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Optimal strategy of electricity and natural gas aggregators in the energy and balance markets

Meysam Khojasteh, Pedro Faria, Fernando Lezama, Zita Vale*

GECAD - Research Group on Intelligent Engineering and Computing for Advanced Innovation and Development, LASI - Intelligent Systems Associate LAboratory, Polytechnic of Porto, P-4200-072, Porto, Portugal

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A B S T R A C T

This paper presents a stochastic two-stage model for energy aggregators (EAs) in the energy and balancing markets to supply electricity and natural gas to end-users equipped with combined heat and power (CHP) units. The suggested model takes into account the battery energy storage (BES) as a self-generating unit of EA. The upper and lower subproblems determine the optimal energy supply strategy of EA and consumption of consumers, respectively. In the lower subproblem, the McCormick relaxation is used to linearize the cost function of the CHP unit. To solve the proposed model, the two-stage problem is transformed into a linear single-stage problem using the KKT conditions of the lower subproblem, the Big M method, and the strong duality theory. The performance and efficiency of the proposed model are evaluated using a case study and three scenarios. According to the simulation results, adding CHP units to the energy-scheduling problem of BES-owned aggregators increases the profit of EA by 5.96% and decreases the cost of consumers by 1.57%.

1. Introduction

Nowadays, modern power systems focus on integrating different sources of energy such as electricity, natural gas, and heat, and coordinating their operations to improve their overall efficiency [1]. The new operation paradigm results in the emergence of a new participant known as the energy aggregator (EA). Various types of energy are provided by EAs, including electricity, natural gas, and heat [2]. EAs could supply the required energy of their consumers from different sources of energy such as day-ahead, bilateral, and balance markets [3]. Moreover, residential resources can provide a part of the required energy. Since the investment costs for large-scale power plants are high, fossil fuel resources have environmental problems, as well as the consumption of final consumers is increasing, small-scale sources such as battery energy storage systems and combined heat and power (CHP) systems have become increasingly popular. Increasing the penetration level of these resources changes end-users from passive to active consumers [4]. A part of the required electricity and heat demands of consumers can be supplied by small CHP units. It shall be noted that residential consumers can sign the energy-supply contract with EAs to give this responsibility to them. To supply the clients' demands, EAs need a framework to specify their optimal strategy. The interactions between EA, consumers, and the grid are shown in Fig. 1.

The uncertain nature of consumption is inevitable. EAs can compensate for unexpected variations in demand by using flexible resources like battery storage systems (BESs). The main goal of EA as a private entity is the maximization of the expected profit. Moreover, energy cost minimization is one of the main priorities of residential consumers. Thus, a comprehensive energy supply strategy must take into account both EA's profit and consumers' cost, which is the main purpose of this paper. Scheduling of small-size energy resources, bidding strategy of aggregators, and coordination between electricity and natural gas markets have been studied in various references. [5] explores active consumers' interactions with aggregators. The model proposes that aggregators control consumers' demand and resources based on the minimization of energy costs. Additionally, the real-time market is used to compensate for unexpected variations in renewable resources and consumption. Accordingly, the proposed model is formulated as a bi-level optimization problem in which the optimal participation levels in the day-ahead and real-time markets are determined in

* Corresponding author.
E-mail address: zav@isep.ipp.pt (Z. Vale).

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A. Index and set

h, H Index and set of time
ω, Ω Index and set of uncertain demand scenarios

B. Parameters

π^{g} Natural gas buying price (EUR/m^3)
λ^{g} Natural gas selling price to consumers (EUR/m^3)
$\pi^{e,DA}$ Electricity price in the day-ahead market (EUR/kWh)
PR_{ω} Probability of scenario ω
$\pi^{e,\uparrow}$ Electricity up-regulation price in the balance market (EUR/kWh)
$\pi^{e,\downarrow}$ Electricity down-regulation price in the balance market (EUR/kWh)
ΔP^{L} Variation of electricity demand (kW)
$\lambda^{e,\min/\max}$ Minimum/maximum selling price of electricity (EUR/kWh)
a, b, c, d Coefficients of CHP cost function
$P^{CHP,\min/\max}$ Minimum/maximum electrical power of CHP unit (kW)
$H^{CHP,\min/\max}$ Minimum/maximum heat power of CHP unit (kW)
$\eta^{CH/DCH}$ Charging/discharging efficiency of BES (%)
$E^{\min/\max}$ Minimum/maximum energy capacity of BES (kWh)
$P^{CH,\max}$ Maximum charging power capacity of BES (kW)
$P^{DCH,\max}$ Maximum discharging power capacity of BES (kW)
$T^{\min/\max}$ Minimum/maximum values of temperature (C)
η^{g} Efficiency of natural gas heating system (%)
m Mass (kg)
C_p Specific heat (Wh/kg·C)
T^{amb} Ambient temperature (C)
θ Heat reduction coefficient (%)
$G^{\min/\max}$ Minimum/maximum capacity of natural gas demand (m^3)
D_h^{L} Electricity consumption (kW)
M_1, M_2, M_3 Big constant values

C. Variables

G^{L} Natural gas consumption (m^3)
$P^{e,DA}$ Purchased power of EA from the day-ahead market (kW)
λ^{e} Electricity selling power (EUR/kWh)
P^{L} Supplied power by EA (kW)
$P^{\uparrow}$ Up-regulation purchased power from balance market (kW)
$P^{\downarrow}$ Down-regulation purchased power from balance market (kW)
P^{CHP} Electricity generation of CHP units (kW)
H^{CHP} Heat generation of CHP units (kW)
Z^{CHP} Auxiliary variable (kW)^2
E Energy of BES (kWh)
P^{CH} Charging power of BES (kW)
$P^{CH,\uparrow(\downarrow)}$ Increasing/decreasing of charging power of BES in the balance market (kW)
P^{DCH} Discharging power of BES (kW)
$P^{DCH,\uparrow(\downarrow)}$ Increasing/decreasing of discharging power of BES in the balance market (kW)
T Temperature of consumers (C)
H^{L} Supplied heat power (kW)
α^{AB}, α^{CD}, α^{BC}, $\alpha^{CHP,\max}$, $\alpha^{CHP,\min}$, $\beta^{CHP,\max}$, $\beta^{CHP,\min}$, $\alpha^{G,\max}$, $\alpha^{G,\min}$, $\delta^{\max}$, $\delta^{\min}$, $\alpha^{CHP,UE}$, $\beta^{CHP,UE}$, $\alpha^{CHP,OE}$ $\beta^{CHP,OE}$ Auxiliary binary variables which are used for the linearization of complementarity constraints

the upper and lower sub-problems, respectively. In Ref. [6], a risk-based model is proposed to determine the bidding strategy of electric vehicle aggregators in the day-ahead and regulation markets. To evaluate the impacts of uncertain day-ahead and regulation prices, a hybrid stochastic/robust approach is addressed. The proposed robust model in Ref. [7] provides an aggregated bidding strategy for the participation of electricity aggregators in the joint energy and reserve markets. The battery energy storage is used as the backup resource to compensate for fluctuations in wind power. In Ref. [8], the interactions between the aggregator and distribution system operator are modeled by a two-stage problem. The upper sub-problems determine the optimal scheduling of aggregator's resources based on cost minimization. The constraints of distribution networks such as voltage and congestion are checked in the lower sub-problem [9]. presents a decentralized method for scheduling aggregator resources. A detailed analysis of the proposed model reveals that the main challenge is to identify the differences between optimal solutions of centralized and decentralized frameworks. However, the authors claim that their model ensures the same optimality as the centralized framework [10]. presents a linear distributed optimization framework that combines PV and energy storage systems to aggregate large-scale active consumers in real-time and day-ahead markets. In Ref. [11], uncertainties of renewable resources and price responsive consumption are modeled by the robust optimization approach. In Ref. [12], battery energy storage is deployed to improve the frequency of distribution grids. However, the role of aggregators and their objective function are neglected in this model. In Refs. [13,14], the impacts of demand response on the optimal strategy of aggregators are studied. Moreover, the evidence theory is proposed to model the uncertainty of consumers' responses [14]. In Ref. [15], a stochastic model is proposed to supply the thermal and electrical load of consumers based on the uncertainties of energy price, wind power, consumption, and solar radiation. The impact of CHP units on the optimal strategy aggregators is studied in Ref. [16]. However, the thermal load and related constraints are neglected in this work. The coordination between CHP units and the price-sensitive

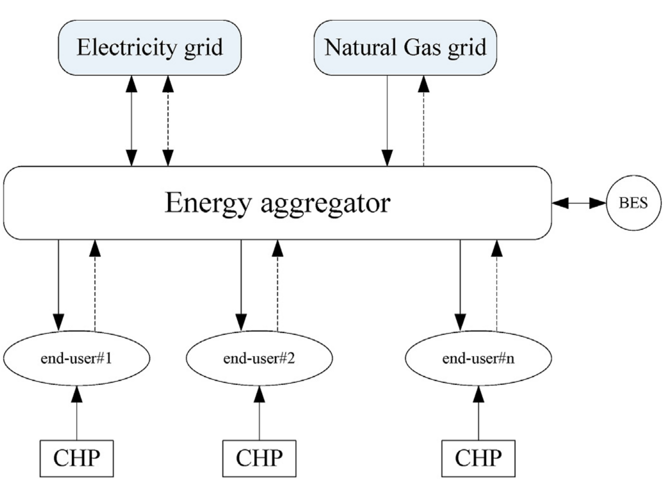

Fig. 1. Interactions between EA, end-users, and grid (solid and dashed lines show energy and money flows).

