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Belief Revision and Computational Argumentation: A Critical Comparison

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Abstract
This paper aims at comparing and relating belief revision and argumentation as approaches to model reasoning processes. Referring to some prominent literature references in both fields, we will discuss their (implicit or explicit) assumptions on the modeled processes and hence commonalities and differences in the forms of reasoning they are suitable to deal with. The intended contribution is on one hand assessing the (not fully explored yet) relationships between two lively research fields in the broad area of defeasible reasoning and on the other hand pointing out open issues and potential directions for future research.

Keywords Argumentation · Theory · Belief revision · Dynamics of knowledge · Knowledge representation · Reasoning

1 Introduction

Belief Revision and *Argumentation* represent two exceedingly rich and fertile areas in Artificial Intelligence and Philosophy; both areas are concerned with how intelligent

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agents manage their beliefs and the process of performing the relevant reasoning activities. As such, they are part of the symbolic approach to AI in the field known as Knowledge Representation and Reasoning (KR). Inside KR, the study of the particular property of non-monotonicity that our reasoning mechanisms exhibit has given place to a large corpus of research: as an example, a prominent handbook in the field (van Harmelen et al. 2008) contains chapters addressing both topics. The analysis of human reasoning in Philosophy has a long tradition that spans millennia and has contributed to the AI effort with a distinguished perspective (Strasser and Antonelli 2019; Koons 2017).

The present work should not be considered as an introduction to these two exciting domains of inquiry; nevertheless, in Sect. 3 we provide a bare-bones rendering of the technical level. Our target audience is a researcher who wants to explore one of the areas while being familiar with the other, looking for commonalities and possible connections. Excellent starting points to grasp the formal details in Belief Revision can be found in Peppas (2008), Fermé and Hansson (2011, 2018), Dubois et al. (2020), while for an introduction to the field of Computational Argumentation the reader may refer to Bench-Capon and Dunne (2007), Besnard and Hunter (2008), Rahwan and Simari (2009), Baroni et al. (2018), among others.

We will examine the common ground and consider the differences that can be found between these two research fields, focusing our efforts on finding stimuli for future research in their confluence.

Belief Revision studies the process of changing the content of a belief repository when considering the arrival of a new piece of information. The intricacies of obtaining intuitively satisfactory solutions for this task have been the focus of research in the Computer Science areas of Artificial Intelligence and Databases. However, the inquiry was started as a philosophical question that later got the attention of logicians.[1]

In Argumentation, the primary interest is centered on studying the acceptance of claims backed by arguments representing a form of justification for holding those claims; these justifications are obtained by reasoning from accepted premises. The acceptance process is carried through in the context of disagreement, where the support and interference of other arguments are considered.

When considering settings where autonomous agents are involved, Belief Revision describes how an agent is supposed to change its beliefs when new information arrives, restoring its belief state's consistency.[2] In three and a half decades of intense and fructiferous research, some extensions were introduced that propose the use of belief bases, a construct which distinguishes between fundamental and derived beliefs (Hansson 1998), and paraconsistent belief revision models that distinguish between incoherence and inconsistency (see for instance Rott 2001, Chapter 3 and Casini et al. 2018; Testa et al. 2017). Argumentation is concerned with establishing the agent's current beliefs from a potentially incomplete and inconsistent knowledge base, ensuring that these beliefs satisfy some desirable properties (for an overview see Baroni et al. 2011, 2018).

[1] For the impact of AGM belief revision in the Artificial Intelligence community, see Carnota and Rodríguez (2011).

[2] Here, we are assuming the classical AGM theory (Alchourrón et al. 1985).

The relationships and distinctions between belief revision and argumentation go back to Epistemology, where the classical AGM Theory of Belief Revision (Alchourrón et al. 1985) represents a coherentist approach and argumentation a foundationalist approach (Sosa 1980; Olsson 2017). In Computer Science, the origins of computational argumentation could be found in Jon Doyle's Truth Maintenance Systems (Doyle 1979). Later, Jon Doyle studied the relation between belief revision and argumentation in Doyle (1992), where he contrasted the foundationalist approach and the coherentist approach:

> Recent years have seen considerable work on two approaches to belief revision: the so-called foundations and coherence approaches. The foundations approach supposes that a rational agent derives its beliefs from justifications or reasons for these beliefs: in particular, that the agent holds some belief if and only if it possesses a satisfactory reason for that belief. According to the foundations approach, beliefs change as the agent adopts or abandons reasons. The coherence approach, in contrast, maintains that pedigrees do not matter for rational beliefs, but that the agent instead holds some belief just as long as it logically coheres with the agent's other beliefs. More specifically, the coherence approach supposes that revisions conform to minimal change principles and conserve as many beliefs as possible as specific beliefs are added or removed.

It is interesting to observe that Jon Doyle was persuaded that there existed similarities in both approaches that could produce significant advances in the effort to provide an autonomous agent with effective knowledge representation and reasoning abilities. In what follows, we will explore both approaches with the intention of introducing several research questions that could bring more understanding of the issues involved in belief maintenance and reasoning, much in the spirit of the work presented in Gärdenfors (1990) which was focused on the same problem. [3]

The paper is organized as follows. Section 2 informally introduces belief revision and argumentation discussing some of their different facets at an intuitive level, while Sect. 3 provides a succinct technical background. Sect. 4 discusses the principles underlying the processes of belief revision and argumentation, while Sect. 5 provides a general analysis of these processes. Their drivers and goals are the subject of Sect. 6, while the process objects are examined in Sect. 7 and Sect. 8 concludes the paper.

2 Belief Revision and Argumentation: The Intuitive Level

From a very general perspective, both belief revision and argumentation can be seen as processes carried out by a reasoner having the goal (or the need) to modify its cognitive state when facing some change.

As a preliminary step in our analysis, we will examine some informal definitions of the two processes which are available in the literature with the goal of identifying commonalities, specificities, and, possibly, significant differences between both areas.

[3] For a discussion see also Del Val (1997), Hansson and Olsson (1999).

Starting with belief revision, let us introduce the following example (Fermé 1992; Fermé and Hansson 2018):

Example 1 We consider the following set of sentences in natural language "Juan was born in Puerto Carreño" (α), "José was born in Puerto Ayacucho" (β), and "Two people are compatriots if they were born in the same country" (γ). We assume that this set represents all the currently available information about Juan and José. Suppose that we receive the following piece of new information: "Juan and José are compatriots" (δ). If we add the new information to our corpus of beliefs, then we obtain a new set of beliefs that contains the sentences α, β, γ and δ. We can define an operation of addition as one that takes a sentence and a set of previous beliefs and returns the minimal set that includes both the previous beliefs and the new sentence. This operation exemplifies the simplest and unproblematic way of changing a set of sentences. There are other types of change that are not so simple.

For example, suppose that upon consulting an atlas we discover to our surprise that Puerto Carreño is in Colombia (ϵ) and Puerto Ayacucho is in Venezuela (ϕ). If we add ϵ and ϕ to the set $\{\alpha, \beta, \gamma, \delta\}$, the result will be a set with contradictory information: Juan and José are compatriots but Puerto Carreño and Puerto Ayacucho do not belong to the same country. The addition does not satisfactorily reflect the notion of a *consistent revision*. If we wish to retain consistency, then some subset of the original set must be discarded or perhaps a part of the new information has to be rejected. In our example, there are several possible alternatives. The information about Juan's or José's birthplace could be wrong, and so could the atlas. Finally the fact that Juan and José are compatriots could be wrong. Any of these three options, either individually or combined, will allow us to solve the problem of the incompatibility among the original and the new information or beliefs. Consequently, we can specify an operation that takes a set and a sentence and returns a new consistent set. The new set includes parts (or all) of the beliefs in the original set and it also includes the new sentence (if we are willing to accept it). The outcome of a revision can be expressed as a consistent subset of the outcome of the addition. This operation is based on two notions: *consistency* and a *selection* among the possible ways to perform the change.

The use of the term belief revision predates the AGM paper (Alchourrón et al. 1985); in particular, the earliest reference to it came from the first writings of Gärdenfors (Gärdenfors 1981, 1982, 1984), while Alchourrón and Makinson were interested initially in changes in logical theories and in norms (Alchourrón and Makinson 1981, 1982). Later, both approaches (postulates and meet functions) were joined in the AGM seminal paper.[4]

In Artificial Intelligence, a survey article of 1980 by Doyle and London (1980) provides a selected bibliography of about 250 papers related to this area. In its introduction, it states that "Belief Revision concentrates on the issue of revising systems of beliefs to reflect perceived changes in the environment or acquisition of new information. Additionally, belief revision research includes the study of methods for representing models of environments as collections of beliefs and the development of formal theories of belief.". This definition adopts the perspective of an intelligent agent dealing

[4] For details see Makinson (1996), Gärdenfors (2011).

with an external environment and suggests a loosely specified distinction between different forms of change ("perceived changes in the environment or acquisition of new information").

