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MSc THESIS

VoLTE: Fundamentals and Investment under Uncertainty by analogy with the Real Options Theory – A real case application in Greek Telecommunications market

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ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

VoLTE: Βασικές αρχές και Επένδυση υπό Αβεβαιότητα σε αναλογία της Θεωρίας των Πραγματικών Δικαιωμάτων Προαίρεσης – Εφαρμογή στην Ελληνική Αγορά Τηλεπικοινωνιών

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ABSTRACT

With the emergence of Telecommunications, the voice broke all physical borders and could be transferred worldwide in real-time. Soon, voice services became a core business for the providers and the trigger for the development of Fixed and Mobile Telecommunications. The state of the art for mobile voice delivery is Voice over LTE (VoLTE), which is an important network capability to significantly improve the service performance and radio capacity while reducing operating costs. This study thesis presents the fundamental principles of VoLTE architecture and provides an analysis of the VoLTE solution as an investment opportunity.

In the first part, a historical review is given regarding the evolution of the Cellular Mobile Telecommunication systems since their first generation. An overview of the VoLTE architecture is included with an analysis of the main subsystems and the core components based on the technical specifications. Moreover, the basic functionalities of the VoLTE technology are presented. The next part describes the benefits and challenges of deploying the VoLTE solution from technical and market perspectives. In order to mitigate the risks, the Real Options theory from the financial market is introduced for evaluating the VoLTE investment according to modern literature. The VoLTE demand is modelled using the Geometric Brownian Motion process and the dynamic programming is used to structure a Real Options-based framework for calculating optimal investment rules and opportunity cost. In the sixth chapter, a real case application of the proposed framework in the Greek Mobile Telecommunications market is presented. The results are compared with the traditional tools and analyzed by performing Monte Carlo simulations. Conclusions and interesting insights are provided in the last chapter.

SUBJECT AREA: VoLTE Investment under Uncertainty

KEYWORDS: VoLTE, Real Options, Geometric Brownian Motion, Dynamic Programming, Monte Carlo simulation.

ΠΕΡΙΛΗΨΗ

Με την εμφάνιση των Τηλεπικοινωνιών, η φωνή έσπασε όλα τα φυσικά όρια και έγινε πραγματικότητα η μετάδοσή της παγκοσμίως σε πραγματικό χρόνο. Σύντομα, οι υπηρεσίες φωνής μετατράπηκαν σε βασική δραστηριότητα για τους παρόχους και το έναυσμα για την ανάπτυξη των Σταθερών και Κινητών Τηλεπικοινωνιών. Η τελευταία λέξη της τεχνολογίας για την Κινητή μετάδοση φωνής είναι το Voice over LTE (VoLTE), το οποίο αποτελεί μια σημαντική δυνατότητα του δικτύου για τη ουσιαστική βελτίωση της απόδοσης της φωνής και της χωρητικότητας του ραδιοδικτύου με ταυτόχρονη μείωση στα λειτουργικά κόστη. Στην παρούσα διπλωματική εργασία παρουσιάζονται οι βασικές αρχές της αρχιτεκτονικής VoLTE και παρέχεται μία ανάλυση της τεχνολογίας VoLTE ως επενδυτική ευκαιρία.

Στο πρώτο μέρος παρατίθεται μια ιστορική αναδρομή σχετικά με την εξέλιξη των συστημάτων φωνής των Κινητής Τηλεπικοινωνιών από την πρώτη γενιά έως σήμερα. Περιλαμβάνεται μια επισκόπηση της αρχιτεκτονικής VoLTE με ανάλυση των κύριων υποσυστημάτων και των βασικών τους στοιχείων με βάση τις τεχνικές προδιαγραφές. Επιπλέον, παρουσιάζονται οι βασικές λειτουργίες μετάδοσης φωνής μέσω της τεχνολογίας VoLTE. Στο επόμενο μέρος περιγράφονται τα οφέλη και οι προκλήσεις που γεννιούνται από την ανάπτυξη της λύσης VoLTE, από τεχνικής άποψης αλλά και από πλευράς αγοράς. Για τον μετριασμό των κινδύνων, προτείνεται από τις χρηματοπιστωτικές αγορές η θεωρία των για την αξιολόγηση της επένδυσης, με βάση τη σύγχρονη βιβλιογραφία. Η ζήτηση της υπηρεσίας VoLTE μοντελοποιείται χρησιμοποιώντας τη γεωμετρική κίνηση Brown και αναπτύσσεται μια μεθοδολογία βασισμένη στα πραγματικά δικαιώματα προαίρεσης μέσω του δυναμικού προγραμματισμού, για τον υπολογισμό των βέλτιστων επενδυτικών κανόνων και του κόστους ευκαιρίας. Στο έκτο κεφάλαιο, παρουσιάζεται ένα παράδειγμα πραγματικής περίπτωσης επένδυσης VoLTE στην ελληνική αγορά Κινητών Τηλεπικοινωνιών, χρησιμοποιώντας την προτεινόμενη μεθοδολογία των πραγματικών δικαιωμάτων προαίρεσης. Τα αποτελέσματα συγκρίνονται με την παραδοσιακή προσέγγιση και αναλύονται με την χρήση προσομοιώσεων Monte Carlo. Συμπεράσματα και ενδιαφέροντα ευρήματα παρέχονται στο τελευταίο κεφάλαιο.

ΘΕΜΑΤΙΚΗ ΠΕΡΙΟΧΗ: Επένδυση VoLTE υπό αβεβαιότητα

ΛΕΞΕΙΣ ΚΛΕΙΔΙΑ: VoLTE, Πραγματικά Δικαιώματα Προαίρεσης, Γεωμετρική κίνηση Brown, Δυναμικός προγραμματισμός, Προσομοίωση Monte Carlo.

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PROLOGUE

The present Master of Science Thesis was carried out in Athens in the context of my attendance in the Interdisciplinary Postgraduate Degree Program in “Management and Economics of Telecommunication Networks” of the Informatics and Telecommunications department of National and Kapodistrian University.

The thesis is to the best of my knowledge original except where acknowledgements are made to previous work or technical standards.

1. INTRODUCTION

Telecommunications is “the science and technology of communication at a distance by electronic transmission of impulses, as by telegraph, cable, telephone, radio, or television” [1]. For many years, the Telecommunications have been the key driver for the global economic development and the business processes. Especially the voice services triggered positive changes and remained for decades the dominant business for the Telecom Operators. According to GSMA (2020) [2], in 2019 the contribution of mobile industry to the Gross Domestic Product (GDP) globally was 4.1$ trillion.

Technology is evolving, and more services have been launched to meet the extended needs of the subscribers and the industries. Moreover, the increased market competition and the unpredictability of the demand created uncertainty into the market, which led Telecom Operators to upgrade and optimize their networks and seek for more cost-efficient and enhanced solutions.

Currently, the most advanced Cellular Mobile systems support broadband services of fourth-generation releases and beyond over packet-switched network architectures. The cellular voice service may not be the leading service anymore, but because it remains one of the cores, it has to evolve and adapt to the rapidly changing environment.

Technical working groups, specialized in cellular technology, have designed different mobile generations to cope with the increasing demand for better communication and higher speech quality. Also, multiple features and capabilities have been defined, while network operators invest considerable funds to adopt the most innovative and profitable solutions before their competitors.

Speech is the easiest and singularly the most efficient way for humans to intuitively convey and share thoughts and ideas to other people. Thus, the contribution of Telecommunications to social and economic development has become widely acknowledged. The first generation (1G) mobile Telecommunication system was introduced in the early 1980s. It was an analogue voice-only transmission system, used mostly by exclusive business customers due to the expensive and sizable cell phones. Its capacity was limited, and frequency utilization wasn’t efficient, as the channels for the user traffic were few, and the cells were quite large, about 10 to 20 km [3].