demand response resource is presented in Refs. [17,18]. For the aggregation of CHPs in the transactive electricity market [19], proposes a linear decentralized model. A portfolio strategy that compensates for wind unit fluctuations with a CHP system is presented in Refs. [20,21]. The effect of heating constraints on the optimal level of participation of CHP units in the reserve market is studied in Ref. [22]. The coordination between the owners of CHP units and the distribution system operators to provide the required flexibility of the distribution network is investigated in Ref. [23]. In Ref. [24], a model is presented for simultaneous scheduling of residential photovoltaic panels, battery storage, and the CHP unit. The robust optimization approach is addressed in Ref. [25] to model the uncertainty of market-clearing price in the scheduling problem of CHP units. In Ref. [26], robust optimization is addressed to model the renewable uncertainty in the unit commitment problem. The coordination between the planning of PV charging stations and demand response is studied in Ref. [27]. The coordination between CHP units and BES to supply the electrical and thermal loads is investigated in Ref. [28]. However, the clearing procedure of the proposed market is not studied. The strategic bidding of CHP units in the electrical and thermal markets is studied in Ref. [29]. In Ref. [30], the Stackelberg-based single-leader multi-follower problem is presented to model the interactions between the micro-grid operator and consumers, and scheduling demand-side resources. The optimal scheduling of power-to-gas storage in the electricity and natural gas markets is presented in Refs. [31,32]. In Ref. [33], the electricity and natural gas market clearing problem is modeled as a bi-level problem. In that model, scheduling of resources and optimal clearing prices are determined in the upper and lower sub-problems, respectively. The impacts of uncertain wind power on electricity and natural gas clearing prices are studied in Ref. [34]. The marginal price of integrated electricity and natural gas markets is calculated in Ref. [35]. The joint gas and electricity market based on equilibrium problem with equilibrium constraints (EPEC) and mathematical program with equilibrium constraints (MPEC) are investigated in Refs. [36,37], respectively.

The main research gap, which is covered in this work, is presenting an integrated scheduling strategy for EA to supply the required electricity and natural gas to consumers based on their electrical and thermal constraints. The electrical and thermal constraints of CHP units and allowable temperature range are the main constraints at the end-user level. Moreover, the balance, power and energy constraints of BES are considered at the EA level. Profit maximization and cost minimization are the main objectives of EA and consumers, respectively. Accordingly, the strategy of EA is determined in a way that optimizes the objectives of EA and consumers, simultaneously. As mentioned before, the energy price and energy consumption are determined by the EA and consumers, respectively. Increasing the selling price decreases consumption, and vice-versa. As shown in Fig. 2, EA faces a two-stage optimization problem. In the upper subproblem, the optimal participation level of EA in the day-ahead and balance markets, and selling prices are determined. The lower linear sub-problem specifies the optimal consumption and scheduling of CHP units based on the thermal and electrical constraints. To compensate for variations in electrical demand, the BES is used in this work. Moreover, the stochastic programming approach is used to model the uncertainty of consumption and determine the expected participation level of EA in the balancing market. At the lower level, a temperature margin is considered to determine the heating load. To calculate the supply cost of consumers, the fuel cost of CHP units is approximated by the quadratic function. The cost function of CHP units contains a bilinear term that is linearized by the McCormick relaxation approach. It shall be noted that the McCormick relaxation method is a fast and efficient approach for linearizing the bilinear terms [38,39]. Finally, the lower sub-problem is replaced by KKT conditions to transform the bi-level optimization problem into a single-level problem. To linearize the single-level problem, the strong duality theory is used. The linearized single-level problem that is formulated as a mixed-integer linear programming (MILP) problem, can be solved by common optimization solvers.

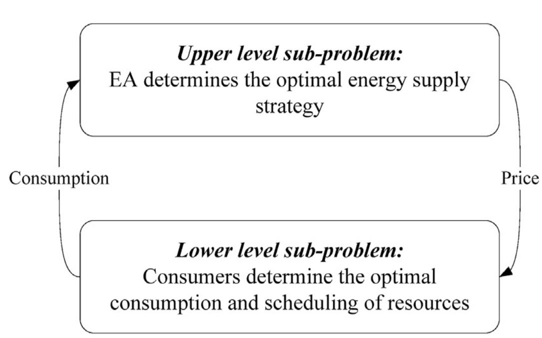

Fig. 2. Structure of proposed bi-level optimization problem.

The main contributions of this work can be summarized as follows:

- Proposing a two-stage model to supply the required energy of end-users by EA: the proposed model determines the optimal strategy of EA in the day-ahead and balancing markets to meet end users' electricity and natural gas demand. Accordingly, both the thermal and electrical constraints of the consumers are considered in the proposed model. The optimal strategy is determined to maximize the profit of EA while minimizing the consumers' supply cost.
- Using a novel solving procedure: the proposed two-stage model is reformulated as a mixed-integer linear programming (MILP) problem, which can be solved using the standard optimization solvers. Accordingly, the McCormick relaxation approach is used to linearize the cost function of CHP units. The two-stage optimization problem is recast as a single-level linear problem by replacing the lower level with the KKT conditions. The single-level optimization problem is linearized by the strong duality theory and the Big-M theory.

The rest of this paper is organized as follows: Section II provides the structure of the proposed problem. In Section III, the solving procedure is presented. The performance and effectiveness of the model are evaluated in Section IV. Finally, conclusions and remarks are presented in section V.

2. Bi-level model for the integrated energy scheduling problem

This section presents the proposed bi-level problem. As mentioned before, the upper and lower sub-problems specify the optimal strategy of EA and consumers, respectively.

2.1. Upper sub-problem

Currently, the supply of natural gas is based on a fixed price that is specified by suppliers, and there is no daily or hourly competitive market. Therefore, EA's role in the supply of natural gas is to bridge the gap between suppliers and consumers by purchasing and reselling it to clients. In the electricity section, EA could supply the electrical demand by BES units or participation in the day-ahead or balance markets. Therefore, the objective function of EA can be represented as follows:

$$
\mathrm{O.F.}^{EA} = \min_{\substack{\lambda_h^e, P_h^{DA}, P_{h,\omega}^{\uparrow}, P_{h,\omega}^{\downarrow}, P_h^{CH}, \\ P_h^{DCH}, P_{h,\omega}^{CH,\uparrow}, P_{h,\omega}^{CH,\downarrow}, P_{h,\omega}^{DCH,\uparrow} \\ , P_{h,\omega}^{DCH,\downarrow}, \nu_h}} \sum_{h\in H}\left(\pi_h^g - \lambda_h^g\right)G_h^L + \sum_{h\in H}\left(\pi_h^{e,DA}.P_h^{e,DA} + \rho^B.\left(P_h^{CH} + P_h^{DCH}\right)\right) - \sum_{h\in H}\lambda_h^e.P_h^L + \sum_{h\in H}\sum_{\omega\in\Omega} PR_{h,\omega}.\left(\pi_h^{e,\uparrow}.P_{h,\omega}^{\uparrow} - \pi_h^{e,\downarrow}.P_{h,\omega}^{\downarrow} + \rho^B.\left(P_{h,\omega}^{CH,\uparrow} - P_{h,\omega}^{CH,\downarrow} + P_{h,\omega}^{DCH,\uparrow} - P_{h,\omega}^{DCH,\downarrow}\right) - \lambda_h^e.\Delta P_{h,\omega}^L\right) \quad (1)
$$

In (1), the first term represents the minus value of profit of gas supply. Also, EA can supply the electrical demand to consumers by BES or participation in the day-ahead market that is represented by the second term. The third term shows the income from selling electricity to consumers. The expected cost of buying/selling power from/to balance market and regulation cost of BES is formulated by the fourth term. In (1), it is supposed that consumers pay the same price for the electrical demand variations, which is represented by the fifth term.