An entirely different perspective expressing a distinct kind of change was considered in another paper of the early eighties (Fagin et al. 1983) that has played a significant role in the subsequent development of the belief revision area. In that paper, the problem under study was that of updating a database that is equipped with integrity constraints over its content and rules for deriving explicit information. The reasons that led to the update are left unspecified, and there is no notion of an environment; the attention is instead focused on the requirements over the update operation: "... we consider the problem of updating arbitrary theories by inserting into them or deleting from them arbitrary sentences. The solution involves two key ideas: when replacing an old theory by a new one we wish to minimize the change in the theory, and when several theories involve minimal changes, we look for a new theory that reflects that ambiguity."

Thus, since the beginning of the field, we witness the coexistence of two views of belief revision. On one hand, an agent-oriented one, where an agent holds beliefs that are referred to an external world, hence some attention is paid to the features of the agent-environment interaction. And, on the other hand, a theory-oriented one, where the term belief is used in a rather generic sense (a currently true proposition in some information repository), and attention is given to the change operations to be carried out and their properties.

Gärdenfors's foundational book (Gärdenfors 1988) considers the two visions together, as stated in the preface: "This is a book about how to change your mind. More precisely, I present a theory of rational changes of belief. The epistemic changes that are at focus are revisions that occur when the agent receives new information that is inconsistent with the present epistemic state." While adopting an agent-oriented perspective explicitly, attention is focused since the beginning on the formal property of consistency and its preservation.

Sampling the literature, we can identify examples of definitions that can be assimilated to each of the three perspectives described below. Notably, alternative terms like *belief change* or *belief revision* have been sometimes used.

Agent-oriented

- Any intelligent agent has to account for a changing environment and the fact that its own beliefs might be inaccurate. For this reason, belief revision is a central task for any form of intelligent behavior (Nebel 1989).
- The study of belief change has been an active area in Philosophy and Artificial Intelligence, and, more recently, in game theory. The focus of this research is to understand how an agent should revise his beliefs as a result of getting new information (Friedman and Halpern 1994).

Theory-oriented

- Belief Revision is the process of incorporating new information into a knowledge base while preserving consistency (Nebel 1992).

– In the logic of belief revision, a belief state (or database) is represented by a set of sentences. The most significant change operations are those consisting of the introduction or removal of a belief-representing sentence (Hansson 2017).

Combined

– The capability of gathering information about the world and revising its beliefs based on the new information is crucial for an intelligent agent; therefore, belief revision is a central topic in Artificial Intelligence. Technically, belief revision is the process of changing the beliefs of an agent to accommodate new, more precise, or more reliable evidence that is possibly inconsistent with the existing beliefs (Jin and Thielscher 2007).
– Belief revision is the area of knowledge representation that is concerned with how an agent may incorporate new information about a domain into its set of beliefs. It is assumed that the agent has some corpus of beliefs K which are accepted as being true or holding in the domain of application. A new formula α is given, which the agent is to incorporate into its set of beliefs. Since consistency is to be maintained wherever possible if α conflicts with K some beliefs will have to be dropped from K before α can be added (Delgrande 2012).

All the above definitions have in common the reference to the process of managing changes to an information base in front of some input. As to the underlying differences, at a general level it is possible to point out that the theory-oriented view puts more emphasis on the form of the process, with no specific attention to the nature of the input and of the object of the process itself (*e.g.,* the terms belief set, belief base, knowledge base, logical theory, and database are regarded as somehow interchangeable). On the other hand, the agent-oriented view has, at first reading, a narrower focus and some consideration for different sorts of input, while leaving formal properties of the process in the background.

In computational argumentation too it is possible to identify a probably most cited and most influential paper, namely Dung's paper on abstract argumentation frameworks (Dung 1995) but interest in using computational models of arguments in AI predates Dungs's paper by at least fifteen years. The earliest works (Birnbaum et al. 1980; Cohen 1981; Birnbaum 1982) arose from the area of computational linguistics, where argumentation is regarded as a dialogical process ("a specific kind of conversation", quoting Cohen 1981) between two opponents which hold different opinions and try to defeat each other through utterances of attacks and counterattacks. While being very preliminary (and compact), these works introduce several fundamental notions and issues, in particular, argument defeasibility, the relation of attack among them, and the problem of identifying the prevailing arguments which are key elements of all subsequent works in the field.

While building (more or less explicitly) on these key elements, other definitions describe argumentation as a kind of reasoning process, where non-conclusive reasons are used to derive conclusions. For instance, Pollock (1992) states that "Defeasible reasoning is, *a fortiori*, reasoning. Reasoning proceeds by constructing arguments, where reasons provide the atomic links in arguments. Conclusive reasons logically

entail their conclusions. Defeasibility arises from the fact that not all reasons are conclusive. Those that are not are prima facie reasons. Prima facie reasons create a presumption in favor of their conclusion, but it is defeasible."

Thus, also in informal descriptions of computational argumentation, we can identify (at least) two non-incompatible views: a dialogue-oriented one, where arguments are moves in a game concerning the acceptance of some claim, and a reasoning-oriented one, where arguments are the results of some inference process, conflicts among them may arise due to the limits of the information and/or knowledge used in the process, and the reasoner needs to take a stance on the conflicts themselves. Again, definitions combining the two views can also be found, as evidenced by the following literature sampling.

- *Dialogue-oriented*
  - Argument is a social and verbal means of trying to resolve, or at least to contend with, a conflict or difference that has arisen or exists between two (or more) parties. An argument necessarily involves a claim that is advanced by at least one of the parties (Walton 1990).
  - The study of argumentation may, informally, be considered as concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held (Bench-Capon and Dunne 2007).
  - To the extent that agents are autonomous, no one agent can impose its will on another. To the extent that agents are intelligent, they will need to persuade one another to adopt particular beliefs or courses of action, or negotiate with one another to divide scarce resources between them. Such activities are examples of argument, which we might define as rational, or reason-based, interaction between autonomous and intelligent agents to achieve particular goals (Rahwan and McBurney 2007).
- *Reasoning-oriented*
  - Arguments are *prima facie* proofs that may make use of assertions that one sentence is (defeasible) reason for another. They indicate support for a proposition, but do not establish warrant once and for all; it matters what other counterarguments there may be (Simari and Loui 1992).
  - This chapter surveys logics for a particular group of patterns of inference, namely those where arguments for and against a certain claim are produced and evaluated, to test the tenability of the claim. Such reasoning processes are usually analysed under the common term 'defeasible argumentation' (Prakken and Vreeswijk 2001).
- *Combined*
  - In multi-agents systems, conflicts of interest are inevitable. To address these problems, agents can use argumentation, a process based on the exchange and valuation of arguments for and against opinions, proposals, claims and decisions. Argumentation, in its essence, can be seen as a particularly useful and

intuitive paradigm for doing nonmonotonic reasoning (Caminada and Amgoud 2007).
- The word argument may recall several intuitive meanings, like the ones of line of reasoning leading from some premise to a conclusion or of utterance in a dispute (Baroni and Giacomin 2009).
- In a sense, monological argumentation is a static form of argumentation. It captures the net result of collating and analyzing some conflicting information. In contrast, dialogical argumentation is a dynamic form of argumentation that captures the intermediate stages of exchanges in the dialogue(s) between the agents and/or entities involved (Besnard and Hunter 2008).

All of the above definitions have in common the reference to the process of managing some form of conflict. The dialogue-oriented view encompasses the presence of different actors and emphasizes the dynamic aspects of the process, while the reasoning-oriented view is not concerned with process actors and is more focused on defeasibility and the evaluation of argument acceptability.

Due in part to the development of the relevant scientific literature along the last 20 years, another important distinction concerns the abstraction level of theoretical models of argumentation. Rooted in the seminal paper by Dung (1995), *abstract argumentation* models adopt the notions of argument and attack as primitive concepts, the underlying nature, and structures of arguments and attacks being abstracted entirely away. Abstract argumentation models are therefore solely focused on the issues of conflict management and argument justification. As such they cover only a part (though a very important one) of the whole argumentation process. Models, where argument construction and attack definition are explicitly taken into account, can be referred to generically as *instantiated* or *structured* argumentation models (Besnard et al. 2014), though it may well be the case that these models are abstract in some respect and far from real instantiations (*e.g.,* they may leave the logical language for argument construction unspecified). It is not uncommon that structured models are bridged with abstract argumentation as far as the issues of conflict management and argument justification are concerned. Clearly, the distinction between abstract and structured models is orthogonal to the separation of dialogical and reasoning-oriented argumentation and the literature is rich in works covering all four possible combinations. This variety cannot be illustrated by a single example, which of course, may cover only some of the various aspects of argumentation. With this warning, in the following, we revisit Example 1 in argumentation terms, noting that it belongs to the reasoning-oriented view.

