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Hybrids of Uniform Test and Sequential Uniform Designs with "Intersection" Method for Multi-objective Optimization

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Abstract: For multi-objective optimization under condition of complicated objective function, the data processing in the evaluation is sometime tediously long, special algorithm is needed to be adopted. Since the remarkable features of uniform distribution of test points within the test domain and the small number of tests, fully representative of each point, and easy to perform regression analysis, the uniform test design method is hybrid with the "intersection" method for multi-objective optimization to simplify the complicated data process in evaluation first. Furthermore, the "intersection" multi-objective optimization methodology is combined with sequential uniform design so as to get a more precise approximation for solving multi-objective optimization problem, the procedure for searching optimum of the "intersection" multi-objective optimization methodology with sequential uniform design algorithm is put forward. A multi-objective optimization of linear programming problem with three variables is taken as our example, which involves a maximum for one objective and a minimum for another objective. The result for applying the novel approach to the example indicates the effectiveness of current hybrids.

Keywords: hybrid; "intersection" multi-objective optimization; sequential uniform design; simplification in evaluation; uniform test design method

1 INTRODUCTION

Multi-objective optimization is a perpetual project in daily life and practical industry production, which ranges almost all fields. It likely involves several performances or attributes which must be simultaneously considered in the analysis. An appropriately recommended alternative is an optimal one that needs to meet requirement of compromised optimization of various response variables (performances) simultaneously, they are even conflicting each other. Various techniques have been proposed, which include the technique for order preference by similarity to ideal solution (TOPSIS), VIšekriterijumsko KOmpromisno Rangiranje (VIKOR), multi attribute decision-making (MADM), Analytical Hierarchy Process (AHP) and Multi-objective Optimization based on Ratio Analysis (MOORA), etc. [1]. The inherent problems of "additive" algorithm and personal / subjective factors in the above multi-object optimizations make them puzzled [1].

Recently, a new approach named "intersection" method for multi-objective optimization was proposed in the viewpoints of probability theory [1]. It tries to overcome the shortcomings of personal and subjective factors in the above multi-object optimizations by introducing a new idea of favorable probability and the corresponding assessment. The favorable probability is used to reflect the favorable degree of the candidate in the optimization, all performance utility indicators of candidates are divided into beneficial or unbeneficial types according to their features in the selection, and each performance utility indicator of the candidate contributes to a partial favorable probability quantitatively; the total favorable probability is the product of all partial favorable probabilities in the viewpoints of probability theory and "intersection" of set theory, which is the overall consideration owing to the compromised optimization of various response variables simultaneously; the total favorable probability is the unique decisive index in the competitive selection process. Appropriate achievements have been obtained [1].

In general [2, 3], the difficulty in solving the multi-objective optimization problem is due to the complexity of the multi-objective optimization problem itself. The primary problem of multi-objective optimization is to generate a subset of non-inferior solutions, and then find the final ideal and effective solution from it according to the intention of the decision maker. There are three main types of methods to find the final solution: a) Generative method, which first finds a large number of non-inferior solutions to form a subset of non-inferior solutions, and then finds the final solution according to the intention of the decision maker; b) Interactive method, which does not need to find a lot of non-inferior solutions first, but gradually find the final solution through dialogue between the analyst and the decision-maker; c) Request the decision-maker to provide the relative importance of the goals in advance, and use this as a basis to combine multiple goals. The problem is converted to a single objective problem to be solved. These are mainly achieved through algorithms. Many experts and scholars have used different algorithms to solve multi-objective optimization problems, such as multi-objective evolutionary algorithm, multi-objective particle swarm algorithm and ant colony algorithm, simulated annealing algorithm and artificial immune system. It indicates that more serious algorithms are involved in the solving of multi-objective optimization problem [2, 3].

In 1978, Fang and Wang developed a novel type of experimental design that is known as uniform design [4, 5]. Since the remarkable features of uniform distribution of test points within the test domain and the small number of tests, fully representative of each point, and easy to perform regression analysis, this method had been successfully applied to the design of Chinese missiles [4, 5].

After their articles were published in the early 1980s, uniform design has been widely applied in China and has gotten great achievements [4, 5].