The balance constraints in the day-ahead and balance markets are formulated by (2) and (3), respectively. Furthermore, constraint (4) ensures that the price of electricity charged to consumers always falls within the allowable range [40].

$$
P_h^{e,DA} + P_h^{DCH} - P_h^{CH} = P_h^L : \ \forall h\in H \quad (2)
$$

$$
P_{h,\omega}^{\uparrow} + P_{h,\omega}^{DCH,\uparrow} + P_{h,\omega}^{CH,\downarrow} - P_{h,\omega}^{DCH,\downarrow} - P_{h,\omega}^{CH,\uparrow} - P_{h,\omega}^{\downarrow} = \Delta P_{h,\omega}^L \ : \ \forall h\in H, \forall \omega\in\Omega \quad (3)
$$

$$
\lambda^{e,\min} \le \lambda_h^e \le \lambda^{e,\max} : \ \forall h\in H \quad (4)
$$

The stored energy of BES in each scenario is calculated by (5). The main constraints of the BES units are energy, charging, and discharging power capacities that are represented by (6), (7)-(10), and (11)-(14), respectively.

$$
E_{h,\omega} = E_{h-1\omega} + \eta^{CH}.\left(P_h^{CH} + P_{h,\omega}^{CH,\uparrow} - P_{h,\omega}^{CH,\downarrow}\right) - \left(P_h^{DCH} + P_{h,\omega}^{DCH,\uparrow} - P_{h,\omega}^{DCH,\downarrow}\right)\Big/\eta^{DCH} \ : \forall h\in H, \forall\omega\in\Omega \quad (5)
$$

$$
E^{\min} \le E_{h,\omega} \le E^{\max} : \forall h\in H, \forall\omega\in\Omega \quad (6)
$$

$$
0 \le P_h^{CH} \le P^{CH,\max}.(1-\nu_h) : \forall h\in H \quad (7)
$$

$$
0 \le P_{h,\omega}^{CH,\uparrow} \le P^{CH,\max}.(1-\nu_h) : \forall h\in H \quad (8)
$$

$$
0 \le P_{h,\omega}^{CH,\downarrow} \le P^{CH,\max}.(1-\nu_h) : \forall h\in H \quad (9)
$$

$$
0 \le P_h^{CH} + P_{h,\omega}^{CH,\uparrow} - P_{h,\omega}^{CH,\downarrow} \le P^{CH,\max}.(1-\nu_h) : \forall h\in H \quad (10)
$$

$$
0 \le P_h^{DCH} \le P^{DCH,\max}.\nu_h : \forall h\in H \quad (11)
$$

$$
0 \le P_{h,\omega}^{DCH,\uparrow} \le P^{DCH,\max}.\nu_h : \forall h\in H \quad (12)
$$

$$
0 \le P_{h,\omega}^{DCH,\downarrow} \le P^{DCH,\max}.\nu_h : \forall h\in H \quad (13)
$$

$$
0 \le P_h^{DCH} + P_{h,\omega}^{DCH,\uparrow} - P_{h,\omega}^{DCH,\downarrow} \le P^{DCH,\max}.\nu_h : \forall h\in H \quad (14)
$$

2.2. Lower sub-problem

As mentioned before, in this work, it is supposed that consumers can use CHP units to provide a part of the required energy. Fig. 3 shows the feasible operational region (FOR) of the CHP unit [41]. Accordingly, the linear constraints (15)–(17) are used to model FOR. Furthermore, the generating power and heat ranges of the CHP unit are represented by (18) and (19), respectively.

$$
P_h^{CHP} - \left(\frac{P^{CHP,B} - P^{CHP,A}}{H^{CHP,B}}\right).H_h^{CHP} \le P^{CHP,A} : \mathrm{p}_h^{AB}, \forall h\in H \quad (15)
$$

$$
P_h^{CHP} - \left(\frac{P^{CHP,C} - P^{CHP,D}}{H^{CHP,C}}\right).H_h^{CHP} \ge P^{CHP,D} : \mathrm{p}_h^{CD}, \forall h\in H \quad (16)
$$

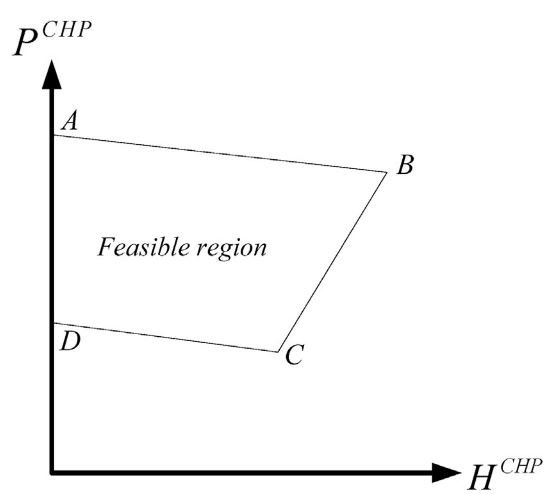

Fig. 3. Feasible operational region of the CHP unit.

$$\left(H^{CHP,B}-H^{CHP,C}\right).P_h^{CHP} \le \left(P^{CHP,B}-P^{CHP,C}\right).H_h^{CHP}+P^{CHP,C}.H^{CHP,B}-P^{CHP,B}.H^{CHP,C}: \quad p_h^{BC}, \forall h \in H \quad if \quad \frac{P^{CHP,C}}{P^{CHP,B}} \le \frac{H^{CHP,C}}{H^{CHP,B}} \tag{17a}$$

$$\left(H^{CHP,B}-H^{CHP,C}\right).P_h^{CHP} \ge \left(P^{CHP,B}-P^{CHP,C}\right).H_h^{CHP}+P^{CHP,C}.H^{CHP,B}-P^{CHP,B}.H^{CHP,C}: \quad p_h^{BC}, \forall h \in H \quad if \quad \frac{P^{CHP,C}}{P^{CHP,B}} > \frac{H^{CHP,C}}{H^{CHP,B}} \tag{17b}$$

$$P^{CHP,\min} \le P_h^{CHP} \le P^{CHP,\max} : p_h^{CHP,\max}, p_h^{CHP,\min}, \forall h \in H \tag{18}$$

$$H^{CHP,\min} \le H_h^{CHP} \le H^{CHP,\max} : h_h^{CHP,\max}, h_h^{CHP,\min}, \forall h \in H \tag{19}$$

Using the quadratic function, the cost function of the CHP unit is approximated as follows [41]:

$$C_h^{CHP} = a + b.P_h^{CHP} + c.H_h^{CHP} + d.P_h^{CHP}.H_h^{CHP} : \quad \forall h \in H \tag{20}$$

The McCormick relaxation approach is an efficient and fast method to linearize bilinear terms. In the McCormick envelop optimization, the auxiliary variable Z is defined as follows:

$$Z_h^{CHP} = P_h^{CHP}.H_h^{CHP} : \quad \forall h \in H \tag{21}$$

The variable Z is approximated by linear under ((22–23)) and over ((24)–(25)) estimators, as follows:

$$Z_h^{CHP} \ge P^{CHP,\min}.H_h^{CHP} + H^{CHP,\min}.P_h^{CHP} - P^{CHP,\min}.H^{CHP,\min} : z_h^{CHP,UE}, \forall h \in H \tag{22}$$

$$Z_h^{CHP} \ge P^{CHP,\max}.H_h^{CHP} + H^{CHP,\max}.P_h^{CHP} - P^{CHP,\max}.H^{CHP,\max} : y_h^{CHP,UE}, \forall h \in H \tag{23}$$

$$O.F.^{CS} = \min_{P_h^{CHP},H_h^{CHP},Z_h^{CHP},G_h^L,P_h^L,T_h} \sum_{h\in H}\left(\lambda_h^e.P_h^L + \lambda_h^g.G_h^L\right) + \sum_{h \in H}\left(a + b.P_h^{CHP} + c.H_h^{CHP} + d.Z_h^{CHP}\right) + \sum_{h\in H}\sum_{\omega\in\Omega} PR_{h,\omega}.\lambda_h^e.\Delta P_{h,\omega}^L \quad s.t.: (15)-(19), (22)-(28) \tag{29}$$

$$Z_h^{CHP} \le P^{CHP,\max}.H_h^{CHP} + H^{CHP,\min}.P_h^{CHP} - P^{CHP,\max}.H^{CHP,\min} : z_h^{CHP,OE}, \forall h \in H \tag{24}$$

$$Z_h^{CHP} \le P^{CHP,\min}.H_h^{CHP} + H^{CHP,\max}.P_h^{CHP} - P^{CHP,\min}.H^{CHP,\max} : y_h^{CHP,OE}, \forall h \in H \tag{25}$$

To calculate the consumed natural gas, the allowable temperature interval of consumers is represented by (26):

$$T_h^{\min} \le T_h \le T_h^{\max} : T_h^{\max}, T_h^{\min}, \forall h \in H \tag{26}$$

Equality constraint (27) demonstrates the relation between temperature and heat. The variation of hourly temperature depends on the total heat that contains the generated heat of CHP units and heat of the natural gas system, ambient temperature, mass, and the specific heat capacity. It shall be noted that in this work, a linear relation is considered between the generated heat of the natural gas system (H_h^L) and the consumed natural gas (G_h^L). The capacity constraint of the natural gas system is represented by (28).

$$T_h = \frac{\left(H_h^L + H_h^{CHP}\right)}{m.C_p} + T_{h-1}.\theta + T_h^{amb} = \frac{\left(\eta^g.G_h^L + H_h^{CHP}\right)}{mC_p} + T_{h-1}.\theta + T_h^{amb} : T_h, \forall h \in H \tag{27}$$

$$G^{\min} \le G_h^L \le G^{\max} : g_h^{G,\max}, g_h^{G,\min}, \forall h \in H \tag{28}$$

The main objective of consumers is the minimization of electricity and gas costs. The first, second, and third terms of (29) show the costs of purchasing electricity and natural gas, the CHP unit, and load variations, respectively. Moreover, (30) shows the day-ahead load that shall be supplied by the EA.

$$D_h^L - P_h^{CHP} = P_h^L : \rho_h, \forall h \in H \tag{30}$$

3. Solving procedure

To solve the proposed bi-level problem, the lower sub-problem is replaced by the KKT conditions, which are represented by (31)-

