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Transcription and translation of Nicole Oresme: Quaestiones super geometricam Euclidis: Questio 2

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Quaestiones super geometriam Euclidis (Questions concerning Euclid's geometry)

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by Nicole Oresme (1323–1382)

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Questio 2

Consequenter queritur: *Utrum magnitudini possit fieri addicio in infinitum per partes proporcionales.*

Arguitur primo quod non, quis sequitur, quod magnitudo sit actu augmentabilis in infinitum. Consequencia est contra Aristotelem 3^o *Physicorum* et contra Campanum in principio huius, ubi ponit differenciam inter magnitudinem et numerum, quia numerus crescit in infinitum et non decrescit et magnitudo e contrario. Probatur consequencia: ex quo fit addicio in infinitum, cum ex addicione augmentata est, augetur in infinitum.

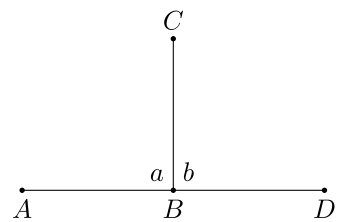
Oppositum arguitur: quidquid potest dirimi ab aliqua magnitudine, potest alteri addi, sed ab aliqua magnitudine potest fieri detractio in infinitum per tales partes, igitur ex hoc potest probari, quod sit augmentabilis in infinitum.

Question 2

Consequently, it is asked: *Whether a magnitude can be added to ad infinitum by proportional parts?*

First, it is asserted that what follows from this is: it is not the case that a magnitude is in fact infinitely augmentable. This consequence is contrary to Aristotle, *Physics* 3^{rd}, and to Campanus at the beginning of [his Commentary to Euclid's *Elements*], where he sets forth the difference between magnitude and number, because number may increase to infinity but does not decrease [indefinitely], whereas magnitude, to the contrary, [may decrease indefinitely but does not increase to infinity]. The consequence is proved: from the fact that the addition is performed to infinity, what is increased by the addition is being increased to infinity.

The opposite is argued: whatever can be removed from a certain magnitude can be added to another, but from a certain magnitude subtraction can be made of an infinite number of such [proportional] parts, therefore it can be proved from this that it may be increased *ad infinitum*.



Exemplum ponitur de angulo recto et de angulo acuto vel de duobus rectis et sit una linea super aliam faciens duos angulos rectos, qui sunt a et b. Deinde declinat una linea [c] versus alteram extremitatem, que est d, tunc arguitur sit: quantum decrescit angulus per talem motum, tantum crescit angulus a. Patet, quia quidquid removebitur a b angulo, addetur angulo a, sed angulus b descrescit in duplo, 3^o, 4^o et sic in infintum, igitur angulus a crescit in infinitum.

The example is given of a right angle and an acute angle, or of two straight lines: let there be one line [placed] upon another making two right angles, which are a [$= \angle ABC$] and b [$= \angle CBD$]. Then the one line [CB] declines towards the other's extremity, which is D, whence it is argued that: as much as the angle [b] decreases by such a movement, so much does the angle a increase. It is clear that whatever is removed from the angle b will be added to the angle a, but the angle b decreases in doubled, tripled, quadrupled [proportion], and so on to infinity, therefore the angle a increases infinitely.

Pro questione est primo notandum, quod quedam est proporcio equalitatis et est inter equalia; alia est proporcio maioris inequalitatis, que est maioris ad minus, sicut quattuor ad duo; et ista nomina differunt sicut relativi [sc. termini] positicionis et supposicionis, ut patet in predictis et secundum hoc potest tribus modis fieri addicio alicui quantitati.

As to the question, it must first be noted that there is a certain *proportion of equality* and this holds among equals; another is the *proportion of the greater inequality*, which is the greater to the lesser, as four to two; further, these nouns differ as relative [terms] of position and supposition, as is clear from the aforesaid, and according to this, addition by any quantity can be made in three ways.

Notandum secundo quod, si fiat addicio in infinitum per partes proporcionales in proporcione equalitatis vel maioris inequalitatis, totum fierit infinitum; si non, fiat hoc secundum proporcionem minoris inequalitatis, nunquam fierit infinitum, etsi fierit addicio in infinitum. Hec causa est, quia totum habebit certam proporcionem finitam ad primum assumptum, ad illud, cui fit addicio, sicut postea declarabitur.