In 1982, pan-European mobile technology introduced the second generation (2G) digital system, called Global System for Mobile Communications (GSM), which became the most popular system for 2G technology worldwide. The first GSM network was launched in 1991 from Radiolinja in Finland. GSM had better spectrum usage than 1G systems, offered digitally encrypted calls at the air interface and introduced circuit-switched (CS) data, text and multimedia messages. Additionally, with the 2.5 General Packet Radio Services (GPRS) and 2.75 Enhanced Data Rates for GSM Evolution (EDGE) generations, we had for the very first time data transfer up to 384 Kbit/s over an IP-based packet-switched core network through the same 2G base stations. Meanwhile, more 2G systems were developed, like CDMA (IS-95), designed by Qualcomm, which was deployed mainly in the USA as cdmaOne, and it became the base air technology for the next generation.

Due to the advent of the Internet and the foreseen demand for both enhanced voice and high-speed data services, the International Telecommunication Union (ITU) started the research program IMT-2000 (International Mobile Telecommunications-2000) in 1992, in order to define the requirements for the global third-generation technology (3G). However, any system did not fulfill the technical expectations until 1999, with the introduction of the Universal Mobile Telecommunication System (UMTS) by the 3rd Generation Partnership Project (3GPP)/Release 99. UMTS introduced a new radio access technology, the W-CDMA (Wideband Code Division Multiple Access), designed by Ericsson, which utilized the same circuit-switched core network for the voice services and the same packet-switched core network for the IP-based data traffic as GSM. This fact allowed the maintenance and development of the existing GSM/GPRS networks, along with the introduction of the High-Speed Packet Access (HSPA) for 3,5G and the Evolved High-Speed Packet Access (HSPA+) for 3.75G, which could achieve data rates up to 42.2 Mbit/s. UMTS was used primarily in Europe, while in the USA they deployed CDMA2000 as a 3G equivalent system, the successor of cdmaOne [4].

In the second half of the 2000s, it was evident that mobile broadband data would be the future dominant service in the telecom world. Based on GSMA, there were over 120 WCDMA, 85 HSPA networks and 2.7 billion mobile subscriptions until 2006 in more than 50 countries [5]. The trends from 3G data services, Internet and Wi-Fi networks showed an explosive bandwidth demand for the next decades. In 2007, “smartphones” were launched and adopted fast from the market, while GSM and UMTS networks started to get overloaded and met design limitations.

Again, ITU began a study group to define the new standards set for the following generation networks, called IMT-Advanced (International Mobile Telecommunications-Advanced). IMT-Advanced was published in November of 2008, defining the requirements for the fourth generation systems (4G) (Table 1). Some of the key features of the 4G architecture evolution were the enhanced peak data rates, the low latency, the interworking with other radio access systems and the roaming capabilities.

Table 1: IMT-Advanced Requirements

Item	IMT-Advanced
Peak Data Rate (DL)	1 Gbps
Peak Data Rate (UL)	500 Mbps
Spectrum Allocation	>40 MHz
Latency (User Plane)	10 ms
Latency (Control Plane)	100 ms

In December of 2008, 3GPP introduced Release 8 for the Evolved Packet System (EPS) specifications, an all-IP based packet-switched network for broadband mobile services. EPS specifications include two new items, the radio access network, called Evolved UMTS (E-UTRAN) for the Long Term Evolution (LTE) of UMTS, and the packet core network, called Evolved Packet Core (EPC). EPS, commonly referred to as just LTE, was compatible with other 3GPP and non-3GPP networks, while interoperating with legacy CS domain for messaging and voice calls, through Circuit-Switch Fallback (CSFB) feature. However, 3GPP Release 8 EPS didn’t meet all the IMT-Advanced requirements and was considered a 3.9 generation system.

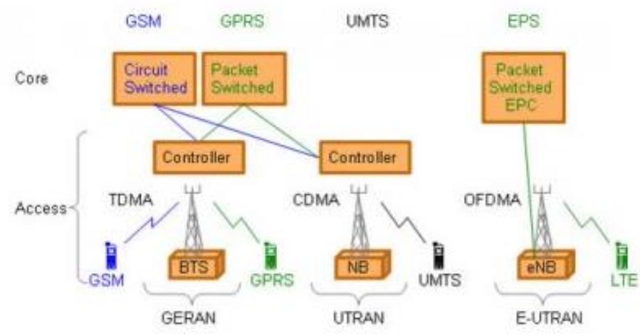

Figure 1: Network Solutions from GSM to LTE [6]

In 2010, mobile data took the leadership on telecom traffic. In the same year and upon the "One Voice" initiative for an IP Multimedia subsystem (IMS) voice-based LTE network, 3GPP and GSMA released and specified the new LTE-Advanced (LTE-A) Release 10, the first standard release complied with ITU's advanced requirements for Fourth Generation systems. The LTE-A remained an all-IP network which was interworking with the IMS domain to provide cellular Voice over LTE (VoLTE) service [7].

VoLTE architecture is state of the art for cellular mobile voice delivery. The first working trial for the VoLTE network occurred in 2012 in Dallas, Texas, from MetroPCS Operator and the first commercial launch in South Korea in the same year. According to GSA, at the end of December 2021, 282 Operators are investing in VoLTE architecture of which 227 have already launched commercially in 109 countries [8].

1.1 Motivation

Currently, there is a large interest in Voice over LTE technology by the Telecom Operators. Overall, the voice service is perceived as an important field for the Operators due to the basic and vital functionality it serves to the people. However, before starting to invest heavily to enhance the voice network, a project evaluation and a risk analysis should be performed in the context of the decision-making process regarding the technology readiness, the deployment challenges and the financial benefits.

The motivation for conducting a study on the topic of Voice over LTE was to perform an extensive research on this state of the art technology specifications and requirements; moreover to investigate the impact of uncertainty in Telecommunication projects and study new investment evaluation techniques according to the modern approach of the Real Options theory.

1.2 Objectives and Contribution

The main objectives of this thesis are on the one hand to provide a clear view of the VoLTE Technology principles and analyze the benefits and challenges that this service offers. On the other hand to develop a tool that fits the VoLTE Telecommunication project and with which the optimal investment rules can be acquired.

As a result of this work, the fundamentals of the technology and its basic functionalities are presented in a concise but detailed way based on the technical specifications. Additionally, an investment evaluation tool based on the Real Options Approach (ROA) is built, which can provide project's optimal investment rules. The tool is applied in a real-world example in the Greek Telecommunications market. Its precision and contribution to uncertainty management are compared with the existing traditional NPV tools and both performances are evaluated with the help of a Monte Carlo simulation. Finally, we are optimistic that this study becomes one-step forward for further analysis of investments in VoLTE solution based on ROA.

1.3 Structure of the thesis

The thesis is structured as follows:

Second chapter provides an overview of the VoLTE solution and the high level architecture based on the technical specifications. The three main subsystems of the VoLTE technology are defined. For each of these subsystems, the core components are analyzed. At the end of the chapter, the basic functionalities of the VoLTE service are presented and specifically the LTE Attach procedure, the IMS Registration procedure, the Mobile Originating Call and the IMS Deregistration.

Third chapter presents the VoLTE solution as an investment opportunity discussing the challenges and opportunities facing Mobile Operators from market and technical points of view.

Fourth chapter introduces the Real Options theory from financial markets to embrace some important characteristics into the project evaluation. More precisely, these characteristics correspond to the uncertainty, the irreversibility and the flexibility that both options and investments actually face. While the Real Options are used to identify the uncertainty, the Geometric Brownian Motion is proposed to capture the risk as a suggested stochastic modeling tool.

In Fifth chapter, a proposed framework is deployed for the calculations of the optimal investment rules and the opportunity costs based on the Real Options theory in the context of valuing the Entry and Exit strategies and the Sudden Death Option. The dynamic programming methodology is used, incorporating the Geometric Brownian Motion process to model the stochastic variable of this scenario which is the service demand.

Sixth chapter provides an application of the proposed Real Options framework for a VoLTE investment in the Greek Mobile Telecommunications market. To demonstrate the different logic and the benefits offered by the Real Options framework the traditional approach is applied for the same project and with the same variables in the context of investment analysis. The results are compared and a Monte Carlo simulation is performed to evaluate the performance of both approaches.

Finally, Conclusions are drawn in chapter Seventh.

2. Voice over LTE - STATE OF THE ART

In this chapter, the fundamentals of the VoLTE concept are given based on published technical specifications and guidelines. The VoLTE solution is mainly built on three domains and more precisely the LTE, the EPC and the IMS. In addition, the core nodes of these subsystems and their main tasks are analyzed. At the end of this chapter, the basic functionalities of the VoLTE service are presented.

