></tr>
<tr><th colspan="2">The current component is in the major defective state or failed</th><th>Action and respective cost structure</th></tr>
<tr><td>Case 2</td><td> </td><td>NOT REUSE: when the component is correctly classified with probability q:
$C_{r\_nr} = (R_{items})C_{ritem} + (1 - R_{items})C_{nitem} + C_d$
*NOT REUSE: when the component is mistakenly classified with probability $(1 - q)$:
$C_{r\_nr\_e} = (R_{items})C_{ritem} + (1 - R_{items})C_{nitem} + C_d + C_{error}$</td></tr>
<tr><td>Case 3</td><td> </td><td>NOT REUSE: the component is failed.
$C_f = C_{r\_nr} + C_{pen}$</td></tr>
<tr><td colspan="3">* The component is initially classified as reusable but then in the repairing area it is found to be not reusable anymore. For this reason there is an additional cost C_{error} due to the judgement error.</td></tr>
</table>

Note that in case 1, the current component is in the minor defective state. So, when it is correctly classified with probability p, the replacement cost C_{r_r} has the benefit of the bonus B_r (related to practical benefits of reuse actions, such as, the reduction in the quantity of disposed components and, consequently, the reduction in negative environmental impact). This is due to the refurbishment of the current component that costs less than a brand-new component and the fact that the reuse does not incur discharging cost. When it is mistakenly classified with probability $(1 - p)$, so the component is discharged and a new one should replace the defective component, in this way the replacement cost C_{r_nr} takes into consideration the discharging cost C_d and the acquisition of a new component. In case 2, the current component is in the major defective state. So, when it is correctly classified with probability q, the replacement cost C_{r_nr} takes into consideration the discharging cost C_d. However, when it is mistakenly classified with probability $(1-q)$, the replacement cost $C_{r\_nr\_e}$ adds the error cost C_{error} of sending one non-reusable component to the in-house maintenance. In case 3, the component fails so that there is an addition of the penalty cost of the failure C_{pen} to the replacement cost of a component that is discharged C_{r_nr}. The probability, the expected cost of a cycle and the expected length of a cycle for each case are as follows.

Case 1: The probability of a cycle that ends at a positive inspection of a minor defective state is shown in Eq. (2).

$$P_1(T)=\sum_{i=1}^{\infty}\int_{(i-1)T}^{iT} f_x(x)\int_{iT-x}^{\infty} f_y(y)dydx \tag{2}$$

The expected cost of a cycle that ends at a positive inspection of a minor defective state is given by Eq. (3).

$$\begin{aligned}U_1(T)=&\,p\left[\sum_{i=1}^{\infty}\left[iC_i+C_{r\_r}\right]\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx\right]\\&+(1-p)\left[\sum_{i=1}^{\infty}\left[iC_i+C_{r\_nr}\right]\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx\right]\end{aligned} \tag{3}$$

The expected length of a cycle that ends at a positive inspection of a minor defective state is given by Eq. (4).

$$V_1(T)=\sum_{i=1}^{\infty}\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty}\left[iT\right]f_x(x)f_y(y)dydx \tag{4}$$

Case 2: The probability of a cycle that ends at a positive inspection of a major defective state is shown in Eq. (5).

$$P_2(T)=\sum_{i=1}^{\infty}\int_{(i-1)T}^{iT} f_x(x)\int_{0}^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx \tag{5}$$

The expected cost of a cycle that ends at a positive inspection of a major defective state is given by Eq. (6).

$$\begin{aligned}U_2(T)=&\,q\left[\sum_{i=1}^{\infty}\left[iC_i+C_{r\_nr}\right]\int_{(i-1)T}^{iT} f_x(x)\int_{0}^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx\right]\\&+(1-q)\left[\sum_{i=1}^{\infty}\left[iC_i+C_{r\_nr\_e}\right]\int_{(i-1)T}^{iT} f_x(x)\int_{0}^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx\right]\end{aligned} \tag{6}$$

The expected length of a cycle that ends at a positive inspection of a major defective state is given by Eq. (7).

$$V_2(T)=\sum_{i=1}^{\infty}\int_{(i-1)T}^{iT}\int_{0}^{iT-x}\int_{iT-x-y}^{\infty}\left[iT\right]f_x(x)f_y(y)f_h(h)dhdydx \tag{7}$$

Case 3: The probability of a cycle that ends due to a failure is shown in Eq. (8).

$$P_3(T)=\sum_{i=1}^{\infty}\int_{(i-1)T}^{iT}f_x(x)\int_{0}^{iT-x}f_y(y)\int_{0}^{iT-x-y}f_h(h)dhdydx \tag{8}$$

The expected cost of a cycle that ends due to a failure is given by Eq. (9).

$$U_3(T)=\sum_{i=1}^{\infty}\left[(i-1)C_i+C_f\right]\int_{(i-1)T}^{iT}f_x(x)\int_{0}^{iT-x}f_y(y)\int_{0}^{iT-x-y}f_h(h)dhdydx \tag{9}$$

The expected length of a cycle that ends at a failure is given by Eq. (10).

$$V_3(T)=\sum_{i=1}^{\infty}\int_{(i-1)T}^{iT}\int_{0}^{iT-x}\int_{0}^{iT-x-y}(x+y+h)f_x(x)f_y(y)f_h(h)dhdydx \tag{10}$$

Since all possible cases were defined and $\sum_{i=1}^{3}P_i(T)=1$. This provides a validation on the exhaustiveness of the cases. Eq. (11) shows the long run cost per unit of time $C(T)$ and the next section presents the numerical examples.

$$C(T)=\frac{\sum_{i=1}^{3}U_i(T)}{\sum_{i=1}^{3}V_i(T)} \tag{11}$$

4. Bounds of the expected total cost

In Section 3, the expected costs of a cycle that ends at a positive inspection of a minor defective state, major defective state and failed state are listed respectively. In practice, especially in project planning and evaluation, practitioners may want to know that the bounds of $\sum_{i=1}^{3}U_i(T)$ for uncertainty analysis. More importantly, under the circumstance where the exact and precise value of costs is difficult to obtain (probably involving arduous calculations such as integrals), it is essential to derive an appropriate upper and lower bound of the costs. Hence, this section derives the lower and upper bounds of $\sum_{i=1}^{3}U_i(T)$ when sojourn times in the good state, in the minor defective state and in the major defective state follow Weibull distributions.