Example 2 From an argumentation perspective, the available knowledge and information give rise to the production of arguments, which, in many approaches, can be classified as defeasible or strict, depending on whether they contain elements that are considered fallible, *e.g.,* uncertain premises or rules which admit exceptions. While the acceptance of a strict argument is undebatable, the acceptance of a defeasible argument may vary, being subject to confrontation with other possibly conflicting arguments. In the example, at the start, we would have two (probably defeasible) arguments corresponding to the pieces of evidence held by the agent that "Juan was born in Puerto Carreño" (A_1), "José was born in Puerto Ayacucho" (A_2). The rule that "Two people

are compatriots if they were born in the same country" (R_1) would, in many structured argumentation approaches, belong to a rule base used for argument construction. One may assume that this rule does not admit exceptions. Clearly, in this situation, there are no conflicts between the two arguments A_1 and A_2, which can be regarded as acceptable, together with their claims.

The new evidence "Juan and José are compatriots" gives rise to a corresponding (again probably defeasible) argument (A_3) and then also, using R_1, to the production of one or more other arguments (the details depending on the knowledge representation and inference mechanism adopted) including one (A_4) whose conclusion is "Puerto Carreño and Puerto Ayacucho are in the same country". Also, there are no conflicts between the produced arguments in this situation, which can be regarded as all acceptable, together with their claims.

After consultation of the atlas, one or more arguments are produced (again, the details depend on the knowledge representation and inference mechanism adopted), including one (A_5) whose conclusion is "Puerto Carreño and Puerto Ayacucho are not in the same country".

Now, given the presence of a contradiction between the conclusions of arguments A_4 and A_5, a relation of attack among arguments has to be identified (this step was needed, in principle, also in the previous situations but produced an empty relation). While the details on attack identification depend, again, on the specific argumentation formalism adopted, we observe that this step is not trivial (see, for instance, the various notions of attack considered in Gorogiannis and Hunter 2011 in the context of logic-based argumentation and the general discussion on attack relations and their properties in Baroni et al. 2018). In particular, while the existence of a conflict is revealed by the contradictory conclusions of A_4 and A_5, the attack identification procedure may look for the roots of this conflict in the inference process and thus identify attacks involving A_1, A_2 and A_3. Also, the direction of these attacks will typically take into account the difference between strict and defeasible arguments.

Based on the attack relation, the acceptability of arguments can be assessed through a formal criterion (typically called *argumentation semantics*), leading to identify several alternative sets of arguments (typically called *extensions*) which can stand together and correspond to a defensible position. For instance, assuming that all pieces of evidence (including the one coming from the atlas) are considered equally uncertain and that the adopted semantics tends to maximize the considered sets of acceptable arguments, the alternative defensible positions would correspond to the following sets of arguments $E_1 = \{A_1, A_2, A_3, A_4\}$ (corresponding to the case where the argument based on the evidence of the atlas is rejected), $E_2 = \{A_1, A_2, A_5\}$ (corresponding to the case where the argument about Juan and José being compatriots and the derived one about Puerto Carreño and Puerto Ayacucho being in the same country are rejected), $E_3 = \{A_2, A_3, A_5\}$ and $E_4 = \{A_1, A_3, A_5\}$ (corresponding respectively to the cases where the argument concerning the birthplace of Juan or José is rejected and hence is also the one about Puerto Carreño and Puerto Ayacucho being in the same country).

If instead, one assumes, for instance, that the evidence-based on the atlas has to prevail on the other ones, the identified attack relation would be different, and as a consequence, the set E_1 would not be included among the defensible positions.

Armed with this (clearly non-exhaustive) overview of basic concepts and intuitions in belief revision and argumentation, we are now ready to undertake a more detailed analysis and comparison from a process modeling perspective. As to belief revision, we will use as main reference the AGM approach. As to argumentation, we will refer to the structure of a generic argumentation system as conceptualized in Prakken and Vreeswijk (2001), and use as a formal counterpart Dung's theory of abstract argumentation frameworks.

Before diving into the matter, to make the article self-contained, we will provide a summary of the necessary basic technical background and terminology in the following section.

3 Belief Revision and Argumentation: The Technical Level

We will now succinctly introduce the necessary technical details regarding belief revision and argumentation systems.

3.1 The AGM Approach to Belief Revision

In the AGM approach (Alchourrón et al. 1985; Gärdenfors 1988), beliefs are represented as sets of sentences in some language $\mathcal{L}$ including the standard connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ and constants $\top$, $\bot$. The language is governed by a logic expressed by a consequence relation $Cn \subset 2^{\mathcal{L}} \times 2^{\mathcal{L}}$ from sets of sentences to sets of sentences. Given a set of sentences A, $Cn(A)$ is the set of sentences which are logical consequences of A. For a sentence x, $x \in Cn(A)$ is also written as $A \vdash x$ and $x \notin Cn(A)$ as $A \nvdash x$. A set A of sentences is consistent if $A \nvdash \bot$. It is assumed that, for any sets of sentences A and B, Cn satisfies the following three conditions: $A \subseteq Cn(A)$, $Cn(A) = Cn(Cn(A))$, if $A \subseteq B$ then $Cn(A) \subseteq Cn(B)$. Furthermore we will assume that Cn satisfies *supraclassicality* ($Cn(A)$ includes what can be derived from A using classical logic), *compactness* (if $x \in Cn(A)$ then $x \in Cn(A')$ for some finite $A' \subseteq A$) and *deduction* ($x \in Cn(A \cup \{y\})$ if and only if $(y \rightarrow x) \in Cn(A)$).

A *belief set* is a set of sentences K closed under the consequence relation, namely $K = Cn(K)$. Given a consistent belief set K, a sentence x is accepted with respect to K if $x \in K$, is rejected with respect to K if $\neg x \in K$, is undetermined if neither of the above cases holds.

The following three basic operations of change on a belief set K are considered:

- *Expansion* (denoted as $K^+(x)$) consists in adding to K a new sentence x which is consistent with K;
- *Contraction* (denoted as $K^-(x)$) consists in removing from K a sentence which was previously in K;
- *Revision* (denoted as $K^*(x)$) consists in adding to K a new sentence x which is not consistent with K.

Expansion can be univocally defined as $K^+(x) = Cn(K \cup \{x\})$; but, that is not the case for contraction and revision. The AGM approach provides a set of postulates for

the operations of contraction and revision that constrain the space of possible choices to satisfy the rationality criteria.

An alternative to the AGM approach is the *Levi identity*, which states that the revision operation can be expressed in terms of contraction and expansion: revising a belief set K by adding an inconsistent sentence x is equivalent to contract K with respect to $\neg x$ and then expanding the result with x, formally:

$$K^*(x) = (K^-(\neg x))^+(x) \text{ (Levi identity)}$$

Dually, it is possible to express contraction in terms of revision through the *Harper identity*, which states that contracting of a belief set K with respect to a sentence x is equivalent to intersecting K and the revision of K with respect to $\neg x$, formally:

$$K^-(x) = K \cap K^*(\neg x) \text{ (Harper identity)}$$

We will provide further details and comments on these operations and their properties and relationships throughout the paper. It is interesting to remark that the term *revision* is overloaded as it is used to denote both a specific operation and the general activity of managing any change operation. This situation justifies the use of the term *belief change* (or similar ones) in many works expressing the latter meaning.

3.2 Argumentation Systems

There is consensus in the argumentation community over what are the required steps to define an argumentation system (Prakken and Vreeswijk 2001); namely, five main conceptual components are necessary:

(a) an underlying "logical" language,
(b) the definition of what an argument is,
(c) the definition of a conflict relation between arguments,
(d) the definition of a defeat relation between arguments,
(e) the definition of the justification status of arguments.

Some quick comments on these components are in order. First, it is assumed that arguments are constructed using some language (several alternatives are possible) which is typically referred to in the literature as "logical" in the sense that it encompasses some mechanism to represent argument claims and premises and to derive claims from premises. The actual language used may vary widely, ranging from propositional logic to rule-based systems. Second, the construction of arguments is typically assumed to be monotonic: new arguments do not suppress previous arguments, but instead, they may be in conflict with them. The presence of conflicts does not give rise to pathological situations like the production of any possible argument but is regarded as an essential feature of the system. Several types of argument conflicts can be conceived, the main idea being, in any case, that conflicting arguments cannot be accepted together. Not all conflicts are effective, the ones which are give rise to the defeat relation. To give a simple example, if two arguments α and β have incompatible conclusions, *e.g.,* α concludes that tomorrow will be sunny and β concludes that

tomorrow will be rainy, they are in mutual conflict. However, the reasons underlying α might be stronger than the ones underlying β (*e.g.,* a reliable local forecast against the weather report of a national newspaper) so α defeats β but not vice versa. Finally, given the defeat relation, the justification status of the arguments has to be evaluated (*e.g.,* an undefeated argument will probably be accepted, while an argument which is defeated by an undefeated argument will be rejected). Many variants, extensions, and subtle analyses of these basic concepts are available in the literature. The reader is referred to Chesñevar et al. (2000), Prakken and Vreeswijk (2001), Bench-Capon and Dunne (2007), Besnard and Hunter (2008), Rahwan and Simari (2009), Baroni et al. (2011), Atkinson et al. (2017), Baroni et al. (2018) for introductory and/or review material on the field. We will now introduce the two main families of formalisms, called abstract and structured, used in the literature for describing and investigating computational argumentation systems at different levels of abstraction.

