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On standard imbeddings of hyperbolic spaces in the Minkowski space

Dong-Soo Kim [a,1], Young Ho Kim [b,2], Dae Won Yoon [c]

[a] *Department of Mathematics, Chonnam National University, Gwangju 500-757, South Korea*
[b] *Department of Mathematics, Kyungpook National University, Daegu 702-701, South Korea*
[c] *Department of Mathematics Education and RINS, Gyeongsang National University, Jinju 660-701, South Korea*

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ABSTRACT

We establish some characterizations of the standard imbeddings of hyperbolic spaces in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$ with intrinsic and extrinsic properties such as the n-dimensional area of the sections cut off by hyperplanes, the $(n+1)$-dimensional volume of regions between parallel hyperplanes, and the n-dimensional surface area of regions between parallel hyperplanes. In the same manner, we give an affirmatively partial answer to Question A suggested in [6], which is for the characterization of hyperspheres in the $(n+1)$-dimensional Euclidean space $\mathbb{E}^{n+1}$.

RÉSUMÉ

Nous établissons quelques caractérisations des plongements standard d'espaces hyperboliques dans l'espace de Minkowski $\mathbb{L}^{n+1}$ de dimension $n+1$, avec des propriétés intrinsèques et extrinsèques comme la surface n-dimensionnelle des sections coupées par des hyperplans, le volume en $n+1$ dimensions de régions entre des hyperplans parallèles et la surface n-dimensionnelle de régions entre des hyperplans parallèles. De la même façon, nous donnons une réponse affirmative partielle à la question A suggérée dans [6], qui concerne la caractérisation d'hypersphères dans l'espace Euclidien $\mathbb{E}^{n+1}$ de dimension $n+1$.

1. Introduction

We consider the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1} = \mathbb{R}^{n+1}_1$ with metric $ds^2 = dx_1^2 + \cdots + dx_n^2 - dx_{n+1}^2$ for $x = (x_1, \cdots, x_{n+1})$. Let us denote by $H^n(r) \subset \mathbb{L}^{n+1}$ the spacelike hyperquadric defined by $\langle p, p\rangle = -r^2$, $p_{n+1} > 0$. Then $H^n(r)$

E-mail addresses: dosokim@chonnam.ac.kr (D.-S. Kim), yhkim@knu.ac.kr (Y.H. Kim), dwyoon@gnu.ac.kr (D.W. Yoon).

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is a Riemannian space form with constant sectional curvature $K = -\frac{1}{r^2}$, which is called the standard imbedding of the hyperbolic space of curvature $K = -\frac{1}{r^2}$, or simply the hyperbolic space.

For a fixed point $p \in H^n(r)$, $N(p) = -p/r$ is the timelike unit normal to $H^n(r)$ at p pointing to the concave side of $H^n(r)$. If $t > 0$ is sufficiently small, let us denote by Φ_t the hyperplane passing through the point $p - tN(p)$, which is parallel to the tangent hyperplane Φ_0 of $H^n(r)$ at p.

We denote by $A_p(t)$, $V_p(t)$ and $S_p(t)$ the n-dimensional area of the section in Φ_t enclosed by $\Phi_t \cap H^n(r)$, the $(n+1)$-dimensional volume of the region bounded by the hyperbolic space and the hyperplane Φ_t, and the n-dimensional surface area of the region of $H^n(r)$ between the two hyperplanes Φ_t and Φ_0, respectively.

Then, for sufficiently small $t > 0$, we have the following properties of the hyperbolic space $H^n(r)$.

Proposition 1. The hyperbolic space $H^n(r)$ satisfies the following conditions:

(V): the $(n+1)$-dimensional volume $V_p(t)$ of the region is independent of the point p,
(A): the n-dimensional area $A_p(t)$ of the section is independent of the point p,
(S): the n-dimensional surface area $S_p(t)$ of the region is independent of the point p.

Proof. For two points p and q on the hyperbolic space $H^n(r)$, there exists a linear isometry $\phi: \mathbb{L}^{n+1} \to \mathbb{L}^{n+1}$ carrying $H^n(r)$ isometrically to itself, with $\phi(p) = q$ ([10, pp. 113–114]).

