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Regulation, Capital, and Margining: Quant Angle

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Table of Contents - Part I

Overview of Regulation & Capital Basics

Table of Contents - Part II

IMM

Table of Contents - Part III

Various Aspects of IMM

Table of Contents - Part IV

Basel 3+4, Clearinghouses, IA/IM

Table of Contents – Part V

Related Topics: “XVA”, Capital Management, Collateral,..

- CVA/DVA
- Symmetric and Asymmetric FVA
- KVA, MVA
- Management of Cost and Capital Metrics

I: Overview of Regulation & Capital Basics

Regulation and Quant/Strats - (1)

- Just about the only "growth industry" in the banking world these days is regulation-based: regulatory compliance and reporting, regulatory examinations, regulatory "optimization" (margins, collateral, CSAs, capital hedges, funding,...).
- Fortunately, for quants/strats, much new regulation is complex and, for larger banks at least, requires profound amounts of new analytics development, documentation, and deployment.
- While not revenue generating per se, these activities are very visible at the top of the house, as capital numbers are typically in the \$100BN’s, versus exotics trading book revenues in the \$10MM’s.
- Fees, charges, excess capital requirements, lawsuits,... are big concerns these days – and avoidance of them involve (shadow) revenues much larger than what quants typically work on.

Regulation and Quant/Strats - (2)

- At BofA, regulatory compliance currently is more than 50% of the work the quant/strat group, and not likely to diminish.
- We have launched significant learning programs to “reschool” quants and to teach them capital basics.
- Quants (including myself) are not meant to be regulatory experts (legal and capital groups have this responsibility), so the quant focus is necessarily selective and targeted to those areas that involve analytics.
- My "tour" of regulation is therefore highly biased in its coverage (and my knowledge somewhat limited...)

Basel Reg Capital (and Margin)

- Basel Committee on Banking Supervision (BCBS) is a Basel-based committee tasked with developing international policy guidelines for banking supervision, especially around capital adequacy
- Founded by the Central Banks of the G10 countries in 1974. Current members include most developed nations, such as Sweden. Denmark not on the membership list!?
- Role is to formulate broad principles, issued in documents available on their web-site. It is up to the individual nations' Central Banks (and/or other bodies, such as the FDIC and OCC in the US) to turn recommendations into concrete regulation and supervision.
- Currently, banks have to simultaneously (!) worry about 5 different capital adequacy accords: Basel 1, Basel 2, Basel 2.5, Basel 3, Basel 4.

Dodd-Frank

- The Dodd-Frank Act (which leans on Basel 3) was signed into law in the US in July 2010. Based on a G20 agreement that means to stabilize the financial system through a variety of measures (clearing, transparency,..)
- The part that is most relevant for Wall Street is Title VII ("Wall Street Transparency and Accountability")
- Subindex: *centralized clearing*, submission of derivatives data to a *central repository*, *SEFs*, supervision through SEC and CFTC, *Volcker rule*, *Collins floor*.
- The European "version" of the law is through EU directives: European Markets and Infrastructure Regulation ("EMIR").

Capital for Market & Credit Risk - (1)

- Capital is essentially a buffer that banks set aside to protect them from going insolvent when faced with losses.
- These losses can come from *market risk exposure* – i.e., movements in financial variables that affect the value of the bank's holding of securities.
- Or these losses can come from *counterparty credit risk exposure* – i.e., from failures of the bank's counterparties to pay on their obligations.
- As mentioned, capital adequacy regulations are written by agencies such as the Bank for International Settlements (BIS) and local regulators such the FRB, the FSA, etc. BIS drafts the so-called *Basel Accords*, the key methodology prescriptions.

Capital for Market & Credit Risk - (2)

- From a regulatory standpoint, capital held by a bank is tiered into various “quality grades”.
  - Tier 1: Common Stock and Retained Earnings.
  - Tier 2: Supplementary bank capital that includes items such as revaluation reserves, undisclosed reserves, hybrid instruments and subordinated term debt.
  - Tier 3: “Everything else” – a greater number of subordinated issues, undisclosed reserves and general loss reserves.
- The regulatory formulas compute “RWA” – Risk Weighted Asset – values for both market risk (RWA$_M$), credit risk (RWA$_C$), and operational risk (RWA$_O$).

Capital for Market & Credit Risk - (3)

- The regulatory capital (RC) requirements then take the form

$$\frac{RC}{\text{RWA}_M + \text{RWA}_C + \text{RWA}_O} \geq X\% \tag{1}$$

  where X depends on the Accord: 8% for Basel 2, 10.5% for Basel 3.

- There are also restriction on Tier 1 capital alone. Certain “globally systemically important” banks (G-SIBs) need to post more Tier 1 capital than other banks. BofA is on the black list.

- I’ll ignore RWA_O going forward.

Market Risk RWA in a Nutshell - (1)

- Under regulatory capital rules, capital requirements for market risk exposure leans heavily on value-at-risk (VaR) computations, supplemented by add-on charges for so-called *specific risk* (= idiosyncratic event risk for individual firms).
- These computations are fairly "standard", involving 99th percentile return distributions over 10-day horizons.
- Results can often be pulled from banks' regular VaR market risk systems and the computations have traditionally not been complex enough to involve front office quants (they are generally handled by risk management).

Market Risk RWA in a Nutshell - (2)

- While our focus here shall be on credit risk capital, it is worth noticing that the various financial crisis has caused BIS to recently refine/revise its market risk capital requirements substantially.
- Some of these changes has meant an increase in complexity that in many banks has required implementation support by the quant teams.
- In a nutshell, market risk requirement evolution is something like this:

Market Risk RWA in a Nutshell - (3)

- **Basel 1 (1988):** RWA = 12.5 * m * VaR + specific risk add-on. Here, m = supervisory multiplier > 3.
- **Basel 2 and 2.5 (2004 and 2010):** Add “stressed” VaR to overall VaR requirement. For credit derivatives, add two new measures – Incremental Risk Charge (IRC) and Comprehensive Risk Measure (CRM) – to better measure risk associated with defaults and credit spread dynamics.
- **Basel 3 (2011):** Add VaR on CVA (Credit Value Adjustment). Also, provisions for liquidity risk, leverage ratios, etc.
- **Basel 4 (draft):** Replace VaR with CVaR, avoid double-counting VaR and “stressed VaR”, eliminate IRC/CRM,...

IRC/CRM - (1)

- IRC (for CDSs) and CRM (for CDOs) were introduced following the financial crisis, which had regulators concerned about the effects of credit derivatives on financial stability.
- BCBS appeared especially worried about lacking liquidity, and require VaR-type calculations on a 1-year horizon.
- In practice this requires simulation of *all* components that go into valuation of structured credit derivatives books; enough samples to estimate the 99.9% confidence level.
- This requires modeling of: joint dynamics of spreads, defaults, recovery, basis spreads, correlations, ratings, etc. Done by quants in most banks.

IRC/CRM - (2)

- The specification and implementation (and defense) of the simulation model and the portfolio “aging” assumptions was an expensive and lengthy exercise for most US banks.
- AND NOW: Basel 4 eliminates these new charges (starting officially in 2017).

Credit Capital - (1)

- *Regulatory credit risk capital*: standardized regulatory requirements for how much capital banks should hold to protect themselves against counterparty defaults.
- As mentioned, imposed by regulatory agencies such as BIS (Basel), FDIC, FRB, FSA, and so forth.
- *Economic capital*: how much capital a firm should rationally set aside to protect against insolvency from economic credit losses, in the absence of regulatory capital.
- Economic capital is a risk measure (it does not equal actual capital set aside), and is used internally by banks for decision making purposes.

Credit Capital - (2)

- In Basel 1, regulatory credit risk capital is computed in a very simplistic fashion, based on deal notionals and categorization of trades into various buckets. There is no explicit recognition of the rating and recovery of the counterparty.
- This rule, which is still the law in the US, is known as the *Current Exposure Method (CEM)*.

CEM Method - (1)

- Consider a counterparty with several netting sets (NS). Trade j with the counterparty is assumed to have value v_j to the bank.
- CEM method (Basel 1, 1988) writes RWA = 12.5 · EAD · RW, where the risk weight RW is given "in a table", and EAD is

$$EAD = CE + PFE$$

- Here CE (current exposure) is

$$CE = \sum_k \left(\sum_{j \in NS_k} v_j \right)^+$$

- PFE (potential future exposure) is (N_j: notional of trade j)

$$PFE = (0.4 + 0.6 \cdot NGR) \cdot \sum_{\forall j} \alpha_j N_j$$

CEM Method - (2)

- Here *net-gross ratio NGR* is

$$NGR = \frac{CE}{\sum_{\forall j} v_j^+}.$$

- The α_j are heuristic trade-level add-ons, to be looked up in table provided by BCBS. They depend on asset class and maturity.
- Note that CEM, besides being quite heuristic, only recognizes maximum 60% diversification

CEM Metnod - (3)

- CEM has been criticized – even by BCBS itself – on many grounds, including (from BCBS docs):
  - It does not differentiate between margined and unmargined transactions;
  - The supervisory add-on factors do not sufficiently capture the level of volatilities as observed over the recent stress periods; and
  - The recognition of hedging and netting benefits through NGR is too simplistic and does not reflect economically meaningful relationships between the derivative positions.
- Basel 2 was introduced in large part to address these issues and to make regulatory credit capital more resemble *economic* capital.

Economic Credit Capital for Loans - (1)

- Assume that a bank has a portfolio of loans with B counterparties. For counterparty i, the net loan notional is assumed to be N_i and the loss-given-default percentage (LGD) is l_i.
- The primary economic risk of interest is here *credit risk*, due to counterparty default exposure.
- Over a period $[0, T]$, the cumulative economic loss due to defaults is

$$L(T) = \sum_{i=1}^{B} N_i l_i 1_{\tau_i \leq T},$$

where τ_i is the default time of counterparty i, and 1_A is an indicator for the event A (1 if A happens, 0 if it does not).

- The expected value (in the actual probability measure $\mathbb{P}$) of $L(T)$ is denoted the *credit reserve* or the *expected loss* (EL).

Economic Capital for Loans - (2)

- Hence, if $\mathrm{E}(\cdot)$ denotes expectations in $\mathbb{P}$,

$$EL = \mathrm{E}\left(L(T)\right) = \sum_{i=1}^{B} N_i l_i \cdot p_i, \quad p_i = \mathbb{P}\left(\tau_i \leq T\right),$$

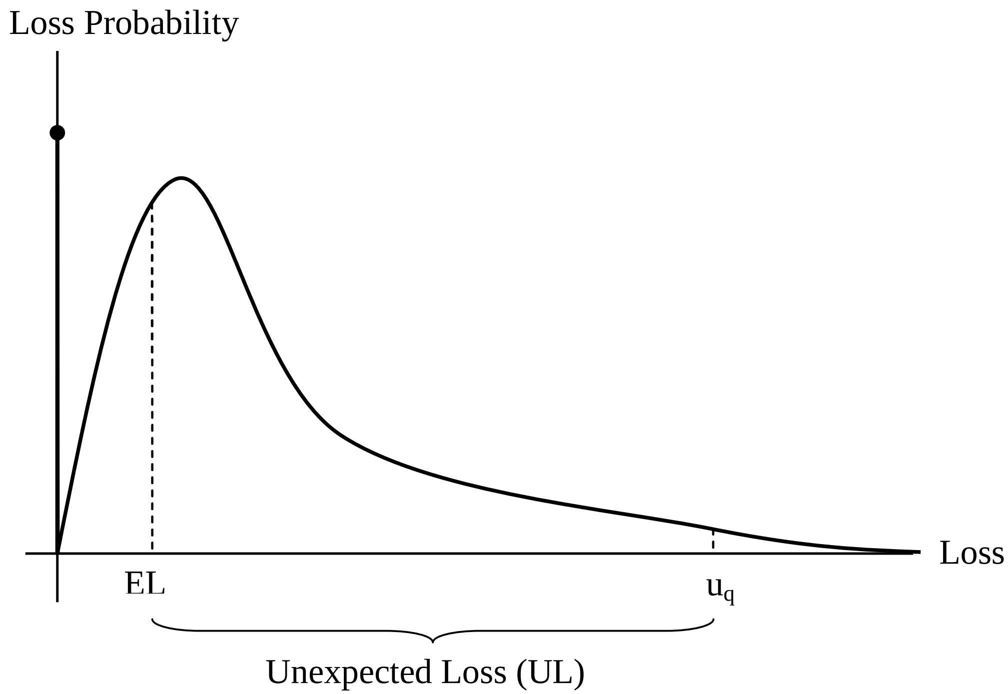
where p_i is the default probability of counterparty i on $[0, T]$.