(36), as follows:

$$\lambda_h^e - \rho_h = 0 : \quad \forall h \in H \tag{31}$$

$$\lambda_h^g + \mathrm{g}_h^{G,\max} - \mathrm{g}_h^{G,\min} + \frac{\eta^g}{mC_p}.\mathrm{T}_h = 0 : \quad \forall h \in H \tag{32}$$

$$\begin{cases} b - \rho_h + \mathrm{p}_h^{AB} - \mathrm{p}_h^{CD} + \left(H^{CHP,B} - H^{CHP,C}\right).\mathrm{p}_h^{BC} + \mathrm{p}_h^{CHP,\max} - \mathrm{p}_h^{CHP,\min} + H^{CHP,\min}.\mathrm{z}_h^{CHP,UE} + \\ H^{CHP,\max}.\mathrm{y}_h^{CHP,UE} - H^{CHP,\min}.\mathrm{z}_h^{CHP,OE} - H^{CHP,\max}.\mathrm{y}_h^{CHP,OE} = 0 : for\ \frac{P^{CHP,C}}{P^{CHP,B}} \le \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \\ b - \rho_h + \mathrm{p}_h^{AB} - \mathrm{p}_h^{CD} - \left(H^{CHP,B} - H^{CHP,C}\right).\mathrm{p}_h^{BC} + \mathrm{p}_h^{CHP,\max} - \mathrm{p}_h^{CHP,\min} + H^{CHP,\min}.\mathrm{z}_h^{CHP,UE} + \\ H^{CHP,\max}.\mathrm{y}_h^{CHP,UE} - H^{CHP,\min}.\mathrm{z}_h^{CHP,OE} - H^{CHP,\max}.\mathrm{y}_h^{CHP,OE} = 0 : for\ \frac{P^{CHP,C}}{P^{CHP,B}} > \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \end{cases} \tag{33}$$

$$\begin{cases} c - \left(\frac{P^{CHP,B} - P^{CHP,A}}{H^{CHP,B}}\right).\ \mathrm{p}_h^{AB} + \left(\frac{P^{CHP,C} - P^{CHP,D}}{H^{CHP,C}}\right).\ \mathrm{p}_h^{CD} - \left(P^{CHP,B} - P^{CHP,C}\right).\ \mathrm{p}_h^{BC} \\ + \mathrm{h}_h^{CHP,\max} - \mathrm{h}_h^{CHP,\min} + P^{CHP,\min}.\ \mathrm{z}_h^{CHP,UE} + P^{CHP,\max}.\ \mathrm{y}_h^{CHP,UE} - P^{CHP,\max}.\ \mathrm{z}_h^{CHP,OE} \\ - P^{CHP,\min}.\ \mathrm{y}_h^{CHP,OE} + \frac{1}{mC_p}.\mathrm{T}_h = 0 : for\ \frac{P^{CHP,C}}{P^{CHP,B}} \le \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \\ c - \left(\frac{P^{CHP,B} - P^{CHP,A}}{H^{CHP,B}}\right).\ \mathrm{p}_h^{AB} + \left(\frac{P^{CHP,C} - P^{CHP,D}}{H^{CHP,C}}\right).\ \mathrm{p}_h^{CD} + \left(P^{CHP,B} - P^{CHP,C}\right).\ \mathrm{p}_h^{BC} \\ + \mathrm{h}_h^{CHP,\max} - \mathrm{h}_h^{CHP,\min} + P^{CHP,\min}.\ \mathrm{z}_h^{CHP,UE} + P^{CHP,\max}.\ \mathrm{y}_h^{CHP,UE} - P^{CHP,\max}.\ \mathrm{z}_h^{CHP,OE} \\ - P^{CHP,\min}.\ \mathrm{y}_h^{CHP,OE} + \frac{1}{mC_p}.\mathrm{T}_h = 0 : for\ \frac{P^{CHP,C}}{P^{CHP,B}} > \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \end{cases} \tag{34}$$

$$d - \mathrm{z}_h^{CHP,UE} - \mathrm{y}_h^{CHP,UE} + \mathrm{z}_h^{CHP,OE} + \mathrm{y}_h^{CHP,OE} = 0 : \quad \forall h \in H \tag{35}$$

$$\begin{cases} \mathrm{T}_h^{\max} - \mathrm{T}_h^{\min} - \mathrm{T}_{h+1}.\theta + \mathrm{T}_h = 0 : \quad \forall h : 1, \ldots, H-1 \\ \mathrm{T}_H^{\max} - \mathrm{T}_H^{\min} + \mathrm{T}_H = 0 : \quad \forall h = H \end{cases} \tag{36}$$

Moreover, the complementarity conditions for the lower sub-problem are as follows:

$$0 \le \left(P^{CHP,A} - P_h^{CHP} + \left(\frac{P^{CHP,B} - P^{CHP,A}}{H^{CHP,B}}\right).H_h^{CHP}\right) \perp \mathrm{p}_h^{AB} \ge 0 \quad : \quad \forall h \in H \tag{37}$$

$$0 \le \left(P_h^{CHP} - \left(\frac{P^{CHP,C} - P^{CHP,D}}{H^{CHP,C}}\right).H_h^{CHP} - P^{CHP,D}\right) \perp \mathrm{p}_h^{CD} \ge 0 \quad : \quad \forall h \in H \tag{38}$$

$$\begin{cases} 0 \le \begin{pmatrix} \left(P^{CHP,B} - P^{CHP,C}\right)H_h^{CHP} + P^{CHP,C}.H^{CHP,B} \\ -P^{CHP,B}.H^{CHP,C} - \left(H^{CHP,B} - H^{CHP,C}\right)P_h^{CHP} \end{pmatrix} \perp \mathrm{p}_h^{BC} \ge 0 : \\ for\ \frac{P^{CHP,C}}{P^{CHP,B}} \le \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \\ 0 \le \begin{pmatrix} \left(H^{CHP,B} - H^{CHP,C}\right)P_h^{CHP} - P^{CHP,C}.H^{CHP,B} \\ +P^{CHP,B}.H^{CHP,C} - \left(P^{CHP,B} - P^{CHP,C}\right)H_h^{CHP} \end{pmatrix} \perp \mathrm{p}_h^{BC} \ge 0 : \\ for\ \frac{P^{CHP,C}}{P^{CHP,B}} > \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \end{cases} \tag{39}$$

$$0 \le \left(P^{CHP,\max} - P_h^{CHP}\right) \perp \mathrm{p}_h^{CHP,\max} \ge 0 : \forall h \in H \tag{40}$$

$$0 \le \left(P_h^{CHP} - P^{CHP,\min}\right) \perp \mathrm{p}_h^{CHP,\min} \ge 0 : \forall h \in H \tag{41}$$

$$0 \le \left(H^{CHP,\max} - H_h^{CHP}\right) \perp \mathrm{h}_h^{CHP,\max} \ge 0 : \forall h \in H \tag{42}$$

$$0 \le \left(H_h^{CHP} - H^{CHP,\min}\right) \perp \mathrm{h}_h^{CHP,\min} \ge 0 : \forall h \in H \tag{43}$$

$$0 \le \left(Z_h^{CHP} - \left(P^{CHP,\min}.H_h^{CHP} + H^{CHP,\min}.P_h^{CHP} - P^{CHP,\min}.H^{CHP,\min}\right)\right) \perp \mathrm{z}_h^{CHP,UE} \ge 0 : \forall h \in H \tag{44}$$

$$0 \le \left(Z_h^{CHP} - \left(P^{CHP,\max}.H_h^{CHP} + H^{CHP,\max}.P_h^{CHP} - P^{CHP,\max}.H^{CHP,\max}\right)\right) \perp \mathrm{y}_h^{CHP,UE} \ge 0 : \forall h \in H \tag{45}$$

$$0 \leq \left(P^{CHP,\max}.H_h^{CHP} + H^{CHP,\min}.P_h^{CHP} - P^{CHP,\max}.H^{CHP,\min} - Z_h^{CHP} \right) \perp \mathrm{z}_h^{CHP,OE} \geq 0 : \forall h \in H \tag{46}$$

$$0 \leq \left(P^{CHP,\min}.H_h^{CHP} + H^{CHP,\max}.P_h^{CHP} - P^{CHP,\min}.H^{CHP,\max} - Z_h^{CHP} \right) \perp \mathrm{y}_h^{CHP,OE} \geq 0 : \forall h \in H \tag{47}$$

$$\mathrm{O.F.}^{EA} = \min_{\substack{\lambda_h^e, P_h^{DA}, P_{h,\omega}^{\uparrow}, P_{h,\omega}^{\downarrow}, P_h^{CH}, \\ P_h^{DCH}, P_{h,\omega}^{CH,\uparrow}, P_{h,\omega}^{CH,\downarrow}, P_{h,\omega}^{DCH,\uparrow} \\ , P_{h,\omega}^{DCH,\downarrow}, \nu_h}} \left(\sum_{h \in H} \left(\pi_h^g - \lambda_h^g \right) G_h^L + \sum_{h \in H} \left(\pi_h^{e,DA}.P_h^{e,DA} + \rho^B.\left(P_h^{CH} + P_h^{DCH} \right) \right) \right) - \sum_{h \in H} \overbrace{\lambda_h^e.P_h^L}^{\text{Bilinear Term}}$$
$$+ \sum_{h \in H} \sum_{\omega \in \Omega} PR_{h,\omega} \cdot \left(\pi_h^{e,\uparrow}.P_{h,\omega}^{\uparrow} - \pi_h^{e,\downarrow}.P_{h,\omega}^{\downarrow} + \rho^B.\left(P_{h,\omega}^{CH,\uparrow} - P_{h,\omega}^{CH,\downarrow} + P_{h,\omega}^{DCH,\uparrow} - P_{h,\omega}^{DCH,\downarrow} \right) - \lambda_h^e.\Delta P_{h,\omega}^L \right) \tag{52}$$

