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C. R. Acad. Sci. Paris, Ser. I 346 (2008) 839–844

COMPTES RENDUS MATHEMATIQUE

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Partial Differential Equations/Optimal Control

# Detecting a moving obstacle in an ideal fluid by a boundary measurement

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Received 7 January 2008; accepted 18 June 2008

Available online 25 July 2008

Presented by Gilles Lebeau

**Abstract**

In this Note we investigate the problem of the detection of a moving obstacle in a perfect fluid occupying a bounded domain in $\mathbb{R}^2$ from the measurement of the velocity of the fluid on one part of the boundary. We show that when the obstacle is a ball, we may identify the position and the velocity of its center of mass from a single boundary measurement. Linear stability estimates are also established by using shape differentiation techniques. ***To cite this article: C. Conca et al., C. R. Acad. Sci. Paris, Ser. I 346 (2008).***



**Résumé**

**Détection d'un obstacle en mouvement dans un fluide parfait à partir d'une mesure sur le bord du domaine.** Dans cette Note, on s'intéresse au problème de la détection d'un obstacle en mouvement dans un fluide parfait incompressible à partir de la mesure de la vitesse du fluide sur une partie du bord du domaine. Lorsque l'obstacle est une boule, on montre que la position et la vitesse de son centre de gravité peuvent être identifiées à l'aide d'une seule mesure. La stabilité linéaire par rapport à la mesure est prouvée par des techniques de différentiation par rapport au domaine. ***Pour citer cet article : C. Conca et al., C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

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## Version française abrégée

Dans cette Note, on étudie le problème inverse consistant à déterminer la position et la vitesse d'un obstacle en *mouvement* dans un fluide incompressible à partir d'une mesure localisée de la vitesse du fluide. La détermination de la forme d'un obstacle *fixe* dans un domaine borné $\Omega \subset \mathbb{R}^d$ a été étudiée récemment dans [2,4] et [5] pour un fluide

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1631-073X/$ – see front matter 
doi:10.1016/j.crma.2008.06.007

réel (de type Navier–Stokes) et $d = 2, 3$. Dans le cas d'un fluide parfait potentiel, la question se ramène à un problème classique (cf. [1] pour la détection de fissures en dimension deux).

Nous considérons dans cette Note un fluide parfait potentiel occupant un ouvert borné régulier $\Omega \subset \mathbb{R}^2$. Pour simplifier, on suppose que l'obstacle est la boule $B_1(h(t))$ centrée en $h(t)$ et de rayon 1. On obtient le système

$$\Delta\varphi = 0 \quad \text{dans } \Omega \setminus \overline{B_1(h(t))}, \tag{1}$$

$$\frac{\partial\varphi}{\partial n} = l \cdot n \quad \text{sur } \partial B_1(h(t)), \tag{2}$$

$$\frac{\partial\varphi}{\partial n} = g \quad \text{sur } \partial\Omega \tag{3}$$

où $v(x,t) = \nabla\varphi(x,t)$ est la vitesse du fluide, $l(t) = h'(t)$ est la vitesse du centre de la boule, et $g \in H^s(\partial\Omega)$ $(s \geqslant 0)$ est le flux sortant à travers $\partial\Omega$. On suppose que $\varphi$ est mesuré sur une partie $\Gamma_m$ de $\partial\Omega$, et on se demande si la connaissance de cette mesure à un instant donné $t$ suffit à identifier $(h, l)$. Il est clair que la donnée $g(x) = l \cdot n(x)$, où $l \in \mathbb{R}^2$ est un vecteur fixe, est à exclure, car la boule peut se mouvoir à la même vitesse $l$ que le fluide et être indiscernable. Le résultat principal de cette Note montre qu'en dehors de cette situation, l'identifiabilité est possible. On note $(e_1, e_2)$ la base canonique de $\mathbb{R}^2$, $V = \text{span}(e_1 \cdot n, e_2 \cdot n) \subset L^\infty(\partial\Omega)$, et pour tout $(h_i, l_i) \in \mathbb{R}^4$, $\varphi_i$ la solution (définie à une constante près) de (1)–(3) associée à $(h, l) = (h_i, l_i)$ et $g$.