- The credit reserve covering EL should be priced into the loans from the outset, and is *not* counted in economic capital. (Regulators do keep an eye on whether banks have adequate provisions for EL).
- Instead, economic capital is designed to sit as a buffer against *unexpected* large losses.

Economic Capital for Loans - (3)

- To characterize large losses, let q be some small probability (i.e. 0.05%), and define the corresponding loss percentile u_q:

$$\mathbb{P}\left(L(T) \geq u_q\right) = q.$$



Economic Capital for Loans - (4)

- Economic Capital is set to protect against unexpected losses (UL), so

$$\boxed{EC_q = UL = u_q - EL.} \tag{2}$$

- For a given horizon T (often 1 year), a reasonable way to set q would be based on historical default rates for a target rating or internal grade.
- For instance, to reach an “AA” rating, one equates q to the T-year historical default probability of AA-rated firms ($\approx 0.03\%$, if $T = 1$).
- With this amount of capital, the probability of credit losses wiping out all capital and reserves (and causing bank to go insolvent) would theoretically equal a AA default probability.
- Note: some ratings agencies rely on this principle to rate ring-fenced subsidiaries. To be conservative, it is common to use a worst-case analysis on multiple horizons T.

Computation of EC - (1)

- In principle, the computation of EC_q in (2) can be done by Monte Carlo simulations, where we draw correlated default times for the pool of loan counterparties.
- Default correlation is frequently generated with a one-factor Gaussian copula. Specifically, if the copula correlation is ρ, we set

$$1_{\tau_i \le T} = 1_{Z_i \le H_i}, \tag{3}$$

where

$$Z_i = \sqrt{\rho} X + \sqrt{1-\rho}\, \epsilon_i,$$

X is a common Gaussian $N(0,1)$ economy-wide factor, and ϵ_i an i.d. $N(0,1)$ idiosyncratic random variable.

Computation of EC - (2)

- In (3), we obviously need

$$\mathbb{P}\left(Z_i \le H_i\right) = \mathbb{P}\left(\tau_i \le T\right) = p_i,$$

or

$$H_i = \Phi^{-1}(p_i),$$

where $\Phi(\cdot)$ is the cumulative Gaussian distribution function.

- With this, we can generate outcomes of $1_{\tau_i \le T}$ for all i, which allows us to simulate $L(T)$.
- This, in turn, allows us to compute EC_q from (2)

Large-Portfolio Limits - (1)

- It is common to try to avoid Monte Carlo simulations by using various approximations or by (exact) Panjer recursions.
- For capital computations, the most important technique is the simple *Vasicek large-portfolio limit*.
- The assumption here is simple: there are an *infinite* number B of counterparties.
- In this setup, consider the X-conditional expectation of *per-counterparty* loss (L/B):

$$\lim_{B\to\infty} \mathrm{E}\left(B^{-1}L(T)|X\right) = \lim_{B\to\infty} B^{-1}\sum N_i l_i \mathrm{E}\left(p_i|X\right),$$

$$\mathrm{E}\left(p_i|X\right) = \mathbb{P}\left(Z_i \le H_i|X\right) = \Phi\left(\frac{H_i - \sqrt{\rho}X}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\Phi^{-1}(p_i) - \sqrt{\rho}X}{\sqrt{1-\rho}}\right)$$

Large-Portfolio Limits - (2)

- We need some assumptions about the portfolio composition to allow us to form a meaningful large-B limit.
- For instance, if the portfolio is *homogeneous* with all $p_i = p$ and $l_i = l$ identical, the limit exists and we get

$$\lim_{B\to\infty} \mathrm{E}\left(B^{-1}L(T)|X\right) = Nl\Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}X}{\sqrt{1-\rho}}\right) = Nl \cdot h(X).$$

- In the homogeneous case, it is easily seen that

$$\lim_{B\to\infty} \mathrm{Var}\left(B^{-1}L(T)|X\right) = 0,$$

so in large-B limit, we diversify away all idiosyncratic risk not originating from X. And therefore we simply have:

$$\lim_{B\to\infty} L(T)/B = Nl \cdot h(X).$$

Large-Portfolio Limits - (3)

- We therefore have, for large B,

$$\mathbb{P}\left(L(T)/B \geq x\right) = \mathbb{P}\left(h(X) \geq \frac{x}{Nl}\right) = \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{1-\rho}\,\Phi^{-1}\left(\frac{x}{Nl}\right)}{\sqrt{\rho}}\right) \tag{4}$$

- We can compute EC_q *per counterparty* as

$$EC_q/B = u_q - EL/B = u_q - pNl,$$

where the percentile u_q is given by $\mathbb{P}\left(L(T)/B \geq u_q\right) = q$.

- Using (4), we get the *large-portfolio economic capital formula*

$$\boxed{EC_q/B = Nl \cdot \left\{\Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}\,\Phi^{-1}(q)}{\sqrt{1-\rho}}\right) - p\right\}}. \tag{5}$$

Non-Loan Portfolios - (1)

- We emphasize that (5) is only an approximation of per-counterparty EC, and does not (so far) cover anything other than loans.
- We now up the ante and consider the more challenging situation where our exposure to a counterparty is not generated by loans alone, but by a complex portfolio of securities.
- Let $V_i(t)$ be the promised (default-free) time t value to the bank of counterparty portfolio i, and let $C_i(t)$ be the (stochastic) collateral value posted by the counterparty.
- The stochastic *exposure* at any t is (ignoring close-out risk, for now)

$$E_i(t) = (V_i(t) - C_i(t))^+.$$

Unlike CVA, for capital we only consider the bank's exposure to counterparty (not vice versa).

Non-Loan Portfolios - (2)

- Positive exposure combined with a default of counterparty i will lead to a credit loss of $l_i E_i(\tau_i)$.
- Therefore,

$$L(T) = \sum_{i=1}^{B} l_i E_i(\tau_i) 1_{\tau_i \leq T}. \quad (6)$$

- This is similar to the loan setting from earlier, except that the loan notionals (N_i) are now random numbers (E_i).
- Economic capital is still computed as before, $EC_q = u_q - EL$, but both u_q and EL are now more complicated to compute.

Non-Loan Portfolios - (3)

- A (naive) simulation algorithm could work like this:
  1. Simulate in $\mathbb{P}$ a path of correlated market data (rates, equities, commodities, spreads, FX,...) out to time T. Let $\omega(t)$ be the market data state at time t and prior.
  2. Generate a set of correlated default times τ_i, $i = 1, \ldots, B$.
  3. At time τ_i (if less than T), use $\omega(\tau_i)$ with pricing analytics to establish $V_i(\tau_i)$ and $C(\tau_i)$, $i = 1, \ldots, B$.
  4. Establish $L(T)$ from (6).
  5. Repeat for many paths, to uncover full distribution of $L(T)$.
- Equipped with the simulated loss distribution, we can establish EC_q.
- This can be a very challenging/time-consuming exercise, especially if q is small and if the portfolio is complex and expensive to price.

Basel 2 and IRB - (1)

- Key objective of the *internal ratings-based* methodology (IRB) in Basel 2 is to provide a framework for regulatory credit risk capital that is spiritually similar to economic capital.
- However, the IRB needs to be sufficiently simple and transparent to be used in a regulatory setting.
- A special requirement by regulators is *portfolio invariance*: the capital treatment given to a loan position with a given counterparty should be identical from one bank to the next, and should not depend on exposure to other counterparties.
- This is accomplished by assuming *infinite diversification*, in the same manner as we did for the large-portfolio EC result.

Basel 2 and IRB - (2)

- In addition, to avoid the complexities of joint market and default simulations, regulators wish to decouple exposure and default simulations by introducing the concept of *loan-equivalent notional* (LEN) a.k.a. *exposure-at-default* (EAD).
- The idea behind EAD is to take a securities portfolio and replace it in some fashion with a simple loan. After this, the Vasicek formula for loans is applied directly.
- We shall return to how EAD is computed later. Let us first discuss some modifications that Basel 2 makes to the Vasicek economic capital formula (5).

Basel 2 and IRB - (3)

- First, correlation is made a decaying function of default probability p, in an attempt to fit historical observations for asset correlations across various economic cycles. (???)
- Second, regulators wish to introduce a component of *transition risk*, i.e. the risk of market value losses due to counterparty ratings deterioration (i.e. spread increases) over the interval $[0, T]$, even when there are no outright defaults.
- Such transition risk increases with the spread duration of the portfolio exposure, so Basel 2 also needs a methodology for computing a loan equivalent *effective maturity* (M). We discuss this later.
- The transition risk adjustment to regulatory capital (RC) takes the form of a scale function $k(M, p)$ that depends on effective maturity M and default probability p.

Basel 2 and IRB - (4)

- As in Basel 1, one writes RWA$_C = 12.5 \cdot RC$ where the *key formula* for regulatory capital (RC) for a counterparty-level trading position:

$$RC_q = EAD \cdot RW, \tag{7}$$

$$\boxed{RW = l \cdot \left\{ \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho(p)}\Phi^{-1}(q)}{\sqrt{1-\rho(p)}} \right) - p \right\} \cdot k(M,p)}, \tag{8}$$

and:

- p: 1-year probability of default (PD);
- l: loss-given-default percentage (LGD);
- q: 0.001 (“once in a thousand years”);
- M: effective maturity;
- $\rho(p) = 0.24 - 0.12(1 - e^{-50p})$ (ignoring small-firm terms)

Basel 2 and IRB - (5)

- The transition risk adjustment function k is:

$$k(x,y) = \frac{1 + (x - 2.5)b(y)}{1 - 1.5b(y)},$$
$$b(y) = (0.11852 - 0.05478 \ln y)^2.$$

- The function k is complex, and the way it has been arrived at is not completely transparent.
- BIS documents hint at the usage of VaR computations using a ratings-based MtM credit risk system similar to KMV PortfolioManager, but details are not disclosed. “Black Box”.

Basel 2 and IRB - (6)

- The inputs to the RC computation in (7) are: EAD, LGD, PD, and M.
- Because EL and UL emerge in a clean portfolio-invariant fashion from obligor-specific characteristic (PD and LGD), the RC approach is considered purely *ratings-based*, hence the IRB moniker.
- At most banks, a dedicated capital management team is responsible for the estimation of LGD and PD. The methodologies are actuarial in nature, and must be approved by regulators.
- The capital management teams are generally also responsible for the ultimate reporting of RC numbers produced by (7).
- However, it is becoming increasingly necessary to have quant teams execute the computations for EAD and M, a topic that we return to shortly.

FIRB vs AIRB

- Depending on where EAD, LGD, PD and M come from, we have two IRB approaches: Foundation IRB and Advanced IRB (FIRB and AIRB)
- In FIRB, EAD is computed by CEM (see earlier slide) and LGD, M are provided by regulatory rules. PD must be estimated by bank itself.
- In AIRB, all quantities are provided by the bank itself, subject to examinations (for each quantity separately) by regulators.
- All large banks are expected to use AIRB.

PD - (1)

- It would be tempting to pick PDs from traded securities, such as CDSs. Apart from the fact that such information is only available to a very small set of obligors, recall that we need default probabilities in the actual probability measure $\mathbb{P}$, not in a risk-neutral measure.
- PD formally is:

  "*[T]he long-run average one-year default rate for the rating grade assigned [...] to the obligor, capturing the average default experience for obligors in the rating grade over a mix of economic conditions..*"

- Specialized risk rating teams at banks are charged with assigning each counterparty to an internal *obligor risk rating* (ORR), an internal scale (e.g., from 1 to 10).

PD - (2)

- Assigning an ORR to an obligor is often based on fundamental analysis (“scorecards”) similar to that undertaken by rating agencies.
- In fact, rating agency ratings (when they exist), are taken into account in the ORR, but supplemented by bank’s own data and methodologies.
- PD is not easy to estimate, and bank methodologies vary. There have been controversies, for instance (Risk Magazine, June 2013)

  “*Danske Bank and its regulator were pitched into open conflict in mid-June, when the Danish Financial Supervisory Agency told the bank to hold more capital for corporate loans. [...] The primary driver of the seemingly anomalous risk weights is the bank’s consistently low PD estimates, [...] although low LGD numbers also play a role.* ”

LGD

- “*A bank must estimate an LGD for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks. This LGD cannot be less than the long-run default-weighted average loss rate given default [...]*”
- Regulators want banks to use LGDs that are higher than average, to reflect the fact that LGDs tend to increase in a systemic crisis. Can be done by emphasizing data from periods where credit losses are higher than normal.
- “Downturn” LGDs are estimated by capital management teams using historical default and recovery data.
- The estimation often involves several factors, primarily the *collateral type* and *line of business*.