If we denote by Φ_0' and Φ_t' the tangent hyperplane of $H^n(r)$ at q and the hyperplane passing through $q - tN(q)$, which is parallel to Φ_0', respectively, then we also have:

$$\phi\big(N(p)\big) = N(q), \qquad \phi(\Phi_0) = \Phi_0', \qquad \phi(\Phi_t) = \Phi_t'.$$

These relations complete the proof. □

Conversely, it is natural to ask the following question.

Question 2. Are there any other convex spacelike hypersurfaces in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$ that satisfy the above properties?

We will say that a convex spacelike hypersurface in $\mathbb{L}^{n+1}$ is *strictly convex* if the shape operator of the hypersurface is positive definite with respect to the unit normal N pointing to the concave side.

In this article, we study strictly convex spacelike hypersurfaces M in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$ of which parallel hypersurfaces satisfy the above-mentioned properties. For a constant s, the parallel hypersurface M_s is defined by the set consisting of the points $p - sN(p)$, $p \in M$. It is a well-defined hypersurface, provided s is small enough.

We assume that for each $s \in (-\epsilon, \epsilon)$, $\epsilon > 0$, the parallel hypersurface M_s is strictly convex. Then, for a fixed point $p \in M_s \subset \mathbb{L}^{n+1}$, $A_p(t)$, $V_p(t)$ and $S_p(t)$ are defined as above.

In Section 3, as a result, we prove the following theorem.

Theorem 3. Let M be a strictly convex spacelike hypersurface in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$. We assume that for each $s \in (-\epsilon, \epsilon)$, $\epsilon > 0$, the parallel hypersurface M_s is strictly convex. Suppose that each M_s, $s \in (-\epsilon, \epsilon)$ satisfies one of the following conditions:

(A): the n-dimensional area $A_p(t)$ of the section is independent of the point $p \in M_s$,
(V): the $(n+1)$-dimensional volume $V_p(t)$ of the region is independent of the point $p \in M_s$,
(S): the n-dimensional surface area $S_p(t)$ of the region is independent of the point $p \in M_s$.

Then, up to isometries of $\mathbb{L}^{n+1}$, the hypersurface M is an open part of the hyperbolic space $H^n(r)$.

Conversely, for the hyperbolic space $M = H^n(r)$ and sufficiently small s, the parallel hypersurface M_s is nothing but the hyperbolic space $M_s = H^n(r+s)$. Hence, it follows from Proposition 1 that each M_s satisfies the conditions (A), (V) and (S), respectively.

Remark 4. It is well known that the convex and complete hypersurfaces in the Euclidean space with positive constant Gauss–Kronecker curvature with respect to the unit normal pointing to the convex side are the hyperspheres [3]. Using this fact, the first and second authors established some characterizations of Euclidean hyperspheres [5,6]. On the other hand, in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$, there exist a lot of convex and complete spacelike hypersurfaces with positive constant Gauss–Kronecker curvature with respect to the unit normal pointing to the concave side [2,4,7].

Finally, if we use the same argument as in the proof of Theorem 3, we are able to give a partial affirmative answer to Question A in [6] as follows.

Theorem 5. Let M be a strictly convex hypersurface in the $(n+1)$-dimensional Euclidean space $\mathbb{E}^{n+1}$. We assume that for each $s \in (-\epsilon, \epsilon)$, $\epsilon > 0$, the parallel hypersurface M_s is strictly convex. Suppose that each M_s, $s \in (-\epsilon, \epsilon)$ satisfies one of the following conditions:

(A) : the n-dimensional area $A_p(t)$ of the section is independent of the point $p \in M_s$,
(V) : the $(n+1)$-dimensional volume $V_p(t)$ of the region is independent of the point $p \in M_s$,
(S) : the n-dimensional surface area $S_p(t)$ of the region is independent of the point $p \in M_s$.

Then, up to isometries of $\mathbb{E}^{n+1}$, the hypersurface M is an open part of the round hypersphere $S^n(r)$.

Throughout this article, all objects are smooth and connected, unless otherwise mentioned.

2. Preliminaries

Suppose that M is a strictly convex spacelike hypersurface in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$ with the timelike unit normal N pointing to the concave side.

We may assume that M is locally the graph of a non-negative convex function $f : \mathbb{R}^n \to \mathbb{R}$. Since M is spacelike, we have $|\nabla f(x)| < 1$, where ∇f denotes the gradient vector of the function f. Hence the unit normal N to M is

$$N\big(x, f(x)\big) = \frac{-1}{W(x)}\big(f_1(x), \cdots, f_n(x), 1\big) = \frac{-1}{W(x)}\big(\nabla f(x), 1\big), \tag{2.1}$$

where $f_i(x)$ denotes the partial derivative of f with respect to x_i, $i = 1, \cdots, n$ and $W(x) = (1 - |\nabla f(x)|^2)^{1/2}$.