2.1 Overview

Long Term Evolution (LTE) was an iteration of the GSM and UMTS systems. However, in contrast with the previous generations of 2G and 3G, it was designed to support only Packet-Switch services through the EPC network (Figure 2). The evolution of LTE, LTE-Advanced (LTE-A) specified in Release 10, targets increasing capacity and offering heavy mobile data rates. LTE-A, mostly known as the Fourth Generation Technology (4G), fulfilled the IMT-Advanced requirements set by ITU and introduced **VoLTE** as the solution for the voice services.

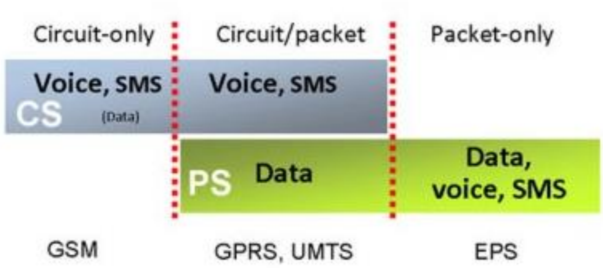

Figure 2: Circuit vs. Packet system domains [9]

LTE-A and EPC were designed to establish data pipes, which are called bearers, and, regardless of the application or the content, to transport the data packets to the destination over IP streams. Voice over LTE solution relies on these characteristics to deliver voice services over EPS (Figure 3).

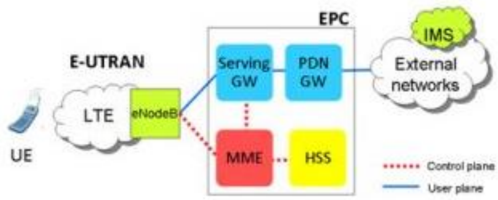

Figure 3: Basic EPS architecture with E-UTRAN access [9]

VoLTE is a standard-compliant solution of LTE-A architecture, which utilizes an IP Multimedia subsystem (IMS) over the packet-switched EPC network. It is comprised of two types of traffic; the Session Initiation Protocol (SIP) based traffic and the Real-Time Protocol (RTP) based traffic. The EPC is used to transfer signalling and user plane traffic as IP packets over Internet protocols towards the IMS. IMS is neither part of LTE nor EPC but a separate external network, defined a long time before the LTE, in 2002. However, the solution places IMS functional elements at the centre of the 4G LTE Voice Core network.

IP Multimedia Subsystem (IMS) is a subsystem for delivering advanced multimedia services and applications. It was initially introduced in 1999 as an access-agnostic core network called 3G.IP. Later in 2002, 3GPP Release 5 defined IMS as the supportive GPRS network for real-time multimedia services, like instant messaging. The specifications attracted a great deal of interest and were later enhanced to support more access technologies. Although there were some early implementations, mainly in fixed networks, mobile Operators initially concluded that there were no viable business cases to justify the expenses of deploying an IMS. The 3G circuit-switched domain could handle voice and video calls so the IMS would be limited to peripheral services.

As stated previously, the project "One Voice" designed LTE to support only packet services without any circuit-switched functionality, which was adopted by many study groups, like 3GPP and GSMA. The goal for the voice was to be transported over IP, which was the ideal application to adopt the IMS solution. IMS was introduced to carry voice signalling and audio packets natively over the EPS network. Therefore, it was proposed as the long-term approach for the Voice over LTE core network without being part of the LTE system. Voice services through 2G and 3G circuit-switched systems were still possible for the non-VoLTE users, based on the CSFB feature, and for the VoLTE users in low LTE coverage areas, with the Handover capability

By those specifications, IMS mainly provides the service control layer for VoLTE services. IMS is defined by a complex set of specifications applying the SIP. The EPC doesn't process the SIP signalling messages that travel between the device and the IMS, and it doesn't need to (Figure 4). It transfers them on top of the LTE user plane through a default EPS bearer, which it is installed before the device registers to the IMS. The EPC also transfers the device's voice traffic using the RTP, through a dedicated EPS bearer, which it is set up at the begging of the call and it is turned down at the end of it [10].

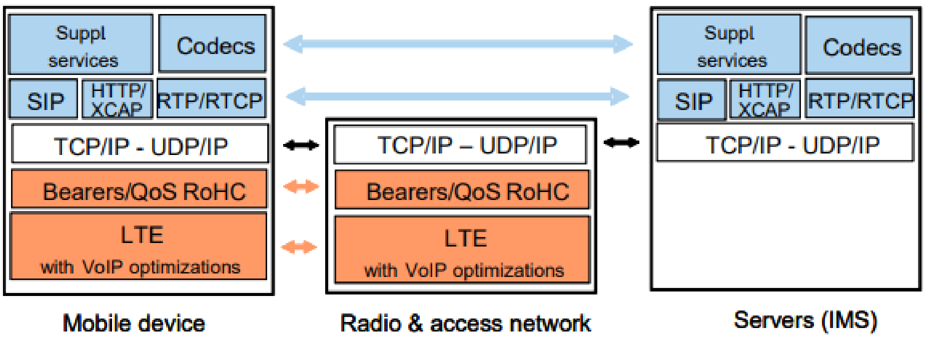

Figure 4: Depiction of UE and Network Protocol Stacks in IMS Voice Profile [11]

2.2 High Level Architecture

In Figure 5, the high-level architecture of the VoLTE deployment is being presented. The complete solution for the VoLTE service consists of three main domains:

- the LTE access radio,
- the EPC data core network,
- the IMS domain.

The main components of these subsystems are depicted logically with the related interfaces below; however, vendors can combine different functional nodes into a single physical node and implement the interfaces as internal.

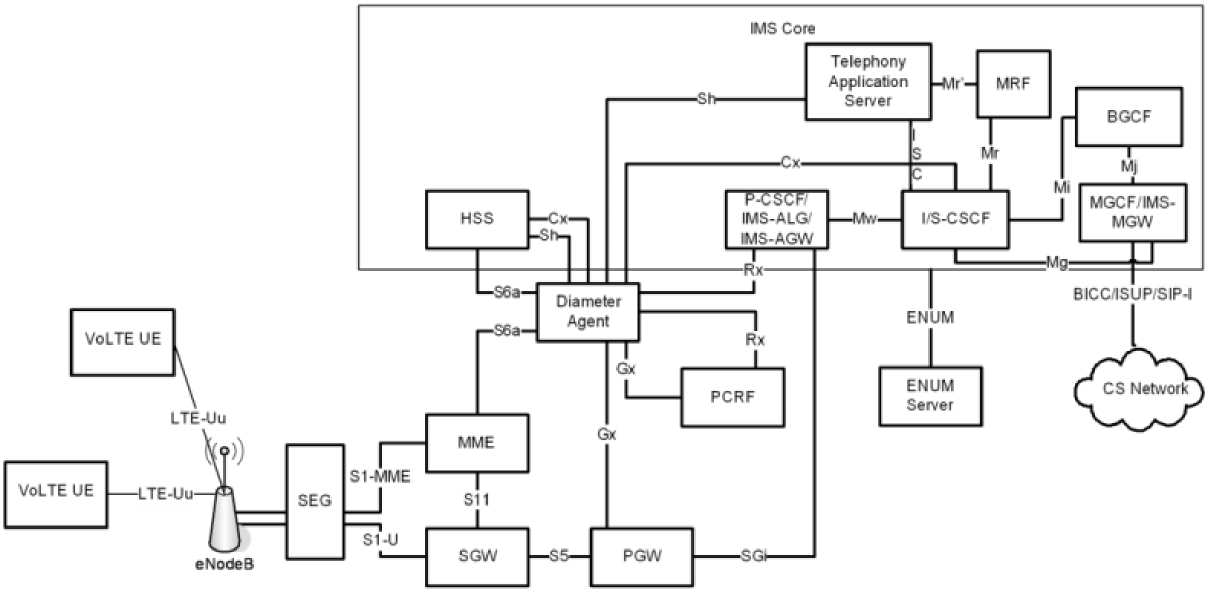

Figure 5: VoLTE Deployment [12]

2.3 User Equipment for Mobile Broadband Access

Before describing the network domains, it is important to mention the basic technology of the handsets that the subscribers are using to access the mobile networks. The device is called **User Equipment** (UE) and is used to connect the user to the IMS through the EPC via the EUTRAN LTE-Uu air radio interface. Based on 3GPP standard the UE can be splitted into two logical components as is depicted in Figure 6; the Terminal Equipment, which terminates all the data flows, and the Mobile Termination, which handles all the communication functions [13].