- For **Case 1**, we have

$$\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx$$
$$= R_{items}\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_1(x)f_y(y)dydx + (1-R_{items})\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_2(x)f_y(y)dydx$$
$$= \frac{R_{items}\beta_{x_1}}{\eta_{x_1}}\int_{(i-1)T}^{iT}\left(\frac{x}{\eta_{x_1}}\right)^{\beta_{x_1}-1}\exp\left(-\left(\frac{x}{\eta_{x_1}}\right)^{\beta_{x_1}}-\left(\frac{iT-x}{\eta_y}\right)^{\beta_y}\right)dx$$
$$+\frac{(1-R_{items})\beta_{x_2}}{\eta_{x_2}}\int_{(i-1)T}^{iT}\left(\frac{x}{\eta_{x_2}}\right)^{\beta_{x_2}-1}\exp\left(-\left(\frac{x}{\eta_{x_2}}\right)^{\beta_{x_2}}-\left(\frac{iT-x}{\eta_y}\right)^{\beta_y}\right)dx \quad (12)$$

Because $\exp\left(-\left(\frac{iT-x}{\eta_y}\right)^{\beta_y}\right)$ is an increasing function of x, given $x \in [(i-1)T, iT]$, we have

$$\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right) \le \exp\left(-\left(\frac{iT-x}{\eta_y}\right)^{\beta_y}\right) \le 1 \quad (13)$$

Therefore, we obtain the following inequality,

$$R_{items}\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right)\int_{(i-1)T}^{iT}\frac{\beta_{x_1}}{\eta_{x_1}}\left(\frac{x}{\eta_{x_1}}\right)^{\beta_{x_1}-1}\exp\left(-\left(\frac{x}{\eta_{x_1}}\right)^{\beta_{x_1}}\right)dx + (1$$
$$-R_{items})\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right)\int_{(i-1)T}^{iT}\frac{\beta_{x_2}}{\eta_{x_2}}\left(\frac{x}{\eta_{x_2}}\right)^{\beta_{x_2}-1}\exp\left(-\left(\frac{x}{\eta_{x_2}}\right)^{\beta_{x_2}}\right)dx$$
$$\le \int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx$$
$$\le R_{items}\int_{(i-1)T}^{iT}\frac{\beta_{x_1}}{\eta_{x_1}}\left(\frac{x}{\eta_{x_1}}\right)^{\beta_{x_1}-1}\exp\left(-\left(\frac{x}{\eta_{x_1}}\right)^{\beta_{x_1}}\right)dx + (1$$
$$-R_{items})\int_{(i-1)T}^{iT}\frac{\beta_{x_2}}{\eta_{x_2}}\left(\frac{x}{\eta_{x_2}}\right)^{\beta_{x_2}-1}\exp\left(-\left(\frac{x}{\eta_{x_2}}\right)^{\beta_{x_2}}\right)dx \quad (14)$$

At the same time, we have

$$\int_{(i-1)T}^{iT}\frac{\beta_{x_k}}{\eta_{x_k}}\left(\frac{x}{\eta_{x_k}}\right)^{\beta_{x_k}-1}\exp\left(-\left(\frac{x}{\eta_{x_k}}\right)^{\beta_{x_k}}\right)dx = \left(1-\exp\left(-\left(\frac{x}{\eta_{x_k}}\right)^{\beta_{x_k}}\right)\right)\Big|_{(i-1)T}^{iT}$$
$$= \exp\left(-\left(\frac{(i-1)T}{\eta_{x_k}}\right)^{\beta_{x_k}}\right)-\exp\left(-\left(\frac{iT}{\eta_{x_k}}\right)^{\beta_{x_k}}\right), k = 1,2. \quad (15)$$

The result of Eq. (15) can be denoted as $W\left(T;\beta_{x_k},\eta_{x_k}\right), k = 1,2$. Substituting Eq. (15) to Eq. (14), we can denote

$$R_{items}\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right)W(T;\beta_{x_1},\eta_{x_1}) + (1-R_{items})\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right)W(T;\beta_{x_2},\eta_{x_2})$$

as $Lower(T)$, and

$$R_{items}W(T;\beta_{x_1},\eta_{x_1}) + (1-R_{items})W(T;\beta_{x_2},\eta_{x_2})$$

as $Upper(T)$, which are the lower and upper bounds of $\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx$ respectively. Thus, the upper and lower bounds of the expected cost $U_1(T)$ have the following forms.

$$\begin{aligned} p\left[\sum_{i=1}^{\infty}\left[iC_i + C_{r\_r}\right]Lower(T)\right] + (1-p)\left[\sum_{i=1}^{\infty}\left[iC_i + C_{r\_nr}\right]Lower(T)\right] \le U_1(T) \\ \le p\left[\sum_{i=1}^{\infty}\left[iC_i + C_{r\_r}\ \right]Upper(T)\right] + (1-p)\left[\sum_{i=1}^{\infty}\left[iC_i + C_{r\_nr}\ \right]Upper(T)\right] \qquad (16) \end{aligned}$$