EAD and M

- The computation of EAD and M is governed by a regulatory framework called *internal models methodology* (IMM) and is normally handled by a counterparty credit risk function, along with assistance from technology and, increasingly, front office quants.
- EAD computations are complicated and model intensive, so banks have to establish very robust controls and oversights around their computations.
- In addition, rigorous backtesting procedures are required to prove that the models used for exposure computations are realistic and conservative. Models must be approved by model validation.
- Formal submission to regulators and passing an examination with the OCC/FRB is required to be approved for IMM and Basel 2.

II: IMM

IMM & EAD Basics

- Starting with EAD, the purpose of IMM is to find a way to take an arbitrary derivatives portfolio (including its collateral) and replace it with a "representative" loan notional.
- The loan notional need only be representative for a one-year period, since IRB works with this horizon.
- The cumulative loss on $[0, T]$ generated for a portfolio $V(t)$ with collateral $C(t)$ is

$$L(T) = l \cdot (V(\tau) - C(\tau))^{+} 1_{\tau \leq T} = l \cdot E(\tau) 1_{\tau \leq T}, \tag{9}$$

where τ is the counterparty default time and $E(t) = (V(t) - C(t))^{+}$ is the *exposure* at time t.

EAD by Expectations - (1)

- If we only had a loan with notional N (and maturity $> T$), we would have

$$L_N(T) = l \cdot N 1_{\tau \le T}. \tag{10}$$

- How do we pick N such that (9) and (10) are “close”?
- We could try to align expectations $(p$ is default probability, $T = 1$yr):

$$N \mathrm{E}\left(1_{\tau \le T}\right) = \mathrm{E}\left(E(\tau) 1_{\tau \le T}\right) \Rightarrow N = \frac{\mathrm{E}\left(E(\tau) 1_{\tau \le T}\right)}{p},$$

where we have assumed that l is non-random.

- This can be rewritten as

$$N = \frac{\mathrm{E}\left(E(\tau) | \tau \le T\right) \mathrm{E}\left(1_{\tau \le T}\right)}{p} = \mathrm{E}\left(E(\tau) | \tau \le T\right). \tag{11}$$

EAD by Expectations - (2)

- In the computation of the r.h.s. of (11), it is clear that *right- and wrong-way risk matters:* conditioned on an early default ($\tau \leq T$), is the exposure higher (wrong-way) or lower (right-way) than normal?
- Building models that cleanly allow for dependence between exposure and τ is non-trivial, so regulators want a way out of this.
- For this, suppose that exposure and τ are *independent*, in the sense that

$$\begin{aligned} \mathrm{E}\left(E(\tau)|\tau \leq T\right) &= \int_0^T \mathrm{E}\left(E(t)\right)\mathbb{P}\left(\tau \in dt|\tau \leq T\right) \\ &= \int_0^T EE(t)\mathbb{P}\left(\tau \in dt|\tau \leq T\right), \end{aligned}$$

where $EE(t)$, $t \leq T$, is the *expected exposure profile*.

EAD by Expectations - (3)

- What can we make of the term $\mathbb{P}\left(\tau \in dt | \tau \leq T\right)$? In a simple Poisson model where default arrives at an intensity of λ, we have, for $t \leq T$,

$$\mathbb{P}\left(\tau \in dt | \tau \leq T\right) = \frac{\mathbb{P}\left(\tau \in dt, \tau \leq T\right)}{\mathbb{P}\left(\tau \leq T\right)} = \frac{\mathbb{P}\left(\tau \in dt\right)}{\mathbb{P}\left(\tau \leq T\right)} = \frac{e^{-\lambda t}\lambda}{1 - e^{-\lambda T}}\, dt.$$

- If λ is smallish, we have

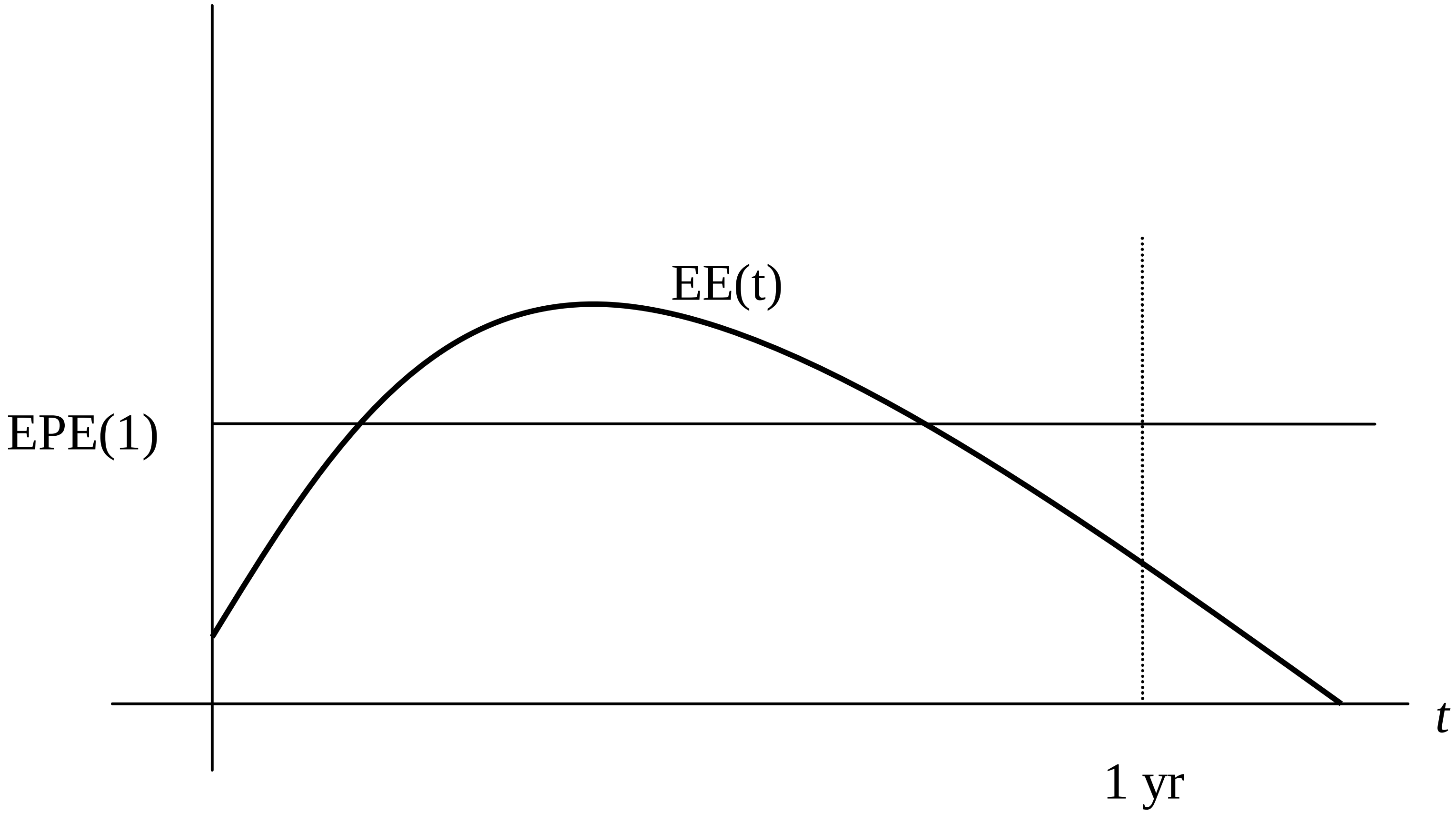
$$\mathbb{P}\left(\tau \in dt | \tau \leq T\right) \approx \frac{(1-\lambda t)\lambda}{\lambda T}\, dt \approx \frac{1}{T}.$$

- So, therefore, with independence,

$$\mathrm{E}\left(E(\tau) | \tau \leq T\right) \approx \frac{1}{T}\int_0^T EE(t)\, dt \triangleq EPE(T). \qquad (12)$$

- EPE ("expected positive exposure") is the *time-average* of EE.

EPE



- Note that EPE is an intuitive definition of loan-equivalent notional.

Reinvestment Risk - (1)

- Combining (11) and (12) would suggest that a reasonable (and intuitive) estimate for EAD could be 1-yr EPE.
- In reality, things are more complicated. First, we ignored wrong-way risk. Second, our principle of matching just the first moment would tend to ignore the volatility of the exposure, and could be less than conservative. Third, we have ignored *re-investment risk*.
- Implicit in the way we drew our exposure profile was an assumption that the portfolio is static: after time 0, we just leave it alone to age and, ultimately, die.
- For portfolios with maturities $\ll 1$ year (e.g. repos, or short-dated FX), this is not conservative, since in reality the portfolio will be "refreshed" with new trades as part of the on-going trading business.

Reinvestment Risk - (2)

- To capture this, Basel mandates that for RC computations one does not use the EE profile directly, but instead an alternative profile EE* ("effective EE") that is found as a running maximum of the EE profile:

$$EE^*(t_i) = \max\left(EE^*(t_{i-1}), EE(t_i)\right)$$

where $\{t_i\}$ is some sampling grid.

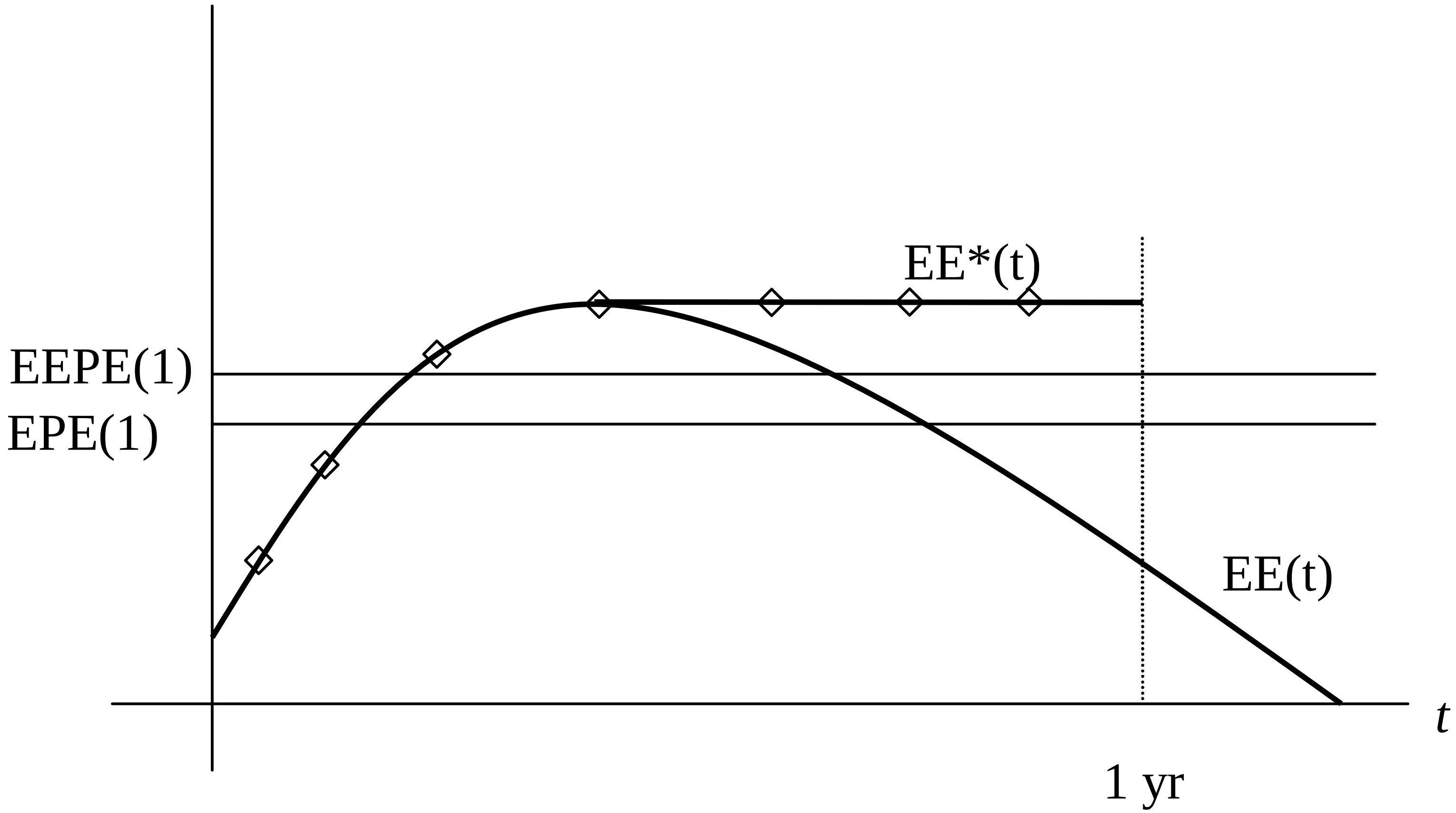
- The time-average of this "modified" EE profile is called the *effective EPE*, or *EEPE*:

$$EEPE(T) = \frac{1}{T}\int_0^T EE^*(t)\,dt.$$

- If the longest deal maturity in the portfolio is less than T, then:

$$EEPE(T) = \frac{1}{T_{\max}}\int_0^{T_{\max}} EE^*(t)\,dt.$$

Reinvestment Risk - (3)



- Note that EEPE > EPE. Not easy to justify EEPE formally, though.

