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Algebra

Double affine Hecke algebras, conformal coinvariants and Kostka polynomials

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Abstract

We study a class of representations called 'calibrated representations' of the rational and trigonometric double affine Hecke algebras of type GL_n. We give a realization of calibrated irreducible modules as spaces of coinvariants constructed from integrable modules over the affine Lie algebra $\widehat{\mathfrak{gl}}_m$. We also give a character formula of these irreducible modules in terms of a generalization of Kostka polynomials. These results are conjectured by Arakawa, Suzuki and Tsuchiya based on the conformal field theory. The proofs using recent results on the representation theory of the double affine Hecke algebras will be presented in the forthcoming papers. ***To cite this article: T. Suzuki, C. R. Acad. Sci. Paris, Ser. I 343 (2006).***

Résumé

Algèbres de Hecke affine double, co-invariants conformes et polynômes de Kostka. Nous étudions la classe de représentations, qui s'appelle les représentations calibrées, de l'algèbre de Hecke affine double rationnelle/trigonométrique de type GL_n. Nous réalisons les modules simples calibrés comme des espaces de co-invariants construits de modules intégrables sur l'algèbre de Lie affine $\widehat{\mathfrak{gl}}_m$. En plus, nous donnons une formule de caractère de ces modules simples en terme d'une généralisation des polynômes de Kostka. Ces résultats sont conjecturés par Arakawa, Suzuki et Tsuchiya en se basant sur la théorie de champs conformes. Pour leur démonstration, nous utilisons les résultats récents de la théorie des représentations de l'algèbre de Hecke affine double. Les détails seront donnés dans des publications ultérieures. ***Pour citer cet article : T. Suzuki, C. R. Acad. Sci. Paris, Ser. I 343 (2006).***

1. The affine Lie algebra and the double affine Hecke algebras

Throughout this article, we use the notation $[i, j] = \{i, i+1, \ldots, j\}$ for $i, j \in \mathbb{Z}$. Let $\mathbb{F}$ be an algebraically closed field of characteristic 0.

Fix $m \in \mathbb{Z}_{\geqslant 1}$. Let $\mathfrak{g}$ denote the Lie algebra $\mathfrak{gl}_m(\mathbb{F})$ consisting of all the $m \times m$ matrices over $\mathbb{F}$. Let $\hat{\mathfrak{g}} = \mathfrak{g} \otimes \mathbb{F}[t, t^{-1}] \oplus \mathbb{F}c$ be the associated affine Lie algebra with the central element c and the commutation relation $[a \otimes t^i, b \otimes t^j] = [a, b] \otimes t^{i+j} + \mathrm{Tr}(ab) i \delta_{i+j,0} c$ for $a, b \in \mathfrak{g},\ i, j \in \mathbb{Z}$. Put $\mathfrak{g}[t] = \mathfrak{g} \otimes \mathbb{F}[t]$ and $\mathfrak{g}[t, t^{-1}] = \mathfrak{g} \otimes \mathbb{F}[t, t^{-1}]$,

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which are Lie subalgebras of $\hat{\mathfrak{g}}$. Let $\mathfrak{h}$ denote the Cartan subalgebra of $\mathfrak{g}$ consisting of diagonal matrices. Put $\hat{\mathfrak{h}} = \mathfrak{h} \oplus \mathbb{F}c$, which gives a Cartan subalgebra of $\hat{\mathfrak{g}}$. Denote the dual space of $\mathfrak{h}$ (resp. $\hat{\mathfrak{h}}$) by $\mathfrak{h}^*$ (resp. $\hat{\mathfrak{h}}^* = \mathfrak{h}^* \oplus \mathbb{F}c^*$, where c^* is the dual vector of c). A $\hat{\mathfrak{g}}$-module M is said to be of level $\ell \in \mathbb{F}$ if the center c acts as a scalar ℓ on M. For $\kappa \in \mathbb{F}$ and $\lambda \in \mathfrak{h}^*$, let $L_\kappa(\lambda)$ (resp. $L_\kappa^\dagger(\lambda)$) denote the highest weight (resp. lowest weight) irreducible $\hat{\mathfrak{g}}$-module with highest weight $\lambda + (\kappa - m)c^*$ (resp. lowest weight $-\lambda - (\kappa - m)c^*$). We naturally identify $\mathfrak{h}^*$ with $\mathbb{F}^m$, and introduce its subspaces $X_m = \mathbb{Z}^m$, $X_m^+ = \{(\lambda_1, \ldots, \lambda_m) \in X_m \mid \lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_m\}$ and $X_m^+(\kappa) = \{(\lambda_1, \ldots, \lambda_m) \in X_m^+ \mid \kappa - m - \lambda_1 + \lambda_m \in \mathbb{Z}_{\geqslant 0}\}$. Note that $L_\kappa(\lambda)$ and $L_\kappa^\dagger(\lambda)$ are integrable for $\lambda \in X_m^+(\kappa)$, and that $X_m^+(\kappa)$ is empty unless $\kappa - m \in \mathbb{Z}_{\geqslant 0}$.