- Similarly, for **Case 2**, the lower and upper bounds of the integral

$\int_{(i-1)T}^{iT} f_k(x)\int_0^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx\,, k = 1{,}2$ can be derived as follows. (where $f_h(h) \sim Weibull(\beta_h,\eta_h)$)

$$\begin{aligned} 0 &\le \int_{(i-1)T}^{iT} f_k(x)\exp\left(-\frac{iT-x}{\eta_h}\right)^{\beta_h}\int_0^{iT-x} f_y(y)\,dydx \\ &\le \int_{(i-1)T}^{iT} f_k(x)\int_0^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx \le \int_{(i-1)T}^{iT} f_k(x)\int_0^{iT-x} f_y(y)\,dydx \\ &\le \left[1-\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right)\right]\int_{(i-1)T}^{iT} f_k(x)dx \quad (17) \end{aligned}$$

where $\int_{(i-1)T}^{iT} f_k(x)dx$ is given in Eq. (15).

- For **Case 3**, we have $(k = 1{,}2)$

$$\begin{aligned} 0 &\le \int_{(i-1)T}^{iT} f_k(x)\int_0^{iT-x} f_y(y)\int_0^{iT-x-y} f_h(h)dhdydx \\ &\le \int_{(i-1)T}^{iT} f_k(x)\left[1-\exp\left(-\left(\frac{iT-x}{\eta_h}\right)^{\beta_h}\right)\right]\int_0^{iT-x} f_y(y)\,dydx \\ &\le \left[1-\exp\left(-\left(\frac{T}{\eta_h}\right)^{\beta_h}\right)\right]\left[1-\exp\left(-\left(\frac{T}{\eta_y}\right)^{\beta_y}\right)\right]\int_{(i-1)T}^{iT} f_k(x)dx \qquad (18) \end{aligned}$$

where $\int_{(i-1)T}^{iT} f_k(x)dx$ is given in Eq. (15).

Substituting the two above inequities (17) and (18) to Eq. (6) and Eq. (9) respectively, the upper bounds of the expected costs in Case 2 and Case 3 can be obtained.

5. Improvements of the classification capability

With the development of engineers' capability of classifying the defective items, the accuracy of classifying the minor and major defective states may increasing gradually when more inspections are performed. Thus, the probability of correctly classifying p or q is a function of the number of inspections.

The learning rates of minor defective state and major defective state detection may be different, which can be described using two completely different probability distributions or the same distribution with different parameters. The practitioners can choose an appropriate discrete distribution based on the real situation.

Here, as an illustrative example, if we assume that the correctly classification probabilities of minor and major defective states follow the same distribution $P(X=j;\Theta)$ with different parameter vectors Θ_1 and Θ_2, namely,

$$p = P(X=j;\Theta_1), q = P(X=j;\Theta_2), \tag{19}$$

the expected costs in Case 1 and Case 2 can be re-written as follows.

$$\begin{aligned} U_1^v(T) = &\sum_{i=1}^{\infty} P(i;\Theta_1)\left[iC_i + C_{r\_r}\ \right] \int_{(i-1)T}^{iT} \int_{iT-x}^{\infty} f_x(x) f_y(y)\,dy\,dx \\ &+ \sum_{i=1}^{\infty} \left(1 - P(i;\Theta_1)\right)\left[iC_i + C_{r\_nr}\ \right] \int_{(i-1)T}^{iT} \int_{iT-x}^{\infty} f_x(x) f_y(y)\,dy\,dx, \end{aligned} \tag{20}$$

$$\begin{aligned} U_2^v(T) = &\sum_{i=1}^{\infty} P(i;\Theta_2)\left[iC_i + C_{r\_nr}\ \right] \int_{(i-1)T}^{iT} f_x(x) \int_{0}^{iT-x} f_y(y) \int_{iT-x-y}^{\infty} f_h(h)\,dh\,dy\,dx \\ &+ \sum_{i=1}^{\infty} \left(1 - P(i;\Theta_2)\right)\left[iC_i \right. \\ &\left. + C_{r\_nr\_e}\ \right] \int_{(i-1)T}^{iT} f_x(x) \int_{0}^{iT-x} f_y(y) \int_{iT-x-y}^{\infty} f_h(h)\,dh\,dy\,dx. \end{aligned} \tag{21}$$

Then, in Eq. (11), replacing the original values $U_1(T)$ and $U_2(T)$ with the above $U_1^v(T)$ and $U_2^v(T)$, the long run cost per unit of time with varying probabilities of correctly classifying a defective item, denoted by $C^v(T)$, can be derived.

Example. The Planck distribution is a discrete form of the exponential distribution, and its Probability Density Function (PDF) denoted by $P(j;\lambda)$ and Cumulative Distribution Function (CDF), denoted by $F(j;\lambda)$ are given below.

$$P(j;\lambda) = (1 - \exp(\lambda)) \exp(-\lambda j)\,, j\lambda \geq 0 \tag{22}$$

$$F(j;\lambda) = 1 - \exp\big(-\lambda(j+1)\big), j\lambda \geq 0 \tag{23}$$

Because the probability p of correctly classifying a defective item is in the interval (0,1) and increases with the increase of the number of inspections, the CDF of the Planck distribution can be used to model the learning process of classifying, the plot of which is shown in Figure 4.