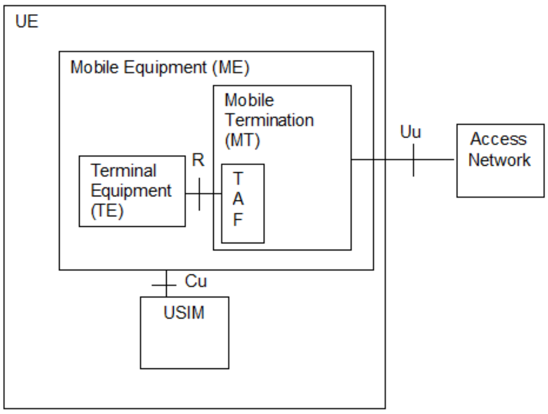

Figure 6: UTRAN Iu mode PLMN Access Reference Configuration [13]

The UE is composed of modules supported a variety of radio and network features, like operating in different frequencies, serving different maximum data rates and supporting various network services. For the registration to the IMS, it uses the IP Multimedia Services Identity Module (ISIM), which is an application stored in the Universal Integrated Circuit Card (UICC), better known as SIM card [14]. UICC and UE, according to the specifications, should operate independently of the respective manufacturer, card issuer or operator, which ensures the portability and interoperability of mobile technology [15]. The most crucial application of UICC is the Universal Subscriber

Identity Module (USIM), which stores subscriber-related, security and authentication information, the PLMN list, temporary location information etc. [16].

2.4 Evolved UMTS Terrestrial Radio Access Network

The UE radio-related communications for LTE are handled by the radio access network, which is called **Evolved UMTS Terrestrial Radio Access Network** (E-UTRAN). The single component of the EUTRAN is the base station, which is called enhanced Node B (eNodeB). UE and eNodeB use the Radio Resource Control (RRC) protocol for radio-related signalling and the Packet Data Convergence Protocol (PDCP) stack for the user plane.

Release 10 of LTE-A proposed a flexible and scalable bandwidth allocation scheme for higher data rates up to 1Gbps and improved spectrum efficiency. This technique is called Carrier Aggregation (CA). With CA, the available bandwidth is increased up to 100MHz by aggregating up to five LTE carriers.

The evolution of UTRAN targets not only to offer higher data rates and improved system capacity but also to reduce the Operator's costs. Unlike GSM and UMTS, there is no controller in the LTE radio network. Therefore, eNodeB combines the radio controlling functionalities of both base station and controller of the previous generations, such as the radio bearer control, the mobility control and the resource handling, by utilizing the existing site locations. According to 3GPP standard, this new architecture of a more simplified network achieves signalling optimization, lower latency and better mobility performance across the cellular network. Moreover, E-UTRAN can reduce the CAPEX and OPEX of the radio network due to multi-vendor equipment interoperability and backhaul infrastructures reuse [17].

2.5 Evolved Packet Core Network

The main network entities of the **LTE Evolved Packet Core** (EPC) are:

- the Mobility Management Entity (MME),
- the Serving Gateway (S-GW) and
- Packet Data Network Gateway (P-GW).

The Policy and Charging Rules Function (PCRF) is a centralized policy control element that can support different Packet Data Networks (PDN), not only the EPC but it will be described in this chapter as part of it.

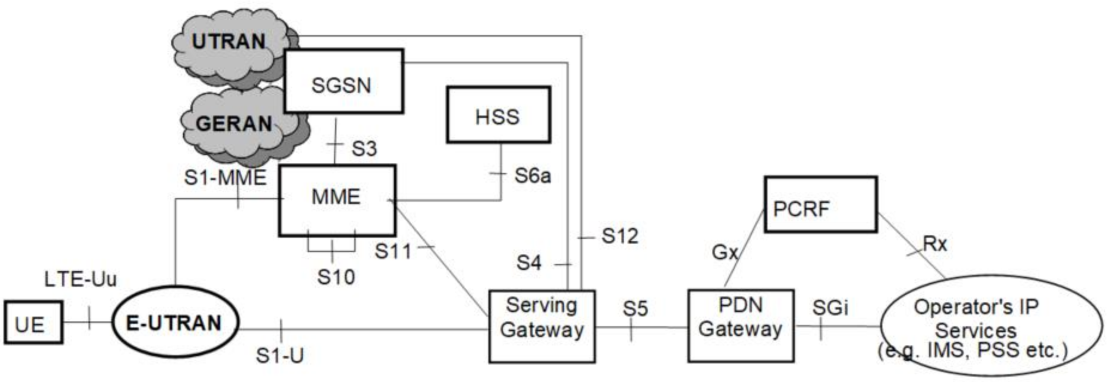

Figure 7: Non-roaming Architecture for 3GPP Accesses [18]

2.5.1 Mobility Management Entity

The **Mobility Management Entity** (MME) is the key control signalling node of the EPC domain, the evolution of the Serving GPRS Support Node (SGSN) of the GPRS domain. The basic functionalities of the MME are:

- Access Control Function
- Session Management Function,
- Mobility Management Function,
- Control Selection of Network Elements (NE) for the UE,
- Roaming Management.

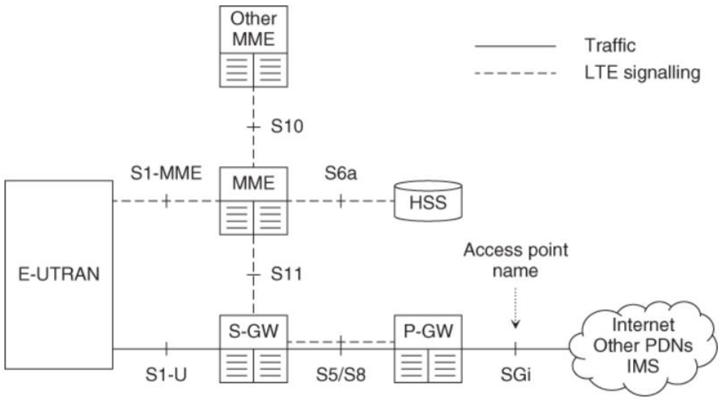

Figure 8: Mobility Management Entity Network Interfaces [19]

It is the node responsible for all signalling exchanges between the UE/eNodeB and the core network. It handles the UE attach and the bearer activation, the idle tracking and the paging procedures, as well as the 2G/3G mobility and intra-handover functionalities through the S1-MME interface between eNodeB and MME. It can select the S-GW at the initial attach, based on geolocation criteria, in order to minimize signalling and reduce latency.

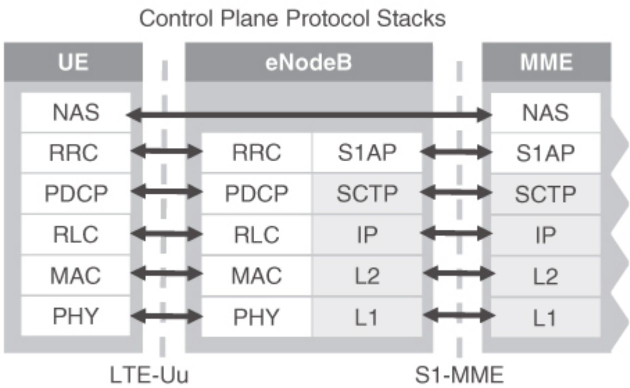

Figure 9: Control Plane Protocol Stacks between UE – eNodeB – MME [20]

For the non-radio related functions, like the temporary identification number assigned to the UE, MME uses the non-access stratum (NAS) signalling towards the UE. It is also responsible for the authentication and authorization of the user by downloading the subscriber profile and the authentication data from the HSS (Home Subscriber Server) through the s6a interface [21].