Let $V = \mathbb{F}^m$ denote the basic representation of $\mathfrak{g}$. Put $V[x, x^{-1}] = V \otimes \mathbb{F}[x, x^{-1}]$ (resp. $V[x] = V \otimes \mathbb{F}[x]$), and regard $V[x, x^{-1}]$ (resp. $V[x]$) as a $\mathfrak{g}[t, t^{-1}]$-module (resp. $\mathfrak{g}[t]$-module) through the correspondence $a \otimes t^i \mapsto a \otimes x^i$ ($a \in \mathfrak{g},\ i \in \mathbb{Z}$).

Fix $n \in \mathbb{Z}_{\geqslant 2}$. In the rest, we write $\mathbb{F}[\underline{z}]$ (resp. $\mathbb{F}[\underline{z}^{\pm 1}]$) for the polynomial ring $\mathbb{F}[z_1, \ldots, z_n]$ (resp. the Laurent polynomial ring $\mathbb{F}[z_1^{\pm 1}, \ldots, z_n^{\pm 1}]$) of n-variables. Let E denote the n-dimensional vector space with the basis $\{\epsilon_i^\vee\}_{i \in [1,n]}$. Introduce the bilinear form $(|)$ on E by $(\epsilon_i^\vee | \epsilon_j^\vee) = \delta_{ij}$. Let $E^* = \bigoplus_{i=1}^n \mathbb{F}\epsilon_i$ be the dual space of E, where ϵ_i is the dual vector of $\epsilon_i^\vee$. The natural pairing is denoted by $\langle | \rangle : E^* \times E \to \mathbb{F}$. Put $\alpha_{ij} = \epsilon_i - \epsilon_j$, $\alpha_{ij}^\vee = \epsilon_i^\vee - \epsilon_j^\vee$ and $\alpha_i = \epsilon_i - \epsilon_{i+1}$, $\alpha_i^\vee = \epsilon_i^\vee - \epsilon_{i+1}^\vee$. Then $R = \{\alpha_{ij} \mid i, j \in [1, n],\ i \neq j\}$ and $R^+ = \{\alpha_{ij} \in R \mid i < j\}$ give a set of roots and a set of positive roots of type A_{n-1} respectively. Let W denote the Weyl group associated with the root system R, which is isomorphic to the symmetric group of degree n. Denote by s_α the reflection in W corresponding to $\alpha \in R$. We write $s_i = s_{\alpha_i}$ and $s_{ij} = s_{\alpha_{ij}}$. Put $P = \bigoplus_{i \in [1,n]} \mathbb{Z}\epsilon_i$. Let $S(E)$ (resp. $S(E^*)$) denote the symmetric algebra of E (resp. E^*), which is identified with $\mathbb{F}[\underline{\epsilon}^\vee]$ (resp. $\mathbb{F}[\underline{\epsilon}]$).