The volume density $\mathrm{d}V$ on the hypersurface M is given by ([1, p. 3])

$$\mathrm{d}V(X_1, \cdots, X_n) = \big|\det\big(\langle X_i, X_j\rangle\big)\big|^{1/2}, \tag{2.2}$$

where $X_i = (e_i, f_i(x))$, $i = 1, \cdots, n$ is a basis for T_pM, $p = (x, f(x))$ and $e_1, \cdots, e_n$ the natural basis of $\mathbb{R}^n$. It is straightforward to show that:

$$\det\big(\langle X_i, X_j\rangle\big) = \det\begin{pmatrix} 1 - f_1^2 & -f_1 f_2 & \cdots & -f_1 f_n \\ -f_2 f_1 & 1 - f_2^2 & \cdots & -f_2 f_n \\ \vdots & \vdots & & \vdots \\ -f_n f_1 & -f_n f_2 & \cdots & 1 - f_n^2 \end{pmatrix} = 1 - |\nabla f|^2 > 0. \tag{2.3}$$

The shape operator $L : T_pM \to T_pM$ with respect to the unit normal N is defined by

$$L(X) = -\bar{\nabla}_X N, \tag{2.4}$$

where $\bar{\nabla}$ is the usual connection on $\mathbb{L}^{n+1}$. We denote by $k_1, \cdots, k_n$ the eigenvalues of the shape operator L, which are called the principal curvatures of M at p with respect to the unit normal N. The Gauss–Kronecker curvature K is then defined by $K = k_1 \cdots k_n$. Hence we get [7]:

$$K = \det(L) = \frac{\det(f_{ij})}{W(x)^{n+2}}. \tag{2.5}$$

For spacelike surfaces in $\mathbb{L}^3$, the intrinsic Gauss curvature is $-K$.

Since M is a strictly convex spacelike hypersurface in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$ with respect to the timelike unit normal N pointing to the concave side, for each $i = 1, 2, \cdots, n$, we have $k_i > 0$.

For a fixed point $p \in M$ and a sufficiently small $t > 0$, consider the hyperplane Φ_t passing through the point $p - tN(p)$, which is parallel to the tangent hyperplane Φ_0 of M at p. Let us again denote by $A_p(t)$, $V_p(t)$ and $S_p(t)$ the n-dimensional area of the section in Φ_t enclosed by $\Phi_t \cap M$, the $(n+1)$-dimensional volume of the region bounded by the hypersurface and the hyperplane Φ_t and the n-dimensional surface area of the region of M between the two hyperplanes Φ_t and Φ_0, respectively.

Now, using a Lorentzian motion of $\mathbb{L}^{n+1}$, we may introduce a coordinate system $(x_1, x_2, \cdots, x_{n+1})$ of $\mathbb{L}^{n+1}$ with the origin p, the tangent space of M at p is the hyperplane $x_{n+1} = 0$. Hence, M is locally the graph of a non-negative convex function $f : \mathbb{R}^n \to \mathbb{R}$ satisfying $f(0) = 0$ and $\nabla f(0) = 0$.

Thus, from $N(p) = -(0, \cdots, 0, 1)$, for a sufficiently small $t > 0$ we obtain

$$A_p(t) = \iint\limits_{f(x) < t} 1\, \mathrm{d}x, \tag{2.6}$$

and

$$V_p(t) = \iint\limits_{f(x)<t} \{t - f(x)\}\,\mathrm{d}x, \tag{2.7}$$

where $x = (x_1, x_2, \cdots, x_n)$ and $\mathrm{d}x = \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_n$. It follows from (2.2) and (2.3) that

$$S_p(t) = \iint\limits_{f(x)<t} \sqrt{1 - \left|\nabla f(x)\right|^2}\,\mathrm{d}x. \tag{2.8}$$

Note that we also have

$$\begin{aligned} V_p(t) &= \iint\limits_{f(x)<t} \{t - f(x)\}\,\mathrm{d}x \\ &= \int_{z=0}^{t} \Bigg\{ \iint\limits_{f(x)<z} 1\,\mathrm{d}x \Bigg\}\,\mathrm{d}z. \end{aligned} \tag{2.9}$$

Together with the fundamental theorem of calculus, Eq. (2.6) shows that

$$V_p'(t) = \iint\limits_{f(x)<t} 1\,\mathrm{d}x = A_p(t). \tag{2.10}$$

First of all, we prove (cf. [5,6]) the following lemma.