We will commence by presenting an important formal approach that has allowed the in-depth study of the interplay of arguments when nothing but their existence and the relation of attack (defeat in the terminology introduced above) is taken into account. Given the abstract nature of the formalism, several of the five items described conflate themselves in two elements: the notion of argument reduces to an atomic entity making unnecessary the specification of a logical representation language, and the notion of attack is expressed as a binary relation between these atoms (arguments); it is crucial to notice that an attack is invariably successful, always resulting in defeat; thus, attack and defeat coincide. Then, we will discuss all five elements required for structuring the arguments, a logical representation language and what an argument is, and defining their interplay, attack, defeat, and final justification status in the full definition of a concrete system.

3.2.1 Dung's Abstract Argumentation Framework

An *argumentation framework* Dung (1995), is simply a pair $\langle \mathcal{A}, \mathcal{R} \rangle$ consisting of a set $\mathcal{A}$ whose elements are called *arguments* and of a binary relation $\mathcal{R} \subset \mathcal{A} \times \mathcal{A}$ called *attack relation*. Given two arguments α, β if $\langle \alpha, \beta \rangle \in \mathcal{R}$ it is said that α attacks β. An argumentation framework has a visible representation as a directed graph where nodes are arguments and edges are drawn from attacking to attacked arguments.

While the word *argument* may recall several intuitive meanings, abstract argumentation frameworks are not (even implicitly or indirectly) bound to any of them: an abstract argument is not assumed to have any specific structure but, roughly speaking, an argument is anything that may attack or be attacked by another argument, where, again, no specific meaning is ascribed to the notion of attack. As such, in abstract argumentation frameworks the first three components of an argumentation system (language, argument definition, and conflict definition) are entirely abstracted away: the (binary) attack relation corresponds to the defeat relation of Prakken and Vreeswijk (2001), and the main focus is on the evaluation of the justification status of arguments.

An *argumentation semantics* is the formal definition of a method (either declarative or procedural) ruling the argument evaluation process. A rich variety of alternative proposals of argumentation semantics is available in the literature (see Baroni et al. 2011;

Baroni and Giacomin 2009). Two main styles of argumentation semantics definition can be identified in the literature: *extension-based* and *labeling-based*.

In the extension-based approach a semantics definition specifies how to derive from an argumentation framework a set of *extensions*, where an extension E of an argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ is just a subset of $\mathcal{A}$, intuitively representing a set of arguments which can "survive together" the conflict or are "collectively acceptable". Putting things more formally, given an extension-based semantics $\mathcal{S}$ and an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ let us denote the set of extensions prescribed by $\mathcal{S}$ for AF as $\mathcal{E}_{\mathcal{S}}(AF) \subseteq 2^{\mathcal{A}}$. The justification state of an argument $\alpha \in \mathcal{A}$ according to an extension-based semantics $\mathcal{S}$ is then a derived concept, defined in terms of the membership of α to the elements of $\mathcal{E}_{\mathcal{S}}(AF)$. In particular, the simplest notions of justification correspond to *credulous justification*, namely membership in at least one extension, and *skeptical justification*, namely membership in all extensions.

In the labeling-based approach, the definition of a semantics specifies how to obtain a set of *labelings* from an argumentation framework, where a labeling L is the assignment to each argument in $\mathcal{A}$ of a label taken from a predefined set $\mathcal{L}$, which corresponds to the possible alternative states of an argument *in the context of a single labeling*. Again, an argumentation semantics prescribes a set of labelings and the justification state of an argument α turns out to be a derived concept, defined in terms of the labels assigned to α in the various labelings. A typical choice for the set of labels is $\mathcal{L} = \{\texttt{in}, \texttt{out}, \texttt{undec}\}$: under this choice, it is possible to translate an extension-based semantics into its labeling-based equivalent formulation and vice versa (see Baroni et al. 2011).

3.2.2 Arguments with Structure

As we have just seen, in an abstract argumentation framework the inner details of the arguments are not considered, *i.e.,* arguments are treated as atomic entities. However, when the issue of actual argument construction becomes pertinent for the formalism the five aspects introduced at the beginning of this section become relevant.

The construction of arguments admits many different possibilities and the research community has produced systems where argument structure has been adequately considered (Pollock 1987; Loui 1987; Simari and Loui 1992; Prakken and Sartor 1997; Bondarenko et al. 1997; Besnard and Hunter 2001; García and Simari 2004; Prakken 2010). In a recent special issue dedicated to structured argumentation (Besnard et al. 2014) tutorial papers on four prominent argumentation formalisms can be found. In all of these proposals, it became necessary to make decisions regarding the representation language, the actual construction of arguments using that underlying language, the specification of the relation of conflict between these arguments, the determination of how the conflict is decided, and the final decision on the justification status of arguments.

All of the five elements mentioned at the beginning of Sect. 3.2 have an impact on the characterizations of revision mechanisms over structured argumentation systems. Every one of them becomes more or less relevant in the different approaches to the definition of systems that combine belief revision and argumentation. In the forthcoming discussion, we will make appropriate comments on these issues.

4 Rationality Criteria and Postulates

Both belief revision and argumentation can be seen as general processes that admit several different realizations. For instance, belief revision can be performed using different operators, while in the argumentation field alternative semantics for argument evaluation are available. General requirements for these general processes are formulated in terms of postulates or principles which can be used to discriminate or evaluate distinct instantiations. In this respect, the historical evolution has been entirely different in the two fields.

In belief revision, AGM postulates (Alchourrón et al. 1985) have played a seminal role, so that postulate (re-)definition has been regarded as a starting (and central) point in all subsequent developments like belief updating, base revision, and other forms of belief dynamics. As we mentioned previously, contraction and revision cannot be defined univocally, but the proposed postulates circumscribe the operations to the compliance of the rationality criteria. For AGM, these criteria listed in order of priority are (Gärdenfors 1988; Dalal 1988):

- *Primacy of new information*: the new information is always accepted.
- *Consistency*: the new epistemic state must be consistent, if possible.
- *Minimal change (informational economy)*: the attempt to retain as much of the old beliefs as possible.
- *Adequacy of representation (categorical matching):* the revised knowledge should have the same representation as the old knowledge.
- *Fairness:* If there are many epistemic states candidates for the outcome of a belief change, then one of them should not be arbitrarily chosen.

In abstract argumentation, the identification and analysis of general principles for semantics evaluation and comparison (Baroni and Giacomin 2007) has been undertaken more than a decade after Dung's seminal paper and checking the properties of a new semantics proposal against these principles is becoming a standard practice. For rule-based argumentation systems a set of so-called *rationality postulates* (Caminada and Amgoud 2007) has been proposed in the same year. A recent comprehensive overview of the developments of the principle-based approach in abstract argumentation is given in van der Torre and Vesic (2018), while for rationality postulates, the reader may refer to Caminada (2018).

Reviewing and comparing in detail postulates in the two fields is far beyond the scope of the present work, therefore we will limit ourselves to some general observations.

First, we note that we can distinguish:

- Informal principles, like informational economy;[5]
- Formal principles not involved with the underlying formal representation and inference mechanism, like semantics evaluation principles of Baroni and Giacomin (2007);

[5] In Belief Change, the postulates that involve minimal change have been the object of controversies. For instance, in contraction, the *recovery* postulate has been extensively debated (Makinson 1987, 1997a; Hansson 1991, 1999a; Fermé 2001), whereas in the case of revision, the AGM revision operator has been questioned regarding the preservation of minimal change (Rott 2000).

– Formal principles concerning the underlying representation formalism and inference mechanism, like consistency, the AGM postulates, and the rationality postulates of Caminada and Amgoud (2007).

Naturally, principles at different levels cannot be compared directly, although in some cases some relations to each other can be found.

Starting with the principle of *informational economy*, it has no direct counterpart in argumentation also because of the underlying differences in the two settings. In belief revision sentences are removed from the belief set, while in argumentation systems the arguments change their justification status according to the adopted semantics, but, roughly speaking, nothing gets lost in the process. It can be noted however that, given the same argumentation framework, different semantics ascribe different justification status to the same arguments. These different outcomes occur because some semantics tend to be very restrictive when producing the set of justified arguments while others are more "liberal". This intuitive notion has a formal counterpart in the definition of *skepticism relations* between semantics (Baroni and Giacomin 2009). While these relations have been introduced and analyzed without reference to the dynamical aspects of argumentation, relating them to the principle of informational economy presents an interesting research issue.

Also, the principles for abstract argumentation semantics, introduced in Baroni and Giacomin (2007), have been conceived for a static setting and are not bound to any underlying logical representation. Some of them, however, bear some resemblance with notions in belief revision. A quick and very partial list of potential links follows.

– The *language independence* principle (semantics evaluation at the abstract level is not influenced by differences in the representation at the language level) can somehow be put in relation with the AGM *extensionality* postulate (logically equivalent sentences give the same revision/contraction result).
– The *I-maximality* principle (no extension is a strict subset of another one) plays a role in the behavior of semantics in a setting of skeptical argument justification and can, therefore, be related with *informational economy*.
– The *conflict-free* property of extensions (an argument in an extension cannot attack another argument in the same extension) can be regarded as a weakening of the consistency requirement.
– The *reinstatement* of an argument (if the attackers of an argument are in turn attacked, then the argument can be accepted) recalls the AGM postulate of *recovery* (if one removes and then adds a sentence, nothing of the initial belief set gets lost).