Table 2: MME 3GPP Interfaces

Interface	A-Side	B-Side
S1-MME	eNodeB	MME
S6a	MME	HSS
S13	EIR	MME
S11	MME	SGW
S10	MME in Pool	MME in Pool
S3/Gn	MME	SGSN
SGi	MME	P-CSCF

2.5.2 Serving Gateway

The **Serving Gateway** (S-GW) is the gateway that terminates the interfaces towards E-UTRAN. The S-GW main tasks are:

- User plane anchor for mobility and handovers
- Packet Routing
- Packet Forwarding

It acts as the local mobility anchor point for intra-LTE and LTE to 2G/3G mobility and handovers. It functions as a router responsible for routing and forwarding the user plane traffic from the eNodeB through the S1-U interface, and towards the Packet Data Network Gateway (P-GW), through the S5 interface.

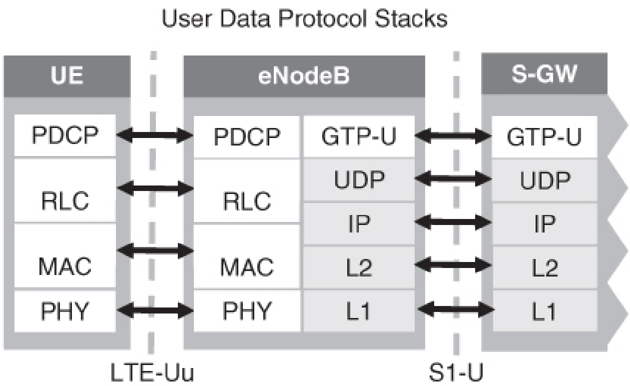

Figure 10: User Plane Protocol Stacks between UE – eNodeB – S-GW [20]

Usually, S-GW and P-GW are collocated and operate into the same physical network element. S-GW is also exchanging signalling packets with MME, in order to support user mobility and bearer management through the S11 interface. It also collects and stores the PDN context and bearer information of the UE [22]**.**

Table 3: S-GW 3GPP Interfaces

Interface	A-Side	B-Side
S11	MME	SGW
S5	HPLMN SGW	HPLMN PGW
S8	VPLMN SGW	HPLMN PGW
S1-U	eNodeB	SGW
S4/Gn	SGSN	SGW

2.5.3 Packet Data Network Gateway

The **Packet Data Network Gateway** (P-GW) basic functionalities are:

- Packet Routing and Forwarding
- Bearer Management
- UE Address Allocation
- PDN Gateway
- Packet Filtering
- Accounting Support
- Charging Support
- Lawful Interception

It is the user plane border gateway, providing connectivity between 3GPP and non-3GPP systems. It is the IP allocation and handling responsible node. It can provide policy, charging and packet filtering functionalities as well. From radio perspective, each UE is registered and has an always-on connectivity with the P-GW, even if it is using the UTRAN or GERAN radio network.

Table 4: P-GW 3GPP Interfaces

Interface	A-Side	B-Side
Gx	P-GW	PCRF
S5	HPLMN SGW	HPLMN PGW
S8	VPLMN SGW	HPLMN PGW
Gy	P-GW	OCS
S4/Gn	SGSN	SGW

It terminates all the connections and operates as the EPC's point of contact with other external PDNs, like the Internet or IMS. Each PDN is identified by an Access Point Name (APN), configured at the P-GW and sent by the UE to inform for the provisioned interesting domain. It uses SGi Interface to exchange control and user plane traffic with other PDNs and Gy for online charging with Online Charging System (OCS) [22].

2.5.4 Policy and Charging Rules Function

The **Policy and Charging Rules Function** (PCRF) is a centralized control node that applies business policy rules and handles resource allocation for the subscribers. It provides dynamic policy control and manages flow-based charging functionalities for PDNs.

Table 5: PCRF 3GPP Interfaces

Interface	A-Side	B-Side
Gx	P-GW	PCRF
Rx	P-CSCF	PCRF

Gx is the diameter interface between PCRF and the Policy and Charging Enforcement Function (PCEF) module of P-GW, which supports packet service activation/deactivation, accounting, default bearer establishment and session management. Rx is the diameter interface between PCRF and Proxy Call Session Control Function (P-CSCF) to establish a session with IMS and bind the VoLTE application to the default bearer [23].

2.6 IP Multimedia Subsystem

The **IMS subsystem** comprises the core elements responsible for supporting IP multimedia services like audio, video, text, chat or a combination of them, delivered over the packet-switched domain. The main entities of the IMS Core are:

- Proxy Call Session Control Function (P-CSCF),
- Serving Call Session Control Function (S-CSCF),
- Interrogating Call Session Control Function (I-CSCF),
- Media Resource Function (MRF),
- Telephony Application Server (TAS),
- Breakout Gateway Control Function (BGCF),
- Home Subscriber Server (HSS).

Each CSCF has its own functionalities, which are described in the following subsections. Common to all three CSCFs is their key role during the registration and the session establishment. TAS and MRF can be roughly classified as service entities. The BGCF provides mainly interworking functions between two operator domains. The HSS can be the database not only for the IMS users but for the EPC as well. However it will be mentioned as part of it.

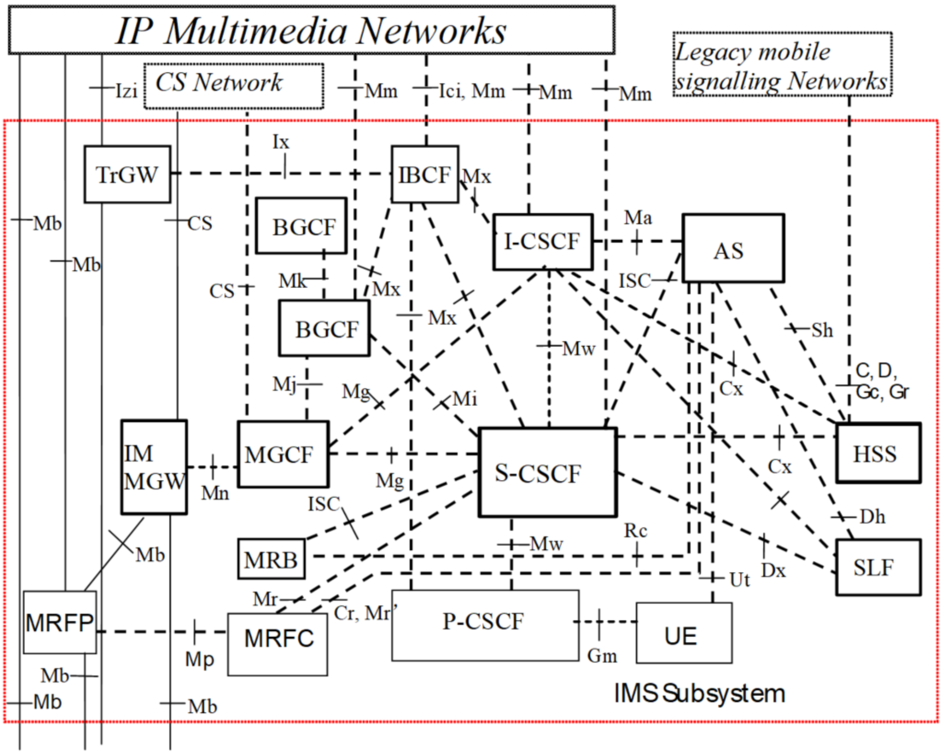

Figure 11: Reference Architecture of the IP Multimedia Core Network Subsystem [24]

Each VoLTE-provisioned user mainly uses the SIP to reach the IMS network through an IP Connectivity Access Network (IP-CAN). An IP-CAN could be an untrusted non-3GPP network like the Wi-Fi, the LTE or the 3G/2G access radio network. In the case of VoLTE service, the IP-CAN is the LTE network [24].