$$0 \leq \left(G^{\max} - G_h^L \right) \perp \mathrm{g}_h^{G,\max} \geq 0 : \forall h \in H \tag{48}$$

$$0 \leq \left(G_h^L - G^{\min} \right) \perp \mathrm{g}_h^{G,\min} \geq 0 : \forall h \in H \tag{49}$$

$$0 \leq \left(T_h^{\max} - T_h \right) \perp \mathrm{T}_h^{\max} \geq 0 : \forall h \in H \tag{50}$$

$$0 \leq \left(T_h - T_h^{\min} \right) \perp \mathrm{T}_h^{\min} \geq 0 : \forall h \in H \tag{51}$$

To linearize the complementarity conditions, the Big M theory is used. The linear form of complementarity conditions is presented in the Appendix of this work. Accordingly, the single-level optimization problem is represented as follows:

Subject to: (2)–(14), (31)–(36), (A.1)-(A.47) (53)

As seen in (52), the objective function of EA contains a bilinear term. To linearize this term, the strong duality function is used, and the lower sub-problem is replaced by the dual function, as follows [42]:

$$\sum_{h \in H} \lambda_h^e.P_h^L = -\sum_{h \in H} \lambda_h^g.G_h^L - \sum_{h \in H} \left(a + b.P_h^{CHP} + c.H_h^{CHP} + d.Z_h^{CHP} \right) - \sum_{h \in H} \sum_{\omega \in \Omega} PR_{h,\omega}.\lambda_h^e.\Delta P_{h,\omega}^L - \sum_{h \in H} \left(P^{CHP,A}.\mathrm{p}_h^{AB} - P^{CHP,D}.\mathrm{p}_h^{CD} \right.$$
$$\left. + \left(P^{CHP,C}.H^{CHP,B} - P^{CHP,B}.H^{CHP,C} \right).\mathrm{p}_h^{BC} \right) - \sum_{h \in H} \left(P^{CHP,\max}.\mathrm{p}_h^{CHP,\max} - P^{CHP,\min}\mathrm{p}_h^{CHP,\min} + H^{CHP,\max}.\mathrm{h}_h^{CHP,\max} - H^{CHP,\min}.\mathrm{h}_h^{CHP,\min} \right)$$
$$- \sum_{h \in H} P^{CHP,\min}.H^{CHP,\min}.\mathrm{z}_h^{CHP,UE} + P^{CHP,\max}.H^{CHP,\max}.\mathrm{y}_h^{CHP,UE} + \sum_{h \in H} P^{CHP,\max}.H^{CHP,\min}.\mathrm{z}_h^{CHP,OE} + P^{CHP,\min}.H^{CHP,\max}.\mathrm{y}_h^{CHP,OE}$$
$$- \sum_{h \in H} T_h^{\max}.\mathrm{T}_h^{\max} - T_h^{\min}\mathrm{T}_h^{\min} + G^{\max}.\mathrm{g}_h^{G,\max} - G^{\min}.\mathrm{g}_h^{G,\min} + T_h^{amb}.\mathrm{T}_h + D_h^L.\rho_h : for \ \frac{P^{CHP,C}}{P^{CHP,B}} \leq \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \tag{54a}$$

$$\sum_{h \in H} \lambda_h^e.P_h^L = -\sum_{h \in H} \lambda_h^g.G_h^L - \sum_{h \in H} \left(a + b.P_h^{CHP} + c.H_h^{CHP} + d.Z_h^{CHP} \right) - \sum_{h \in H} \sum_{\omega \in \Omega} PR_{h,\omega}.\lambda_h^e.\Delta P_{h,\omega}^L - \sum_{h \in H} \left(P^{CHP,A}.\mathrm{p}_h^{AB} - P^{CHP,D}.\mathrm{p}_h^{CD} \right.$$
$$\left. - \left(P^{CHP,C}.H^{CHP,B} - P^{CHP,B}.H^{CHP,C} \right).\mathrm{p}_h^{BC} \right) - \sum_{h \in H} \left(P^{CHP,\max}.\mathrm{p}_h^{CHP,\max} - P^{CHP,\min}\mathrm{p}_h^{CHP,\min} + H^{CHP,\max}.\mathrm{h}_h^{CHP,\max} - H^{CHP,\min}.\mathrm{h}_h^{CHP,\min} \right)$$
$$- \sum_{h \in H} P^{CHP,\min}.H^{CHP,\min}.\mathrm{z}_h^{CHP,UE} + P^{CHP,\max}.H^{CHP,\max}.\mathrm{y}_h^{CHP,UE} + \sum_{h \in H} P^{CHP,\max}.H^{CHP,\min}.\mathrm{z}_h^{CHP,OE} + P^{CHP,\min}.H^{CHP,\max}.\mathrm{y}_h^{CHP,OE}$$
$$- \sum_{h \in H} T_h^{\max}.\mathrm{T}_h^{\max} - T_h^{\min}\mathrm{T}_h^{\min} + G^{\max}.\mathrm{g}_h^{G,\max} - G^{\min}.\mathrm{g}_h^{G,\min} + T_h^{amb}.\mathrm{T}_h + D_h^L.\rho_h : for \ \frac{P^{CHP,C}}{P^{CHP,B}} > \frac{H^{CHP,C}}{H^{CHP,B}} : \forall h \in H \tag{54b}$$

Finally, the bilinear term in (52) is replaced by (54). The construction of the proposed model is summarized in Fig. 4.

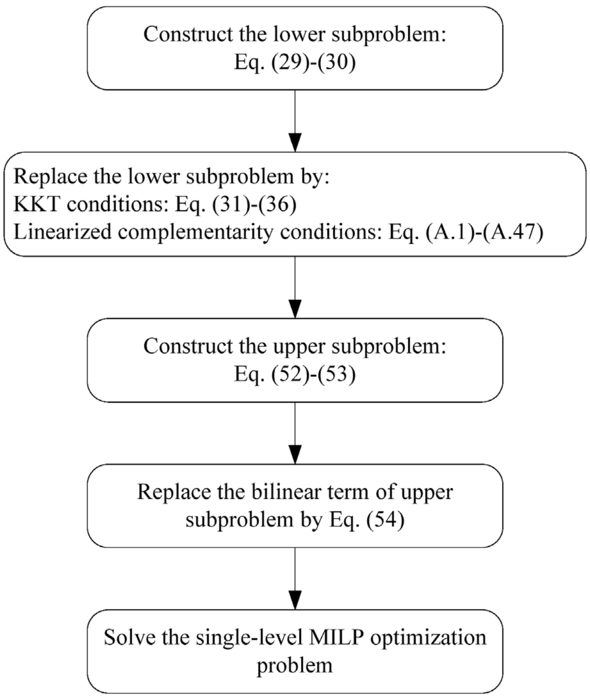

Fig. 4. The construction of the proposed model.

The performance of the proposed model is evaluated in the next section via a case study.

4. Numerical simulations

The simulation results are presented in this section. The proposed model is tested on a system that includes 1 EA with BES and 500 consumers, which are equipped with the CHP units. Fig. 5 shows the total consumption and day-ahead prices [40]. Moreover, the thermal characteristics of the test system and modified data of CHP and BES units are presented in Table 1 [41]. The FOR curve of the CHP unit, the ambient temperature, and the permissible temperature interval of consumers are presented in Figs. 6 and 7, respectively. It shall be noted that, in this work, it is supposed that up and down-regulation prices are equal to 1.19 and 0.95 of day-ahead prices, respectively [40].

To evaluate the performance of the proposed model, three scenarios are considered as follows:

- Scenario I: The uncertainty of consumption and CHP units is neglected, and the optimal strategy of EA in the day-ahead market is studied (Eqs (15)-(25) are neglected and the proposed model is solved for the expected values of electrical consumption that is shown in Fig. 5).
- Scenario II: The effects of CHP units on the performance of the proposed model are analyzed (Eqs (15)-(25) are considered and the proposed model is solved for the expected values of electrical consumption that is shown in Fig. 5).
- Scenario III: The impact of demand variation on the optimal strategy of EA is evaluated. According to the expected value ($\overline{D}_h^L$)

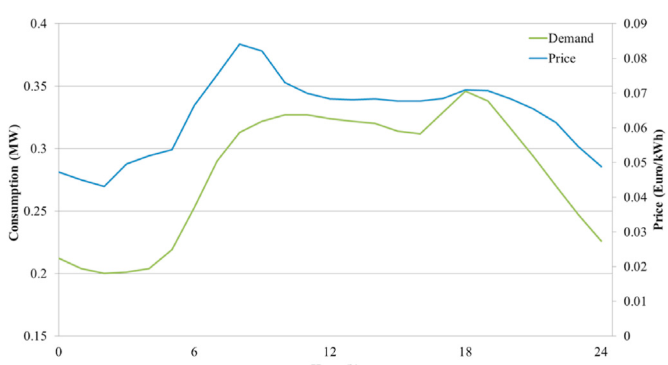

Fig. 5. Data of consumption and day-ahead price.