These relationships, which point forward to a more in-depth inquiry, also suggest the opportunity for belief revision postulates to be reconsidered at a more abstract logic-independent level, while argumentation principles are extended or reformulated to account for reasoning dynamics.

The so called *rationality postulates* for rule-based argumentation (Caminada and Amgoud 2007) have some stricter similarity with the belief revision setting.

The postulate of *closure under strict rules* can be seen as a weakening of the assumption of closure on belief sets. So, on the one hand, it carries within argumentation the kind of "omniscience" problems that will be discussed in Sect. 7, on the other hand, the fact that closure refers only to strict rules raises the issue of distinguishing between

facts and (different types of) rules, and this may represent a significant research point. Similarly, the postulates of *direct* and *indirect consistency* provide a refinement of the consistency property. Direct consistency means that conclusions explicitly justified by the results of the argumentation process are consistent, while indirect consistency means that the closure under strict rules of the explicitly justified conclusions is consistent. Some rule-based argumentation formalisms fail to satisfy closure under strict rules and indirect consistency, but it can be (and it is) debated whether this is "a bug or a feature", in the sense that a "blind" application of closure under strict rules can be regarded as essentially in conflict with the nature of argumentation as an open process, where not only arguments but also the conflicts have to be built incrementally and explicitly.

Leaving apart these conceptual issues, in Caminada and Amgoud (2007) it is shown that postulate satisfaction could be recovered through a proper (re)definition of the notion of rebutting (conflict) between arguments. By analogy, this suggests that the notion of consistency could be reconsidered, so to open the way to some form of inconsistency-tolerant belief revision.[6]

Looking deeper at the notion itself of consistency reveals another significant issue which is worth analyzing. In belief revision, a sentence can be in one of three justification states depending on the membership of the sentence (and of its negation) to the belief set: accepted, rejected, or undecided. Certainly, a belief set cannot include both a sentence and its negation, and this is fundamentally the only constraint for consistency. In extension-based argumentation, the relationship between an argument and an extension (which is essentially a conflict-free set of arguments) gives rise to the same justification states, while, equivalently, labeling-based argumentation assigns one out of three possible labels to each argument. However, most argumentation semantics prescribe multiple alternative extensions (or labelings). If one adopts a skeptical attitude, they can be reduced to the case of a single extension/labeling by taking their intersection (this is done for instance in Caminada and Amgoud (2007)). Still, the existence of multiple extensions introduces the possibility of a less drastic attitude and consider a set of seven justification states (see Baroni et al. 2004; Baroni and Giacomin 2009; Wu and Caminada 2010), which correspond to the possible combinations of memberships/labels of an argument in different extensions/labelings (*i.e.,* to all non-empty subsets of {`in`, `out`, `undec`}).

In this more variegate model of justification, also the notion of consistency can assume a less "traditional" form, allowing various nuances. Whether this vision can be extended with beneficial effects to belief revision is a further intriguing question.

Summing up …

– Postulates and principles are a fundamental element of classical belief revision theory (and of all its variations). They played a lesser role in the initial development

[6] Several different ways for inconsistency-tolerant belief revision can be found in the literature; among them is possible to mention the use of paraconsistent logic instead of classical logic (Priest 2001; Testa et al. 2017, 2018), the use of context sensitivity (Chopra and Parikh 1999; Hansson and Wassermann 2002), and the adoption of belief states (Fermé and Wassermann 2018).

of argumentation theory; however, principles and postulates concerning both the properties of extensions and the rationality of the conclusions drawn have received significant and increasing attention in the last years.
– Postulates and principles formulated at different levels of abstraction and concerning static vs. dynamic reasoning contexts are not directly comparable. It is possible however to identify some analogies and conceptual relationships which (along with non-symmetrical gaps) suggest interesting opportunities of cross-fertilization between the two fields.

5 Belief Revision and Argumentation as Processes

We are now ready to start our analysis of belief revision and argumentation as processes. By process, we mean a well-defined series of operations which, given the initial state of the object of the processing activity and some input, define the transition of the object to a new state, where some further input can be processed in turn. The object of processing activity, its state, the process input, and the transition steps have therefore to be characterized.
In the AGM approach this is, at a first glance, relatively easy:

– The object of the processing activity is a belief set.
– State information is implicit in the belief set itself (the members of the belief sets are accepted).
– The input is a sentence along with the specification of the operation to be carried out: expansion, contraction, or revision (note that a preliminary consistency check is needed to distinguish between expansion and revision).
– There is only one transition step, namely the application of a suitable operator to the belief set to produce a new belief set.

In argumentation, the situation is more articulated. First, within the whole argumentation process (at least) two distinct sub-processes can be identified: the production of arguments (and specifically of counterarguments in dialogical argumentation) and the evaluation of their justification status. The grain of interleaving of the two sub-processes depends on the type of argumentation. In dialogical argumentation at each dialogue step, some arguments (typically one) can be produced. The new argument is bound to be related (typically is a defeater) of a previous one. According to the rules of the dialogue protocol, the newly added argument entails the modification of the acceptance status of some previously produced arguments, and the dialogue can proceed with the next step. In reasoning-oriented argumentation typically no specific constraints are given on how far argument production can go before the relevant conflicts are identified, and argument justification status is evaluated.

Leaving apart interleaving, let us now focus on the two sub-processes. Argument production operates on some knowledge base and produces a set of arguments. In this sense, it differs from belief revision since it does not modify the initial object (the knowledge base) but instead produces a new object (a set of arguments). There are no requirements of consistency or closure on the knowledge base. Indeed, the presence of inconsistencies is one of the leading reasons for using argumentation, where inference

is explicit in the construction of arguments rather than being implicit in the assumption of closure.

In this context, the acquisition of new information corresponds to modifying the knowledge base. At least conceptually, all arguments are then rebuilt from scratch from the new knowledge base. Drawing a parallel with belief revision operators we note that, in the addition of a new element to the knowledge base used to build arguments, there is no need to distinguish between expansion and revision, as inconsistency does not need any special treatment. As to retraction, we note that argument production is a monotonic process: no previous argument needs to be canceled to change your mind since this is the task of the subsequent evaluation phase.

The subprocess of argument evaluation operates on a set of arguments and their defeat relation and attaches a justification status to each of them. There is no direct information acquisition in this process since it operates on already built arguments. However, at an abstract level, one can consider modifications of the set of arguments, of the defeat relation, or both, as triggers of the evaluation process, without considering the causes of these modifications in the argument construction phase explicitly. In any case, at least conceptually, after the modifications argument evaluation is carried out from scratch. As belief revision operators refer to sentences while evaluation operates on arguments, drawing a parallel with this subprocess is not immediate. It can be however noted that, like belief revision operators, argument evaluations concerns justification states[7] and their dynamics. Hence, some parallel could be drawn between the various justification changes in the two areas, with the specific difference that belief revision operators require as a parameter a sentence p, while this is not the case in the dynamics of argument evaluation. These kinds of issues are left for future analysis.

5.1 A "Hidden" State in Belief Revision

While the description above offers a basic and self-contained view of belief revision and argumentation as processes, it has to be remarked that any non-trivial belief revision process requires a further state component, which is not explicitly addressed by its basic definitions and postulates.

In fact, the AGM postulates offer a wide range of choices for the definition of a contraction (or equivalently, revision, as one can be defined in terms of the other) operator, some of the possible choices being regarded as unsatisfactory (*e.g.,* in Alchourrón et al. 1985 it is stated that *maxichoice revision* has *disconcerting properties*). To constrain the choice to a set of more reasonable alternatives, in Alchourrón et al. (1985) the notion of partial meet contraction was introduced. Leaving apart formal details, given a belief set K and a sentence x to be retracted, partial meet contraction is based on a *selection function* γ that picks some of the maximal subsets of K which do not imply x. Thus, the result of a contraction is the intersection of the selected maximal subsets.

With the addition of the selection function as a relevant element, it emerges that the starting point of belief revision is not just the initial belief set: it will also include

[7] The reader may refer to Baroni and Riveret (2019) for a discussion of the evaluation of statement justification based on argument justification.

the choice by the reasoner of a selection function γ. Intuitively, γ has to select the "most important" (Alchourrón et al. 1985) maximal subsets of K that fail to imply x. To fulfill this intuition, the authors of Alchourrón et al. (1985) proposed that the selection function γ will be based on a total preorder among the maximal subsets of K which do not imply x. Gärdenfors (1988), Gärdenfors and Makinson (1988) proved that this is equivalent to define a contraction (resp. revision) operation based on an *epistemic entrenchment* ordering between the sentences of the language; essentially, the epistemic entrenchment ordering is a total preorder among the sentences, where $x \leq y$ if and only if $K^{-}(x \wedge y)$ does not imply x or $\vdash x \wedge y$.[8]

It has to be evidenced that the notion of epistemic entrenchment does not concern the credibility of sentences (since all the sentences in K are credible), the idea is rather that "some propositions are more valuable than others in inquiry and decision making" and then a reasoner is more inclined to give up those propositions which are less useful. Strictly speaking, this "ordering" relation over sets of sentences (Alchourrón et al. 1985) or over sentences (Gärdenfors 1988) determines univocally how to perform the belief change. Another way to express the behaviour of a change operation (*i.e.,* contraction or revision) uses the form $x > y$ with the meaning: "If we revise our belief set by x, then y would be accepted". However, these conditional beliefs cannot be included in the belief set (this result is known as Gärdenfors' impossibility theorem Gärdenfors 1986), therefore a richer structure is needed.