Table 6: IMS Reference Points

Name of Reference Point	A-Side	B-Side	Protocol
Gm	P-CSCF	UE	SIP
Rx	P-CSCF	PCRF	Diameter
Cx	CSCF	HSS	Diameter
Mr	CSCF	MRF	SIP
Mr'/Cr	TAS	MRF	SIP
ISC	TAS	S-CSCF	SIP
Sh	HSS	TAS	Diameter
Ut	UE	TAS	HTTP
Mj	BGCF	MGCF	SIP

2.6.1 Proxy Call Session Control Function

The initial point of contact with the IMS is the **Proxy Call Session Control Function** (P-CSCF), one of the three functions of the CSCF component. As a proxy, the P-CSCF establishes and secures the signalling messages between itself and the UE through the

Gm interface while forwarding the traffic to the Serving CSCF. The P-CSCF also acts as an Application Function (AF) to bind the IMS sessions with the corresponding bearers, and interacts with PCRF in order to apply dynamic policy and charging control and monitor the quality of the media flows [24].

2.6.2 Serving Call Session Control Function

The SIP registration and session control server in the IMS network is the **Serving Call Session Control Function** (S-CSCF), similar to the MME in the EPC network. S-CSCF provides session control functionality, authenticates and queries the HSS for the subscriber profile through the diameter Cx interface and registers the active VoLTE-capable users to its database. It handles calls for the registered UEs, invokes the Telephony Application Server (TAS) based on the Initial Filter Criteria (IFC) received from the HSS at the time of the registration, and generates billing records for each session [24].

2.6.3 Interrogating Call Session Control Function

The **Interrogating Call Session Control Function** (I-CSCF) acts as a distribution function for the incoming requests within the Operator's network. It handles all the connections coming either from the P-CSCF, either from another IMS network or from the CS core, through the Media Gateway Control Function (MGCF) component. During the IMS registration, it queries the HSS for subscriber information through the Cx interface in order to select the suitable S-CSCF to route the initial registration request coming from the user in cases of several S-CSCFs elements. During a terminating call request, it interrogates the HSS to find which S-CSCF the UE is registered on. If the UE doesn't belong to the local domain, it will reject the call [24].

2.6.4 Media Resource Function

The **Media Resource Function** (MRF) provides media plane processing and bearer transcoding functionalities, like playing ringing tones and announcements, mixing media streams for a conference and multiparty calls, under the control of TAS or I/S-CSCF etc. It consists of three components; the processor (MRFP), which manages and processes the media streams; the controller (MRFC), which controls the media resources of the MRFP and handles messages arriving from CSCF through Mr or from TAS through Mr' and Cr interfaces; and the Media Resource Broker (MRB), which decides the resource function that will handle a particular stream, based on the application's requirements [24].

2.6.5 Telephony Application Server

The **Telephony Application Server** (TAS) is a SIP controlling application server (AS) that empowers Operators to offer real-time bidirectional conversational streams and supplementary services across the IMS network. These streams can be high-quality multimedia communications, such as voice and video, and optionally fax transmission, SMS over IMS through IP Short Message Gateway (IP-SMGW) server, voicemail etc.

TAS uses the SIP ISC interface to communicate with the S-CSCF in order to provide location information, execute originating and terminating services for the VoLTE user and receive event notifications. Through the Diameter Sh interface, the HSS provides subscription information for multimedia and supplementary services to the TAS. During a terminating call, TAS receives the Terminating Access Domain Selection (T-ADS) information from the HSS to properly route the call, based on which domain the VoLTE user has camped, either on LTE or 2G/3G network. Moreover, users can manipulate voicemail preferences and supplementary services by using XCAP protocol across direct Ut interface between UE and TAS [25].

2.6.6 Breakout Gateway Control Function

The **Breakout Gateway Control Function** (BGCF) is a routing responsible element that decides the next hop for routing SIP messages between domains based on information within the SIP/SDP and internal routing configuration data. The outcome of the routing process can be either a breakout into a Public Switched Telephone Network (PSTN)/CS domain or a breakout into another IMS domain. If the breakout takes place towards the PSTN/CS domain within the Operators' network, the BGCF selects the responsible MGCF to forward the session through the Mj interface. If the PSTN belongs to another Operator, the BGCF forwards the session to another BGCF in the selected network. If the routing destination is in another IMS network, the BGCF forwards the session signalling to an I-CSCF of the concerned IMS domain [24].

2.6.7 Home Subscriber Server

The subscribers' database is the **Home Subscriber Server** (HSS), which stores static and dynamic data related to the subscriber's profile, initially created by the Operator. The same or several HSSs can be used from IMS and 3GPP access networks to authenticate and authorize the subscribers, retrieve user profile information, perform naming/addressing resolution, store user location information etc. Regarding the IMS user profile, this includes a set of services that are provisioned by the Operator to the user and activated during the registration procedure. It also contains the IFCs, which define the application server based on the service [26].

2.7 VoLTE Functional Scenarios

In this subchapter, the main functional scenarios of a VoLTE enabled UE under a VoLTE capable network will be presented. More specifically, a LTE Attach, an IMS Registration, an IMS Deregistration and a Mobile-originated VoLTE call flow will be demonstrated based on the VoLTE Service Description and Implementation Guidelines of the GSMA. The call flows demonstrate the role and the functions of the elements involved in the service.

2.7.1 VoLTE UE - LTE Attach Procedure

Whether a VoLTE UE switches on and is under LTE coverage, it shall automatically perform an LTE attach procedure and then an IMS registration, if the network has VoLTE capability enabled at that time.

The UE selects the Public Land Mobile Network (PLMN) and a cell that belongs to that specific PLMN to send an RRC Connection request. At the initial dialogue with the eNodeB, the UE advertises the physical layer and the radio capabilities to the EPS to adjust the radio resources accordingly. After establishing a signalling connection with the selected base station, the UE moves from RRC_IDLE to RRC_Connected status and sends an Initial Attach Request to the EPC, with a PDN connectivity request embedded asking the network to establish a logical link called the default EPS bearer for the IMS service.

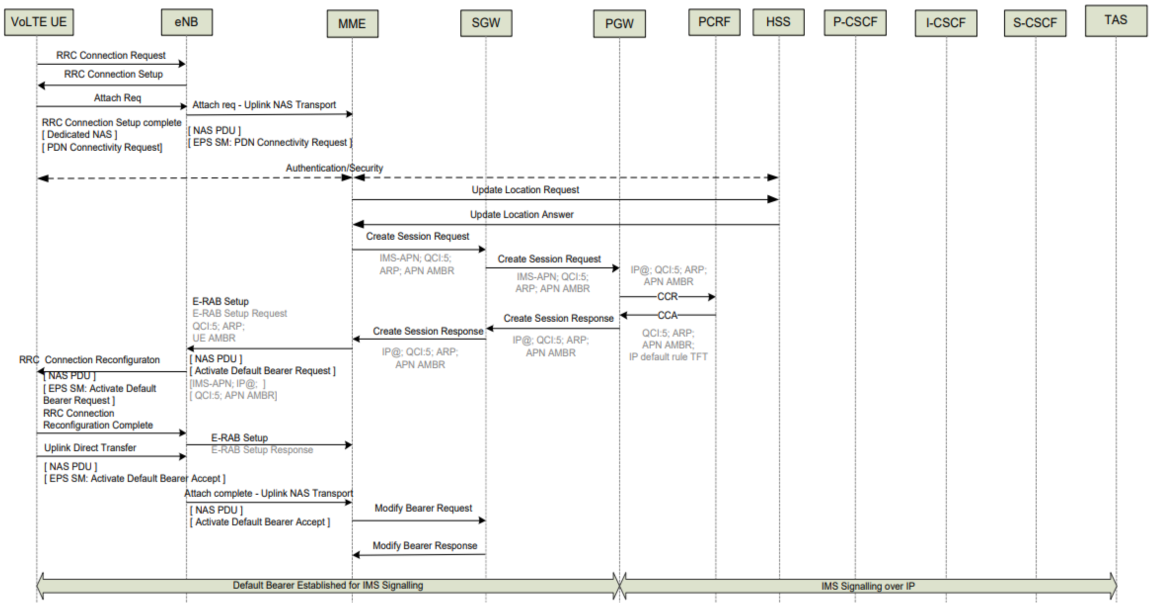

Figure 12: VoLTE UE Attachment message sequence [27]

For each PDN connection, at least one bearer is created, which is called the default bearer. The bearer is an enhancement mechanism enforced into the LTE to carry data packets and provide differentiation in the Quality of Service (QoS) based on the type of the application. All messages within the same bearer are associated with the same QoS Class Identifier (QCI). The default bearer for IMS, which carries only IMS signalling traffic, usually has a QCI value equals 5, based on recommendations, while LTE establishes the dedicated bearer with QCI value equals 1 for the IMS media traffic. For the data traffic, the default bearer has a QCI value equals 9. QCI feature is provided only in packet-switched and not in circuit-switched networks, which supports prioritized handling, packet forwarding, QCI-based charging etc. [23].