**Théorème 0.1.** *Soient $g \in H^s(\partial\Omega) \setminus V$ avec $s > 1/2$ et $\int_{\partial\Omega} g \, \mathrm{d}\sigma = 0$, et pour $i = 1, 2$, $(h_i, l_i) \in \mathbb{R}^4$ avec $\text{dist}(h_i, \partial\Omega) > 1$. Alors*

$$\varphi_1 = \varphi_2 \quad \text{sur } \Gamma_m \quad \Rightarrow \quad h_1 = h_2 \quad \text{et} \quad l_1 = l_2.$$

La preuve du Théorème 0.1 repose sur une analyse des singularités d'un problème de Dirichlet non-homogène sur un domaine à coins.

On peut établir une estimation de stabilité linéaire pour l'application de mesure $\Lambda : (h, l, g) \mapsto \varphi_{|\Gamma_m}$. On fixe $g$ comme dans le Théorème 0.1, avec $s \geqslant 1$, et un couple $(h_0, l_0)$ dans $\mathbb{R}^4$ avec $\text{dist}(h_0, \partial\Omega) > 1$. Le résultat qui suit s'obtient par une technique de différentiation par rapport au domaine.

**Théorème 0.2.** *Il existe deux nombres $\rho > 0$, $c > 0$ tels que pour $\|(h - h_0, l - l_0)\|_{\mathbb{R}^4} < \rho$, on ait*

$$\big\|\Lambda(h, l, g) - \Lambda(h_0, l_0, g)\big\|_{H^{s+1}(\Gamma_m)/\mathbb{R}} \geqslant c\big\|(h - h_0, l - l_0)\big\|_{\mathbb{R}^4}.$$

Les preuves détaillées des résultats de cette Note figurent dans [3].

## 1. Introduction

In this Note we investigate the issue of the detection of a *moving* obstacle in an incompressible fluid filling a bounded domain in $\mathbb{R}^2$ from the measurement of the velocity of the fluid on one part of the boundary of the domain.

The problem of the detection of a *fixed* obstacle in a real fluid of Navier–Stokes type has been tackled in [2,4] and [5]. For an ideal fluid with a potential flow, the identification of fixed obstacles reduces to a classical problem (see e.g. [1] for the detection of multiple cracks). However, for a *moving* obstacle of known form with an unknown velocity, the identifiability of the position of the obstacle seems not to be a direct consequence of the unique continuation property for Laplace equation, nor of topological arguments as in [1].

For the sake of simplicity, we shall assume here that the obstacle is the ball $B_1(h(t))$ of radius one centered at the point $h(t)$, and that the fluid is ideal with a potential velocity $v(x,t) = \nabla\varphi(x,t)$. (Notice that no identification result can be expected for general Eulerian flows as in [8], because of the existence of "ghost" solutions compactly supported in space [7].) Setting $l = h'$, the system reads

$$\Delta\varphi = 0 \quad \text{in } \Omega \setminus \overline{B_1(h(t))}, \tag{4}$$

$$\frac{\partial\varphi}{\partial n} = l \cdot n \quad \text{on } \partial B_1(h(t)), \tag{5}$$

$$\frac{\partial\varphi}{\partial n} = g \quad \text{on } \partial\Omega \tag{6}$$

and we assume that $\varphi$ is measured on an open set $\Gamma_m \subset \partial\Omega$. We wonder whether only one pair $(h, l)$ may be associated with a given measurement.

Clearly, the data $g(x) = l \cdot x$, with $l \in \mathbb{R}^2$ a given fixed vector, has to be excluded, for it may lead to the situation where the ball, which is surrounded by a fluid flowing at the same velocity ($\varphi(x, t) = l \cdot x$), is not identifiable. The first main result in this Note asserts that for any data $g$ which is not of this form, the identifiability property holds true. Let $(e_1, e_2)$ denote the canonical basis in $\mathbb{R}^2$, $V = \operatorname{span}(e_1 \cdot n, e_2 \cdot n) \subset L^\infty(\partial\Omega)$, and for any $(h_i, l_i) \in \mathbb{R}^4$, let $\varphi_i$ be the solution (defined up to an additive constant) of (4)–(6) associated with $(h, l) = (h_i, l_i)$ and $g$.