Alpha Multiplier - (1)

- What about the independence assumption and zero variance?
- Originally, the Basel committee attempted to cover the missing variance by replacing expected exposure (EE) by *potential exposure* (PE), where PE is, say, the 95th percentile of the exposure $E(t)$.
- This, however, was met with protests from industry which demonstrated (using simulation studies) that using PE would lead to large overestimates for economic capital. This is not surprising.
- Instead, the Basel committee proposed to introduce a scale $\alpha > 1$:

$$EAD = \alpha \cdot EEPE(T) \tag{13}$$

- The “Alpha” multiplier is meant to adjust for a variety of effects: wrong-way risk, variance of the exposure, finite granularity of portfolios, noise in $E(t)$ simulation,...

Alpha Multiplier - (2)

- Note: the original draft of Basel 2 (from 2001) had an explicit granularity adjustment, which was subsequently dropped.
- Industry studies on real portfolios have shown that $\alpha \approx 1.1 - 1.2$.
- Basel 2 uses $\alpha = 1.4$, but does allow banks to argue for a lower α:

  "Banks may seek approval from their supervisors to compute internal estimates of alpha subject to a floor of 1.2, where alpha equals the ratio of economic capital from a full simulation of counterparty exposure across counterparties (numerator) and economic capital based on EPE (denominator), assuming they meet certain operating requirements."

- All things considered, $\alpha = 1.4$ is probably reasonable.

But wait, there is more..

- On top of the 1.4 multiplier, the BCBS has issued language to add *another* fudge factor:

  > *"The Committee believes it is important to reiterate its objectives regarding the overall level of minimum capital requirements. These are to broadly maintain the aggregate level of such requirements, while also providing incentives to adopt the more advanced risk-sensitive approaches of the revised Framework. To attain the objective, the Committee applies a scaling factor to the risk-weighted asset amounts for credit risk under the IRB approach. The current best estimate of the scaling factor using quantitative impact study data is 1.06."*

- ???

Effective Maturity - (1)

- IMM also covers the computation of effective maturity M.
- M is meant to represent credit spread duration, so it makes sense to consider a contract that pays out an amount $EE(\tau)$ at time τ, on some interval $[0, T_{\max}]$.
- If the default intensity is λ, we have (ignoring discounting)

$$PV = \int_0^{T_{\max}} EE(t)\lambda e^{-\lambda t} dt$$

such that, for small λ,

$$PV01 = \frac{\partial PV}{\partial \lambda} = \int_0^{T_{\max}} EE(t)e^{-\lambda t} dt + \lambda \int_0^{T_{\max}} EE(t)te^{-\lambda t} dt$$
$$\approx \int_0^{T_{\max}} EE(t)\, dt.$$

Effective Maturity - (2)

- For a (loan) contract with a flat notional of EAD and maturity M, we would have

$$PV01 \approx EAD \int_0^M dt = EAD \cdot M.$$

- This suggests using (ignoring α)

$$M = \frac{\int_0^{T_{\max}} EE(t)\,dt}{EAD} = T\frac{\int_0^{T_{\max}} EE(t)\,dt}{\int_0^{T} EE^*(t)\,dt},$$

where T_{max} is the longest maturity in the book, and T is 1 year.

Effective Maturity - (3)

- The actual formula is similar to this, but a) adds discounting; b) splits the numerator into two pieces (to ensure that $M = T$ if $T_{\max} = T = 1$); and c) imposes a 5-year cap and a 1-year floor:

$$M = \max\left(1, \min\left(5, T\frac{\int_0^1 EE^*(t)\,P(t)dt + \int_1^{T_{\max}} EE(t)\,P(t)dt}{\int_0^1 EE^*(t)\,P(t)dt}\right)\right) \quad (14)$$

  where $P(t)$ is the discount factor to time t.

- The formula for M is quite a **nuisance**, as it requires one to establish EE-profiles all the way out to $T_{\max}$ (which could be 50 years for a swap portfolio). In contrast, computation of EAD only requires establishing out to $T = 1$ year.

- With formulas (14) and (13), the technical part of the IMM framework is **complete** .

EE Computations - (1)

- At the heart of both (14) and (13) lies the expected exposure profile EE – once this has been established, computation of EE* and, finally, EAD and M is straightforward.
- Computation of EE is the *biggest challenge* of IMM.
- Recall that

$$EE(t) = \mathrm{E}\left(\left(V(t) - C(t)\right)^+\right) = \mathrm{E}\left(E(t)\right),$$

where V is the (netted) portfolio value and $C(t)$ the (dynamic) collateral.

EE Computations - (2)

- A possible Monte Carlo algorithm looks like this:
  1. Simulate in $\mathbb{P}$ a path of correlated market data (rates, equities, commodities, spreads, FX,...) out to time $T_{\max}$. Let $\omega(t)$ be the market data state at time t and prior.
  2. At some time-grid $\{t_i\}$, use $\omega(t_i)$ in pricers to establish value of all (netable) trades in the counterparty portfolio.
  3. Aggregate trade values to compute portfolio value $V(t_i)$.
  4. Use CSA and collateral data to establish $C(t_i)$. Compute $E(t_i) = (V(t_i) - C(t_i))^+$.
  5. Repeat for many paths, to establish $EE(t_i)$, $i = 1, \ldots$.

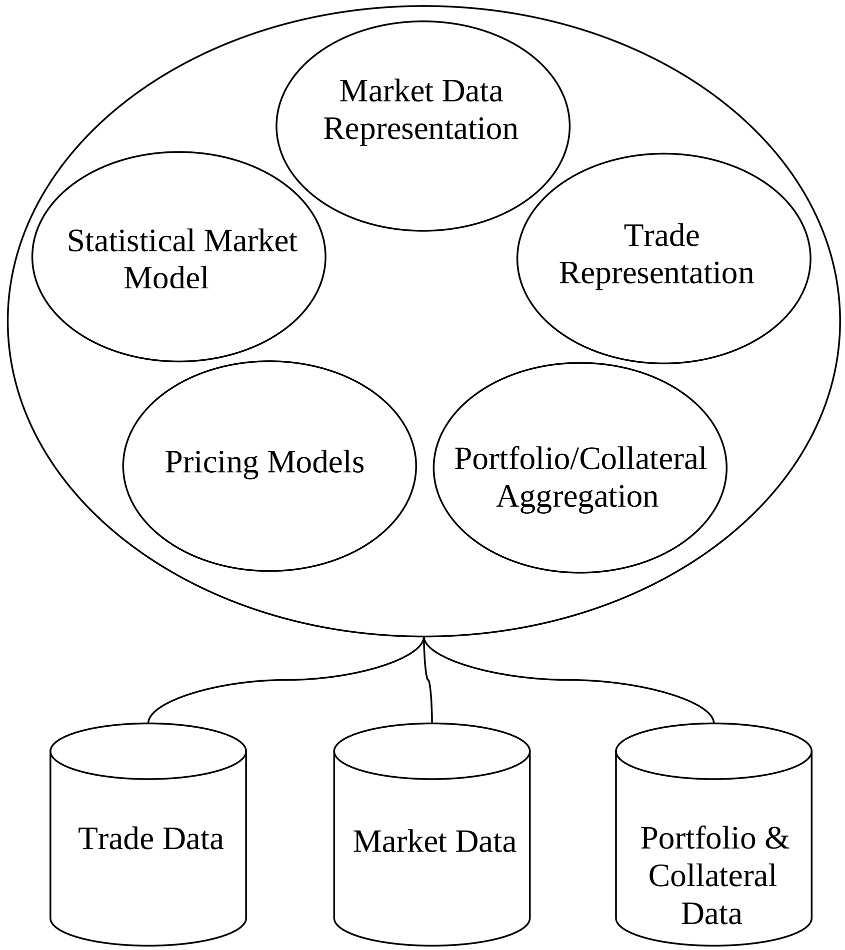
EE Computations - (3)

- When simulating collateral, we should aim to incorporate "margin delay" when financially important.
- Margin delays can mean two things: a) the frequency at which margin/collateral agreements get enforced; b) the delay associated with collateral disputes.
- In the exposure simulation, one can explicitly account for a margin period η. Adding also at δ closeout period, exposure can be written as something like $\max\left(V(t) - V(t-\delta-\eta), 0\right)$, for full collateralization with no thresholds.
- Requires adding extra points to the simulation (to make sure that for each t_i we also have $t_i - \delta - \eta$). There are Brownian Bridge methods available that can sometimes allow one to avoid doubling the grid.

Computer Systems for EE - (1)

- Functionally, a system to compute EE profiles must have:
  1. A representation of market data (rates, vols, spreads, equity prices, FX rates, etc.). Market data values at time 0 is the initial condition for the market data simulation.
  2. A simulation model for the joint evolution of the market data in the statistical measure. Must be supported by historical back-testing.
  3. A representation of trades. This representation must (unlike VaR) be able to **correctly age** trades through time.
  4. Pricing models, taking market and trade date as inputs.
  5. A concept of portfolios, netting sets, and collateral. Combined with the trade data and pricing models, this allows for the computation of exposures on the simulation path generated by the statistical model.

Computer Systems for EE - (2)



Computer Systems for EE - (3)

- In principle, it seems logical to use as many front office (FO) components for EE simulations as possible: FO market data, FO trade data, FO pricing models, and so forth.
- In practice, this may not be feasible due to enormous number of repricings that a capital system needs to perform.
- For instance, if we simulate 1,000,000 trades (BAC has much more than this) for 5,000 paths on a bi-monthly grid for 30 years, we need of the order of 10^{12} trade valuations (!).
- Many security valuations are complex, so typically FO systems can only handle in the order of $10^6 - 10^8$ trades per day.
- So strong simplifications are typically required on both trade representation, market data representation, and pricing models.

Pricing Errors - (1)

- Simplifications can cause problems, if done too coarsely. This can happen if price and/or data models are too simplistic, e.g., if the Capital systems get out of synch with FO developments.
- A key symptom of problems are discrepancies in the current market value (CMV) produced by FO and Capital systems.
- To be more precise, let the counterparty portfolio in question consist of R trades with values $v_1(t), v_2(t), \ldots v_R(t)$. That is,

$$V(t) = \sum_{j=1}^{R} v_j(t)$$

- Pricing errors cause (FO: front office; C: capital system)

$$V^{FO}(0) \neq V^{C}(0)$$

Pricing Errors - (2)

- This causes initial exposures to differ, $E^{FO}(0) \neq E^{C}(0)$. So, the EE profile computed by the capital system has the wrong starting point.
- Regulators like to measure pricing errors at the trade level, by adding absolute trade pricing errors:

$$e = \sum_{j=1}^{R} |v_j^{FO}(0) - v_j^{C}(0)|$$

- Regulators want a low value of e, which involves improving trade and market data fidelity, as well as improving pricing model precision. Very complex accuracy-efficiency tradeoffs are involved here
- Also, some part of e is unavoidable, due to timing effects.