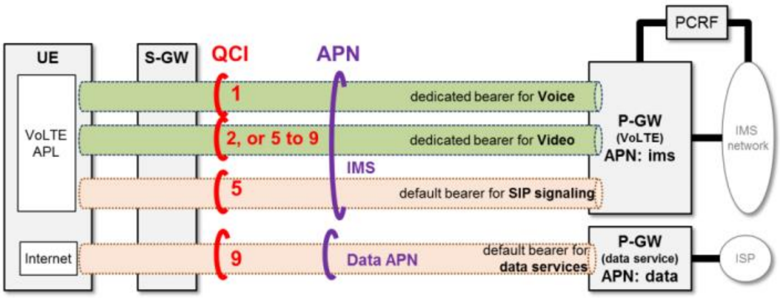

Figure 13: QCI-based and APN-based differentiation [28]

Then, the eNodeB forwards the Attach Request, including the Tracking Area Identity (TAI) and the E-UTRAN Cell Global Identifier (ECGI) location information of the cell, to the MME, which is specified in the RRC parameters. The TAI, formed from the Tracking Area Code (TAC) and the Network Identity, is a globally unique area identifier. The

MME, after performing authentication and security procedure to activate integrity protection and NAS ciphering towards the UE, it sends an Update Location Request (ULR) to the HSS. The HSS accepts the Update Location Request message and replies with an Update Location Answer (ULA), including the subscriber profile, the related International Mobile Subscriber Identity (IMSI) and the alias of the IMS APN. The MME has now all the necessary information to establish the default IMS bearer.

The MME resolves the IMS APN name with the destination IP address of the P-GW, selects a suitable S-GW and forwards the Create Session Request for the IMS signalling bearer establishment with QCI=5. The S-GW creates a new entry in the EPS Bearer table and allocates relevant Tunnel Endpoint Identifiers (TEID) for the control and user plane. TEID enables S-GW to route the control plane traffic between the MME and the P-GW. Then, it forwards the request to the P-GW.

The P-GW allocates either an IPv4 or an IPv6 or both addresses to the UE and utilizes a dynamic Policy and Charging Control (PCC) function to initiate a Credit Control Request (CCR) towards the PCRF to retrieve the PCC rules for the default bearer. The PCRF binds the default policy rules to the IP address of the UE and responds to the P-GW with potentially modified QoS parameters and the default Traffic Flow Template (TFT), which are included in the Credit Control Answer (CCA) message. The TFT is installed at both the UE and the P-GW to determine if a particular traffic stream needs to be traversed by a particular EPS bearer. In the message to the P-GW, the PCRF shall also subscribe itself to modifications related to the default bearer by sending the corresponding Audio Video Profiles (AVP), like the DEFAULT_EPS_BEARER_QOS_CHANGE, RELEASE_OF_BEARER, etc.

The P-GW can now acknowledge the S-GW's request by replying with a Create Session Response, including the UE IP address and the QoS parameters of the default EPS bearer. The S-GW forwards the message to the MME, and the MME can send an Attach Accept to the eNodeB. Then the eNodeB communicates with the UE to update the RRC configuration and include the information received from the EPC as part of the Create Session Request dialogue. The UE sends back the Attach Complete message to the eNodeB by forwarding it to the MME. At this time the UE is capable of sending uplink packets. For the downlink, the MME initiates a Modify Bearer Request to the S-GW, including the EPS Bearer ID, the eNodeB address and the TEID. The S-GW associates and acknowledges the Request with a Modify Bearer Response to the MME. The establishment of the IMS APN default bearer for the IMS signalling is now completed [29].

2.7.2 VoLTE UE - IMS Registration Procedure

Since the UE is now attached to the EPC network, it can perform an IMS registration using the SIP protocol through the default EPS bearer QCI=5 (Figure 14). The UE initiates a SIP Register Request towards the P-CSCF address, which is received during the LTE attach. The P-CSCF inserts a Path header to the message with a SIP Uniform Resource Identifier (SIP-URI) and forwards it to the I-CSCF. The I-CSCF sends a User Authentication Request (UAR) to the HSS, authenticates and authorizes the user, and receives the corresponding S-CSCF hostname to forward the SIP Register Request to. The S-CSCF identifies that the Request is part of the IMS registration with the Authentication and Key Agreement (AKA) security procedure and initiates a Multimedia Authentication Request (MAR) towards the HSS to retrieve the authentication vectors.

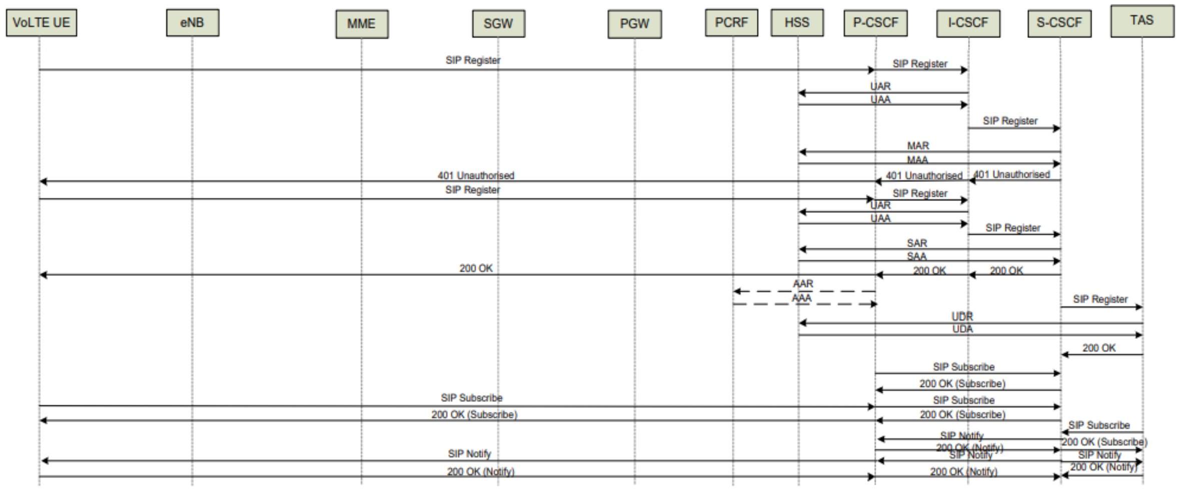

Figure 14: IMS Registration message sequence [27]

Upon receiving the vectors of the IMS-AKA authentication mechanism, the S-CSCF stores the eXpected user RESponse (XRES) and replies with a 401 Unauthorized Response to the UE, indicating the security mechanism which should be used. The UE checks the network's authentication token, calculates its RESponse (RES) value and sends it to the P-CSCF via a second REGISTER Request. The P-CSCF forwards the Request to the I-CSCF, the I-CSCF receives the related S-CSCF name from the User Authentication Answer (UAA) and forwards it to the specific S-CSCF. The S-CSCF checks whether the XRES stored before and the RES received match. If yes, it forwards a Server Assignment Request (SAR) to the HSS to download the relevant user profile, registers the user for VoLTE services and replies with a 200 OK to the UE. At the same time, S-CSCF performs a third party registration to the TAS that is specified in the IFCs. TAS retrieves the user profile by sending a User Data Request (UDR) to the HSS.

Optionally, the P-CSCF can establish an Rx Session towards the PCRF, with an Authorize Authenticate Request (AAR), to perform application binding with the default bearer. P-CSCF requests to be informed in the event of the default bearer loss or disconnection to trigger an IMS de-registration procedure towards the S-CSCF. The PCRF performs the binding and responds with an Authorize Authenticate Answer message to the P-CSCF.

