The Seventh Conference on Information Theory and Complex Systems
TINKOS 2019

BOOK OF ABSTRACTS

Belgrade, Serbia, October 15-16, 2019

Mathematical Institute of the Serbian Academy of Sciences and Arts
Institute of Physics Belgrade

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Editor: Velimir Ilić

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МИНИСТАРСТВО ПРОСВЕТЕ, НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА

The conference is organized under auspices of the Ministry of Education, Science and Technology Developmentof the Republic of Serbia. TINKOS 2019 is particularly grateful to the project No. 451-03-019632017-0904, Secure and energy efficient distributed source coding for sensor networks, "Pavle Savic - Partenariat Hubert Curien" for supporting this event.

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A Stochastic Theory of Wavelets

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Keywords

stochastic processes; time operator; multiresolution; real and p-adic analysis

Summary

Wavelet (in French *ondolette*) is a term originating from Roger Balian, that was finally adopted by Jean Morlet [1, 2]. It implies a function generating base for decomposition of the finite energy signals both in spatial and in frequency domain concurrently. Given a wavelet ψ, the base of $L^2(\mathbb{R})$ is generated through translations and dilatations $\psi_{j,k}(x) = \psi(2^j x - k)$ whereby the integers j and k indicate spatial position and dyadic scale of a basic element. Their emergence corresponds to the base proposed by Alfréd Haar in the doctorial thesis under Hibert's supervision (1909) and his paper published in the *Mathematische Annalen* [3]. The Haar wavelet $\psi(x) = \begin{cases} 1 & 0 < x < 1/2 \\ -1 & 1/2 < x < 1 \end{cases}$ generates a complete orthonormal system of compact support which is not regular in terms of continuous differentiability [4]. Succeeding precursors to wavelets include the Franklin orthonormal system (1927), the Littlewood-Paley theory (1930), the Calderon identity (1960), a modification of the Franklin base given by Strömberg (1981), the Gabor atoms in signal processing (1946), subband coding (1975), pyramidal algorithms (1982), zero-crossings (1982), spline approximations, the Rokhlin multipole algorithms (1985), refinement schemes in computer graphics, coherent states in quantum mechanics, and renormalization in quantum field theory [2]. Construction of wavelets that have compact support and arbitrary high regularity (1988) is ultimately done by Ingrid Daubechies [5].

Independent of the other theories, Karl Gustafson et al. developed a view in which wavelets are regarded to be stochastic processes [6]. The context arose naturally from the time operator formalism of statistical mechanics. Gustafson and Misra looked at models for the decay of quantum particles, having realised that regular stationary stochastic processes imply multiresolution property which was an indication of the time operator [7].

The wavelet theory received a key impetus from interest by mathematicians and physicists cooperating with geologists from the oil companies. In particular, the wavelet transform was developed by Grossmann and Morlet who was the geologist having suggested that seismic traces should be analyzed by translations and dilatations of a suitable function [8]. Grossmann was a theoretical physicist and mentor of investigating coherent states by Ingrid Daubechies wherein wavelets have also emerged, although in her study there was no relation to multiresolution and stochastic processes [9]. In that respect, the quantum theory indubitably played a significant role concerning wavelets [6].

Due to Meyer and Mallat, multiresolution analyses has become an essential tool in exploring wavelets [10, 11]. It corresponds to nested subspaces $\mathcal{A}_j$ of $L^2(\mathbb{R})$ satisfying axioms among which a central one is the property $f(\cdot) \in \mathcal{A}_j \Leftrightarrow f(2\cdot) \in \mathcal{A}_{j+1}$. $\mathcal{A}_j$ is termed the *approximate subspace*, whilst its orthocomplement $\mathcal{D}_j$ such that $\mathcal{A}_{j+1} = \mathcal{A}_j \oplus \mathcal{D}_j$ is the *detail subspace* of a multiresolution. The structure is intimately related to that of the Kolmogorov automorphisms, which belong to the framework of regular stationary stochastic processes [12]. A prime example is induced by the Baker map $B(x, y) = \begin{cases} (2x, y/2) & x < 1/2 \\ (2x-1, y+1/2) & x > 1/2 \end{cases}$ which is a measure preserving transformation of the unit square. The time operator of the Kolmogorov system governed by the evolution $Vf(x) = f(Bx)$ has been explicitly constructed [13].