Let $\kappa \in \mathbb{F}$. The *rational double affine Hecke algebra* (or the *rational Cherednik algebra*) $\mathbf{H}_\kappa$ of type GL_n is the unital associative $\mathbb{F}$-algebra which is generated by the algebras $S(E^*)$, $\mathbb{F}W$ and $S(E)$, and subjects to the following defining relations:

$$s_i h s_i = s_i(h) \quad (i \in [1, n-1],\ h \in E), \qquad s_i \zeta s_i = s_i(\zeta) \quad (i \in [1, n-1],\ \zeta \in E^*),$$
$$[h, \zeta] = \kappa \langle \zeta \mid h \rangle + \sum_{\alpha \in R^+} \langle \alpha \mid h \rangle \langle \zeta \mid \alpha^\vee \rangle s_\alpha \quad (h \in E,\ \zeta \in E^*).$$

It is known by Cherednik that $\mathbf{H}_\kappa \cong S(E^*) \otimes \mathbb{F}W \otimes S(E)$ as a vector space via the natural multiplication.

For $\lambda \in \mathbb{Z}^m = X_m$, we write $\lambda \models n$ when $\sum_{i=1}^m \lambda_i = n$ and $\lambda_i \in \mathbb{Z}_{\geqslant 0}$ for all $i \in [1, m]$. Let $\lambda \in X_m^+$ such that $\lambda \models n$. We sometimes identify λ with the Young diagram $\{(a, b) \in \mathbb{Z}^2 \mid a \in [1, m],\ b \in [1, \lambda_a]\} \subset \mathbb{Z}^2$. Let S_λ denote the irreducible module of W associated with the partition λ. Set $\Delta_\kappa(\lambda) = \mathbf{H}_\kappa \otimes_{\mathbb{F}W S(E)} S_\lambda$, where we let $S(E)$ act on S_λ through the augmentation map given by $\epsilon_i^\vee \mapsto 0$ ($i \in [1, n]$). It is known that the $\mathbf{H}_\kappa$-module $\Delta_\kappa(\lambda)$ has a unique simple quotient [4], which we will denote by $\mathcal{L}_\kappa(\lambda)$.

The *trigonometric double affine Hecke algebra* (or the *degenerate double affine Hecke algebra*) $\widetilde{\mathbf{H}}_\kappa$ of type GL_n is the unital associative $\mathbb{F}$-algebra which is generated by the algebras $\mathbb{F}P$, $\mathbb{F}W$ and $S(E)$, and subjects to the following defining relations:

$$s_i h = s_i(h) s_i - \langle \alpha_i \mid h \rangle \quad (i \in [1, n-1],\ h \in E), \qquad s_i \mathrm{e}^\eta s_i = \mathrm{e}^{s_i(\eta)} \quad (i \in [1, n-1],\ \eta \in P),$$
$$\left[h, \mathrm{e}^\eta\right] = \kappa \langle \eta \mid h \rangle \mathrm{e}^\eta + \sum_{\alpha \in R^+} \langle \alpha \mid h \rangle \frac{\mathrm{e}^\eta - \mathrm{e}^{s_\alpha(\eta)}}{1 - \mathrm{e}^{-\alpha}} s_\alpha \quad (h \in E,\ \eta \in P),$$

where e^η denotes the element of $\mathbb{F}P$ corresponding to $\eta \in P$.

It is known by Cherednik that $\widetilde{\mathbf{H}}_\kappa \cong \mathbb{F}P \otimes \mathbb{F}W \otimes S(E)$. The subalgebra $\mathbb{F}W \cdot S(E) \subset \widetilde{\mathbf{H}}_\kappa$ is called the degenerate affine Hecke algebra $\mathbf{H}^{\mathrm{aff}}$. Let $\lambda, \mu \in X_m^+$ such that $\lambda - \mu \models n$. Let $S_{\lambda/\mu}$ denote the irreducible module of $\mathbf{H}^{\mathrm{aff}}$ associated with the skew diagram λ/μ [3,5]. Set $\tilde{\Delta}_\kappa(\lambda, \mu) = \widetilde{\mathbf{H}}_\kappa \otimes_{\mathbf{H}^{\mathrm{aff}}} S_{\lambda/\mu}$. When $\lambda, \mu \in X_m^+(\kappa)$, it is known that the $\widetilde{\mathbf{H}}_\kappa$-module $\tilde{\Delta}_\kappa(\lambda, \mu)$ has a unique simple quotient [1], which we will denote by $\widetilde{\mathcal{L}}_\kappa(\lambda, \mu)$.