Lemma 6. Suppose that M is a strictly convex spacelike hypersurface in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$. For the unit normal N pointing to the concave side of M, we have the following:

1) $$\lim_{t\to 0} \frac{1}{(\sqrt{t})^n} A_p(t) = \frac{(\sqrt{2})^n \omega_n}{\sqrt{K(p)}}, \tag{2.11}$$

2) $$\lim_{t\to 0} \frac{1}{(\sqrt{t})^{n+2}} V_p(t) = \frac{(\sqrt{2})^{n+2} \omega_n}{(n+2)\sqrt{K(p)}}, \tag{2.12}$$

3) $$\lim_{t\to 0} \frac{1}{(\sqrt{t})^n} S_p(t) = \frac{(\sqrt{2})^n \omega_n}{\sqrt{K(p)}}, \tag{2.13}$$

where ω_n denotes the volume of the n-dimensional unit ball.

Proof. Let us denote by x the column vector $(x_1, x_2, \cdots, x_n)^{\mathrm{t}}$. Then, we have from Taylor's formula of $f(x)$ as follows:

$$f(x) = x^{\mathrm{t}} A x + f_3(x), \tag{2.14}$$

where A is a symmetric $n \times n$ matrix and $f_3(x)$ is an $O(|x|^3)$ function. Then, the Hessian matrix of f at the origin is given by $(f_{ij}(0)) = 2A$. Hence, for the unit normal N to M we have from (2.5)

$$K(p) = \det\big(f_{ij}(0)\big) = 2^n \det A. \tag{2.15}$$

By the assumption, every eigenvalue of A is positive and hence, there exists a nonsingular symmetric matrix B satisfying

$$A = B^{\mathrm{t}} B, \tag{2.16}$$

where B^{t} denotes the transpose of B. Therefore, we get:

$$f(x) = |Bx|^2 + f_3(x). \tag{2.17}$$

We consider the decomposition of $S_p(t)$ as follows:

$$A_p(t) = S_p(t) + T_p(t), \tag{2.18}$$

where

$$A_p(t) = \iint\limits_{f(x)<t} 1\,\mathrm{d}x \tag{2.6}$$

and

$$T_p(t) = \iint\limits_{f(x)<t} \left(1 - \sqrt{1 - |\nabla f|^2}\right) \mathrm{d}x. \tag{2.19}$$

Now, the proof of Lemma 8 in [6] shows that (2.11) and (2.12) hold. Hence, it remains to prove (2.13). In order to prove (2.13), it suffices to show that

$$\lim_{t \to 0} \frac{1}{(\sqrt{t})^n} T_p(t) = 0. \tag{2.20}$$

Note that the following inequality holds

$$0 \le T_p(t) \le \iint\limits_{f(x)<t} \left|\nabla f(x)\right|^2 \mathrm{d}x. \tag{2.21}$$

The function f satisfies

$$\left|\nabla f(x)\right|^2 = 4|Ax|^2 + h_2(x), \tag{2.22}$$

where $h_2(x)$ is an $O(|x|^2)$ function. Thus, there exists a positive constant C satisfying in a neighborhood of the origin

$$\left|\nabla f(x)\right|^2 \le C|x|^2. \tag{2.23}$$

In the same argument as in the proof of Lemma 8 in [6], putting $t = \epsilon^2$ and $x = \epsilon y$, it follows from (2.21) and (2.23) that

$$0 \le \frac{1}{(\sqrt{t})^n} T_p(t) \le C\epsilon^2 \iint\limits_{|By|^2 + \epsilon g_3(y) < 1} |y|^2 \, \mathrm{d}y. \tag{2.24}$$

Since the integral of the right-hand side in (2.24) tends toward a constant as $\epsilon \to 0$, by letting $t \to 0$ in (2.24), we get (2.20). This completes the proof. □

3. Proofs of Theorems 3 and 5

In this section, first of all, we prove Theorem 3.

Let M be a strictly convex spacelike hypersurface in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$. We assume that for each $s \in (-\epsilon, \epsilon)$, $\epsilon > 0$, the parallel hypersurface M_s is strictly convex.