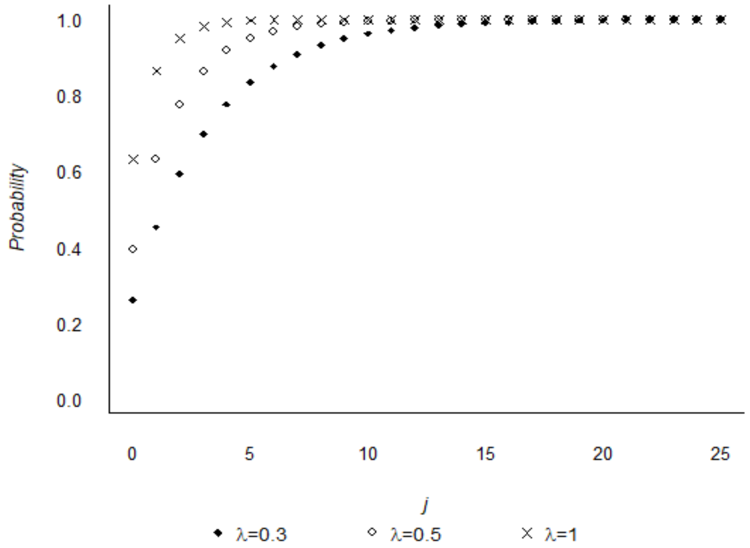


Figure 4. The CDF plot of the Planck distribution with different parameters

Note that $F(0;\lambda)$ is the initial ability of correctly classifying, and the learning progresses rapidly at first, and then gradually becomes stable. The parameter λ is related to the value of the starting point $F(0)$ as well as the speed of learning.

For the minor and major defective state inspection, the learning parameters are denoted as λ_{minor} and λ_{major} respectively. Then, the expected costs for Case 1 and Case 2 are

$$\begin{aligned} U_1^v(T) = &\sum_{i=1}^{\infty}\big[1-\exp\big(-\lambda_{minor}(i+1)\big)\big]\big[iC_i + C_{r\_r}\big]\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx \\ &+ \sum_{i=1}^{\infty}\exp\big(-\lambda_{minor}(i+1)\big)\big[iC_i + C_{r\_nr}\big]\int_{(i-1)T}^{iT}\int_{iT-x}^{\infty} f_x(x)f_y(y)dydx, \end{aligned} \tag{24}$$

and

$$\begin{aligned} U_2^v(T) = &\sum_{i=1}^{\infty}\Big[1-\exp\Big(-\lambda_{major}(i+1)\Big)\Big]\Big[iC_i \\ &+ C_{r\_nr}\Big]\int_{(i-1)T}^{iT} f_x(x)\int_0^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx \\ &+ \sum_{i=1}^{\infty}\exp\Big(-\lambda_{major}(i+1)\Big)\Big[iC_i \\ &+ C_{r\_nr\_e}\Big]\int_{(i-1)T}^{iT} f_x(x)\int_0^{iT-x} f_y(y)\int_{iT-x-y}^{\infty} f_h(h)dhdydx \end{aligned} \tag{25}$$

respectively. Then $C^*(T)$ can be easily obtained based on Eq. (11).

6. Considering the environmental impact of disposed items

Based on the delay time model, all the preceding discussion is relevant to how to minimize the cost per unit of time by recycling and reusing components with acceptable defective states. However, considering the environmental factors, practitioners may want to recycle and reuse as many components as possible on the premise that the total cost would not be excessive. A multi-objective optimization can be applied to describe this problem. Here, according to Assumption (1) listed in Section 2.2, a component with a minor defective state can be renewed and reused. Meanwhile, the probability of a cycle that ends at a positive inspection of a minor defective state, denoted as P_1, is given in Eq. (2). Thus, we have two objectives in this case, including minimizing the long run cost per unit of time $C(T)$ and maximizing the probability $P_1(T)$ at the same time. Considering the different units and scales, we use division to integrate the two objectives and transfer them to the objective

$$\max \frac{P_1(T)}{C(T)}, \tag{26}$$

where $P_1(T)$ and $C(T)$ are given in Eq. (2) and Eq. (11) respectively.

It should be noted that objective function (26) does not need to assume the cost of the environmental impact as estimating the cost is difficult. For example, cost of repairing or replacing a damaged tyre can easily and accurately be estimated. However, estimating the cost of the environmental impact of disposing this tyre may never be accurate.

7. Numerical examples

This section aims to show the application of the proposed method for a single-component system (that can be interpreted as a component and a socket in series, where the socket never fails) subject to two defective states. In the present analysis, we first investigate the effect of different dependability between new and reused components on the optimal inspection interval T (decision variable) and on its respective cost rate $C(T)$, the decision criterion used to determine the inspection policy.

We verify up to which level the dependability of the reused component can be different than the new one, and still be economically viable, given different percentages of reuse in the long run. Then, we analyse the influence of misclassification of minor and major defective states on T and $C(T)$. The case with varying probabilities of correctly classifying a defective item and the multi-objective optimization considering the environmental impact are also involved. The results were obtained numerically and the computing language *R* was used for programming.

7.1 The expected cost with constant p and q

The parameters considered in this analysis are presented in Table 3.

Table 3. Parameters of the model.