**Theorem 1.1.** *Let $g \in H^s(\partial\Omega) \setminus V$ with $s > 1/2$ and $\int_{\partial\Omega} g \, \mathrm{d}\sigma = 0$, and for $i = 1, 2$, $(h_i, l_i) \in \mathbb{R}^4$ with $\operatorname{dist}(h_i, \partial\Omega) > 1$. Then*

$$\varphi_1 = \varphi_2 \quad \text{on } \Gamma_m \quad \Rightarrow \quad h_1 = h_2 \quad \text{and} \quad l_1 = l_2.$$

The proof of Theorem 1.1 rests on the analysis of the singularities for a non-homogeneous Dirichlet problem in an open set with corners.

A linear stability estimate for the measurement map $\Lambda : (h, l, g) \mapsto \varphi_{|\Gamma_m}$ can also be derived. Fix $g$ as in Theorem 1.1 with $s \geqslant 1$, and a pair $(h_0, l_0)$ in $\mathbb{R}^4$ with $\operatorname{dist}(h_0, \partial\Omega) > 1$. The following estimate is obtained in using a shape differentiation result due to Simon [9]:

**Theorem 1.2.** *There exist two numbers $\rho > 0$, $c > 0$ such that for $\|(h - h_0, l - l_0)\|_{\mathbb{R}^4} < \rho$, we have*

$$\big\| \Lambda(h, l, g) - \Lambda(h_0, l_0, g) \big\|_{H^{s+1}(\Gamma_m)/\mathbb{R}} \geqslant c \big\| (h - h_0, l - l_0) \big\|_{\mathbb{R}^4}.$$

The proofs of Theorems 1.1 and 1.2 are sketched in the next sections. Detailed proofs are given in [3].

## 2. Sketch of the proof of Theorem 1.1

We shall use in this section complex analysis, denoting the coordinates by $(x, y)$ instead of $(x_1, x_2)$, and identifying a couple $(x, y)$ of real numbers with the complex number $z = x + \mathrm{i}y$.

For $i = 1, 2$, let $\varphi_i$, $h_i$ and $l_i$ be as in the statement of Theorem 1.1, and let $B_i = B_1(h_i(t))$. Since $\varphi_1 = \varphi_2$ on $\Gamma_m$, and $\frac{\partial\varphi_1}{\partial n} = g = \frac{\partial\varphi_2}{\partial n}$ on $\Gamma_m$, we infer by unique continuation that $\varphi_1 = \varphi_2$ on $\Omega \setminus \overline{B_1 \cup B_2}$. Define a function $\varphi : \Omega \setminus \overline{B_1 \cap B_2} \to \mathbb{R}$ by

$$\varphi(x, y) := \begin{cases} \varphi_1(x, y) & \text{if } (x, y) \in \Omega \setminus \overline{B_1}, \\ \varphi_2(x, y) & \text{if } (x, y) \in \Omega \setminus \overline{B_2}. \end{cases} \tag{7}$$

If $B_1 \cap B_2 = \emptyset$, or if $l_1 = l_2$ and $h_1 \neq h_2$, then it is easily seen that $g \in V$, which is excluded. The non-trivial case is the one for which $0 < \|h_1 - h_2\| < 2$ and $l_1 \neq l_2$. An application of Green's formula gives $(l_2 - l_1) \cdot (h_2 - h_1) = 0$. After simple transformations, we may assume that $h_1 = (0, \delta) = -h_2$, with $0 < \delta < 1$, and that $l_1 = e_1 = (1, 0) = -l_2$. Thus the function $\varphi : \Omega \setminus \overline{B_1 \cap B_2} \to \mathbb{R}$ satisfies the system

$$\Delta\varphi = 0 \quad \text{in } \Omega \setminus \overline{B_1 \cap B_2}, \tag{8}$$

$$\frac{\partial\varphi}{\partial n} = g \quad \text{on } \partial\Omega, \tag{9}$$

$$\frac{\partial\varphi}{\partial n} = e_1 \cdot n \quad \text{on } \partial B_1, \tag{10}$$

$$\frac{\partial\varphi}{\partial n} = -e_1 \cdot n \quad \text{on } \partial B_2. \tag{11}$$