Independently of the technical details, the relevant point is that in this context the initial state of the belief revision process does not consist in the belief set only but also includes the reasoner's attitude towards the selection of what sentences should survive contraction, possibly expressed by an entrenchment ordering. This extra-logical information is necessary to determine a reasonable revision strategy.

Altogether, the reasoner's belief set and this additional "strategical" information are called *epistemic state* in Darwiche and Pearl (1997); Boutilier (1998) and *belief state* in Jin and Thielscher (2007) (however, it has to be noted that these terms have no consistent usage in the literature: for instance, the term epistemic state is used with a different meaning in Gärdenfors (1988) and belief state is used with a different meaning in Boutilier (1998)). Here, we will adopt the term epistemic state.

5.2 Iteration in Argumentation and Belief Revision

Having laid out this basis for discussion, an important observation concerning the iteration of these processes becomes necessary; clearly, dialogical argumentation is iterative by nature. Similarly, in reasoning-oriented argumentation, the processes of argument production and argument evaluation can be iterated at will using the result of the previous iteration as a starting point of the subsequent one, without posing any specific conceptual problem. Actually, studies on argumentation dynamics in Dung's framework (Cayrol et al. 2010; Liao et al. 2011) have been largely focused on computational rather than fundamental conceptual/modeling aspects, the problem being the one of minimizing recomputation when only a part of the framework changes

[8] The epistemic entrenchment ordering is based on a set of postulates, for the sake of readability we left out the formal details; see Gärdenfors (1988), Gärdenfors and Makinson (1988).

by characterizing the effects and scope of different kinds of change. This approach can also be useful in a multi-agent context to efficiently identify which argument to use in a dialogue to obtain a particular desired effect. At a less abstract level, dynamics can be explicitly modeled in the underlying logic for argument construction, with resulting effects on the set of arguments and attacks which have to be considered at a given step of the process evolution (Rotstein et al. 2010).[9] The notion of dynamics in argumentation has also been considered with richer and more articulated meanings, involving the change of the dialogue protocol. For instance, in the context of dialogical argumentation an approach has been proposed to model the fact that "at any point in the argumentation, participants can start a meta-level debate, that is, the rules of order can be made the topic of discussion" (Brewka 2001).

The situation is very different for the basic AGM approach, which does not lend itself to iteration: in a sense, *iterated belief revision* is a research subfield on its own (for instance, see Boutilier 1993; Nayak 1994; Darwiche and Pearl 1997; Fermé and Rott 2004; Booth and Meyer 2006; Jin and Thielscher 2007; Hunter and Delgrande 2011; Rott 2009; Peppas 2014). In fact, given that the starting point of the process of belief revision is actually a full epistemic state, not just a belief set, it follows that when iterating the process for a further step a new full epistemic state is needed.

The revision operator produces the belief set component of this new state, but the "strategic" component (as introduced in the previous subsection) is problematic; indeed, it can not stay the same, since the revision strategy also needs to take into account the new information acquired, but the AGM theory does not give any account on how to revise the revision strategy; so, it stops after the first step. One possible solution to this aporia would be to encode the strategical information within the belief set in the form of conditional beliefs. However, as we mentioned previously, Gardenfors's impossibility theorem (Gärdenfors 1988) shows that including conditional beliefs within the belief set leads to a trivial revision model.

To circumvent the problem just described, it is necessary to define a distinct revision mechanism for the strategical component of the epistemic state and redefine the whole belief revision process so that it operates on the whole epistemic state and produces a new epistemic state. Recent works show that the Levi and Harper identities, *i.e.,* the inter-definition of contraction and revision, are no more valid (Booth and Chandler 2016; Konieczny and Pino Perez 2017). Since our focus is the relation with argumentation in the rest of the paper, we will only focus on iterated revision. This gives concrete evidence that the question on what is the object of the revision (and, as we will see, also of the argumentation) process, is more open than what the traditional models suggest. This will be the subject of Sect. 7, while the next one will deal with process drivers and goals.

[9] In fact, this approach opens an interesting perspective on a potential interplay between belief change and argument change, which is however out of the main scope of the present article. A discussion on some recent approaches using belief revision notions inside argumentation is provided in Sect. 8.

Summing up …

- The process of belief revision involves three kinds of change operations in front of new information: expansion, contraction, and revision. Expansion not being problematic, attention is focused on contraction and revision. Under some assumptions, contraction can be defined in terms of revision and vice versa. At a general level, in argumentation, different kinds of change operations do not need to be identified explicitly: the addition of new information does not require specific operators but is dealt with through regular argumentation activities. It may then give rise to the generation of new arguments and conflicts, then to a different argumentation framework, and, finally, the justification status of all arguments and their claims is re-evaluated.
- In belief revision, the justification status of sentences is defined implicitly by the membership to the belief set, in argumentation the justification status is the result of an explicit evaluation. In both theories, the result, though subject to some general constraints, is not predetermined since there are different choices for the process mechanisms. In belief revision, the choice concerns the actual operator to apply. In argumentation, the choice mainly concerns the semantics for argument evaluation and then the derivation of sentence justification from argument justification. Given a revision operator, the result is a univocal belief set. Given an argumentation semantics, the result is a set of extensions or labelings.
- Iteration is problematic in classical belief revision since the revision process requires some extra-logical information driving the revision strategy but does not specify how to revise it, so this information is lost after the first step. An extended model is needed to account for iteration. Iteration is explicit in dialogical argumentation, implicit in reasoning-oriented argumentation, but does not pose specific conceptual problems.

6 Process Drivers and Goals

In this section, we consider the question of what are the causes of the change process and what are its goals. In belief revision, these issues are highly abstracted. As to the causes, the acquisition of new information (or the necessity to retract something) is taken as the starting point without additional considerations on why and from where the new information arises. Essentially, the goal is to incorporate the new information, preserving consistency (by suppressing some information, which is incompatible with the new one).

However, it has been observed, that different causes for the introduction of the new information require different processes. Classical belief revision implicitly assumes that the starting point is a new acquisition of information about a static world which has not changed. It has been observed (Katsuno and Mendelzon 1991) that the revision process, which is appropriate for this kind of context, is not suitable for the case where the new information acquired is an effect of a change in a dynamic world. In this case, the process is called *belief updating* and needs to satisfy different requirements. The difference between revision and update was originally pointed out by Keller and

Winslett (1985) (in the context of relational databases) and is captured in the following example (Winslett 1988):
"Initially an agent knows that there is either a book on the table (p) or a magazine on the table (q), but not both.

> Case 1: The agent is told that there is a book on the table. She concludes that there is no magazine on the table. This is revision.
> Case 2: The agent is told that subsequently a book has been put on the table.

In this case, she should not conclude that there is no magazine on the table. This is update."

As a matter of fact, while belief revision and belief updating are conceptually distinct processes, it is quite evident that in most real contexts an agent has to manage them "simultaneously" so that an approach that is able to encompass both is highly desirable and has been considered in some of the literature (*e.g.,* see the notion of belief change in Friedman and Halpern 1994, of generalized update in Boutilier 1998, and of belief change with actions and observations in Hunter and Delgrande 2011).

Orthogonal to the distinction between revision and update is the fact that the change involves factual rather than generic knowledge (see Sect. 7). As to our knowledge, this has received limited attention in the literature (Dubois and Prade 1997; Wassermann and Fermé 1999), with the exception of the application of belief revision in dynamics of ontologies, in particular by using Description Logics.[10]

A common feature of the processes considered above is that the new information has absolute priority over the previous beliefs: this corresponds to a *prioritized* revision process. While prioritized revision is the only choice in some contexts (*e.g.,* when including the latest amendments in a normative system) it has been observed that in other contexts (*e.g.,* acquisition of information from possibly unreliable sensors) the new information not necessarily prevails over the previous one. In this case, non-prioritized revision is more appropriate; this form of revision in turn may appear in a variety of forms (see Hansson 1999b). For instance in *screened revision* (Makinson 1997b) a decision is made at the beginning of the process whether to accept or not the input on the basis of its consistency with some previous *core beliefs*, while *selective revision* (Fermé and Hansson 1999) encompasses the case of partial input acceptance.