Weibull distributions								Reuse	Error		Costs						
β_{x1}	η_{x1}	β_{x2}	η_{x2}	β_y	η_y	β_h	η_h	R_{items}	p	q	C_i	C_{nitem}	C_{ritem}	C_d	B_r	C_{error}	C_{pen}
2.5	varied	3	5	2.5	1	2.5	1	varied	0	0	0.05	1	0.5	0.1	0.1	0.05	5

Regarding the parameters of Weibull distributions, the shape parameter β_{x1} of the distribution of the arrival of minor defects in a reused component is slightly smaller than the shape parameter β_{x2} for a new component, due to its larger dispersion in the arrival of the minor defect. This consideration is based on the practical fact that the reused component may not be as dependable as the new one, having a more dispersed time for the arrival of the minor defect. In addition, the sojourn time in the good state is expected to be shorter in the reused component than in the new one and that is the main effect in terms of different dependability between them. For this reason, we vary the scale parameter of the distribution of the arrival of minor defects in reused components, η_{x1}, as a percentage of the same parameter for new components, η_{x2}. By doing so, we can verify the effect of a shorter life in the reused component on T and $C(T)$, depending on the R_{items}, which is also considered as variable values. The other parameters that characterise the arrival of the major defective state and the arrival of the failure are the same for new and reused components and represent that, in these states, the component has a more dispersed and shortened time, compared to the time in the good state of a new component. The error parameters related to the probability of misclassification of minor and major defects are initially set to zero, because the analysis of misclassification will be presented separately afterwards. Concerning the cost parameters, the cost of acquisition of a new component $C_{nitem} = 1$ monetary unit was taken as a reference for the definition of the other cost values, all of them as a proportion of this value, based on the benefit or on the inconvenient associated. For instance, the penalty cost due to a failure is five time the cost of acquisition of a new component and 10 times the cost of using a reused component in the replacement of a defective one. Also, the discharging cost has the same value of the bonus for a component being reintroduced into the system.

Regarding the influence of different dependability levels of a reused component in comparison with a new one, the analysis contemplates reductions on η_{x1} up to 80% of η_{x2}, varying at a step of 10%. The objective is to quantify the variations on the optimal inspection interval T and on its respective cost rate $C(T)$ for different R_{items}. In Table 4, considering cases 1, 10, 19 and 28, compared to case 0 that represents the non-reuse action, the higher the percentage of reused components, the best is the benefit in terms of cost when there are no significant changes in terms of dependability, reaching a maximum cost reduction of 46.35%, for 100% of reuse. In fact, the reuse alternative is less expensive than using a new component when the dependability between reuse and new components are similar. This is quite logical because the company uses a reused component similar to a new one, with a discounted cost.

Table 4. Effect of different dependability between new and reused components.

	$Case$	η_{x1}	$Red\eta_{x1}$	T	$C(T)$		$Case$	η_{x1}	$Red\eta_{x1}$	T	$C(T)$
Non-reuse	0	5	0	1.0632	0.2835	Non-reuse	0	5	0	1.0632	0.2835
100% Reused	1	5	0	0.9543	0.1521	75% Reused	10	5	0	0.9636	0.1774
	2	4.5	10	0.9396	0.1620		11	4.5	10	0.9536	0.1864
	3	4	20	0.9245	0.1739		12	4	20	0.9435	0.1969
	4	3.5	30	0.9092	0.1888		13	3.5	30	0.9335	0.2091
	5	3	40	0.8942	0.2078		14	3	40	0.9238	0.2236
	6	2.5	50	0.8804	0.2331		15	2.5	50	0.9147	0.2409
	7	2	60	0.8700	0.2683		16	2	60	0.9070	0.2622
	8	1.5	70	0.8683	0.3213		17	1.5	70	0.9020	0.2890
	9	1	80	0.8860	0.4100		18	1	80	0.9031	0.3237
50% Reused	19	5	0	0.9729	0.2025	25% Reused	28	5	0	0.9821	0.2276
	20	4.5	10	0.9668	0.2096		29	4.5	10	0.9794	0.2316
	21	4	20	0.9608	0.2174		30	4	20	0.9766	0.2359
	22	3.5	30	0.9548	0.2260		31	3.5	30	0.9740	0.2403
	23	3	40	0.9491	0.2356		32	3	40	0.9713	0.2450
	24	2.5	50	0.9436	0.2462		33	2.5	50	0.9688	0.2499
	25	2	60	0.9387	0.2581		34	2	60	0.9665	0.2550
	26	1.5	70	0.9349	0.2715		35	1.5	70	0.9646	0.2604
	27	1	80	0.9364	0.2868		36	1	80	0.9666	0.2660

On the other hand, when the dependability between reused and new components starts to become very different, the benefits in terms of cost may not be enough to counterbalance the worst performance of the system due to a short period of life, consequently a more likely failure. As a result, the higher the utilization of reused components in the system, the biggest can be the increase in the cost when the dependability of a reused component is far different from a new one. Comparing cases 9, 18, 27 and 36 with case 0, we note that there is a significant increase in the cost for high levels of reuse, 100% and 75%; no significant increase for 50% of reuse and there is still a small reduction in terms of cost for a low percentage of reuse of 25%. This result leads us to an important conclusion. Small percentages of reuse are the ones that result in the lowest benefits in terms of cost but they are the ones that interfere less in the system when dependability of a reused component is far distinct from the dependability of a new one. This behaviour can be better visualized in Figure 5.