Given the evolutionary operator V of a system, the time is defined to be the operator T satisfying $[T, V] = V$, i.e., $[T, V^t] = tV^t$. If the evolution $V^t f(x) = f(B^t x)$ is induced by a measure preserving group B^t, it is equivalent to the uncertainty relation $[T, L] = iI$ whereat

[*] The first author is supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia through the projects OI 174014 and III 44006

[†] The second author is supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia through the project OI 179048.

$V^t = e^{-iLt}$, i.e., the Liouvillian L is an infinitesimal generator of $V^{t\dagger} = e^{iLt}$ in regard to the Stone theorem. Since wavelets on the real line are not related to preservation of any finite measure, one requires reducing their domain onto the interval $\mathbb{I} = [0, 1]$ which is done through periodization $\tilde{\psi_{j,k}}(x) = \sum_l \psi_{j,k}(x + l)$ [14]. A multiresolution on the interval $\mathbb{I} = [0, 1]$ corresponds to the Renyi map R inducing the exact system governed by the evolutionary operator $Uf(x) = f(Rx)$. It is extended naturally to the Kolmogorov system $Vf(x) = f(Bx)$ induced by the Baker map that is measure preserving. The time operator of the system U is determined by the Haar wavelets due to the eigenequation $T\psi_{j,k} = j\psi_{j,k}$ having natural extension to that of V [12].

In that manner, detail subspaces of the multiresolution analyses are regarded to be the age eignestates wandering in terms of the evolution. However, only the Haar wavelet constitutes a multiresolution on the interval since it is undisturbed by periodization. Other ones satisfy the multiresolution property just approximately considering that they are partially localized in the period. Nevertheless, the wavelet domain hidden Markov model concerning statistics of the detail coefficients fits as well to all of them [15]. The Markovian structure $\mathbf{S} = (S_{j,k})$, composed by hidden variables of the model, represents causal states whose informational content $H(\mathbf{S})$ is termed to be the global complexity of a system. It indicates an increase of local complexity $H(S_{j,k})$ in the temporal domain corresponding to eigenvalues j of the time operator T, which is the definition of self-organization by Shalizi [16]. The complexity is proven to be a measure of the decomposition optimality, which is also evident by a superior denoising related to the optimal wavelet [17].

The statistical model regards a signal and its coefficients to be random realizations, which is achieved through natural extension of the unilateral shift U to the bilateral one V. It actually maps a unilateral sequence of binary digits $.i_0 i_1...$ from $\mathbb{I}$, which is the domain of R, to the bilateral string $...i_1 i_0.i_{-1}...$ that represents an element of $\mathbb{I} \times \mathbb{I}$ whereon B acts shifting the representation right. Such a shift corresponds to division by 2 in terms of dyadic numbers whose only multiresolution analysis is the Haar one, although there are many other wavelets generating it [18]. In that manner, dyadic analyses should dissolve the problem concerning a lack of the multiresolution property due to periodizing wavelets on the interval. A usage of p-adic probabilities on the other hand makes irrelevant the problem of positivity preservation which is the main discordance between multiresolution analyses and stochastic processes [6]. The negative probabilities – that correspond to their stabilization in a p-adic norm – is crucial contradistinction of quantum and classical viewpoints [19]. Considering that, the quantum theory plays once again a major role in conjunction to the p-adic numbers whose interrelationship should be elucidated by the wavelet theory which is regarded to be a p-adic spectral analysis [20].

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CIP - Каталогизација у публикацији
Народна библиотека Србије, Београд

519.72(048)(0.034.2)
519.876(048)(0.034.2)
519.21(048)(0.034.2)
51-7:53(048)(0.034.2)

CONFERENCE on Information Theory and Complex Systems (7 ; 2019 ; Beograd)
Book of abstracts [Elektronski izvor] / The Seventh Conference on Information Theory and Complex Systems, TINKOS 2019, Belgrade, Serbia, October 15-16, 2019 ; editor Velimir Ilić. - Belgrade : Mathematical Institute SASA, 2020 (Belgrade : Mathematical Institute SASA). - 1 elektronski optički disk (CD-ROM) ; 12 cm

Sistemski zahtevi: Nisu navedeni. - Nasl. sa naslovne strane dokumenta. - Tiraž 50. - Bibliografija uz svaki apstrakt.

ISBN 978-86-80593-68-5

а) Теорија информација -- Апстракти
б) Теорија система -- Апстракти
в) Стохастички процеси -- Апстракти
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COBISS.SR-ID 27401737

Publisher: Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia.

Printed by Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia.

Number Of Copies: 50. **ISBN: 978-86-80593-68-5** . **Publishing Year:** 2020.