By Schwarz Reflection Principle [6], $\varphi$ may be extended as an harmonic function on a neighborhood of $\partial B_1$ and $\partial B_2$. If we prove that $\varphi$ can be extended as an harmonic function on $B_1 \cap B_2$, then we are done since it follows from (8)–(10) that $g \in V$. We introduce the harmonic conjugate function $\psi$, which is defined (up to a constant) by $\nabla\psi = (\nabla\varphi)^\perp$; it is analytic on the same set as $\varphi$. By an appropriate choice of the constant, $\psi$ satisfies

$$\Delta\psi = 0 \quad \text{in } D_1 := B_1 \setminus \overline{B_2}, \tag{12}$$
$$\psi = y \quad \text{on } \Gamma_1 := (\partial B_1) \setminus B_2, \tag{13}$$
$$\psi = -y \quad \text{on } \gamma_2 := (\partial B_2) \cap B_1. \tag{14}$$

A similar Dirichlet problem is satisfied by $\psi$ on $-D_1 = B_2 \setminus \overline{B_1}$, and from the uniqueness of the solution we infer that $\psi(x,-y) = \psi(x,y) = \psi(-x,y)$.

To prove that $\varphi$ has no singularity in $B_1 \cap B_2$, it is therefore sufficient to check that $\psi$ does not have any singularity in the set $B_2 \cap \{z = x + \mathrm{i}y;\ y \geqslant 0\}$. We first transform problem (12)–(14) into a Dirichlet problem in a convex cone.

We introduce the points $M_\pm = (\pm\sqrt{1-\delta^2}, 0)$ located at the intersection of the circles $\partial B_1$ and $\partial B_2$.

Let $T_1 : z \mapsto z_1 = x_1 + \mathrm{i}y_1 = (z + \sqrt{1-\delta^2})^{-1}$ denote the inversion of pole $M_- = -\sqrt{1-\delta^2}$. As $T_1$ is a Moebius transformation, it carries circles into circles or lines (see [6]). Since $M_-$ is mapped to $\infty$, $T_1$ maps $D_1$ onto a convex cone $C_1$. For notational convenience, we translate and rotate the cone $C_1$. We let $C_2 = T_2(C_1)$, where $T_2(z_1) := z_2 = x_2 + \mathrm{i}y_2 = -(z_1 - (2\sqrt{1-\delta^2})^{-1})$. Then

$$C_2 = \left\{ z_2 \in \mathbb{C}^*;\ \frac{\pi - \theta}{2} < \arg z_2 < \frac{\pi + \theta}{2} \right\},$$

where $\theta \in (0, \pi)$ stands for the angle of $C_1$ at $T_1(M_+)$, or of $\partial D_1$ at $M_+$ by conformal invariance.

Let $\psi_2(z_2) := \psi(z)$. Then $\psi_2$ solves the system

$$\Delta\psi_2 = 0 \quad \text{in } C_2, \tag{15}$$
$$\psi_2(z_2) = \frac{y_2}{(x_2 + c)^2 + y_2^2} \quad \text{on } d_{-1}, \tag{16}$$
$$\psi_2(z_2) = -\frac{y_2}{(x_2 + c)^2 + y_2^2} \quad \text{on } d_0, \tag{17}$$
$$\psi_2(z_2) \to 0 \quad \text{as } z_2 \to \infty,\ z_2 \in C_2, \tag{18}$$

where $c := -(2\sqrt{1-\delta^2})^{-1}$, and for any $k \in \mathbb{Z}$, $d_k$ denotes the half-line

$$d_k = \left\{ z_2 \in \mathbb{C}^*;\ \arg z_2 = \theta_k := \frac{\pi + (2k+1)\theta}{2} \right\}.$$

Notice that $(T_2 \circ T_1)(B_2 \cap \{y \geqslant 0\})$ is the corner $C = \{z_2 \in \mathbb{C}^*;\ \theta_0 < \arg z_2 \leqslant \pi\}$. To prove that $\psi$ does not have any singularity in $B_2 \cap \{y \geqslant 0\}$, it is therefore sufficient to check that $\psi_2$ can be extended as a harmonic function on $C$. This is done by applying several times the following reflection principle for harmonic functions proved in [3].