Intuitive as it may be, non-prioritized revision is not the only way to go to take the possible unreliability of input into account. In fact, one can replace the "uncertain" fact that the room temperature is 28 degrees, with the "certain" fact that the sensor reading is 28 degrees and then consider a more complex reasoning scheme where the fact that the room temperature is 28 degrees is derived (and could not be accepted) from the fact (which is accepted with certainty) that the sensor reading is 28 degrees. In this perspective, non-prioritized belief revision is unnecessary provided that one adopts a "proper" modeling. This poses however two problems: first, the "proper" model may turn out excessively and unnaturally complicated, second, it opens a serious question about what can be considered definitely certain, given that one may doubt about anything (in the example, one can, in turn, be uncertain about the process of reading from the sensor, about its memory of what has been read, and so on). As

[10] For example, see Flouris (2006), Flouris et al. (2008).

a matter of fact, these problems do not arise if the input is regarded rather as an assumption of the reasoner in a frame of hypothetical reasoning. Clearly, the reasoner is free to ascribe arbitrary strength to the adopted assumptions (even the most absurd ones), but the question then arises on whether this casting to hypothetical reasoning is always sensible.

All the above distinctions did not find room for development in abstract argumentation, at least up to now. At an abstract level, the input gives rise to the addition of new arguments or the relevant attack relations (or both), and the process goal is just to recompute the justification status of the whole set of arguments. At the underlying structured level, this corresponds to a modification of the knowledge base from which arguments and attacks are built. No special role is ascribed to the modified part with respect to the preexisting part. The computation of the upgraded set of arguments and conflicts is just a consequence of modifying the knowledge base, which, in a sense, triggers it. Different criteria for argument evaluation are represented by different semantics, but the cause of the modification (for instance the distinction between revision and updating) has not been related to the choice of the semantics. Moreover, abstract argumentation is non-prioritized by nature, since all arguments (either new or old) are subject to the same evaluation method. Recovering consistency is not an issue in argumentation (given that the presence of conflicts lies at its very heart) but, as mentioned previously, the weaker notion of conflict-freeness of extensions (or labelings) captures the same kind of requirement.

Dialogical argumentation is more interesting from this viewpoint since different kinds of dialogue with different goals can be considered. An essential reference in this field (Walton and Krabbe 1995) identifies six main types of dialogue: persuasion, negotiation, inquiry, deliberation, information-seeking, and eristic. They are distinguished by the initial situation, the overall dialogue goal, and participants' specific goals. Different dialogues obey different rules/protocols according to their goal-oriented nature; this is a significant departure from other forms of argumentation and belief revision which are essentially goal-neutral. It has to be observed however that dialogue types appear to be conceived for a static world where only the dialogue participants play an active role: encompassing a notion analogous to belief updating in this context would require further analysis. Further, in argumentation dialogues, the last move is typically regarded as prevailing over previous ones and can be possibly defeated only by the next move. These two fundamental similarities suggest that dialogical argumentation can be closer to belief revision than it appears at first glance.

Argument schemes Walton (1996), Walton et al. (2008) provide another potential starting point for future investigations on different kinds of processes across the two fields. An argument scheme is a predefined semi-formal pattern representing a prototypical way to build presumptive support for a given conclusion in a given context. Examples of these schemes include argument from Expert Opinion, from Position to Know, from Witness Testimony, and so on. For each scheme, a set of *critical questions* are defined. In a nutshell, each critical question represents a challenge for an argument built according to the scheme: whether the presumption supported by an argument stands or falls depends on the answers to the relevant critical questions. So, in a sense, critical questions can be regarded as context-specific guidelines to revise the justification status of arguments from various kinds of input affecting different

aspects of the argument structure. While this is hardly reconcilable with the general context-independent formulation of classical belief revision, one may wonder whether, especially in the context of base revision introduced in Sect. 7, considering different kinds of revision schemes may be useful.

Summing up …

- Classical belief revision is suitable to model the acquisition of new information which has the highest priority and concerns a static world. Removing the priority assumption gives rise to a nonprioritized revision process while considering a dynamic world corresponds to the different process of belief updating. Argumentation is, *per se*, a non-prioritized process where the distinction between revision and updating has not to be considered up to now. Dialogical argumentation has some basic similarities with classical belief revision.
- Classical belief revision has the implicit goal of ensuring consistency, abstract argumentation aims at evaluating argument justification status, so they can be regarded as goal-neutral processes, while argument-based dialogues are goal-oriented processes.
- Argument schemes provide a semi-formal approach to describe, in a context-dependent way, different patterns of presumptive justification and different patterns of revision through the notion of critical question.

7 The Process Objects

As mentioned in Sect. 5, while in the AGM approach the object of the revision process is a belief set, this modeling choice turns out to be unsatisfactory when iteration comes into play. In this section we further elaborate on this point, examining other issues regarding the definition of the process object in belief revision and argumentation.

As already mentioned, classical belief revision theory encompasses the existence of information concerning the revision strategy in addition to the belief set. So the reasoner is endowed with an epistemic state which, at least for iteration purposes, should be the object of the change process rather than the belief set itself. In a sense, the belief set is "too small" as it does not cover the whole epistemic assets of the reasoner.[11]

From another perspective, the belief set has been questioned as being "too large". The reason for this is that the assumption the belief set is closed under the consequence relation corresponds to an unrealistic property of logical omniscience of the reasoner.[12] Isaac Levi has proposed that the belief set K should be interpreted as containing the statements that the agent is committed to believing, rather than those that she truly believes in Levi (1977, 1991). However, since the belief set is infinite except for very simple cases, it has to be acknowledged that actual applications and reasoners can only work on a finite representation. Typically it is possible to identify a finite set of *basic*

[11] For an overview of the literature, see (Fermé and Hansson 2018, Section 5.6).

[12] For an overview of the literature, see (Fermé and Hansson 2018, Section 5.5).

beliefs, which the reasoner holds *a priori* independently of any inference activity and a (usually infinite) set of *merely derived* beliefs, which the reasoner holds as a result of the inference activity. These derived beliefs can, in turn, be distinguished into *explicit beliefs*, that the reasoner has derived up to the current instant and *implicit beliefs* that the reasoner could produce after further inference activity.

Therefore, some approaches to belief revision consider the set of *basic beliefs*, also referred to as a *belief base*, as the object of the change process (Fuhrmann 1991; Nebel 1989, 1992; Hansson 1993). Choosing this alternative has several implications which go beyond the mere issue of practical feasibility.

First, it has to be observed that a belief base induces a unique derived belief set, while different belief bases can induce the same belief set. Moreover, revising different belief bases may produce different results even if the bases correspond to the same belief set. This technical difference evidences that *base revision* and classical belief revision not necessarily have to be regarded as two sides of the same coin. As a matter of fact, in base revision, the actual form of the belief base may make a difference, despite logical equivalence; that is, the content of the belief base carries some meaning concerning what the reasoner "has in mind" (and is revising). To put it in other words, most beliefs are based on reasons, and these reasons may, intuitively, play a role in the revision process. However, as stated by Gärdenfors himself (Gärdenfors 1988): "belief sets cannot be used to express that some beliefs may be reasons for other beliefs". On the other hand, base revision appears to be more suitable to support some form of *reason maintenance* although not including an explicit representation of the reasons underlying the beliefs. In particular, the contents of a belief base can be separated in different classes with different roles in the revision process, *e.g.,* in the database field one can distinguish facts from integrity rules, where, clearly, integrity rules are "less revisable" than facts (Fagin et al. 1983). On the basis of this kind of considerations, Nebel (1989, 1992) develops an analysis on *symbol level vs. knowledge level* belief revision. It is suggested that base revision corresponds to *symbol level* revision, as it depends on the actual representation of the belief base, while classical belief revision is at the *knowledge level* since the revision of a belief set is not affected by choosing a specific representation as a belief base.

Nebel proposes an approach to reconcile these different views, by showing that base revision and belief revision produce the same result under a suitable choice of the selection function underlying belief revision (and provided that one gives up one of the classical revision postulates). Using Nebel's words, in the context of this approach reason maintenance "appears as a side effect".

A more explicit distinction of the objects of revision and, consequently, of the revision processes, has been suggested in Dubois et al. (1996). The authors distinguish two main kinds of information: *factual evidence* and *generic knowledge*. Factual evidence consists of information gathered on the case at hand or the description of the actual world in a given situation. Generic knowledge pertains to a class of situations considered as a whole but does not refer to a particular case. Then the authors distinguish three processes of information acquisition:

– *Focusing* which consists in conditioning the generic knowledge by the factual evidence, namely changing the reference class for an individual (*e.g.,* from bird to penguin);
– *F-revision*, which consists in enforcing a new piece of factual evidence;
– *G-revision*, which consists in enforcing a new piece of generic knowledge.

The authors emphasize that this kind of difference "is most clear in settings where generic knowledge is represented and processed as distinct from factual evidence. Such a distinction is not relevant in propositional logic, for instance, since every piece of information takes the form of a propositional sentence." In this perspective, the distinction on the objects of revision also becomes a difference in the actual revision process, as we have seen in Sect. 6 (see also Baroni et al. 2007).