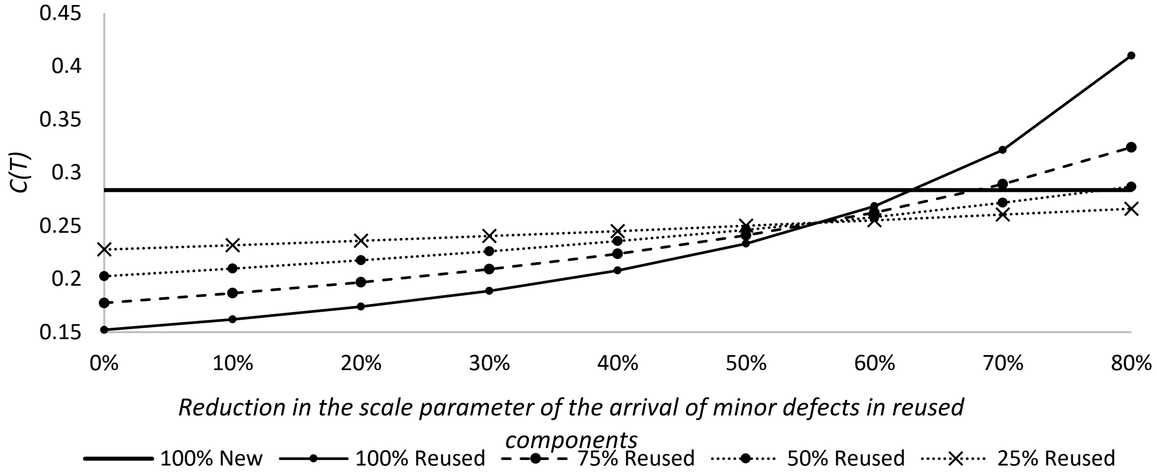
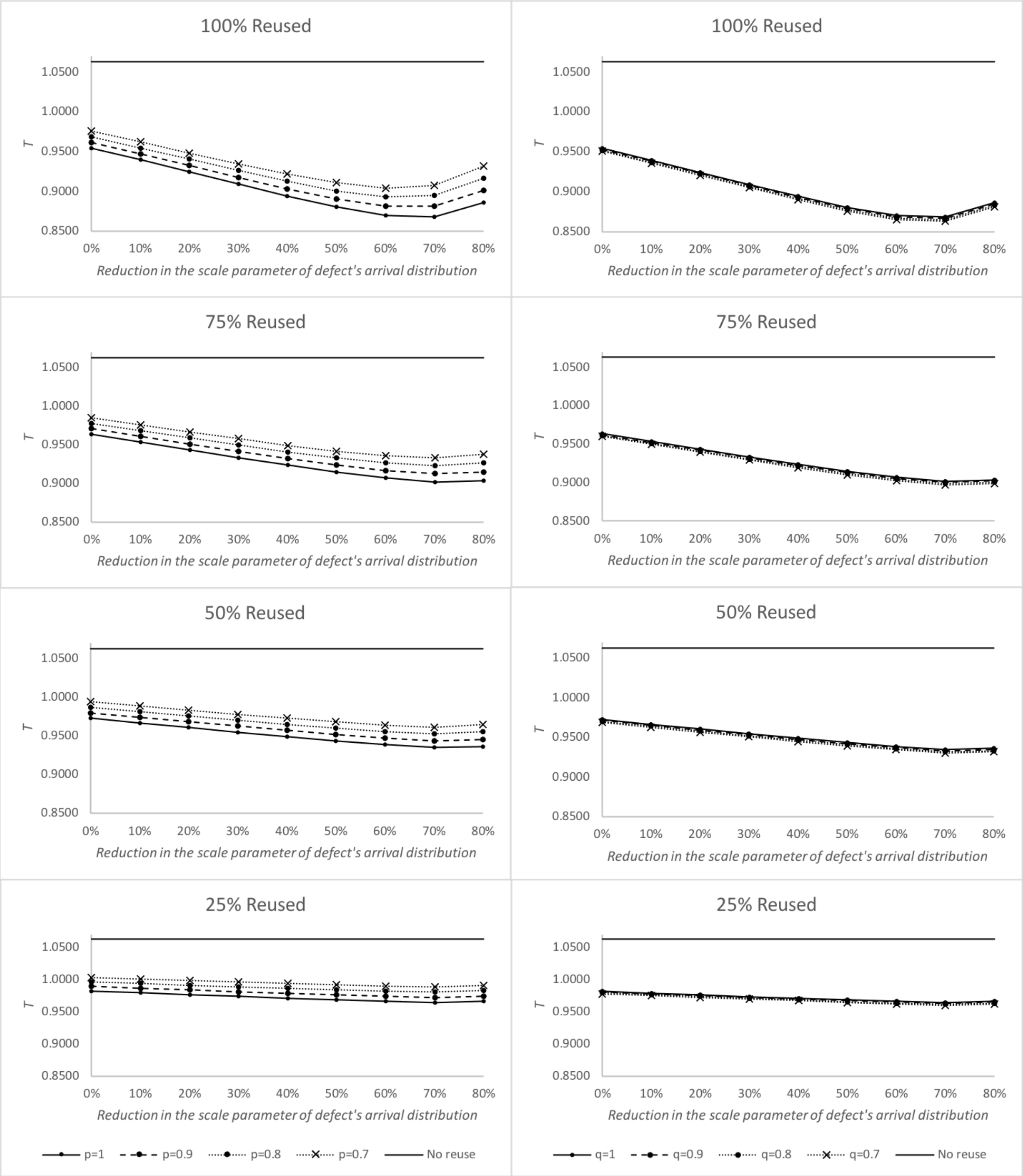


Figure 5. The expected cost of using a determined R_{items} according to the reduction in the scale parameter of the distribution of the arrival of minor defects.

In practical terms, if the company is able to execute a refurbishment process that can guarantee a reused component with a similar dependability of a new one, it is indicated to use a significant percentage of reused components in the long run. Thus, an ideal refurbishment process can enable both economic and environmental benefits. However, if the company is not able to provide an ideal refurbishment process and the dependability of reused and new components are far different, it is indicated that the reuse action not to occur or to be sporadic.