Uniform design belongs to the quasi-Monte Carlo methods or number theoretical methods. In optimization process, as the calculation of a single variable problem is generalized to a multivariable problem, the increase of calculation complexity is increase obviously with the number of variables; as to multi – objective optimization, the increase of complexity in assessment is more serious. Even with the great advances in computational technology, the complexity is still there. Monte Carlo method (i.e., statistical stimulation) is to transfer an analysis problem into a probability problem with the same solution, which use a statistical simulation to deal with the probability problem. This solves some difficulty in analysis of assessment, including the approximate calculation of multiple definite integrals [4-6].

While, the sequential number theory optimization (SNTO) was introduced into the uniform experimental design as a new global optimization approach [7-11].

It seeks for the global extreme value among uniformly distributed points in the space of variables, and the convergence is speeded up by contracting the searching space. In each search only the points close to the extreme value (minimum or maximum) function value are retained among the uniformly scattered points. In order to get the global optimum properly one needs to choose the sufficient number of points for the first search [7-11].

As stated in [7], if $l(D)$ is used to indicate the length of largest edge of the rectangle domain D. Assume that one can find a domain D^* such that $x^* \in D^* \in D$ and $l(D^*) << l(D)$, then the optimization problem of $M = Maxf(x^*)$ for all $x \in D$ (in domain D) is reduced to an optimization problem in the region D^*, and thus the same sized NT-net could lead to a much more precise approximation to x^* normally. This idea was initiated by Niederreiter and Peart in 1986 and by Fang and Wang in 1990 [7]. More precisely, a sequential algorithm for optimization (SNTO) with NT-nets was suggested by Fang and Wang [7]. This method has been used in solving optimum problem with single objective [7-11].

In this paper, a hybrid of uniform test design method with the new "intersection" method for multi-objective optimization is developed to simplify the complicated data process in evaluation first. Then the new "intersection" multi-objective optimization method is combined with sequential uniform design to get a more precise approximation for solving multi-objective optimization problem.

2 PROCEDURES OF HYBRIDS OF THE "INTERSECTION" METHOD FOR MULTI-OBJECTIVE OPTIMIZATION WITH UNIFORM TEST DESIGNS

2.1 Hybrid of "Intersection" Method for Multi-objective Optimization with Uniform Test Design

The remarkable features of the uniform test design (UTD) includes, uniformly distributed "representative points" of designed tests with deterministic positions within the domain of variables, small number of designed tests to reflect the whole feature of the responses within the domain of variables, and fully representative of each point. So the UTD method can be used to get a hybrid with the "intersection" method for multi-objective optimization to simplify the complicated data process in evaluation.

Additionally, in the "intersection" method for multi-objective optimization, the total favorable probability is the unique and decisive indicator of the alternative; therefore, the final assessment including regression analysis should be focused on this determinant indicator with limited number of discrete test points by means of uniform test design.

2.2 Hybrid of Intersection Method for Multi-objective Optimization with Sequential Uniform Design

In reference to the procedure proposed by Fang and Wang for sequential algorithm for optimization (SNTO) with NT-nets [7-11], we could develop the operation process for the combination of sequential uniform design with the "intersection" method for multi-objective optimization.

If SNTO for D being a rectangle $[\boldsymbol{a}, \boldsymbol{b}]$. In our case, the maximum value of total favorable probability P_t is assessed for the point set in each step. Thus, the operation process of SNTO algorithm for combination of sequential uniform design with "intersection" method for multi-objective optimization is as follows:

0th Step: Initialization.

At moment $t = 0$, $D^{(O)} = D$, $\boldsymbol{a}^{(O)} = \boldsymbol{a}$ and $\boldsymbol{b}^{(O)} = \boldsymbol{b}$.

1st Step: Generate an NT - net.

Number-theoretic method is used to generate a n_t points $\boldsymbol{P}^{(t)}$ uniformly distributed on $D^{(t)} =[\boldsymbol{a}^{(t)}, \boldsymbol{b}^{(t)}]$. $P_t(\boldsymbol{x}^{(t)})$ is the maximum value of total favorable probability at moment t of the alternative in the point set.

2nd Step: Calculate a novel approximate value.

Assume $\boldsymbol{x}^{(t)} \in G^{(t)} \cup \{\boldsymbol{x}^{(t-1)}\}$ and $M^{(t)}$ such that $M^{(t)} = P_t(\boldsymbol{x}^{(t)}) \le P_t(\boldsymbol{y})$ for number of points with characteristic of $n_{t-1} = n_t = \ldots$, $\forall \boldsymbol{y} \in G^{(t)} \cup \{\boldsymbol{x}^{(t-1)}\}$, where $\boldsymbol{x}^{(-1)}$ is the empty set, $\boldsymbol{x}^{(t)}$ and $M^{(t)}$ are the best approximations to $\boldsymbol{x}^*$ and M temporarily.