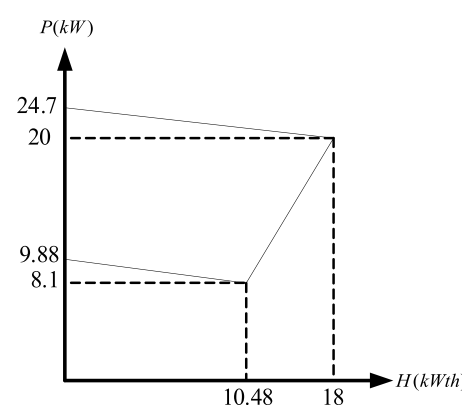

Fig. 6. FOR curve of CHP unit.

Table 1
Data of CHP and BES units, natural gas system, and thermal characteristics.

CHP			
A	0	C	0.06 Euro/kW
B	0.13 Euro/kW	D	0.0011 $(\text{Euro/kW})^2$
BES			
$E^{\min/\max}$	10-180 kWh	$\eta^{CH/DCH}$	1
$P^{CH,\max}$	30 kWh	ρ^B	0.01 Euro/kW
$P^{DCH,\max}$	30 kWh		
Natural-gas system and thermal characteristics			
$G^{\min/\max}$	0–7.08 m^3/h	θ	0.8
η^g	1	C_p	0.27917 Wh/kg·C
π^g	1.09 Euro/m^3	Conversion factor of natural gas	11.1868 kWh/m^3
λ^g	1.1 × 1.09 Euro/m^3	Initial Temperature	20.5
m	3675 kg (for area 1200 m^2, height 2.5 m, air density = 1.225 kg/m^3)		

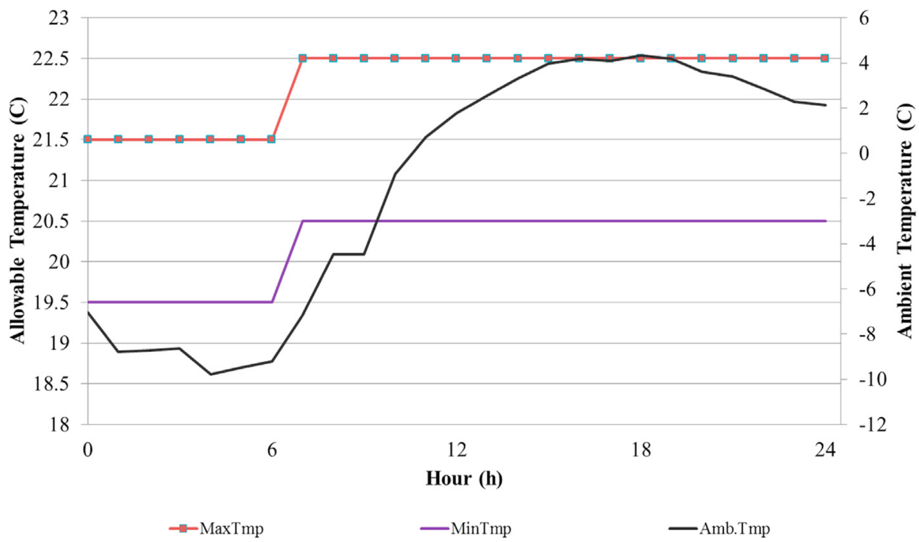

Fig. 7. Permissible range and ambient temperature.

and the standard deviation of demand (σ_h^L), in each operational period, five realizations are considered, as follows:

$$D_{h,\omega}^L = \left[\overline{D}_h^L - 2\sigma_h^L, \overline{D}_h^L - \sigma_h^L, \overline{D}_h^L, \overline{D}_h^L + \sigma_h^L, \overline{D}_h^L + 2\sigma_h^L\right]$$

The values of hourly standard deviation are equal to 5% and 7.5% of the expected values, which are presented in Fig. 5. Moreover, the probability of each scenario is approximated by the normal distribution function.

Scenario I: The optimal participation level of EA in the day-ahead market, and scheduling of the BES unit are shown in Fig. 8. As can be seen in this figure, within low-price periods, EA purchases power from the grid to charge the BES, and supplies a part of the required energy for its clients by discharging the BES during high-price periods. The maximum profit of EA and the minimum cost of consumers are 597.57 and 1071.2 Euro/day, respectively. The supplied natural gas to maintain the temperature of consumers at the desired interval is shown in Fig. 9. Evidently, the consumed natural gas is increased by decreasing the ambient temperature, and vice versa.

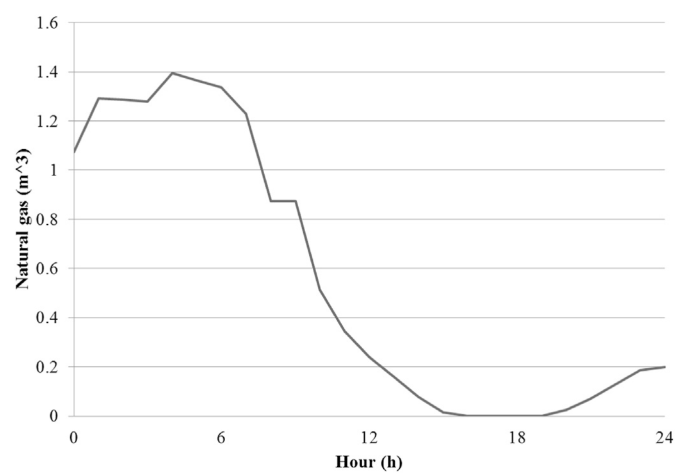

Fig. 9. Supplied natural gas in scenario I.

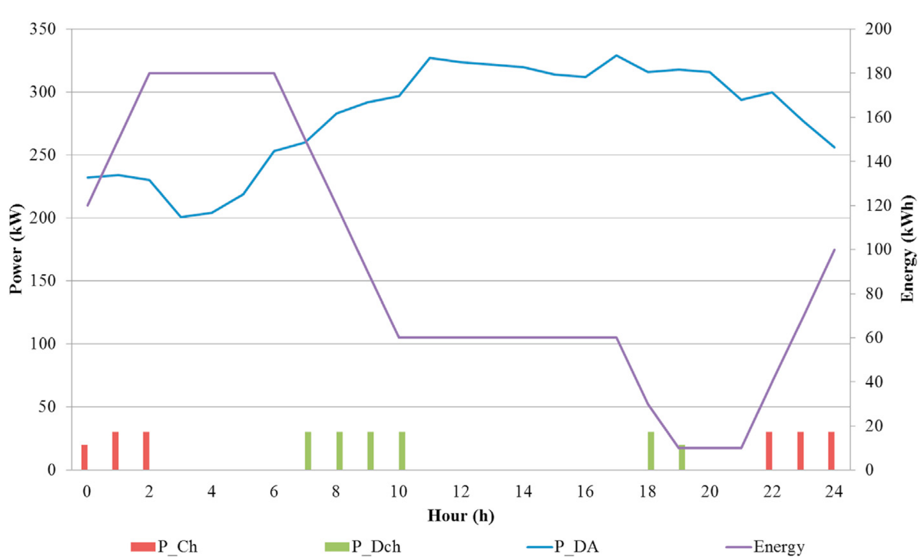

Fig. 8. The Optimal participation level of EA in the day-ahead market and the scheduling of the BES unit in scenario I.

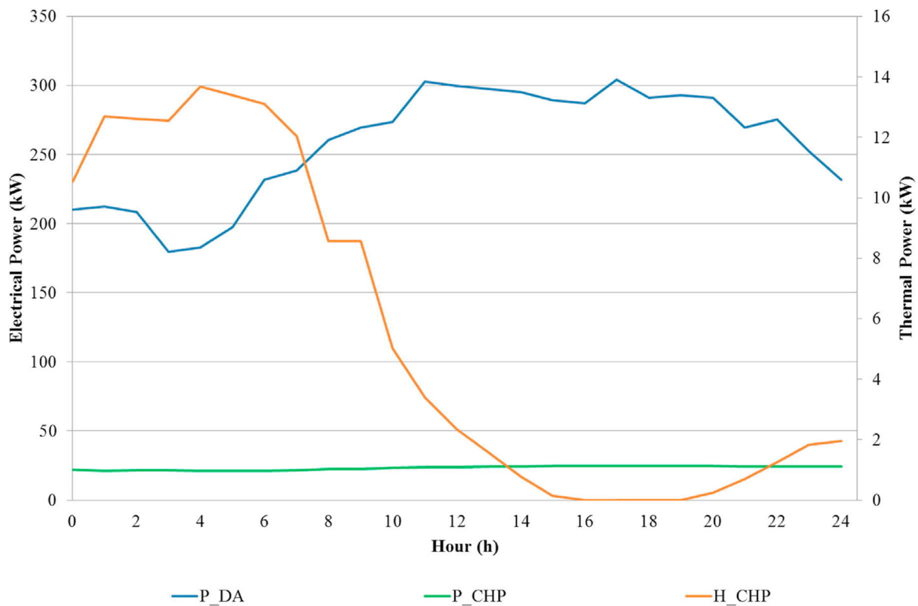

Fig. 10. The optimal participation level of EA in the day-ahead market and the scheduling of the CHP unit in scenario II.