Suppose that each M_s, $s \in (-\epsilon, \epsilon)$ satisfies one of the three conditions (V), (A) and (S). Then Lemma 6 shows that for each $s \in (-\epsilon, \epsilon)$, the Gauss–Kronecker curvature $K(s)$ of the parallel hypersurface M_s is a positive constant.

Now, we prove the following lemma (cf. [8]).

Lemma 7. Suppose that M is a strictly convex spacelike hypersurface in the $(n+1)$-dimensional Minkowski space $\mathbb{L}^{n+1}$. Then the following statements are equivalent.

1) Each parallel hypersurface M_s has constant Gauss–Kronecker curvature.
2) The hypersurface M has constant principal curvatures.

Proof. At a fixed point $p \in M$, let $k_1(p), \cdots, k_n(p)$ denote the principal curvatures with principal vectors $X_1, \cdots, X_n$, which are orthonormal with respect to the metric g of M. Note that the parallel hypersurface M_s is defined by $p - sN(p)$, $p \in M$, which is a hypersurface for sufficiently small s.

With respect to the unit normal N, the shape operator L_s of M_s is given by ([8])

$$L_s = (I - sL)^{-1} L, \tag{3.1}$$

where L and I denote the shape operator of M and the identity operator, respectively. Furthermore, we have

$$L_s\big(X_i(s)\big) = \frac{k_i}{1 - sk_i} X_i(s), \tag{3.2}$$

where $\{X_i(s)\}_{i=1}^n$ are orthonormal frame with respect to the metric g_s of M_s given by

$$X_i(s) = \frac{k_i}{1 - sk_i} X_i. \tag{3.3}$$

Suppose that each parallel hypersurface M_s has constant Gauss–Kronecker curvature $K(s)$. It follows from (3.2) that

$$\prod_{i=1}^{n} \frac{k_i(p)}{1 - sk_i(p)} = K(s). \tag{3.4}$$

Hence, we have

$$\sum_{i=1}^{n} \ln k_i(p) - \sum_{i=1}^{n} \ln\bigl(1 - sk_i(p)\bigr) = \ln K(s). \tag{3.5}$$

By differentiating (3.5) with respect to s, we get:

$$\sum_{i=1}^{n} \frac{k_i(p)}{1 - sk_i(p)} = \frac{K'(s)}{K(s)}, \tag{3.6}$$

which we denote by $k(s)$. Evaluating $k(0), k'(0), \cdots, k^{(n-1)}(0)$, we obtain:

$$\sum_{i=1}^{n} k_i(p) = k(0), \quad \sum_{i=1}^{n} k_i(p)^2 = k'(0), \quad \cdots, \quad \sum_{i=1}^{n} k_i(p)^n = k^{(n-1)}(0). \tag{3.7}$$

These relations show that each principal curvature $k_i(p)$ of M is a constant.

The converse is obvious. □

It follows from Lemma 7 that each principal curvature $k_i(p)$ of M is constant, that is, M is isoparametric. Hence M has at most two distinct constant principal curvatures ([8,9]). Furthermore, if M has two distinct constant principal curvatures, then one of them is zero, which is a contradiction. Thus, we see that the constant principal curvatures are positive and equal to each other. That is, M is totally umbilic, but not totally geodesic. Therefore, it is an open part of the hyperbolic space $H^n(r)$ ([10, pp. 116–117]).

This completes the proof of Theorem 3.

Finally, we prove Theorem 5.

Let M be a strictly convex hypersurface in the $(n+1)$-dimensional Euclidean space $\mathbb{E}^{n+1}$ with respect to the unit normal N pointing to the convex side. We assume that for each $s \in (-\epsilon, \epsilon)$, $\epsilon > 0$, the parallel hypersurface M_s defined by $p + sN(p)$ for $p \in M$ is strictly convex.

Suppose that each M_s, $s \in (-\epsilon, \epsilon)$ satisfies one of the conditions (V), (A) and (S). Then, Lemma 8 in [6] shows that for each $s \in (-\epsilon, \epsilon)$, the Gauss–Kronecker curvature $K(s)$ of the parallel hypersurface M_s is a positive constant. Then, we can use Lemma 7 because it holds for hypersurfaces in an Euclidean space. Hence, the hypersurface M is isoparametric. With the same argument as above, we see that M is totally umbilic, but not totally geodesic. This completes the proof of Theorem 5.

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