Pricing Errors - (3)

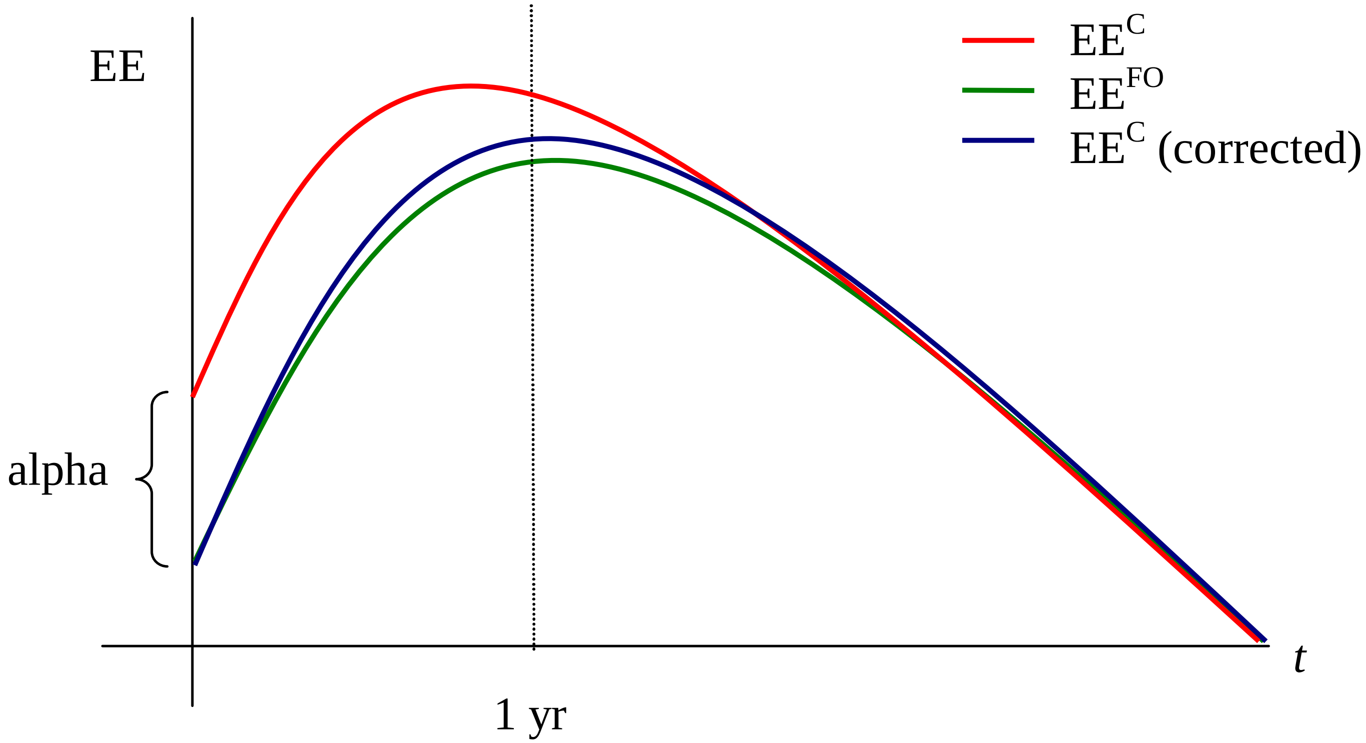
- For systems that generate pricing errors, it is possible to ad-hoc adjust for these errors. Not a substitute for fixing deeper problems, but can work remarkably well if errors are not too big.
- For instance, we can define an improved pricer with guaranteed zero error:

$$v_j^{C*}(t) = v_j^C(t) + \left(v_j^{FO}(0) - v_j^C(0)\right).$$

- This additive adjustment works well for European options, say, where EE profiles are pretty flat.
- For instruments (such as swaps) where errors naturally decline over time, one can amortize away the initial error. For instance, for an instrument with maturity T_j:

$$v_j^{C*}(t) = v_j^C(t) + \frac{T_j - t}{T_j}\left(v_j^{FO}(0) - v_j^C(0)\right)$$

Pricing Errors - (4)



- Error correction procedure for a stylized swap.

But wait: the Collins floor

- As part of the Dodd-Frank Act, US senator Susan Collins instituted a floor on capital that, in effect, makes sure that no future regulation will result in less capital than what would have been required at the time of passing the DF Act (July 2010).
- Effectively, this floors all capital computations at Basel 1 capital levels.
- So, after spending $100MM's to implement the AIRB in Basel 2, the result could be "thrown away" and replaced by a number that everybody, including BCBS, agreed is less than meaningful.
- According to the American Banker's Association:

  "*Imposing a floor that is tied to Basel I rules raises the question of why any bank would want to undertake the expense and effort to convert to the advanced approaches rules if it has the option not to do so. Such rules become, in essence, very expensive risk management exercises.*"

IMM Appendix:

Acronyms and Abbreviations

Appendix - (1)

- α: Basel 2 multiplier on EEPE, in computation of EAD
- CCR: counterparty credit risk
- CMV: current market value
- EAD: exposure-at-default (a.k.a. equivalent loan notional, ELN)
- E: random exposure profile
- EC: economic capital
- EE: expected exposure profile
- EE^*: "effective" exposure profile (accounting for roll-over)
- EEPE: 1-year time average of EE^*
- EL: expected loss (a.k.a. credit reserve, CR)

Appendix - (2)

- EPE: time average of EE
- IRB: Internal ratings-based approach (Basel 2)
- LGD: loss given default (percentage)
- L: credit loss
- M: effective maturity
- PD: 1-year default probability
- RC: regulatory capital
- RW: risk weight on equivalent loan notional
- RWA: risk weighted assets, RWA = RC * 12.5 (so RC = RWA * 8%)
- UL: unexpected loss

3: Various Aspects of IMM

Basel 2 and 2.5 Flashback - (1)

- Regulatory capital under Basel 2.5 is loosely broken into three pieces:
  - A general market risk piece (VaR + stressed VaR) and specific risk
  - IRC and CRM
  - Credit risk capital (the IRB and IMM)
- We recall the Basel 2 Internal Ratings-Based (IRB) formula for credit risk regulatory capital:

$$RC = EAD \cdot RW,$$

$$RW = 1.06 \cdot l \cdot \left\{ \Phi \left(\frac{\Phi^{-1}(p) - \sqrt{\rho(p)}\Phi^{-1}(0.001)}{\sqrt{1-\rho(p)}} \right) - p \right\} \cdot k(M, p),$$

Basel 2 and 2.5 Flashback - (2)

- Where:
  - EAD: exposure-at-default (a.k.a. loan-equivalent notional);
  - p: 1-year probability of default (PD);
  - l: loss-given-default percentage (LGD);
  - M: effective maturity;
  - $\rho(p) = 0.24 - 0.12(1 - e^{-50p})$;
  - k: function to incorporate transition risk.
- IMM (Internal Models Methodology) is the regulatory framework for computing EAD and M.
- A model for joint movements of all financial variables “in the world” are needed to compute expected exposure (EE) profiles, and thereby EAD (which is 1.4·EEPE).

Choice of Measure - (1)

- IMM models have to be validated by model validation and by regulators (FRB, OCC, FSA,..)
- The model choice and parameterization is, rightfully, a key area of focus in IMM examinations
- In theory, all simulations should be set in the actual (aka historical, aka real-life, aka statistical) probability measure.
- Yet regulators allow for leeway:

  "*In theory, the expectations should be taken with respect to the actual probability distribution of future exposure and not the risk-neutral one. Supervisors recognize that practical considerations may make it more feasible to use the risk-neutral one. As a result, supervisors will not mandate which kind of forecasting distribution to employ.*"

Choice of Measure - (2)

- This leeway can be very substantial (probably more than regulators intended), since there is no such thing as a single risk-neutral measure – there is one for each choice of numeraire asset.
- To demonstrate the effects of this, let us consider interest rate modeling (an area of some tension).
- Let $f(t,T)$ be the time t instantaneous forward rate to T, and assume that a vector-valued Brownian Motion $W(t)$ drives the forward curve. Let the (vector-valued) volatility for $f(t,T)$ be $\sigma_f(t,T)$.
- By the HJM result, we know that in the actual measure $\mathbb{P}$,

$$df(t,T) = \mu(t,T)\,dt + \sigma_f(t,T)^\top\,dW(t)$$

$$\mu(t,T) = \sigma_f(t,T)^\top \left(\int_t^T \sigma_f(t,u)\,du + \lambda(t) \right) \triangleq \alpha(t,T) + \sigma_f(t,T)^\top \lambda(t)$$

Choice of Measure - (3)

- Here $\lambda(t)$ (market price of risk) is a vector-valued process independent of T.
- In the risk-neutral measure $\mathbb{Q}$ induced by using a rolling money market account $\beta = exp(\int_0^t r(u)du)$ as numeraire, we have

$$\mu(t,T) = \alpha(t,T).$$

- In the risk-neutral measure $\mathbb{Q}^*$ induced by using a discount bond $P(t,T^*)$ maturing at time T^*,

$$\mu(t,T) = \alpha(t,T) - \alpha(t,T^*).$$

- Range of allowable drifts is very large – in principle we can get $\mu(t,T) = -\infty$ by setting $T^* = \infty$!

Choice of Measure - (4)

- Also, since $\lambda(t)$ is very hard to estimate historically, to boot there is large uncertainty around the theoretically optimal drift.
- All in all, drift terms of stochastic processes used for IMM should be set “pragmatically”. For example, it is not uncommon to set $\mu(t,T) = 0$: “roll up forward curve”.
- Notice that for CVA applications, we are trying to price credit exposure so here there is no ambiguity, since

$$PV(t) = \mathrm{E}^{\mathbb{Q}}\left(V(t)^{+} \frac{1}{\beta(t)} \right) = \mathrm{E}^{\mathbb{Q}^*}\left(V(t)^{+} \frac{P(0,T^*)}{P(t,T^*)} \right).$$

- On the other hand, when computing expected exposure (rather than *present value* of exposure)

$$\mathrm{E}^{\mathbb{Q}}\left(V(t)^{+} \right) \neq \mathrm{E}^{\mathbb{Q}^*}\left(V(t)^{+} \right).$$

Choice of Measure - (5)

- The drift ambiguity, and the use of expected exposure without discounting, are flaws in IMM.
- In addition, there are several securities (e.g. compounding trades) where $\mathrm{E}^{\mathbb{Q}}(V(t)^{+})$ can be extremely large (overflow), whereas $PV(t)$ is perfectly stable.
- Ideally, one should have defined $EE(t) = PV(t)/P(0,t)$.

Backtesting - (1)

- Having covered drifts, how about variances (and co-variances)?
- In theory, these are measure invariant and can often be constructed without ambiguity from observable option prices.
- In practice, however, this is *not* what regulators want. Instead, they want the choice of variances and co-variances to be based on historical data.
- Mathematicians tend to be driven up a wall about this...
- Determining whether a model is suitable for IMM is done through *backtesting* procedures.
- In a nutshell, backtesting is the process of testing whether forecasts done by a stochastic model match realized values.

Backtesting - (2)

- For instance, for some risk factor X and some time grid $\{t_i\}$ we can consider the collection of novations $\epsilon_{ij} = X(t_i + \Delta_j) - X(t_i)$ for $i, j = 1, 2, \ldots$, where Δ_j are time horizons.
- The collection of $\epsilon_{\cdot j}$ for fixed j forms an empirical distribution that can be compared by the distribution generated by a parametric model.
- There is quite a bit of language around backtesting requirements in the Basel rules, as well as a dedicated BIS publication exclusively dealing with backtesting guidance ("Sound Practices for Backtesting Counterparty Credit Risk Models," *Basel Committee on Banking Supervision*, December 2010).
- Testing should be done at both the level of individual risk factors (e.g., an FX rate) and over relevant aggregations, most notably through "representative portfolios".

Backtesting - (3)

- Per regulatory guidance, backtesting must:
  - Be done regularly as part of ongoing model performance measurement, and as part of ongoing model validation
  - Be subject to governance, especially w.r.t. to remediation of exceptions.
  - Test not only the potential exposure percentile but the whole distribution (e.g., 1%, 5%, 25%, 50%, 75%, 95%, 99%)
  - Test a representative sample of time horizons (Δ_j), incl. 1 year or more
  - Test correlations as well as volatilities
  - Be representative of the bank's exposure
  - Monitor not only frequency of exceptions but also severity
  - Consider materiality of exceptions
  - Be designed with statistical significance in mind
  - Be based on historical calibrations using >3 years of data (Basel 3).

Backtesting - (4)

- The complex (and thankless) task of calibrating and statistically backtesting models is normally handled by the credit risk team, rather than by banks. The models in questions have many 100s, if not 1,000s of risk factors.
- Often the statistical framework relies on a “cascade” of tests, starting with BIS “traffic light” test.
- Details are very much bank specific, so let us just describe the traffic light test, the only concrete test that regulators have put forth.
- Assume that our model predicts that ϵ falling outside some range H takes place with probability q:

$$P(\epsilon \notin H) = q.$$

Backtesting - (5)

- In a historical data series with n realizations, we can count the random number of times N that ϵ is outside H ("exceptions")
- IF the model is *correct*, the distribution of N is (under suitable assumptions) binomial:

$$P(N = x) = \left(\begin{array}{c} x \\ n \end{array} \right) q^x (1-q)^{n-x} \triangleq B(q, x).$$

- Suppose we use a cut-off of x_c as a rejection of the model, in the sense that we will discard the model if $N \geq x_c$.
- The probability of rejecting a correct model (a type I error) is

$$p_I = \sum_{x \geq x_c} B(q, x).$$

Backtesting - (6)

- IF the model is wrong, in the sense that really $P(\epsilon \notin H) = w \neq q$, the probability of erroneously accepting a wrong model (a type II error) is

$$p_{II} = \sum_{x<x_c} B(w, x).$$

- Based on tables with various values of q and w, BIS has come up with zones of outcomes for N that it considers green (all OK), yellow (could be a problem), and red (no good).
- The BIS approach is quite simplistic, and really only done for low values of q (1 %).
- One would often supplement the traffic light tests with more sophisticated tests (Kupiec's POF test, Jarque-Bera, Pearson Q, etc).