Scenario II: Fig. 10 shows the participation level of EA in the day-ahead market, and the scheduling of CHP units. Comparing the results in Figs. 8 and 10 demonstrates that considering CHP units reduces the purchased power from the day-ahead market. The maximum profit of EA and the minimum cost of consumer in the second scenario are 633.2 and 1054.4 Euro/day, respectively. In other words, considering CHP units increases EA's profit by 5.96% and decreases consumers' costs by 1.57%.

Scenario III: As mentioned before, in this work, electrical consumption is considered as an uncertain parameter. Table 2 provides the participation level of EA in the day-ahead market and the scheduling level of BES for $\sigma_h^L = 0.05\overline{D}_h^L$. The presented results show that considering fluctuations of consumption decreases the participation level of BES in the day-ahead market and increases the role of BES in the regulation service. According to the simulation results, the scheduled energy of BES in the day-ahead and regulation markets are 142.58 and 317.42 kWh, respectively. It shall be noted that in scenario II, the total scheduled energy of BES in the day-ahead market is 340 kWh. Moreover, the maximum profit of EA and the minimum cost of consumer in the second scenario are 612.7 and 1083.4 Euro/day, respectively.

It shall be noted that more realizations of uncertain parameters are covered by increasing the standard deviation. In other words, the EA's risk preference varies with the variation interval or standard deviation of uncertain parameters. To evaluate the impact of increasing the variation interval of consumption, Table 3 represents

Table 2
The participation level of EA in the day-ahead market and the scheduling of BES $\sigma_h^L = 0.05\overline{D}_h^L$ (kW).

h	$P_h^{e,DA}$	$P_h^{CH,DA}$	$P_h^{DCH,DA}$	$P_h^{CH,Reg}$	$P_h^{DCH,Reg}$	E_h
0	210.05	7.28	0	12.72	0	120
1	212.61	17.76	0	12.24	0	150
2	208.59	18.00	0	12.00	0	180
3	191.64	0	0	0	0	180
4	195.11	0	0	0	0	180
5	210.94	0	0	0	0	180
6	246.90	0	0	0	0	180
7	273.25	0	12.60	0	17.40	150
8	298.10	0	11.22	0	18.78	120
9	308.18	0	10.68	0	19.32	90
10	312.85	0	10.38	0	19.62	60
11	322.81	0	0	30.00	0	90
12	319.35	0	0	30.00	0	120
13	317.03	0	0	30.00	30	90
14	314.70	0	0	0	30	60
15	308.18	0	0	0	0	60
16	306.02	0	0	0	0	60
17	324.04	0	0	0	0	60
18	332.82	0	9.24	0	20.76	30
19	333.58	0	0	0	20.00	10
20	310.33	0	0	20.00	0	10
21	287.12	0	0	0	0	10
22	275.63	13.80	0	16.20	0	40
23	252.78	15.18	0	14.82	0	70
24	231.81	16.44	0	13.56	0	100

Table 3
The Participation level of EA in the day-ahead market and the scheduling of BES for $\sigma_h^L = 0.075\overline{D}_h^L$.

h	$P_h^{e,DA}$	$P_h^{CH,DA}$	$P_h^{DCH,DA}$	$P_h^{CH,Reg}$	$P_h^{DCH,Reg}$	E_h
0	210.05	7.28	0	12.72	0	120
1	212.61	17.76	0	12.24	0	150
2	208.59	18.00	0	12.00	0	180
3	199.68	0	0	0	0	180
4	203.27	0	0	0	0	180
5	219.70	0	0	0	0	180
6	257.02	0	0	0	0	180
7	296.45	0	12.60	0	17.40	150
8	323.14	0	11.22	0	18.78	120
9	333.94	0	10.68	0	19.32	90
10	339.01	0	10.38	0	19.62	60
11	335.89	0	0	30.00	0	90
12	332.31	0	0	30.00	0	120
13	329.91	0	0	30.00	0	90
14	327.50	0	0	0	30	60
15	320.74	0	0	0	30	60
16	318.50	0	0	0	30	60
17	337.20	0	0	0	0	60
18	360.50	0	9.24	0	20.76	30
19	347.10	0	0	0	20.00	10
20	322.97	0	0	20.00	0	10
21	298.88	0	0	0	20.00	10
22	275.63	13.80	0	16.20	0	40
23	252.78	15.18	0	14.82	0	70
24	231.82	16.44	0	13.56	0	100

the participation level of EA in the day-ahead market and the scheduling level of BES for $\sigma_h^L = 0.075\overline{D}_h^L$. Comparing the results in Tables II and III demonstrates that increasing the standard deviation of consumption increases the total scheduled energy of BES in the regulation market to 417.42 kWh. Moreover, the maximum profit of EA and the minimum cost of consumer in scenario III are 599.6 and 1103.2 Euro/day, respectively.

5. Conclusions & future works

In this paper, a bi-level model is proposed for EA to supply the electricity and natural gas of consumers. In the proposed model, the CHP units are considered as self-generating units of consumers that can be controlled by EAs. Moreover, the BES units are deployed to compensate for the demand variations. Accordingly, the upper and lower sub-problems specify the optimal energy supply strategy of EA and consumption of clients, respectively. The proposed model is transformed into a linear single-level problem by KKT optimality conditions, the Big-M method, and the strong duality theory. Moreover, the uncertainty of electrical consumption is modeled by different scenarios. Simulation results show that using CHP units can decrease/increase the total cost/profit of consumers/EA. Moreover, considering the uncertainty of consumption increases the participation level of BES units in the regulation market. According to the presented results, considering the load standard deviation 5% leads to a 3.23% decrease in the profit of EA as well as a 2.75% increase in the minimum cost of consumers. It shall be noted that the relation between EA's profit or consumers' cost and the standard deviation of uncertain demand is not linear. Accordingly, increasing the standard deviation from 5% to 7.5% leads to a decrease of 2.14% in EA's profit and an increase of 1.83% in the minimum cost of consumers.

In addition, it is important to take into account that continuous charging and discharging could reduce the lifetime of BES, and the effect is worth to be explored in the future. Moreover, to reduce the complexity of the bi-level optimization problem, the impacts of uncertain renewable generating power are ignored. Therefore, the authors are going to model renewable resources such as PV and wind units as well as the lifetime of BES units, in future works.

Contributions

Meysam Khojasteh: Data curation, Formal analysis, Investigation, Methodology, software, Validation, Visualization, Writing - original draft, Writing - review & editing.

Pedro Faria: Conceptualization, Methodology, Validation, Writing - review & editing.

Fernando Lezama: Conceptualization, Methodology, Validation, Writing - review & editing.

Zita Vale: Conceptualization, Funding acquisition, Methodology, Project administration, Resources, Supervision, Validation, Writing - review & editing.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix

To linearize the complementarity conditions (37)–(51), the big-M approach is used in this work. It shall be noted that the values of M_1, M_2, and M_3 shall be bigger than the upper bound of $P_h^{CHP}.H_h^{CHP}$, G_h^L, and T_h, respectively [37].

$$P^{CHP,A} - P_h^{CHP} + \left(\left(\frac{P^{CHP,B} - P^{CHP,A}}{H^{CHP,B}}\right)H_h^{CHP}\right) \geq 0 \tag{A.1}$$

$$P^{CHP,A} - P_h^{CHP} + \left(\left(\frac{P^{CHP,B} - P^{CHP,A}}{H^{CHP,B}}\right)H_h^{CHP}\right) \leq \left(1 - \alpha_h^{AB}\right)M_1 \tag{A.2}$$

$$\mathrm{p}_h^{AB} \leq \alpha_h^{AB}.M_1 \tag{A.3}$$

$$P_h^{CHP} - \left(\frac{P^{CHP,C} - P^{CHP,D}}{H^{CHP,C}}\right).H_h^{CHP} - P^{CHP,D} \geq 0 \tag{A.4}$$

$$P_h^{CHP} - \left(\frac{P^{CHP,C} - P^{CHP,D}}{H^{CHP,C}}\right).H_h^{CHP} - P^{CHP,D} \leq \left(1 - \alpha_h^{CD}\right).M_1 \tag{A.5}$$

$$\mathrm{p}_h^{CD} \leq \alpha_h^{CD}.M_1 \tag{A.6}$$

$$\begin{cases} \left(P^{CHP,B}-P^{CHP,C}\right).H_h^{CHP}+P^{CHP,C}.H^{CHP,B}-P^{CHP,B}.H^{CHP,C}-\left(H^{CHP,B}-H^{CHP,C}\right).P_h^{CHP}\geq 0: \\ for\ \frac{P^{CHP,C}}{P^{CHP,B}}\leq\frac{H^{CHP,C}}{P^{CHP,B}}:\ \forall h\in H \\ \left(H^{CHP,B}-H^{CHP,C}\right).P_h^{CHP}-\left(\left(P^{CHP,B}-P^{CHP,C}\right).H_h^{CHP}+P^{CHP,C}.H^{CHP,B}-P^{CHP,B}.H^{CHP,C}\right)\geq 0: \\ for\ \frac{P^{CHP,C}}{P^{CHP,B}}>\frac{H^{CHP,C}}{P^{CHP,B}}:\ \forall h\in H \end{cases} \tag{A.7}$$