It should be noted, secondly, that if an addition is made by infinitely many proportional parts in the proportion of equality, or [in the proportion] of the greater inequality, the whole becomes infinite; if otherwise, let this be done according to the *proportion of the smaller inequality*; then it never becomes infinite, even if the addition takes place to infinity. The reason for this is that the whole will have a certain finite ratio with the first assumed part, that to which the addition is made, as will be explained later.

Ultimo notandum, quod omne altero minus, quod habet ad illud certam propocionem, dicitur ad

Finally, it is to be noted that every [magnitude] smaller than another, which has a certain ratio illud vel respectu illius fraccio vel fracciones vel pars vel partes; et hoc patet in principiis 7^i Euclidis et illud minus denominatur duobus numeris, quorum unus dicitur numerator et alter denominator, ut patet ibidem. Verbi gracia: unum est minus quam duo et sic dicitur de duobus una 2^a et de tribus una 3^a et sic ultra; et duo de tribus dicitur 2^e 3^e et de 5 due quinte; et debent scribi isto modo et li 2^e dicitur numerator li quinte denominator.

with it, is called a fraction or fractions, or a part or parts, of it or with respect to it; and this is clear from Euclid's *Elements* VII; and that the lesser is designated by two numbers, one of which is called the numerator and the other the denominator, as is made clear there. For example: one is less than two and thus it is called one of 2, and of three one of 3, and so on; and two is called 2 of 3 of three, and of 5 two fifths; and they must be written in this way, wherein the 2 is called the numerator and the fifth the denominator.

Prima conclusio est quod, si pedalis quantitas sit assumpta et fiat addicio in infinitum secundum proporcionem subduplam sic, quod addatur ei una 2^a unius pedis, deinde una 4^a, deinde una 8^a et sic in infinitum duplando subduplos, totum precise erit duplum ad primum assumptum. Hoc patet, qui si ab aliquo demerentur iste partes per ordinem, ab illo demeretur precise dupla ad primam, ut patet per primam questionem scilicet precedentem, igitur pare racione, si adderentur.

The first conclusion is that if a quantity of a foot is assumed and addition is made to infinity according to the subduple proportion, so that one of 2 of one foot is added to it, then one of 4, then one of 8, and so on by doubling the subduples to infinity, then the whole will be exactly the double of the first assumed quantity. This is clear, that if the latter parts were to be taken from something in order, there would be removed from it exactly twice as much as the first [removed], as is clear from the first – that is the previous – Question.

Secunda conclusio est ista quod, si aliqua quantitas, ut pedalis, sit assumpta, deinde addatur tercia tanti et postea tercia additi et sic in infinitum, totum erit precise pedale cum dimidio sive in proporcione sesquialtera; et ad hoc sciendum est ista regula, quod nos debemus videre, quantum secunda pars differt a prima et tercia a secunda et sic de aliis, et illa denominare sua denominacione et tunc proporcio tocius aggregati [ad] assumptam erit sicut denominacionis [sc. assumpte] ad denominacionem [sc. differencie]. Verbi gracia: in proposito secunda pars, que est una tercia prime, deficit a prima per duas tercias, ergo proporcio tocius ad primam partem vel ad assumptam est sicut 3 ad duo et hec est sesquialtera.

The second conclusion is that if any quantity, such as a foot, is considered, then a third is added to that amount, and afterwards thirds are added, and so on ad infinitum, then the whole will be precisely a foot and a half, or will be in the sesquialter proportion; and for this we must know the rule, which we must understand: however much the second part differs from the first, and the third from the second, and so on, each from the others, denote this by its proper denomination, and then whatever ratio the whole has with it [the first part] may be taken as the [ratio of] this denomination to the denomination [of the difference with its part]. For example: in what was proposed [earlier], the second part, which is one-third of the first, falls short of the first by two-thirds, whence the ratio [of the whole] to the first part, or to the one considered, is as 3 to two, and this is the sesquialter.