2. The trigonometric double affine Hecke algebra and conformal field theory

For a Lie algebra $\mathfrak{a}$ and an $\mathfrak{a}$-module M, we write $M_{\mathfrak{a}} = M/\mathfrak{a}M$. Let N be a highest weight $\hat{\mathfrak{g}}$-module of level $\kappa - m$ and let M be a lowest weight $\hat{\mathfrak{g}}$-module of level $-\kappa + m$. Define

$$\widetilde{\mathcal{C}}(N, M) = \left(N \otimes V\left[x_1, x_1^{-1}\right] \otimes \ldots \otimes V\left[x_n, x_n^{-1}\right] \otimes M\right)_{\mathfrak{g}[t, t^{-1}]}.$$

Remark 2.1. The ring $\mathbb{F}[\underline{x}^{\pm 1}] = \mathbb{F}[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$ can be identified with the coordinate ring of the affine variety $T = (\mathbb{F} \setminus \{0\})^n$, and acts on $\widetilde{\mathcal{C}}(N, M)$. Let A denote the localization of $\mathbb{F}[\underline{x}^{\pm 1}]$ at the diagonal set $\Delta = \bigcup_{i<j} \{(x_1, \ldots, x_n) \in T \mid x_i = x_j\}$. Then, it follows that the A-module $A \otimes_{\mathbb{F}[\underline{x}^{\pm 1}]} \widetilde{\mathcal{C}}(N, M)$ admits an integrable connection (called the Knizhnik–Zamolodchikov connection), and gives a vector bundle on the affine variety $T \setminus \Delta$ [1,8]. In particular, for $\lambda, \mu \in X_m^+(\kappa)$, it can be shown that the bundle $A \otimes_{\mathbb{F}[\underline{x}^{\pm 1}]} \widetilde{\mathcal{C}}(L_\kappa(\mu), L_\kappa^\dagger(\lambda))$ is equivalent to the vector bundle of conformal coinvariants (the dual of the bundle of conformal blocks) in conformal field theory (see [2, §7.3]).

In [1], an action of the algebra $\widetilde{\mathbf{H}}_\kappa$ on the space $\widetilde{\mathcal{C}}(N, M)$ was constructed through the Knizhnik–Zamolodchikov connection, and some conjectures were proposed concerning the case where N and M are integrable, that is, $N = L_\kappa(\mu)$, $M = L_\kappa^\dagger(\lambda)$ for some $\lambda, \mu \in X_m^+(\kappa)$. Conjecture 5.4.2 in [1] states that $\widetilde{\mathcal{C}}(L_\kappa(\mu), L_\kappa^\dagger(\lambda))$ is isomorphic to $\widetilde{\mathcal{L}}_\kappa(\lambda, \mu)$ when $\lambda, \mu \in X_m^+(\kappa)$ and $\lambda - \mu \models n$, and Conjecture 6.2.6 in [1] gives a decomposition of the W-invariant part $\widetilde{\mathcal{C}}(L_\kappa(\mu), L_\kappa^\dagger(\lambda))^W$ into the weight spaces with respect to the commutative subalgebra $\mathbb{F}[\underline{\epsilon}^\vee]^W$.

The irreducible $\widetilde{\mathbf{H}}_\kappa$-modules $\widetilde{\mathcal{L}}_\kappa(\lambda, \mu)$ $(\lambda, \mu \in X_m^+(\kappa))$ are studied in [7] and their structures are described explicitly in terms of tableaux on periodic skew diagrams. This combinatorial description is applied to prove these two conjectures.

Theorem 2.2. *Conjecture* 5.4.2 *and Conjecture* 6.2.6 *in* [1] *are true.*

The details of the proof will be published in the forthcoming papers. In the rest of this note, we will present the analogous statements for the rational algebra $\mathbf{H}_\kappa$. These statements for $\mathbf{H}_\kappa$ are derived from the corresponding statements for $\widetilde{\mathbf{H}}_\kappa$ through the relation between the category of $\mathbf{H}_\kappa$-modules and that of $\widetilde{\mathbf{H}}_\kappa$-modules established in [6].