**Lemma 1.** *Let $\theta_0 \in \mathbb{R}$ and $\theta \in (0, \pi/2)$. Let $l_\pm = \{z \in \mathbb{C}^*;\ \arg z = \theta_0 \pm \theta\}$, and let $l_0 = \{z \in \mathbb{C}^*;\ \arg z = \theta_0\}$. Let $C_+ = \{z \in \mathbb{C}^*;\ \theta_0 < \arg z < \theta_0 + \theta\}$ (resp. $C_- = \{z \in \mathbb{C}^*;\ \theta_0 - \theta < \arg z < \theta_0\}$) be the sectors bounded by the half-lines $l_0$ and $l_+$ (resp. by the half-lines $l_-$ and $l_0$). Let $\psi$ be a harmonic function on $C_-$ such that*

$$\lim_{z \to Z,\ z \in C_-} \psi(z) = \operatorname{Im} f_-(Z) \quad \forall Z \in l_-, \tag{19}$$
$$\lim_{z \to Z,\ z \in C_-} \psi(z) = \operatorname{Im} f_0(Z) \quad \forall Z \in l_0, \tag{20}$$

*where $f_-$ (resp. $f_0$) is a holomorphic function in a neighborhood of $l_-$ (resp. on $C_- \cup l_0 \cup C_+$). Then $\psi$ can be extended as a harmonic function on the set $C_- \cup l_0 \cup C_+$, and*

$$\lim_{z \to Z,\ z \in C_+} \psi(z) = -\operatorname{Im} f_-\big(\mathrm{e}^{-2\mathrm{i}\theta} Z\big) + \lim_{z \to Z,\ z \in C_+} \operatorname{Im}\big(f_0(z) + f_0\big(\mathrm{e}^{2\mathrm{i}\theta_0}\bar{z}\big)\big) \tag{21}$$

*for each $Z \in l_+$ for which the limit in the right-hand side of* (21) *exists.*

Assume first that $\theta = \pi/(2N+1)$ for some $N \in \mathbb{N}^*$. Then one can prove by induction on $k$ that $\psi_2$ can be extended in an analytic way on the sector $d_{-1}d_k$ for each $k \leqslant N+1$, with

$$\psi_2(z_2) = \operatorname{Im}\left( \frac{1}{z_2 + c} + \sum_{l=1}^{k} \frac{2}{\mathrm{e}^{-2l\mathrm{i}\theta} z_2 + c} \right) \quad \forall z_2 \in d_k. \tag{22}$$

The first singularity encountered in the extension procedure is the point $z_2 = |c|\exp(\mathrm{i}\theta_{N+1}) \in d_{N+1}$. As it is outside $C$, we are done. If $\pi/(2N+3) < \theta < \pi/(2N+1)$ for some $N \in \mathbb{N}$, then $\psi$ can again be extended analytically on the sector $d_{-1}d_{N+1}$, hence on $C$. Therefore, the function $\psi$ does not have any singularity in $B_2 \cap \{y \geqslant 0\}$. The proof of Theorem 1.1 is complete. □

## 3. Sketch of the proof of Theorem 1.2

Let $h_0, l_0$ and $g$ be as in the statement of Theorem 1.2, and let $B_0 = B_1(h_0)$. Without loss of generality, we may assume that $h_0 = (0,0)$. Let $d\Lambda(h_0, l_0, g)$ denote the differential of $\Lambda$ at the point $(h_0, l_0, g)$, and let $L = d\Lambda(h_0, l_0, g)_{|\mathbb{R}^2\times\mathbb{R}^2\times\{0\}}$. It is clearly sufficient to prove that the map $L:\mathbb{R}^4 \to H^{s+1}(\Gamma_m)/\mathbb{R}$ is one-to-one.