BofA Models and Simulation

- Like most banks, BofA's capital systems were built around counterparty credit risk systems.
- For IMM purposes, the models here are generally too simplistic, so a convergence towards the CVA system has taken place.
- Large parts of our CVA and IMM systems are merged, with a branch on the model configuration. CVA: market calibration (quants). IMM: statistical backtesting (quants, CCRA).
- The resulting engine is effectively a large "what-if" machinery, and in principle could handle VaR and IA (through additional configurations).
- Target state for most banks.
- As the models deployed by BofA are not dissimilar to those used at Danske Bank, I will defer to tomorrow's speaker for concrete details.

IMM Carveouts - (1)

- For a variety of reasons, it is likely that some trades in a netting set will not qualify for IMM treatment.
- This can happen if there are no (or inadequate, in the sense of too high pricing errors) pricing models implemented on the IMM platform.
- This, in turn, might be the case if an exotic security pricer is too slow to embed in the IMM simulation loop.
- Note: exotic securities can often be priced efficiently using regression-based methods, but these can take a while to develop, test, and validate – validation tends to be product specific. More on this tomorrow..
- In this case, it becomes necessary to split the portfolio in two pieces: one piece (V_1) that is IMM compliant, and one piece (V_2) that is not.

IMM Carveouts - (2)

- V_1 will attract credit capital according to Basel 2, and V_2 will attract credit capital according to Basel I (CEM). Total credit capital is the sum of the two contributions.
- We notice that EE is always sub-additive when a portfolio and its collateral is split:

$$
\begin{aligned}
EE(t) = E\left((V(t) - C(t))^+\right) &= E\left((V_1(t) + V_2(t) - C_1(t) - C_2(t))^+\right) \\
&\le E\left((V_1(t) - C_1(t))^+\right) + E\left((V_2(t) - C_2(t))^+\right),
\end{aligned}
$$

  if $C(t) = C_1(t) + C_2(t)$ and $V(t) = V_1(t) + V_2(t)$.
- This is easily seen to carry through to EEPE and therefore to EAD. So *EAD is sub-additive*.

IMM Carveouts - (3)

- Note: when carving out trades, a concrete mechanism is required for splitting the collateral. Normally a heuristic is needed, as there is no unique way to do this.
- Is regulatory capital RC sub-additive? For this, we recall that

$$RC = EAD \cdot f(l,p) \cdot k(M,p) \cdot 1.06$$

with $f(l,p)$ being a prescribed function of LGD (l) and PD (p), and $k(M,p)$ being a prescribed “transition risk” function of effective maturity M and PD.

- M depends on EE, so we must write, for the split portfolio,

$$RC_1 + RC_2 = f(l,p) \cdot (EAD_1 \cdot k(M_1,p) + EAD_2 \cdot k(M_2,p)),$$

where we know that $EAD_1 + EAD_2 \geq EAD$.

IMM Carveouts - (4)

- Unfortunately, a careful analysis of the “black-box” function k reveals that it has a design-flaw: it can depend on exposure profiles in such a way that sometimes $RC_1 + RC_2 \leq RC$. Basically a consequence of the cap that sits in the definition of M.
- So regulatory capital is not always sub-additive: splitting the portfolio can result in less capital.
- While this might one doubt the coherence of the formulas, in practice strict subadditivity virtually never happens.
- Moreover, since the non-IMM portfolio gets subject to Basel 1 – which is typically much more “expensive” than Basel 2 – splitting a netting set involves a pretty severe capital cost.
- In Basel 3 this penalty gets extremely high, as we shall see later.

Margin Loans - (1)

- Securities that are subject to margin requirements are currently in a little bit of a vacuum when it comes to regulatory capital.
- One relevant business is margin lending, as executed in, say, the prime brokerage business. In the future most non-cleared securities will require margin posting. More about this later.
- In margin lending, a client holds a portfolio of cash and securities with value $\pi(t)$. Not all of this portfolio has been financed by the client; a certain amount of its value, $D(t)$, has been lent to the client by the bank.
- Writing $\pi(t) = D(t) + E(t)$, the quantity $E(t)$ is the client's *equity*.
- The lending bank can use the entire portfolio $\pi(t)$ as collateral for its loan $D(t)$ if the client defaults. As long as $E(t) > 0$ the position is *over-collateralized*.

Margin Loans - (2)

- To protect itself against the client not repaying the debt amount $D(t)$, the lending bank has a policy where it will issue a margin call if the riskiness of the position is too high.
- The call will require the client to top up the equity position with cash or eligible securities, to some level $E_{\min}(t)$.
- Often, $E_{\min}(t)$ is set by VaR methods. Specifically, if we assume that it will take a period of Δ to liquidate the portfolio after a client default, we might want, for some small number q,

$$P\left(\pi(t+\Delta) < D(t)\right) < q,$$

or

$$P\left(X(t) > 0\right) < q, \quad X(t) = \pi(t) - \pi(t+\Delta) - E_{\min}(t)$$

Margin Loans - (3)

- That is, the probability of the portfolio deterioration exceeding the equity over the liquidation horizon is very small.
- Often q is minuscule – much less than 1%. Besides VaR protection, many margin policies add protection (through add-ons to VaR) against downgrades, concentration risk, liquidity issues, etc.
- How do we define exposure for a margined portfolio?
- If default takes place at time τ, first assume (worst-case) that the client has no excess equity beyond the margin level $E_{\min}$.
- The loss to the lending bank associated with liquidation is

$$L(\tau) = X(\tau)^{+}.$$

- So, the expected exposure is $EE(t) = \mathrm{E}\left(X(t)^{+}\right)$.

Margin Loans - (4)

- Since margined portfolios tend to be quite dynamic with active trading and frequent margin calls, it is a challenge to predict what $\pi(t)$ and $E_{\min}(t)$ will look like at time t. Some simplification is needed.
- One (overly?) sophisticated method would involve using kernel regression to estimate conditional moments of the portfolio, and then to approximate the computation of VaR and, subsequently, $E_{min}(t)$.
- This calculation would assume that the current portfolio is “static’ ’and no new trades would ever be added. Often this is unreasonable – even though it is similar to regular IMM assumptions.
- A better assumption for, say, prime brokerage is that the margin policy aims to keep the tail distribution of the loss-variable X close to constant over time, in the sense that the VaR stays relatively fixed.

Margin Loans - (5)

- Equivalently, assume that the current ($t = 0$) portfolio composition and margin is a “representative” position.
- With this assumption, we write for all t,

$$EE(t) = \mathrm{E}\left(X(0)^+\right) = \mathrm{E}\left(\left(K - \pi(\Delta)\right)^+\right), \quad K = \pi(0) - E_{\min}(0). \quad (15)$$

- So, to compute the entire EE profile (and thereby EEPE and EAD), we “just” need to price a put on $\pi(\Delta)$.
- One complication here is that K is very small due to the overcollateralization feature, so Monte Carlo simulation is difficult to make operational without lots of tricks (importance sampling).
- For speed and clarity, one can use delta-gamma approximations, coupled with a Gaussian distribution assumption for the risk factors behind π. Justified here due to the short time-horizon Δ.

Margin Loans - (6)

- Specifically, we write

$$\pi(\Delta) = D^\top \epsilon + \frac{1}{2}\epsilon^\top \Gamma \epsilon$$

  where $\epsilon \sim N(0, R)$ for some correlation matrix R.
- After a few rotations, it is possible to write the characteristic function for $\pi\,(\Delta)$ in closed form.
- The expectation in (15) can then be written as a Fourier integral.
- Due to the extreme OTM behavior of the integral, in practice we rely on saddlepoint techniques.
- The saddlepoint techniques can be extended to provide efficient hedge and attribution analysis, which is very convenient in practice. (Joint work with J. Kim, 2012).

Margin Loans - (6)

- It should be clear that capital requirements for margin business can be very low. This might be controversial, and some elements of the margin portfolios may, for this reason, stay with Basel I through carve-outs.
- On the other hand, regulators are generally skeptical about cherry-picking of accords, so we shall see.

IV: Basel 3/4, Clearinghouses, IA/IM

Introduction to Basel 3

- Basel 3 was designed in 2010-2011, post crisis.
- Basel 3 is a “mop-up” operation, that primarily aims to plug holes in Basel 2 that were revealed during crisis. Promote a more “resilient” banking system.
- While banks in US have not yet gotten IMM approval, and have not yet moved to Basel 2, new rules for Basel 3 are already being implemented.
- Originally planned for adoption in January 2013, most provisions in Basel 3 are delayed and will not become official standards for several years. Current projection: through 2019.

Some Non-Quant Elements of Basel 3

- Raises quality of capital base, by eliminating Tier 3 capital and gradually increases proportion of Tier 1 capital (while still aiming for 8% capital ratio).
- Adds a 2.5% “capital conservation buffer” to be drawn on in times of stress (to address procyclicality). Effective capital ratio is 10.5%.
- Adds a non-risk weighted Tier 1 Leverage Ratio restriction of 3%, to avoid “excessive leverage in banking industry”.
- Introduces various measures (Liquidity Coverage Ratio and Net Stable Function Ratio) to address liquidity. LCR standard, say, ensures that there are liquid assets to completely cover net cash outflows over a 30 day horizon. (Some quant element to this, actually).

Changes to IRB and IMM - (1)

- EPE must now be computed with a model calibrated to a 3-year stress period. “Stressed EPE”.
- The *maximum* of the stressed and ordinary EPEs must be used in EAD computation.
- The maximum is formed at the total capital level, not at the counterparty level.
- This results in a more conservative estimate, but not necessarily a better capital measure.
- Capital becomes increasingly insensitive to current market conditions and might move in jumps.

Changes to IRB and IMM - (2)

- For collateralized positions, the “close-out” period is lengthened according to a new set of rules.
- For many OTC derivatives positions, and for large netting sets (>5,000 trades per quarter), the close-out period is lengthened from 10 to 20 bdays. (Disturbingly “digital”).
- However, for deals with a clearinghouse, 5 days is sufficient. Basel 3 generally encourages trades with clearinghouses.
- Another change to IRB approach: the asset correlations for financial firms is increased by 25% for large financials as well as for all unregulated financials (hedge funds, say).
- This addresses the fact that financial firms were seen to be more exposed to systemic risk during crisis than non-financials.

CVA VaR Add-On - (1)

- Recall that under Basel 2, capital is the sum of Credit Capital (IRB) and a variety of VaR, stress-VaR, and other charges (e.g., IRC, CRM).
- During crisis, it was noted that a very large part of the variability of bank earnings came from CVA/DVA charges due to moves in credit spreads: “2/3 of CCR losses were due to CVA losses, 1/3 due to actual defaults”.
- In response, a *CVA VaR charge* has been added in Basel 3; it enters into the credit risk RWA part of the overall computation.
- The goal of this charge is to build a buffer against fluctuations in credit-worthiness of counterparties.
- Probably double-counts something (like the ratings transition function in the IRB)...

CVA VaR Add-On - (2)

- Basel 3 always assumes that DVA=0. Reasonable to ignore self-credit when looking for metrics that test ability to avoid default. (Not reasonable from a MTM perspective, obviously).
- Basel 3 defines CVA for a counterparty with known EE profile as:

$$CVA(0) = l_M \sum_n \left(e^{-s_{n-1}t_{n-1}/l_M} - e^{-s_n t_n/l_M} \right) \times \left(\frac{EE(t_{n-1})P(0,t_{n-1}) + EE(t_n)P(0,t_n)}{2} \right) \quad (16)$$

- Here, l_M is the market LGD (not the same as LGD used in IRB) and s_n is the CDS spread observed in the market for tenor t_n.
- Uses the classical intensity approximation $\lambda \approx s/(1-R)$.
- Notice that full EE profile to "longest trade" is needed, not just 1-yr.

CVA VaR Add-On - (3)

- The collection of CVA charges across all N counterparties can be considered a function of N credit spread curves. Using a model for these N curves (including correlation), one can compute the 10-day 99% VaR originating from moves in the curves.
- A regular as well as a stressed VaR number (using stressed EPEs) are to be computed. Both must be added to capital; the sum is a new component of regulatory capital.
- CDS (and index) hedges designated as CVA hedges can be included in the computation (which will likely lower capital). One problem: MTM hedges will be different from capital hedges.
- Worth reiterating: CVA VaR is not charged at the counterparty level, but is a firm-level measure.

CVA VaR Add-On - (4)

- There are issues with the CVA VaR computation:
  - The Basel 3 definition of CVA is not how true CVA is actually computed.
  - Focuses exclusively on the variability of credit spreads, but CVA depends on many other risk factors. Hedges for these risks are left naked, giving wrong incentives.
  - CVA VaR should ideally not be separated out from regular VaR computations – this is not done for any other value component.
  - Why add stressed and non-stressed CVA VaR?
  -
  - ...