Turning to argumentation, the object of the change process is a set of arguments with their justification status. Argument construction and structure are abstracted away in Dung's framework while they are part of the game in more concrete formalisms like the ones mentioned in Sect. 3.2. The distinction between belief set and belief base in belief revision may be put in parallel with the distinction between warrant and justification by Pollock (1992). Pollock conceives reasoning as an activity which "can involve numerous false starts, wherein a belief is adopted, retracted, reinstated, retracted again, and so forth. At each stage of reasoning, if the reasoning is correct then a belief held on the basis of that reasoning is justified, even if subsequent reasoning will mandate its retraction. Epistemic justification, in this sense, is a procedural notion consisting of the correct rules for belief updating having been followed by the system up to the present time in connection with the belief being evaluated." On the other hand, a warrant corresponds to an ideal situation where a "reasoner unconstrained by time or resource limitations would ultimately be led to believe the proposition. Warranted propositions are those that would be justified "in the long run" if the system were able to do all possible relevant reasoning."

The argumentation process deals with justifications rather than warrants, and so is closer to base revision than to classical belief revision. In particular, by operating on arguments rather than directly on sentences, argumentation incorporates, by construction, a form of reason maintenance. In non-abstract argumentation, the distinction between primary and derived beliefs is explicit as it is the reason why an argument is rejected.

In argumentation, the possibility to combine different semantics offers an interesting opportunity of non-monolithic modeling of reasoning activity. In fact, in the context of the same reasoning process (or of the same dialogue), one can identify different sub-contexts or sub-topics where different flavors of reasoning (*e.g.,* more or less skeptical) are appropriate. This has been considered for instance in Prakken (2006) where a proof procedure combining skeptical and credulous reasoning is proposed, based on the standpoint that, in certain contexts, epistemic reasoning (*i.e.,* reasoning about beliefs) is skeptical while practical reasoning (*i.e.,* reasoning about actions) is credulous and that the two forms of reasoning are interleaved in real situations.

Looking at flexible modeling from another perspective, we observe that most of the structured argumentation formalisms are able to accommodate the distinction between factual knowledge (that can be represented by the knowledge base) and generic knowl-

edge (that can be represented by the set of rules). Finer distinctions are then available for both: the knowledge base is partitioned into axioms, premises, assumptions, and issues, while rules can be strict or defeasible. While this provides a basis to support useful distinctions between revision objects and processes of the kind advocated in Dubois et al. (1996), full exploitation of this potential appears to be an open research question. Applying to argumentation an analysis of the kind carried out by Nebel on symbol-level *vs.* knowledge-level representation is also an open issue.

Summing up …

- Classical belief revision refers to an idealized, closed, and possibly infinite belief set as the object of the change process, while in concrete contexts the object of the change is a finite belief base. In argumentation, an analogous idealization corresponds to the notion of warrant, but argumentation models typically refer to the more concrete notion of (revisable) justification.
- Reason maintenance and the distinction between primary and derived beliefs are not encompassed in classical belief revision, while they are dealt with implicitly in base revision. These notions are indeed foundational and explicit in argumentation. Further distinctions on the process objects have been considered in non-abstract (structured) argumentation.
- In belief revision, the idealized setting has been assimilated to the knowledge-level, while base revision to the symbol-level. A similar analysis is lacking in argumentation.

8 Discussion and Conclusions

Modeling reasoning processes is a formidable and fascinating enterprise. Belief revision and argumentation capture partially distinct but surely non-disjoint aspects of this research challenge, with many cross-fertilization opportunities and open questions this article has aimed, at least partially, to illuminate from an original perspective. We believe there are lessons to be learnt and room for improvement in both areas: either can be a source of inspiration for the other in some respects, with mutual benefit.

Note that we focused on the analysis of belief revision and argumentation as general approaches to capture various forms of non-monotonic reasoning. We aimed at analyzing and pointing out differences, commonalities, and cross-fertilization opportunities at a foundational level.

An exciting but essentially different investigation direction concerns the use of belief revision (and, in particular, its postulates) as a reference for managing some change *inside* argumentation. Here it is typically assumed that a given change has to be applied at some level of an argumentation formalism, and AGM-like postulates are used to shape this change process.

For instance, in Coste-Marquis et al. (2014a) a form of revision is considered where a *revision formula* specifying conditions on the acceptance status of arguments is given, and one wants to derive a set of argumentations frameworks (based on the

same set of arguments) such that their extensions comply with the revision formula. A set of postulates inspired by the AGM ones is introduced for revision operators in this context, and a family of operators satisfying these postulates is investigated. Differently, in Baumann and Brewka (2015) the revision process is carried out at the level of argumentation frameworks: the input to the revision process is an argumentation framework to be "added" to a previous one to produce a revised one. Here changes in the sets of arguments are allowed. Similarly to the AGM theory, a notion of expansion for argumentation frameworks is introduced as a basis for the revision operator, where the notions of strong equivalence of argumentation frameworks and the relevant kernels play a key role. AGM-like postulates are introduced for the considered revision operator.

In Diller et al. (2018), two forms of revision of argumentation frameworks are discussed: a first one where a revision formula is used like in Coste-Marquis et al. (2014a), a second one where an argumentation framework (coming from another agent) is given in input. For both cases, an approach is investigated where (differently from Coste-Marquis et al. 2014a) a single argumentation framework is produced by the relevant revision operator. For both families of revision, operators postulates adapted from Katsuno and Mendelzon (1991) are introduced, representation results are obtained, and computational complexity is investigated. In Dupin de Saint-Cyr et al. (2016), another different context is considered where multiple agents are debating by publicly exchanging arguments. Each of them holds a private argumentation framework, and they share a common framework called *target system* which is the object of change. It is observed that in this context, the notion of belief update rather than revision is appropriate, and belief update postulates are adapted to take into account that only some transitions are authorized in the common framework. A suitably defined language called YALLA is used to represent the state of knowledge about exchanged arguments, and an update mechanism reflecting the evolution of the debate is introduced.

While the above approaches refer to cases where some new information is added, in Baumann and Brewka (2019) the case where some extensions need to be removed in an abstract argumentation framework is investigated: this intuitively corresponds to the contraction operator in the AGM theory. Accordingly, some postulates inspired by those of contraction are defined, and the existence of removal operators is investigated for various argumentation semantics. These approaches show how many nuances and variants of the notion of change can be identified inside the field of formal argumentation alone (other works in this vein include Falappa et al. 2002, 2013; Coste-Marquis et al. 2014b; Deagustini et al. 2019, 2021), and the reader may refer for instance to Doutre and Mailly (2018) for a comprehensive survey of the various aspects of dynamics in abstract argumentation. We observe, however, that in all of them, belief revision is essentially regarded as a technical tool in a context where argumentation has been chosen as the primary representation and reasoning approach. On the contrary, we envisage a scenario where belief revision and computational argumentation are both regarded as primary options, and one wants to identify basic commonalities and differences at a conceptual level: in this spirit, this paper is meant to stimulate future research investigating novel perspectives to merge the features of the two approaches, rather than using one of them, so to say, "inside" the other.

As a further indication for future research, we have suggested to both communities the importance of defining a set of reference reasoning cases, let's say of "reasoning benchmarks". The possibility of using common benchmarks to compare the way different formal approaches deal with the same situation would be a very useful step in order to complement conceptual comparisons with concrete case-based counterparts.

We conclude the paper with a set of questions that we propose to the reader as suggestions for further investigation on the topics we addressed in our discussion.

8.1 Questions

Postulates

- Can a sufficiently abstract set of postulates for both static and dynamic aspects of inconsistency management processes be identified so that current belief revision and argumentation theories are seen as special cases of a general framework?
- Is the notion of justification state a mere by-product of the available reasoning process models or can it be taken as a starting point to define requirements or postulates for the reasoning processes themselves?
- Can a spectrum of consistency notions and, consequently, of consistency recovery processes be identified?

Belief Revision and Argumentation as Processes

- Can argumentation borrow distinct kinds of change operations from belief revision for a more detailed modeling of information acquisition? Would this be appropriate?
- Is there any significant relationship between the existence of different operators in belief revisions and of different semantics in argumentation?
- Can contraction be accommodated within argumentation? If not, is anything missing in argumentation?

Process Drivers and Goals

- Is the distinction between revision and updating relevant to argumentation? Do abstract or dialogical argumentation need to be extended to accommodate it?
- Can the notion of a goal-oriented process be suitably and profitably captured in abstract settings like belief revision and argumentation frameworks?
- Is there room for paradigmatic schemes in belief/base revision?

Process Objects

- What is the counterpart in argumentation of the idealized knowledge-level process of belief revision? If it can not be identified, is it worth exploring?
- Which is (or which should be) the counterpart in argumentation of the extra-logical information driving the revision strategy?

- Can reason maintenance be encompassed explicitly in belief revision?
- Can the distinctions on the process objects be exploited to make belief revision (and argumentation itself) a more faithful representation of actual belief change processes?
- Can non-monolithic models of reasoning be adopted in belief revision?
- Can there be a cross-fertilization between the application of multiple argumentation semantics in the same reasoning process and the use of different epistemic states in iterated belief revision?

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