3. The rational double affine Hecke algebra and character formula

Let $\kappa \in \mathbb{F}$ and $m \in \mathbb{Z}_{\geqslant 1}$. For a lowest weight $\hat{\mathfrak{g}}$-module M of level $-\kappa + m$, we set

$$\mathcal{T}(M) = V[x_1] \otimes \ldots \otimes V[x_n] \otimes M, \qquad \mathcal{C}(M) = \mathcal{T}(M)_{\mathfrak{g}[t]}.$$

Put $e_{kl}[p] = e_{kl} \otimes t^p \in \hat{\mathfrak{g}}$ for $k, l \in [1, m]$, $p \in \mathbb{Z}$, where $e_{kl} \in \mathfrak{g}$ denotes the matrix unit with only non-zero entries 1 at (k, l)-th component. For $i \in [1, n]$, define

$$\omega_i = \sum_{p \in \mathbb{Z}_{\geqslant 1}} \sum_{k,l \in [1,m]} 1^{\otimes i-1} \otimes e_{kl}[p-1] \otimes 1^{\otimes n-i} \otimes e_{lk}[-p],$$

which is an element of some completion of $U(\hat{\mathfrak{g}})^{\otimes n+1}$ and defines a well-defined operator on $\mathcal{T}(M)$.

Let $\sigma_{ij} \in \mathrm{End}_{\mathbb{F}}(\mathbb{F}[\underline{x}])$ denote the permutation of x_i and x_j. Let $\pi_{ij} \in \mathrm{End}_{\mathbb{F}}(V^{\otimes n})$ denote the permutation of i-th and j-th component of the tensor product. Note that $\mathcal{T}(M) \cong \mathbb{F}[\underline{x}] \otimes V^{\otimes n} \otimes M$ as a space, through which we regard σ_{ij} and π_{ij} as elements in $\mathrm{End}_{\mathbb{F}}(\mathcal{T}(M))$. Define the operators on $\mathcal{T}(M)$ by

$$D_i = \kappa \frac{\partial}{\partial x_i} + \sum_{j \in [1,n],\ j \neq i} \frac{1}{x_i - x_j}(1 - \sigma_{ij})\pi_{ij} - \omega_i.$$

A parallel argument as in [1, §4] implies the following:

Theorem 3.1. *Let M be a lowest weight $\hat{\mathfrak{g}}$-module of level $-\kappa + m$.*

(i) *There exists a unique algebra homomorphism $\theta : \mathbf{H}_\kappa \to \mathrm{End}_{\mathbb{F}}(\mathcal{T}(M))$ such that*

$$\theta(\epsilon_i^\vee) = D_i \quad (i \in [1, n]), \qquad \theta(\epsilon_i) = x_i \quad (i \in [1, n]), \qquad \theta(s_i) = \pi_{i\,i+1}\sigma_{i\,i+1} \quad (i \in [1, n-1]).$$

(ii) *The $\mathbf{H}_\kappa$-action on $\mathcal{T}(M)$ above preserves the subspace $\mathfrak{g}[t]\mathcal{T}(M)$. Therefore, θ induces an $\mathbf{H}_\kappa$-module structure on $\mathcal{C}(M)$.*

Define the elements u_i $(i \in [1,n])$ of $\mathbf{H}_\kappa$ by $u_i = \epsilon_i \epsilon_i^\vee + \sum_{j=1}^{i-1} s_{ji}$. It follows that $u_1, \ldots, u_n$ are pairwise commutative. Now, let $\kappa \in \mathbb{Z}_{\geqslant 1}$ and set $\Lambda_\kappa^+(m,n) = \{\lambda \in X_m^+(\kappa) \mid \lambda \models n,\ \lambda_m \geqslant 1\}$. As a rational analogue of [1, Conjecture 5.4.2], we have

Theorem 3.2. *Let $\lambda \in \Lambda_\kappa^+(m,n)$. Then $\mathcal{C}(L_\kappa^\dagger(\lambda)) \cong \mathcal{L}_\kappa(\lambda)$, and moreover it is semisimple over $\mathbb{F}[\underline{u}]$.*