Tercia conclusio est ista, quod possibile est, quod alicui quantitati fiat addicio secundum proporciones minoris inequalitatis inproporcionaliter et

The third conclusion is that it is possible for an addition to any quantity to be made disproportionately according to proportions of the smaller tamen totum fieret infinitum; sed si fiet proporcionaliter, fieret finitum sicut dictum est. Verbi gracia: sit pedalis quantitas assumpta, cui addatur in prima parte proporcionali hore una medietas pedis, deinde una tercia in alia et deinde una quarta, deinde quinta et sic in infinitum secundum ordinem numerorum, dico, quod totum fieret infinitum, quod probatur sic: ibi existunt infinite partes, quarum quelibet erit maior quam medietas pedis, ergo totum erit infinitum. Antecedens patet, quia 4^a et 3^a sunt plus quam una medietas, similter de 5^a usque ad 8^am et usque ad 16^am et sic infinitum.

inequality, and still the whole may become infinite; however, if it be done proportionately, it would become finite as has been said. For example: let a quantity of a foot be assumed, to which is added in the first part of a proportional hour one-half of a foot, then one-third at another, and then one-fourth, then one-fifth, and so on to infinity according to the order of the numbers. I claim that the whole would become infinite, which is proved as follows: there exist an infinite number of parts, each of which will be greater than half a foot, therefore the whole will be infinite. The antecedent is clear, because a 4^{th} and a 3^{rd} are more than one-half; similarly, from a 5^{th} up to an 8^{th}, and then up to a 16^{th}, and so on to infinity.

Ad raciones in oppositum: sequitur quod magnitudo etc., potest distingui: magnitudo augmentabitur in infitnitum. Unus sensus potest esse referendo li infinitum ad actum augendi; et sic potest concedi, quod infinitis modis vicibus potest fieri talus actus, dum tamen continuetur, sed iste [sensus] est improprius; et bene sequitur ex questione: alius sensus est proprius. [Iste sensus est improprius], quia [dicitur]: augmentabitur in duplo, quadruplo, etc. in infinitum et hoc est falsum, ut sequitur ex questione. Ad aliam racione, que erit ad oppositum [sc.] cum arguitur de angulo: tantum augmentabitur etc., dico, quod ista est dinstinguenda, quia li tantum et quantum possunt denominare proporcionem arithmeticam, que attenditur penes quantitates excessum; et sic concedo maiorem et nego minorem, quia non fit assumpcio solucio. Aliter possunt denominare proporcionem geometricam; et sic nego maiorem, quia non in tanta proporcione augmentabitur a, quanta b diminuetur, licet de tanta re augmentabitur a, quanta b diminuetur; et isto modo potest argui de qualibet alia quantitatis et qualitatis et hoc patet, quia cum angulus b minuitur in duplo, angulus a non augetur in duplo, imo a tunc diceretur augeri [in duplo], quando perveniret ad d linea c et ille erit, [quando] angulus b esset totum diminutus et corruptus; et sic patet responsio de questione. Sic est finis [huius questionis].

As to arguments for the contrary: it follows that the magnitude, etc., can be determined: a magnitude may be increased to infinity. One meaning may be that this infinity refers to the act of increasing; and thus it would be granted that such an act would be performed in an infinite number of times – as long as it continues. But this [sense] is more improper; and it follows well from the question that another sense is proper. [This sense is more improper] because [it is said]: it will be increased into its double, quadruple, etc., to infinity, but this is false as follows from the question. To another argument which will be to the contrary, [namely] when it is argued about an angle: it will only be increased, etc., I say that this is determinable, because however much it may be [that is added to a or taken from b], this much can denominate an arithmetical proportion, which is taken into account in relation to the excess quantities; and thus I grant the major [premise] but deny the lesser, because what is assumed does not result in an explanation. In another way, these [parts] can form a geometrical proportion; and so I deny the greater [premise], because a will not be increased in the same proportion as b is decreased, although a will be increased by the same amount as b is decreased; and in the same way it can be argued about any other quantity and quality, and this is clear because when the angle b is reduced by a factor of two, the angle a is not increased by a factor of two; nay, a would be said to be increased [doubly only] when C on the line reached D, and that would be [when] the angle b is entirely diminished and corrupted; and thus the answer to the question is clear. Such is the end [of this question].