Regarding the effects of having a misclassification of minor and major defects, the analysis shows that the misclassification problem has a higher effect in the model when it is related to the minor defect. Regarding the optimal time to perform inspections, T, the model suggests larger interval between inspections when the probability p decreases. This is an expected behaviour because the model is trying to reduce the impact of the cost related to the discharge of a reusable component. However, when the misclassification error refers to the major defect, the model indicates a very slight reduction in T, which is also an expected behaviour because the model is established to emphasise reused actions. Also, the indication of reducing T as q decreases is a good strategy to reduce the negative effects of misclassification of the major defect because when inspections are performed earlier, there exists a higher chance for the system to be in the previous defective state (minor defective state). The effect of misclassification on the optimal inspection interval is illustrated in Figure 6.


100% Reused

75% Reused

50% Reused

25% Reused

Figure 6. Effect of misclassification on the optimal inspection interval. On the left, variation on the probability of misclassification of minor defects. On the right, variations on the probability of misclassification of major defects.

Regarding the optimal long run cost per unit of time $C(T)$ (Figure 7), the highest increment in the cost rate is given by the misclassification of the minor defect. Even with prior inspections suggested by the model in order to try to lessen the impact of misclassification on the cost, there is an expected increase in terms of cost for lower values of p. The effect of misclassification of the major defect is lesser, especially because the penalty cost for sending a non-reusable component to the spare parts is considerably lower compared to the cost of not reusing a reusable one.

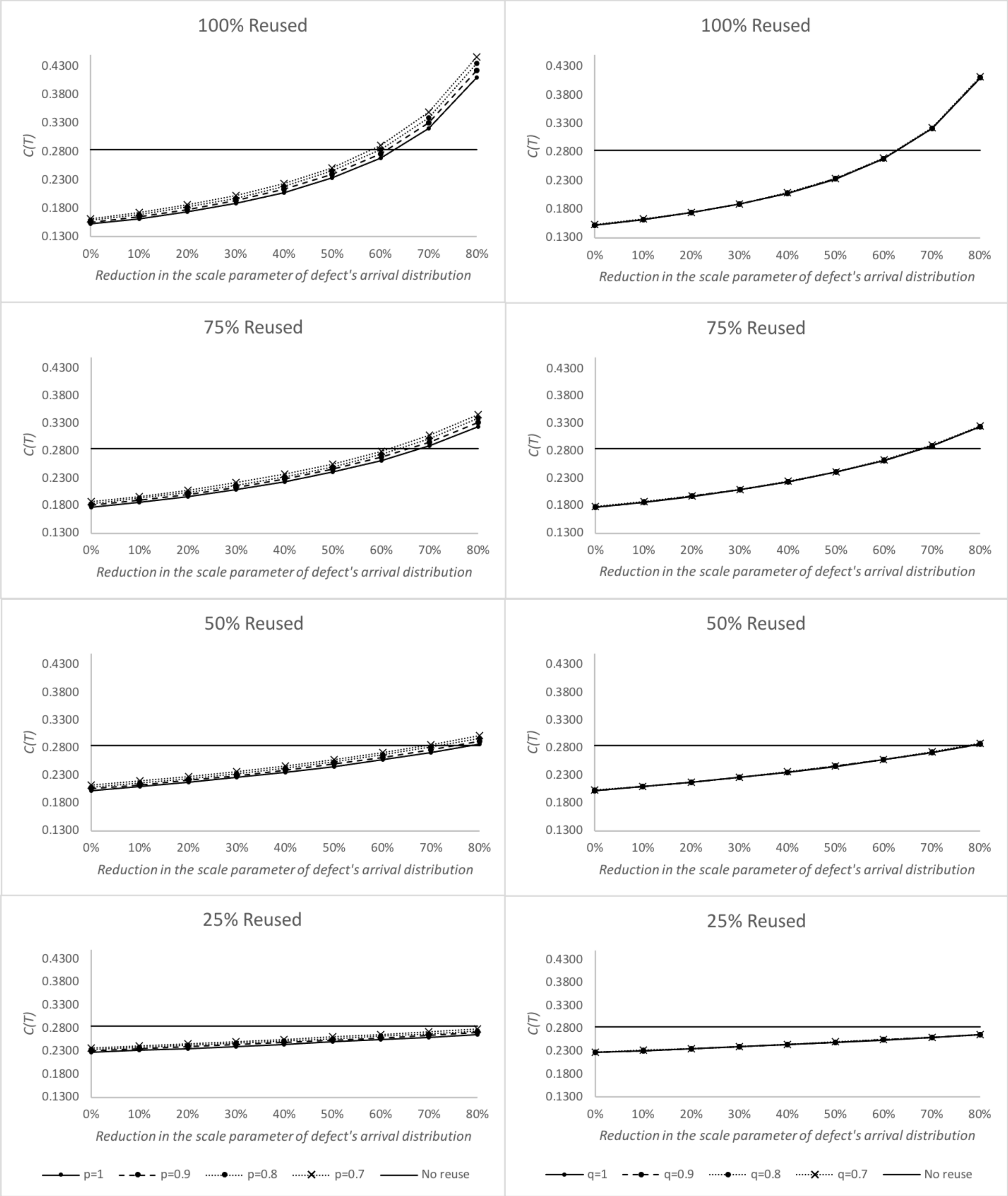


Figure 7. Effect of misclassification on the optimal inspection interval. On the left, variation on the probability of misclassification of minor defects. On the right, variations on the probability of misclassification of major defects.

In practical terms, companies should put emphasis on training actions to reduce the probability of errors in the classification of defects, prioritizing the correct classification of the minor defect, the one that implies on the greatest change in the maintenance policy and on the highest cost rates.