Remark 3.3. It is shown in [6] that the assignment $\lambda \mapsto \mathcal{L}_\kappa(\lambda)$ induces a bijection between the set $\bigsqcup_{m\in[1,\kappa]} \Lambda_\kappa^+(m,n)$ and the set of isomorphism classes of finitely generated irreducible $\mathbf{H}_\kappa$-modules which are semisimple over $\mathbb{F}[\underline{u}]$ and locally nilpotent for the subalgebra $\sum_{i\in[1,n]} \epsilon_i^\vee \mathbb{F}[\underline{\epsilon}^\vee]$. In particular, any irreducible module of this class is realized as the space $\mathcal{C}(L_\kappa^\dagger(\lambda))$ of coinvariants for some $\lambda \in \Lambda_\kappa^+(m,n)$.

For an $\mathbf{H}_\kappa$-module M, put $M^W = \{v \in M \mid wv = v\ \forall w \in W\}$, which we call the symmetric part (or the spherical subspace) of M, and regard as an $\mathbb{F}[\underline{u}]^W$-module. Recall that $\mathbf{H}_\kappa$ has the grading operator $\partial := \kappa^{-1} \sum_{i=1}^n u_i \in \mathbb{F}[\underline{u}]^W$, satisfying $[\partial, \epsilon_i] = \epsilon_i$, $[\partial, \epsilon_i^\vee] = -\epsilon_i^\vee$ and $[\partial, w] = 0$. Analogously to [1, Corollary 6.2.6], a decomposition of $\mathcal{L}_\kappa(\lambda)^W$ $(\lambda \in \Lambda_\kappa^+(m,n))$ into weight spaces with respect to the algebra $\mathbb{F}[\underline{u}]^W$ is described explicitly. This gives a simple and remarkable formula for the trace $\mathrm{Tr}_{\mathcal{L}_\kappa(\lambda)^W} q^\partial$, which we will describe in the sequel.

Let λ be a partition of n and let T be a standard tableau on the Young diagram corresponding to λ; namely, T is a bijection from the diagram λ (viewed as a subset of $\mathbb{Z}^2$) to the set $[1,n]$ which is strictly increasing in both row and column directions. Define $\lambda_T^{(i)}$ $(i \in [1,n])$ as the element of X_m^+ corresponding to the Young diagram $T^{-1}([1,i]) \subset \mathbb{Z}^2$. For $\ell \in \mathbb{Z}_{\geqslant 1}$, a standard tableau T is called an *ℓ-restricted standard tableau* if $\lambda_T^{(i)} \in X_m^+(\ell + m)$ for all $i \in [1,n]$. Let $\mathrm{St}_{(\ell)}(\lambda)$ denote the set of ℓ-restricted standard tableaux on λ. For $T \in \mathrm{St}_{(\ell)}(\lambda)$ and $i \in [1,n]$, define

$$h_i(T) = \begin{cases} 1 & \text{if } a < a', \\ 0 & \text{if } a \geqslant a', \end{cases} \quad \text{where } (a,b) = T^{-1}(i),\ (a',b') = T^{-1}(i+1), \tag{1}$$

and define

$$\check{K}^{(\ell)}_{\lambda\,(1^n)}(q) = \sum_{T\in \mathrm{St}_{(\ell)}(\lambda)} q^{\sum_{i=1}^n (n-i) h_i(T)}. \tag{2}$$

The polynomial given by $K^{(\ell)}_{\lambda\,(1^n)}(q) := \check{K}^{(\ell)}_{\lambda'\,(1^n)}(q)$ is called the ℓ-restricted Kostka polynomial associated with the partitions λ and (1^n), where λ' is the conjugate of λ.

Theorem 3.4. *Let $m, \kappa \in \mathbb{Z}_{\geqslant 1}$ and $\lambda \in \Lambda_\kappa^+(m,n)$. Then*

$$\mathrm{Tr}_{\mathcal{L}_\kappa(\lambda)^W} q^\partial = \frac{q^{\frac{1}{2\kappa}\sum_{i=1}^m \lambda_i(\lambda_i - 2i + 1)}}{(1-q)(1-q^2)\cdots(1-q^n)} \check{K}^{(\kappa - m)}_{\lambda\,(1^n)}(q).$$

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