CVA VaR Add-On - (6)

From Risk Magazine, Oct 31, 2013:

“*Critics of Basel 3’s credit valuation adjustment (CVA) capital charge have long warned it would produce perverse incentives. Now, in the form of a string of quarterly losses in Deutsche Bank’s CVA hedging programme, they believe they are being proved right.*”

“*How much should a bank pay to cut capital? [...] Deutsche Bank has given some answers to that in recent quarterly statements. In the first half of the year, the bank cut the risk-weighted assets (RWAs) generated by Basel 3’s charge for derivatives counterparty risk – or credit valuation adjustment (CVA) – from EUR28 billion to EUR14 billion.*”

CVA VaR Add-On - (7)

"*The hedging strategy that produced those savings also lost the bank EUR94 million – a result of a mismatch between the regulatory and accounting treatments of CVA, which forces banks to choose which regime is most important. If a dealer chooses to hedge accounting CVA, it may not earn capital relief; if it chooses to mitigate the capital numbers, it may be stuck with profit-and-loss (P&L) volatility.*"

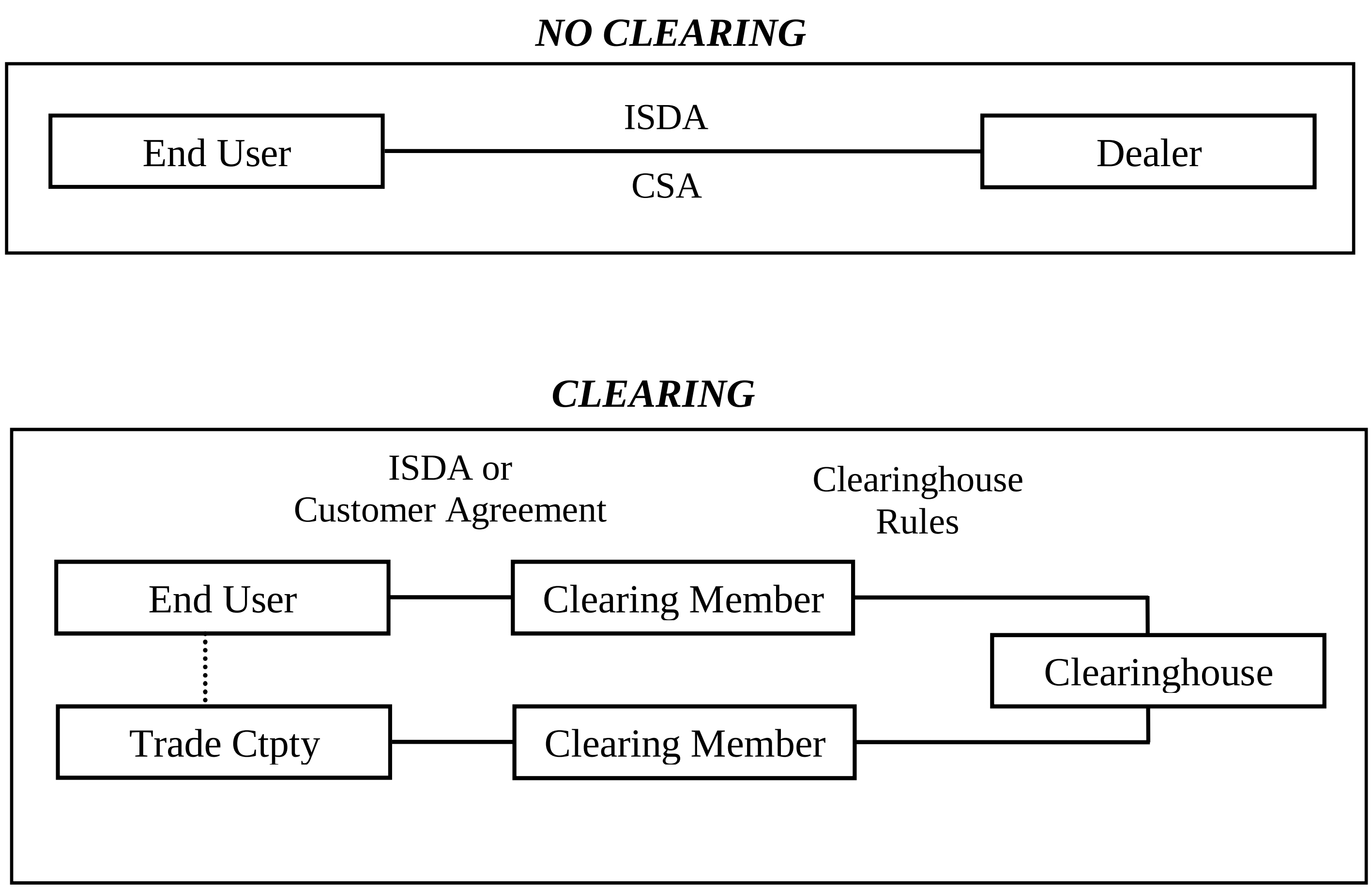
- In the US, the Volcker rule in Dodd-Frank – and especially the recently reaffirmed ban on "portfolio hedging" – will likely make hedging of CVA VaR illegal.
- Recall that "portfolio hedging" is hedging that does not explicitly involve risk mitigation of clearly identified trades.
- Instrumental here: the "London whale" episode where JPM put on a series of partial hedges to minimize the CRM charge.

CVA VaR Add-On - (8)

- The CVA VaR charge is typically very large, often resulting in a doubling of capital relative to Basel 2. (!)
- It is, in fact, so big that Basel 3 capital often exceeds Basel 1, meaning that the Collins floor often does not apply, at least for derivatives portfolios.
- For trades carved out of IMM, Basel 3 requires that non-IMM netting sets use the EAD as computed under CEM (not IMM) as a proxy for EE in (16).
- This can be *very* expensive from a capital perspective, and provides a strong incentive to either simplify portfolios or to increase coverage to CVA systems to full-blown exotics.
- One strategy for exotics: LS regression and a generic payoff language. Details left for tomorrow.

Basel 3: Clearinghouses - (1)

- Recall that the idea of central clearing is to insert a clearinghouse, or Central Counterparty (CCP), as a transaction intermediary:



Basel 3: Clearinghouses - (2)

- Under the Dod-Frank law, the act of matching buyers and sellers pre-clearing must be done on a *Swap Execution Facility (SEF)*.
- SEF's can loosely be thought of as electronic exchanges operating on a "many-to-many" basis. They operate centralized electronic trading screen on which market participant can post bids and offers for everybody else to see – i.e., an "order book".
- A SEF may also offer a request-for-quote (RFQ) system where participants can ask other market participants (at least 3) for quotes.
- The SEF's must report trades to a *swap data depository* (SDR) either for real-time public dissemination or confidential regulatory use.

Basel 3: Clearinghouses - (3)

- The CCP intermediates and settles P&L daily (*variation margin*), and has no market risk. However, it will have close-out risk on default, so it will insist on an additional buffer (*initial margin*).
- Each CCP will have its own margin policies, and they often compete with each other on the specifics – especially on *cross-margining*.
- Typically variation margin is settled in cash, whereas initial margin might be in security form subject to CCP-specific hair cuts.
- To avoid lowering standards, BIS has issued a number of documents related to CCPs, including guidance to how CCPs themselves should be capitalized (default fund waterfall approach) and charge initial margin (5 days, 99%-ile).
- There is guidance on how a CCP can become a *qualifying CCP*, which triggers lenient capital requirements for trading partners.

Basel 3: Clearinghouses - (4)

- For Clearing Member (CM) exposure to an End User (EU), the CCP initial margin requirements are always passed through to the EU, so the exposure to the EU is small and of the same "overcollateralized" type as for prime brokerage. A similar "fixed upper tail" approach can be used, along with usual IRB machinery. Or so we hope.
- For EU or CM ("house account") exposure to the CCP itself, the initial margin sits only with the CCP, and is often considered by lawyers to be at risk on a CCP default due to "co-mingling" of collateral. As such, the initial margin **adds** to the exposure – indeed, it is often the lion's share of the EAD.
- More precisely, the EE to the CCP will be the sum of initial margin exposure and that of a regular collateralized position with no initial margin (but with variation margin), for a 5-day close-out horizon.

Basel 3: Clearinghouses - (5)

- If we can compute the EAD, how do we turn this into regulatory capital? What PD/LGD/correlation/etc does one use for a CCP?
- To understand the BIS capital guidance for CCPs, recall that under IRB for a regular counterparty:

$$RWA = 12.5 \cdot RC = EAD \cdot 12.5 \cdot f(l,p) \cdot k(M,p) \cdot 1.06$$

  or

$$RWA = EAD \cdot RW(l,p)$$

  where RW is a risk weight.
- For trades with a *qualifying* CCP, one sets $RWA = EAD \cdot 2\%$, i.e. the risk weight is prescribed to be simply 2% – low, but not zero.
- No CVA market risk charges for CCPs.

Basel 3: Clearinghouses - (6)

- There are a large number of legal subtleties around CCPs and the exposures they generate to EUs and CMs. Some of this is covered in Andersen, L. (2013), "Exposure and Regulatory Capital: Clearinghouses," BAC Technical Paper.
- Here there are also attempts to build an approach to dynamically model initial margin on the path, should the "fixed VaR" approach be rejected in favor of the usual trade-aging IMM model.
- The basic idea is to use a least squares or kernel regression to estimate the future distribution of portfolio moves over a Δ period, and then use a simple VaR approach to estimate margin. This can be scaled to ensure that the current ($t = 0$) margin is matched.
- Also need to make assumptions/projections about the composition (bonds vs cash) of the initial margin.

Independent Amount - (1)

- BIS in September 2012 issued a consultative document: “Margin Requirements for Non-Centrally Cleared Derivatives”. Rules were made near-final in Feb 2013, to be implemented by 2015.
- Proposal essentially is to require dealers to charge each other initial margin (aka Independent Amount, or IA) for all non-cleared products.
- Postings will, fundamentally, involve a third-party custodian, to prevent hypothecation of margin (and to ensure its availability on default).
- Controversial, since liquidity requirements can be enormous ($1 Trillion).
- Following industry complaints, a threshold of EUR50MM was instituted below which margin need not be posted (BCBS242.pdf). Supposedly cuts liquidity req’s in half.
- There are ample opportunities for disputes on margin. (BCBS requires “agreement to common methodology” on transaction onset.)

Independent Amount - (2)

- ISDA is currently attempting to come up with an industry-wide “minimal methodology” document to at least set a floor under the margin requirements. SIMM: Standard Initial Margin Model.
- The methodology will have to be simple, to make sure that all banks, large and small, can implement it easily
- Delta-based VaR (either historical or parametric) is most likely candidate.
- Substantial work remains in defining the (factor-based, very likely) statistical reference models for all asset classes.
- SIMM data model might be similar to what is used in NIMM (Not-IMM), with common factors as well as idiosyncratic risk components.
- Capital can be computed by same methods as for margin loans.

Basel 4 - (1)

- In 2012, the BCBS released a consultative document, "Fundamental Review of the Trading Book" (FRTB). This is known as Basel 4.
- The FRTB acknowledges that rules have become unwieldy, costly, and difficult to regulate. Some proposals:
  - Replace all calibrations with stressed calibrations. Do not double count ordinary and stressed market risk RWA.
  - Replace VaR with CVaR/ES: $ES_\alpha = \mathrm{E}(X|X < VaR_\alpha)$.
  - Use different horizons for ES calculations to better reflect liquidity in various asset classes. From 10 days to one year.
  - Take steps to minimize the diversification benefits of IMM
  - Establish closer link between IMM and CEM methods, with CEM-type calculations being mandatory and possibly a floor.
  - NIMM ("Not IMM") methodology to replace CEM.

Basel 4 - (2)

- Industry reception of FRTB has been quite mixed. For instance, David Rowe in Risk Magazine (January 2014)

  "*One inappropriate innovation [...] is replacing VAR with expected shortfall – the expected value of all losses greater than a specific threshold [...]. Besides adding little useful information in practice, expected shortfall is impossible to back-test, since actual realised values are never observed.*"

  "*Most disturbingly, it is now proposed that the revised, and now highly complex, standardised approach must be implemented by all banks, including those with approved internal models. It appears those behind the proposal have little or no appreciation of how complex, costly and error-prone such an effort would be. Hopefully [...] the Basel Committee will put this idea where it belongs – on the* ***regulatory scrapheap****.*

Related Topics: “XVA”, Capital Management, Collateral,..