7.2 The expected cost with varying *p* and *q*

Following the above examples using the Plank distribution to model varying values of the probabilities of correctly classifying a defective item, this subsection conducts numerical experiments for the expected cost with varying p and q. Because the major defective state may cause some operation indicators of the system to be obviously abnormal, it is easier to be spotted. Thus, we assume $\lambda_{major} > \lambda_{minor}$, and $\lambda_{major} = 0.5,\ \lambda_{minor} = 0.3$.

Other parameters are set the same as those listed in Table 3, where R_{items} is equal to 0.5 and η_{x_1} takes 1 and 5 respectively. The plot of expected cost is presented in Figure 8.

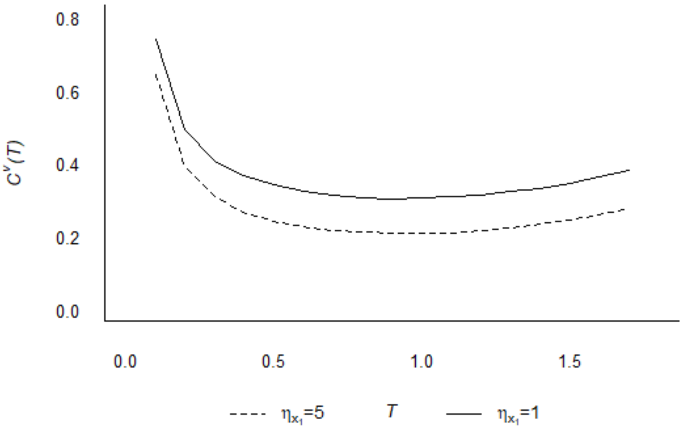


Figure 8. The expected cost per unit of time with varying probabilities p of correctly classifying a defective item under two different values of the scale parameter η_{x_1}.

It can be observed that the optimal inspection interval T is around 1, and the minimum expected costs are approximately 0.2 and 0.3 for the two cases with $\eta_{x_1} = 5$ and $\eta_{x_1} = 1$ respectively.

7.3 With the consideration of the environmental impact of disposed items

This subsection simulates the situation where the environmental impact of disposed items is considered and the objective function given by Eq. (26) is implemented. With the same parameter setting as shown in Table 3 and $R_{items} = 0.5, \eta_{x_1} = 1, 5$, the plot of the change of the objective with different T is given in Figure 9.

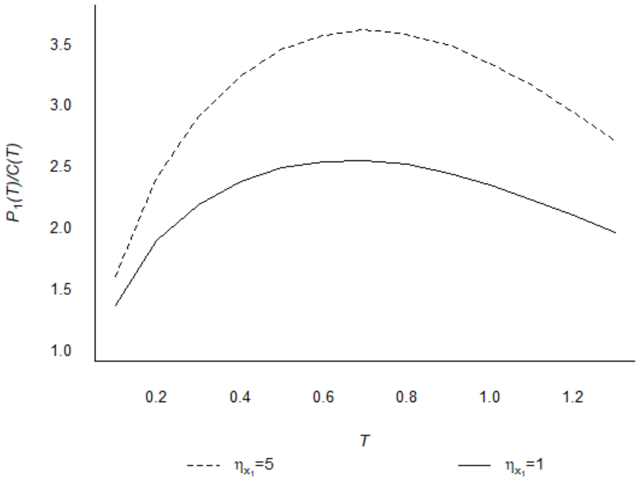


Figure 9. The changing trend of the objective $P_1(T)/C(T)$under two different values of the scale parameter η_{x_1}.

In this case, the inspection interval can take the value around 0.7 to maximize our objective.

7.4 Suggestions of new perspectives, models and analyses

As can be seen in the previous analyses, the focus of this paper is on the optimisation of the interval between inspections and its associated cost, for different proportions of reused items adopted in the long run. In this context, some important scenarios such as, a system with two defective states, misclassification errors at inspections and the environmental impact of disposed items, were evaluated.

This paper did not address the optimization of the percentage of new and reused items in the stock. However, different levels of percentage of new and reused items were considered. In practice, a mix of weak and strong components can be generally considered during the quality control stage. During this stage, manufacturers need to ensure the quality and reliability of their product items. As such, they screen out weak components and keep strong ones. Alternatively, it is motivating to investigate when a weak component may be still economically and environmentally used. In this paper, we extended the operational activities to the operation and maintenance stage and investigate this issue by considering and showing that the reused items (weak items) can also be adopted, within certain limits, with economic and environmental benefits.

Our future research will consider scenarios that are closer to practical scenarios, especially regarding the stock of spare parts. For example, if the inspection cannot be executed in the defined optimal time, by some external interference, the number of new and reused items in the stock could considerably change over time. In this perspective, the optimal inspection period can be considered as an impact factor of R_{items}. Aimed at this new perspective, the current model can serve as an initial base. However, due to a very distinct set of assumptions to be considered, new models, methods of investigation and analyses will be required in our future research.

8. Conclusions

This paper applied the delay time model to the context of reuse of components. The paper emphasised the importance of reuse of industrial deteriorating components and investigated two important practical characteristics: component heterogeneity and misclassification errors. It also presented a practical context for application and a numerical analysis that points out some practical insights into this area.

Finally, the new considerations in the present reuse method enables analyses of important practical characteristics found in reality and it has not been investigated in the literature to a large extent yet. The method proposed in this paper provides an effective way to investigate the possibilities of reuse of an industrial component, based on an analysis of important issues and also taking into consideration the multiple costs involved with the process of reusing or not. A limitation is that the model is only applicable to single-component systems and a suggestion for further investigations is to extend it to a multicomponent-system. Nevertheless, further contributions can also be included in single-component systems, such as the one mentioned in Section 7.4. In addition, applications on different practical examples would enhance practical insights for different particular cases.

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