$$\begin{cases} \left(P^{CHP,B}-P^{CHP,C}\right).H_h^{CHP}+P^{CHP,C}.H^{CHP,B}-P^{CHP,B}.H^{CHP,C}-\left(H^{CHP,B}-H^{CHP,C}\right).P_h^{CHP}\leq\left(1-\alpha_h^{BC}\right).M_1: \\ for\ \frac{P^{CHP,C}}{P^{CHP,B}}\leq\frac{H^{CHP,C}}{P^{CHP,B}}:\ \forall h\in H \\ \left(H^{CHP,B}-H^{CHP,C}\right).P_h^{CHP}-\left(\left(P^{CHP,B}-P^{CHP,C}\right).H_h^{CHP}+P^{CHP,C}.H^{CHP,B}-P^{CHP,B}.H^{CHP,C}\right)\leq\left(1-\alpha_h^{BC}\right).M_1: \\ for\ \frac{P^{CHP,C}}{P^{CHP,B}}>\frac{H^{CHP,C}}{P^{CHP,B}}:\ \forall h\in H \end{cases} \tag{A.8}$$

$$\mathrm{p}_h^{BC}\leq\alpha_h^{BC}.M_1:\quad\forall h\in H \tag{A.9}$$

$$P^{CHP,\max}-P_h^{CHP}\geq 0:\quad\forall h\in H \tag{A.10}$$

$$P^{CHP,\max}-P_h^{CHP}\leq\left(1-\alpha_h^{CHP,\max}\right).M_1:\quad\forall h\in H \tag{A.11}$$

$$\mathrm{p}_h^{CHP,\max}\leq\alpha_h^{CHP,\max}.M_1:\quad\forall h\in H \tag{A.12}$$

$$P_h^{CHP}-P^{CHP,\min}\geq 0:\quad\forall h\in H \tag{A.13}$$

$$P_h^{CHP}-P^{CHP,\min}\leq\left(1-\alpha_h^{CHP,\min}\right).M_1:\quad\forall h\in H \tag{A.14}$$

$$\mathrm{p}_h^{CHP,\min}\leq\alpha_h^{CHP,\min}.M_1:\quad\forall h\in H \tag{A.15}$$

$$H^{CHP,\max}-H_h^{CHP}\geq 0:\quad\forall h\in H \tag{A.16}$$

$$H^{CHP,\max}-H_h^{CHP}\leq\left(1-\beta_h^{CHP,\max}\right).M_1:\quad\forall h\in H \tag{A.17}$$

$$\mathrm{h}_h^{CHP,\max}\leq\beta_h^{CHP,\max}.M_1:\quad\forall h\in H \tag{A.18}$$

$$H_h^{CHP}-H^{CHP,\min}\geq 0:\quad\forall h\in H \tag{A.19}$$

$$H_h^{CHP}-H^{CHP,\min}\leq\left(1-\beta_h^{CHP,\min}\right).M_1:\quad\forall h\in H \tag{A.20}$$

$$\mathrm{h}_h^{CHP,\min}\leq\beta_h^{CHP,\min}.M_1:\quad\forall h\in H \tag{A.21}$$

$$Z_h^{CHP}-\left(P^{CHP,\min}.H_h^{CHP}+H^{CHP,\min}.P_h^{CHP}-P^{CHP,\min}.H^{CHP,\min}\right)\geq 0\quad:\quad\forall h\in H \tag{A.22}$$

$$Z_h^{CHP}-\left(P^{CHP,\min}.H_h^{CHP}+H^{CHP,\min}.P_h^{CHP}-P^{CHP,\min}.H^{CHP,\min}\right)\leq\left(1-\alpha_h^{CHP,UE}\right).M_1:\quad\forall h\in H \tag{A.23}$$

$$\mathrm{z}_h^{CHP,UE}\leq\alpha_h^{CHP,UE}.M_1:\quad\forall h\in H \tag{A.24}$$

$$Z_h^{CHP}-\left(P^{CHP,\max}.H_h^{CHP}+H^{CHP,\max}.P_h^{CHP}-P^{CHP,\max}.H^{CHP,\max}\right)\geq 0\quad:\quad\forall h\in H \tag{A.25}$$

$$Z_h^{CHP}-\left(P^{CHP,\max}.H_h^{CHP}+H^{CHP,\max}.P_h^{CHP}-P^{CHP,\max}.H^{CHP,\max}\right)\leq\left(1-\beta_h^{CHP,UE}\right).M_1:\quad\forall h\in H \tag{A.26}$$

$$\mathrm{y}_h^{CHP,UE}\leq\beta_h^{CHP,UE}.M_1:\quad\forall h\in H \tag{A.27}$$

$$P^{CHP,\max}.H_h^{CHP}+H^{CHP,\min}.P_h^{CHP}-P^{CHP,\max}.H^{CHP,\min}-Z_h^{CHP}\geq 0\quad:\quad\forall h\in H \tag{A.28}$$

$$P^{CHP,\max}.H_h^{CHP} + H^{CHP,\min}.P_h^{CHP} - P^{CHP,\max}.H^{CHP,\min} - Z_h^{CHP} \le \left(1 - \alpha_h^{CHP,OE}\right).M_1 : \quad \forall h \in H \tag{A.29}$$

$$\text{Binary variables} = \left\{\alpha_h^{AB}, \alpha_h^{CD}, \alpha_h^{BC}, \alpha_h^{CHP,\max}, \alpha_h^{CHP,\min}, \beta_h^{CHP,\max}, \beta_h^{CHP,\min}, \alpha_h^{CHP,UE}, \beta_h^{CHP,UE}, \alpha_h^{CHP,OE}, \beta_h^{CHP,OE}, \alpha_h^{G,\max}, \alpha_h^{G,\min}, \delta_h^{\max}, \delta_h^{\min}\right\} : \quad \forall h \in H \tag{A.47}$$

$$z_h^{CHP,OE} \le \alpha_h^{CHP,OE}.M_1 : \quad \forall h \in H \tag{A.30}$$

$$H^{CHP,\max}.P_h^{CHP} + P^{CHP,\min}.H_h^{CHP} - P^{CHP,\min}.H^{CHP,\max} - Z_h^{CHP} \ge 0 : \quad \forall h \in H \tag{A.31}$$

$$H^{CHP,\max}.P_h^{CHP} + P^{CHP,\min}.H_h^{CHP} - P^{CHP,\min}.H^{CHP,\max} - Z_h^{CHP} \le \left(1 - \beta_h^{CHP,OE}\right).M_1 : \quad \forall h \in H \tag{A.32}$$

$$y_h^{CHP,OE} \le \beta_h^{CHP,OE}.M_1 : \quad \forall h \in H \tag{A.33}$$

$$G^{\max} - G_h^L \ge 0 : \quad \forall h \in H \tag{A.34}$$

$$G^{\max} - G_h^L \le \left(1 - \alpha_h^{G,\max}\right).M_2 : \quad \forall h \in H \tag{A.35}$$

$$g_h^{G,\max} \le \alpha_h^{G,\max}.M_2 : \quad \forall h \in H \tag{A.36}$$

$$G_h^L - G^{\min} \ge 0 : \quad \forall h \in H \tag{A.37}$$

$$G_h^L - G^{\min} \le \left(1 - \alpha_h^{G,\min}\right).M_2 : \quad \forall h \in H \tag{A.38}$$

$$g_h^{G,\min} \le \alpha_h^{G,\min}.M_2 : \quad \forall h \in H \tag{A.39}$$

$$T_h^{\max} - T_h \ge 0 : \quad \forall h \in H \tag{A.40}$$

$$T_h^{\max} - T_h \le (1 - \delta_h^{\max}).M_3 : \quad \forall h \in H \tag{A.41}$$

$$T_h^{\max} \le \delta_h^{\max}.M_3 : \quad \forall h \in H \tag{A.42}$$

$$T_h - T_h^{\min} \ge 0 : \quad \forall h \in H \tag{A.43}$$

$$T_h - T_h^{\min} \le \left(1 - \delta_h^{\min}\right).M_3 : \quad \forall h \in H \tag{A.44}$$

$$T_h^{\min} \le \delta_h^{\min}.M_3 : \quad \forall h \in H \tag{A.45}$$

$$p_h^{AB}, p_h^{BC}, p_h^{CD}, p_h^{CHP,\max}, p_h^{CHP,\min}, h_h^{CHP,\max}, g_h^{G,\max}, g_h^{G,\min}, T_h^{\max}, T_h^{\min}, h_h^{CHP,\min}, z_h^{CHP,UE}, y_h^{CHP,UE}, z_h^{CHP,OE}, y_h^{CHP,OE} \ge 0 : \quad \forall h \in H \tag{A.46}$$

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