3rd Step: Termination condition.

Let $c^{(t)} = (P_t^{(t)} - P_t^{(t-1)})/P_t^{(t-1)}$. If Max $c^{(t)} < \delta$, a pre-assigned small number, then $x^{(t)}$ and $M^{(t)}$ are acceptable; terminate algorithm. Otherwise, proceed to next step.

4th Step: Domain contraction.

A new domain is formed like this: $D(t+1) = [\boldsymbol{a}^{(t+1)}, \boldsymbol{b}^{(t+1)}]$ as follows: $a_i^{(t+1)} = \max (x_i^{(t)} - \beta c_i^{(t)}, a_i)$ and $b_i^{(t+1)} = \min (x_i^{(t)} + \beta c_i^{(t)}, b_i)$, where β is a predefined contraction ratio. Set $t = t + 1$. Go to Step 1.

According to Fang and Wang's experience, they suggested $n_1 > n_2 = n_3 = \ldots$ for the processing. The contraction ratio β can be taken as 0.5. While Niederreiter and Peart (1986) suggested using $\beta_i = \beta^i$ as a contraction ratio at the i^{th} step with $\beta > 0$ as constant.

Remarks: in our case, at i^{th} step, $P_t(\boldsymbol{x}^{(i)}) \le P_t(\boldsymbol{x}^{(i-1)})$ in general for $i > 2$ only if $n_2 = n_3 = \ldots$. Or else, check the domain contraction process or stop the process of domain contraction, and take the $P_t(\boldsymbol{x}^{(i-1)})$ and the corresponding $\boldsymbol{x}^{(i-1)}$ as the optimal results.

3 APPLICATIONS

3.1 Discreterization Treatment of the Intersection Multi-objective Optimization for a Linear Programming Problem by Means of UTD

Take a multi-objective optimization of linear programming problem with three variables as our example.

The problem is written as following form,

$$\text{Max } f_1 = 9x_1 + 10x_2 + 14x_3,$$
$$\text{Min } f_2 = 4x_1 + 5x_2 + 8x_3. \quad (1)$$

The domain is [0, 12] × [0, 5] × [0, 7].

In this problem, f_1 belongs to beneficial type of performance, and f_2 is attributed to unbeneficial type of performance.

Table 1 Design and values of f_1 and f_2 of the multi – objective optimization of linear programming problem with three variables due to $U^*_{25}(25^{11})$

No.	x_1	x_2	x_3	f_1	f_2
1	1.20	0.90	6.86	115.84	64.18
2	2.64	1.90	6.58	134.88	72.70
3	4.08	2.90	6.30	153.92	81.22
4	5.52	3.90	6.02	172.96	89.74
5	6.96	4.90	5.74	192.00	98.26
6	8.40	0.70	5.46	159.04	80.78
7	9.84	1.70	5.18	178.08	89.30
8	11.28	2.70	4.90	197.12	97.82
9	0.24	3.70	4.62	103.84	56.42
10	1.68	4.70	4.34	122.88	64.94
11	3.12	0.50	4.06	89.92	47.46
12	4.56	1.50	3.78	108.96	55.98
13	6.00	2.50	3.50	128.00	64.5
14	7.44	3.50	3.22	147.04	73.02
15	8.88	4.50	2.94	166.08	81.54
16	10.32	0.30	2.66	133.12	64.06
17	11.76	1.30	2.38	152.16	72.58
18	0.72	2.30	2.10	58.88	31.18
19	2.16	3.30	1.82	77.92	39.70
20	3.60	4.30	1.54	96.96	48.22
21	5.04	0.10	1.26	64.00	30.74
22	6.48	1.10	0.98	83.04	39.26
23	7.92	2.10	0.70	102.08	47.78
24	9.36	3.10	0.42	121.12	56.30
25	**10.80**	**4.10**	**0.14**	**140.16**	**64.82**

The uniform table $U^*_{25}(25^{11})$ is employed to conduct the discretization of this multi-objective optimization of linear programming problem with three variables, the design together with the values of f_1 and f_2 are shown in Tab. 1 from Fang's book [12]. The assessment results of the partial favorable probabilities for f_1 and f_2 and the total favorable probabilities for each discrete point are presented in Tab. 2.