If $(\hat h, \hat l) \in \mathbb{R}^4$ is given, then by a classical result due to Simon [9] we have that $L(\hat h, \hat l) = \psi_{|\Gamma_m}$, where $\psi$ denotes the solution (defined up to a constant) of

$$\Delta\psi = 0 \quad \text{in } \Omega \setminus \overline{B_0}, \tag{23}$$

$$\frac{\partial\psi}{\partial n} = 0 \quad \text{on } \partial\Omega, \tag{24}$$

$$\frac{\partial\psi}{\partial n} = -\hat h \cdot n \frac{\partial^2\varphi_0}{\partial n^2} + (\nabla\varphi_0 - l_0)\cdot \mathrm{grad}_{\partial\Omega}(\hat h \cdot n) + \hat l \cdot n \quad \text{on } \partial B_0. \tag{25}$$

In the above system, $\varphi_0$ denotes the solution of (4)–(6) with $(h,l) = (h_0, l_0)$, and $\mathrm{grad}_{\partial\Omega}$ stands for the tangential gradient, defined as $\mathrm{grad}_{\partial\Omega} f := \nabla f - (\nabla f \cdot n)n$. If the map $L$ is not one-to-one, then we can pick a pair $(\hat h, \hat l) \neq (0,0)$ such that $L(\hat h, \hat l) = 0$, i.e. $\psi_{|\Gamma_m} = Const$. Since $\frac{\partial\psi}{\partial n}_{|\Gamma_m} = 0$ and $\Delta\psi = 0$ in $\Omega \setminus \overline{B_0}$, we infer that $\psi \equiv Const$ in $\Omega \setminus \overline{B_0}$ by unique continuation. Therefore, (25) gives

$$0 = -\hat h \cdot n \frac{\partial^2\varphi_0}{\partial n^2} + (\nabla\varphi_0 - l_0)\cdot \mathrm{grad}_{\partial\Omega}(\hat h \cdot n) + \hat l \cdot n \quad \text{on } \partial B_0. \tag{26}$$

Notice that $\hat h \neq 0$, otherwise $\hat l = 0$ by (26). Let $(r,\theta)$ denote the polar coordinates with respect to the origin, and let $e_r := (\cos\theta, \sin\theta)$ and $e_\theta := e_r^\perp = (-\sin\theta, \cos\theta)$. Since $g \in H^1(\partial\Omega)$, it follows from a classical regularity result for elliptic problems that $\varphi_0 \in H^{\frac{5}{2}}(\Omega)$, hence

$$0 = \Delta\varphi_0 = \frac{\partial^2\varphi_0}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_0}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi_0}{\partial\theta^2} \quad \text{on } \partial B_0. \tag{27}$$

Using (26)–(27), we arrive to

$$\frac{\partial}{\partial\theta}\left((\hat h \cdot e_r)\frac{\partial\varphi_0}{\partial\theta}\right) = (\hat h \cdot e_\theta)(l_0 \cdot e_\theta) - (\hat h \cdot e_r)(l_0 \cdot e_r) - \hat l \cdot e_r. \tag{28}$$

Integrating with respect to $\theta$ in (28), we infer that $\hat l = \lambda \hat h^\perp$ for some constant $\lambda$, and that $\frac{\partial\varphi_0}{\partial\theta} = l_0 \cdot e_\theta + \lambda$. It follows that $\varphi_0(x) = l_0 \cdot x + \lambda\theta + Const$, and that $g \in V$, which contradicts the assumptions. □

## Acknowledgements

This work was achieved while the last author (LR) was visiting the Centro de Modelamiento Matemático at the Universidad de Chile (UMI CNRS 2807). He thanks this institution for its hospitality, and the CNRS for its support. The first author thanks the Millennium ICDB for partial support through grant ICM P05-001-F. The second author was partially supported by CONICYT-FONDECYT grant 3070040. The authors also thank the Chilean and French Governments through Ecos-Conicyt Grant C07 E05.

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