CVA/DVA Basics - (1)

- The incorporation of credit-related charges into market pricing is a well-established practice, supported by US accounting laws.
- Consider a portfolio traded between the bank (B) and a counterparty (C). Let the portfolio have a no-default price of $V_0(t)$ to bank B.
- Let the default times of B and C be τ_B and τ_C, respectively.
- Assuming a universal recovery rate of R and the so-called *market quotation* method (ISDA 1992) for default settlement, we write for the true value $V(t)$ (assuming $\tau_B, \tau_C > t$):

$$\begin{aligned} V(t) = V_0(t) - \mathrm{E}_t\left((1-R)e^{-\int_t^{\tau_C} r(u)\,du}\, V_0(\tau_C)^+ 1_{\tau_C<\tau_B}\right) \\ + \mathrm{E}_t\left((1-R)e^{-\int_t^{\tau_B} r(u)\,du}\, (-V_0(\tau_B))^+ 1_{\tau_B<\tau_C}\right) \\ \triangleq V_0(t) - CVA(t) + DVA(t). \end{aligned}$$

CVA/DVA Basics - (2)

- Both the CVA and DVA defined here are bilateral computations. A unilateral (but incorrect) computation dispenses of the indicator functions inside the risk-neutral expectations.
- In a Cox process setting with intensities λ_B and λ_C we may write

$$CVA(t) = (1-R)\mathrm{E}_t\left(\int_t^\infty \lambda_C(u)e^{-\int_t^u(\lambda_C(s)+\lambda_B(s)+r(s))\,ds}\,V_0(u)^+\,du\right)$$
$$DVA(t) = (1-R)\mathrm{E}_t\left(\int_t^\infty \lambda_B(u)e^{-\int_t^u(\lambda_B(s)+\lambda_C(s)+r(s))\,ds}\,(-V_0(u))^+\,du\right).$$

- Or, for the unilateral "version"

$$CVA_U(t) = (1-R)\mathrm{E}_t\left(\int_t^\infty \lambda_C(u)e^{-\int_t^u(\lambda_C(s)+r(s))\,ds}\,V_0(u)^+\,du\right)$$
$$DVA_U(t) = (1-R)\mathrm{E}_t\left(\int_t^\infty \lambda_B(u)e^{-\int_t^u(\lambda_B(s)+r(s))\,ds}\,(-V_0(u))^+\,du\right). \tag{17}$$

CVA/DVA Basics - (3)

- Traders generally dismiss the concept of DVA, as does the general public ("Why do firms make money when their credit rating deteriorates??"). In particular, traders dislike the fact that they cannot hedge their own spread.
- Traders also don't like bilateral CVA, for the same reason.
- In addition, they dislike the fact that hedging bilateral CVA will cause a mismatch with CCAR stress tests, which require unilateral definitions.
- Same issue for the Basel 3 CVA VaR (although there are additional issues here).
- Tomorrow, we shall hear a lot about how to model and calculate the various forms of DVA and CVA in practice.

FVA - (1)

- Besides CVA/DVA, the frictions that appear in world of regulation, margin, and expensive funding have led some (= traders) to suggest a plethora of new “valuation adjustments”.
- The most common new measure is ‘funding valuation adjustment” – FVA.
- This measure, which is quite controversial, is meant to measure the ‘cost” of funding uncollateralized (or partially collateralized) positions.
- To understand the notion, consider a trade where a bank’s treasury issues an uncollateralized bond into the market, paying a coupon of c.
- The full proceeds of the issuance is invested into a low-risk security that pays a coupon of d. Assume that the security cannot be financed in repo markets, but has a well-known market (auction) price, which equals the price of the issued bond.

FVA - (2)

- Since the security has lower risk than the issued bond, we would have $c > d$. The bank would "leak money" all the way to the default of the bank or to the maturity of the bond, whatever comes first.
- Financial theory suggests that this is not a problem: the fact that $c > d$ is because there is a risk that the notional on the bank-issued bond will not be paid back. Therefore, the "leakage" is just payment for the cash-flow that takes place at the default time of the bank.
- From the perspective of the bank traders, however, their books will leak until the firm defaults, at which point they will loose their jobs. So trade is a bad one, from their perspective.
- The equity holders of the bank will see it the same way: trade was a bad one.

FVA - (3)

- But the bond holders of the bank will be happy: at the time of default of the bank, they will have the purchased security in their possession and will not have to pay back the (full) notional of the issued bond.
- Market value = equity holder value + debt holder value. Total market value impact of transaction is zero, but positive to debt holder and negative to equity holders.
- FVA adjustments takes a trader-centric view, and will insist that the market value of the transaction is negative and therefore should not be done. Effectively we are told to ignore the cash flow taking place at default because it won't matter to traders.
- Concretely, conjecture the existence of a universal funding curve for uncollateralized bank bonds. Let the rate earned on funds be $r(t) + s_F(t)$ at time t, for some spread $s(t)$.

FVA - (4)

- For an uncollateralized derivative with value V, the funding cost/benefit over the period $[t, t+dt]$ is interpreted by traders to be $-V(t)s_F(t)\,dt$. Taking this at face value it is tempting to define a *funding valuation adjustment* (FVA) of

$$FVA(t) = -\mathrm{E}_t\left(\int_t^T e^{-\int_t^u r(u)} V(u)s_F(u)\,du\right).$$

- Assuming that V represents the value of a European payout at time T, and adding the funding cost/benefit to the value of the payout, we get (using Feynman-Kac or the dividend formula)

$$\begin{aligned} V(t) &= \mathrm{E}_t\left(e^{-\int_t^T r(u)\,du} V(T)\right) + FVA(t) \\ &= \mathrm{E}_t\left(e^{-\int_t^T (r(u)+s_F(u))\,du} V(T)\right). \end{aligned}$$

FVA - (5)

- So basically, all uncollateralized derivatives should be discounted at a higher discount rate. (Piterbarg (2011))
- One immediate issue: derivatives have different values for different firms, and the market will not clear!
- Another issue: for securities where the counterparty owes us money (when $V(T)$ is positive), we end up discounting by our spread, rather than with the spread of our counterparty. When CVA is later added (which traders still want to do), we double-count.
- A third issue: for securities where the counterparty is owed money (when $V(T)$ is negative) FVA and unilateral DVA will be about the same, so we will double-count unless we turn off DVA. (See (17).)

FVA - (6)

- To at least avoid the overlap with DVA, some researchers define FVA as an asymmetric metric:

$$FVA^* = \mathrm{E}_t \left(\int_t^\infty s_F(u) e^{-\int_t^u s_F(s)\,ds} e^{-\int_t^u r(s)\,ds} V(u)^+ \, du \right).$$

- This metric can no longer be applied at the trade-level, only to portfolios. It effectively assumes that excess funds is an unstable source of funding and will only earn interest at OIS.
- There is great controversy around the topic of FVA. Academics disregard it, traders love it.
- At the moment, unclear who is getting the upper hand, but yesterday's JPM report shows that FVA is showing up in accounting statements:

FVA - (7)

- From www.libertyinvestor.com, January 14, 2014:

  "*The biggest surprise in JPM's Q4 earnings release was not the firmâĂŹs legal troubles: those are well-known [...] No, the biggest surprise by far was that as of this quarter in addition to its trusty use of DVA or a Debt Valuation Adjustment (the old fudge when a bank "benefits" when its credit spreads blow out) JPM also added the use of a Funding Valuation Adjustment or FVA. The amount of the FVA benefit? A whopping USD1.5 billion add-back to GAAP EPS, which together with DVA, resulted in a USD2.0 billion pretax loss [...]* "

- Looking at JPM's statement, it appears that they are using the asymmetric version of FVA – and only charge for receivables.
- JPM claims that they see evidence in market prices for FVA adjustments.

FVA - (7)

- My opinion: FVA is a useful decision **metric** (it is roughly a measure of “shareholder value”), and can be used to screen trades to see if they benefit share holders.
- It is not a properly defined measure of market **value**, but potentially a useful measure to minimize. Should be FVM, not FVA.
- Current accounting laws, to my reading, do not support FVA and any other “entity specific” cost allocation into a market price. “Market value” is not the same as “production cost”.
- Of course, if every firm started using FVA, the auction principles of accounting laws would suggest a “minimum funder” valuation adjustment. This could differ significantly from FVA and would be similar to a classic asset-specific liquidity adjustment.

KVA - (1)

- Suppose that a trade (or group of trades) is deemed to be associated with a regulatory capital amount of $C(t)$. The capital amount varies with time, dependent on the exposure generated by the trade over time.
- Equity holders that put up capital want a (high) return $r_E(t) = r(t) + s_E(t)$ on their investment.
- Like for FVA, we can define a *k(c)apital valuation adjustment*, KVA, to capture the “cost” of having to put capital against a position:

$$KVA = -\mathrm{E}\left(\int_0^T e^{-\int_0^t r(u)\,du} C(t) s_E(t)\,dt\right).$$

KVA - (2)

- The main complication here is the fact that **future** capital $C(t)$ is difficult to simulate: we need to be able to do a full capital computation at each point in the future.
- And to see the marginal cost impact of a new trade, we need to run two simulations: with and without the trade.
- For credit capital, we need to execute something like (simplified a bit)

$$C(t) = RW(p,d) \cdot 1.4 \cdot EEPE(t, t+\delta) \cdot 1.06$$

where RW is a risk weight computed by the Vasicek large-portfolio result, $\delta = 1$, and (V is the portfolio value)

$$EEPE(t, t+\delta) = \frac{1}{\delta} \int_t^{t+\delta} \max_{s \in [t,u]} \mathrm{E}_t \left(V(s)^+ \right) \, du.$$

KVA - (3)

- But how do we compute $\mathrm{E}_t(V(s))$, $s > t$, for some future time t? A "simulation within a simulation" is normally too expensive.
- A typical approach involves writing

$$\mathrm{E}_t\left(V(s)^+\right) = \mathrm{E}\left(V(s)^+|\mathcal{F}_t\right) \approx \mathrm{E}\left(V(s)^+|V(t)\right)$$

- There are standard methods for estimating this quantity, either by least-squares regression, or by kernel regression methods.
- For the latter, when doing the expectation of $\mathrm{E}(X|Y)$ we can rely on the following n-sample estimator (Nadaraya-Watson kernel estimator)

$$\mathrm{E}(X|Y=y) \approx \frac{\sum_{i=1}^{n} X_i \cdot K\left(\frac{Y_i - y}{h}\right)}{\sum_{i=1}^{n} K\left(\frac{Y_i - y}{h}\right)}$$

KVA - (4)

- Here K is a measure of “closeness” known as a *kernel*. h is a smoothing parameter known as the *bandwidth*.
- Technically, a kernel is just a real function satisfying

$$\int_{-\infty}^{\infty} K(x)\,dx = 1, \quad K(x) = K(-x).$$

- Gaussian: $K(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$. Uniform: $K(x) = \frac{1}{2} 1_{x \in [-1,1]}$. Epanechnikov: $K(x) = \frac{3}{4}\left(1 - x^2\right) 1_{x \in [-1,1]}$.
- The bigger h is, the smoother (and less noisy) the kernel regression estimate becomes. But a big h also implies a big bias.
- A low h will give noisy, but less biased, results.

KVA - (5)

- There are rules for picking a good h that gives a good compromise between noise and bias. For instance, for the Gaussian kernel,

$$h \approx 1.06 \cdot n^{-1/5} \cdot \sqrt{\mathrm{Var}\,(Y)}$$

- Even if we can pull of a good estimation, there are considerable practical obstacles in the KVA concept. F.ex.:
  - Capital is computed at the portfolio level, not the trade level.
  - How do we estimate the “cost of capital/equity”?
  - How do we project changes in Basel accords? And Basel 6, 7, 8,..
  - What about the other elements of regulatory capital (VaR, CVA Var (Basel 3), maturity adjustment, etc.)?
- And we can also define “XVA“ adjustments for other frictions, such as initial margin – MVA anybody?

How to manage all this??

- In this talk, we have outlined a (very) large range of regulatory requirements that are introduced through regulation and through funding imbalances.
- Many of these impact the bottom line, directly or indirectly.
- While regular quants produce the numbers, there is now increasingly the need for another function: capital structurers/strats.
- Here, the problem is how to manage around all the capital and margin requirements in a rational/optimal way.
- Increasingly this is done by a central desk in the bank, typically inside the CVA trading group.
- Supported by diagnostic tools: where are exposures/capital/margin/liquidity coming from?

Management

- This is a new function, but an important one
- Tools in quiver include:
  - Trading on multiple CCP venues in optimal manner (to minimize margin)
  - Merging legal entities (to compactify netting sets)
  - Intermediation/novation
  - Innovative CSAs that the clients can tolerate (e.g., caps on postings)
  - Trade consolidation/compression
- There is a lot more to come (and say) in this space, I'm sure..