From the assessed results in Tab. 2, it can be seen that the maximum total favorable probability is at the point No. 25 with $x_1^* = 10.80$, $x_2^* = 4.10$ and $x_3^* = 0.14$.

Table 2 Assessment results of the partial favorable probabilities for f_1 and f_2 and the total favorable probabilities for each discrete point

No.	P_{f1}	P_{f2}	$P_t \times 10^3$
1	0.0362	0.0402	1.4543
2	0.0422	0.0355	1.4967
3	0.0481	0.0308	1.4834
4	0.0541	0.0262	1.4147
5	0.0600	0.0215	1.2904
6	0.0497	0.0311	1.5448
7	0.0557	0.0264	1.4700
8	0.0616	0.0217	1.3396
9	0.0325	0.0444	1.4416
10	0.0384	0.0398	1.5267
11	0.0281	0.0493	1.3863
12	0.0341	0.0447	1.5209
13	0.0400	0.0400	1.6000
14	0.0460	0.0353	1.6235
15	0.0519	0.0307	1.5915
16	0.0416	0.0402	1.6740
17	0.0476	0.0356	1.6915
18	0.0184	0.0583	1.0718
19	0.0244	0.0536	1.3048
20	0.0303	0.0489	1.4822
21	0.0200	0.0585	1.1699
22	0.0260	0.0538	1.3968
23	0.0319	0.0492	1.5682
24	0.0379	0.0445	1.6840
25	**0.0438**	**0.0398**	**1.7443**

3.2 Hybrid of the "Intersection" Multi-objective Optimization with SNTO in Treating the Linear Programming Problem

According to the procedure described in previous section, subsequent processing is used to contract domain to conduct further evaluations. Continue to deal with the problem of linear programming problem that was raised in the last section further for assessment that is more precious.

The uniform table $U_{19}^*(19^7)$ from Fang's book is used to perform the succeeding assessments [12]. Tab. 3 shows the consequences of the succeeding evaluations.

Tab. 3 displays that the $c^{(t)}$ value at the 5th step is 0.19%, if we set $\delta = 0.2\%$ as the pre-assigned small number for engineering application, then the final optimal consequences for this multi – objective optimization problem are $f_{1\text{Opt.}} = 135.2026$ and $f_{2\text{Opt.}} = 61.6421$ at $x_1^* = 11.9342$, $x_2^* = 2.7684$ and $x_3^* = 0.0079$.

Table 3 Consequences of the succeeding evaluations by using $U_{19}^*(19^7)$

Step	Domain	Optimum location			$f_{1\text{Opt.}}$	$f_{2\text{Opt.}}$	Max. total favorable probability $P_t \times 10^3$	$c^{(t)}$
		x_1^*	x_2^*	x_3^*				
0	[0, 12] × [0, 5] × [0, 7]	10.8000	4.1000	0.1400	140.1600	64.8200	1.7443	
1	[5, 12] × [1.8, 4.8] × [0, 4]	11.0790	2.8263	0.1053	129.4474	59.2850	2.8906	
2	[8, 12] × [2.5, 4] × [0, 2]	11.4737	3.0132	0.0526	134.1316	61.3816	2.8435	0.0163
3	[11, 12] × [2.6, 3.2] × [0, 1.0]	11.8684	2.8053	0.0263	135.2368	11.8684	2.8016	0.0147
4	[11.5, 12] × [2.7, 3] × [0, 0.5]	11.9342	2.8026	0.0132	135.6184	61.8553	2.7871	0.0052
5	[11.7, 12] × [2.7, 2.9] × [0, 0.3]	11.9342	2.7684	0.0079	135.2026	61.6421	2.7817	0.0019

4 CONCLUSION

From above discussion, it obtains following conclusions:

1) The hybrid of the uniform test design with the "intersection" method for multi – objective optimization is proposed, which simplify the complicated data processing of the evaluation. Therefore, the complicated data processing is successfully simplified as assessments of discrete points, which are evenly distributed within the test domain;
2) The "intersection" multi-objective optimization is combined with sequential uniform design, the detailed

procedure for searching optimum of the "intersection" multi-objective optimization with sequential uniform design algorithm is developed, which could be used to obtain a more precise approximation for solving multi – objective optimization problem;
3) The example for applying the novel approach to deal with the multi-objective optimization of linear programming problem with three variables indicates the validity of the current hybrids.

Conflict Statement

There is no conflict of interest.

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