The VoLTE UE, the P-CSCF and the TAS can subscribe to the registration event package of the S-CSCF by using the SIP SUBSCRIBE message in order to be notified of any change in the registration status. In turn, the S-CSCF shall send a SIP NOTIFY message to the aforementioned subscribing entities in order to update them with the active registration status [29].

2.7.3 VoLTE UE - Originating Side Call Establishment

A VoLTE UE, in order to perform a Voice over LTE call, performs the normal mobile origination procedure, as defined in the 3GPP TS 23.228. As pointed before, the IMS signalling shall be sent over the default bearer and the audio media traffic over a new dedicated bearer. The UE starts the message sequence by sending a SIP INVITE Request towards the S-CSCF through the P-CSCF. The Request includes the local preconditions for media QoS from the originating side and the Session Description Protocol (SDP) offer, with the IMS audio capabilities of the UE. The P-CSCF adds the P-Charging-Vector header at the Request and forwards it to the S-CSCF. The S-CSCF reviews the subscriber profile for the VoLTE services upon receiving it. Since the user has VoLTE service enabled, the S-CSCF forwards the Request to the TAS for the supplementary services. The TAS invokes any supplementary service logic of the originating user and routes the SIP INVITE Request to the S-CSCF.

The S-CSCF queries the DNS for the Called-Party VoLTE UE and identifies if it belongs to the home network as well. Then, it routes the Request to the I-CSCF to discover the terminating S-CSCF for the Called-Party UE. The Called-Party UE will return an SDP answer in a SIP 183 Session Progress message, containing only one audio codec and preconditions desired from the terminating side. Still, it does not establish the dedicated bearer yet. P-CSCF reviews the SDP answer and sends an Authorize/Authenticate Request to the PCRF with the related service information. The PCRF authorizes the Request and confirms the allowed services, the affected IP-CAN, the QoS and the PCC Rules information. Then, it sends a Re-Auth-Request towards the P-GW to initiate the creation of the dedicated bearer with a QCI value equals 1, for the media traffic, with the related QoS parameters.

The P-GW acknowledges the Re-Auth-Request from the PCRF and sends a Create Bearer Request to the S-GW in order to create the dedicated bearer for the VoLTE call. S-GW forwards the Request to the MME, and the MME establishes an E-RAB Setup with the serving eNodeB and replies back to the S-GW with a Create Bearer Response message. The P-CSCF forwards a SIP 183 Session Progress Response of the initial SIP INVITE Request to the UE, in which the 100rel option is included. The 100rel option is used to indicate that the preconditions are supported or required end to end. The UE shall generate a Provisional Response ACKnowledgment (PRACK) to acknowledge the receipt of the Response. The PRACK is transited to the call's terminating side with an associated 200 OK (PRACK) to be sent back to the UE.

After reserving resources, the UE sends a SIP UPDATE message, confirming the previous SDP offer and informing that the media stream is now active. The terminating VoLTE UE is now alerted, replies with a 200 OK (UPDATE) and, as preconditions have been met, a SIP 180 Ringing is sent back to the originating UE through the S-CSCF and the P-CSCF. When the Called-Party answers the call, it sends a 200 OK message towards the Calling-Party UE. The calling UE receives the message and sends a SIP ACK to acknowledge that the call has been established [29].

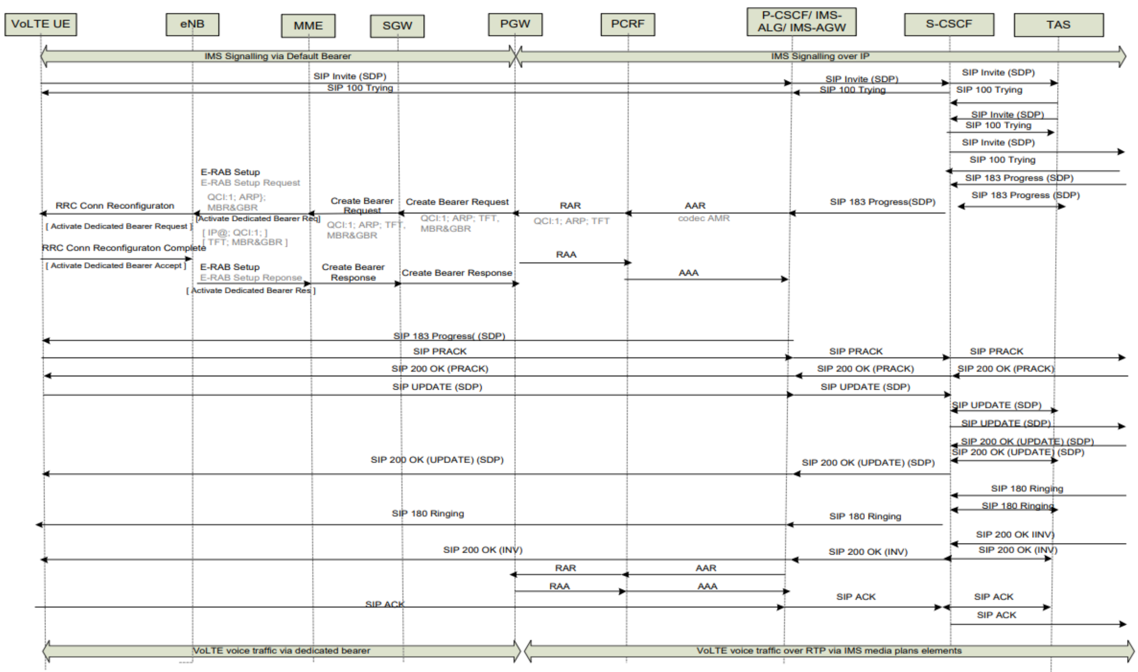

Figure 15: Basic VoLTE UE to VoLTE UE Call Establishment - Originating Side [30]

2.7.4 VoLTE UE - IMS Deregistration

If a VoLTE UE loses the LTE coverage, it should automatically perform IMS Deregistration before the LTE Detach procedure to guarantee the successful execution of the voice terminating services. The procedure of the IMS Deregistration is defined in the 3GPP TS 23.228. The UE initiates a SIP Register Request towards the P-CSCF, with the registration expiration interval timer set to zero. The P-CSCF forwards it to the I-CSCF, asking HSS for the related S-CSCF with a User Authorization Request message.

The S-CSCF initiates a Server Assignment Request procedure towards the HSS, which enables unregistered services for the UE, like voicemail or CS routing, while keeping the same S-CSCF to handle a future terminating INVITE message. Then, the S-CSCF sends a SIP NOTIFY message to all the subscribed elements to inform them for the registration status change of the UE, waiting for a 200 OK Response. After receiving the Responses, S-CSCF sends a 200 OK (REGISTER) towards the UE, confirming successful deregistration [29].

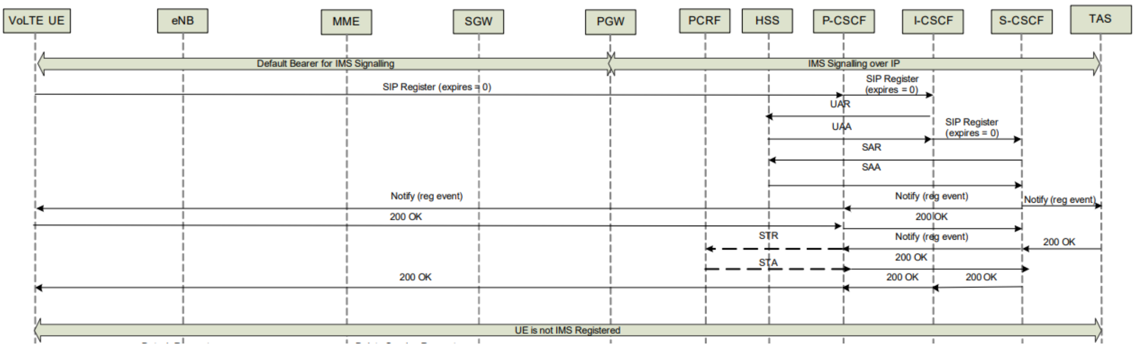

Figure 16: VoLTE UE IMS Deregistration [31]

3. VOLTE AS AN INVESTMENT OPPORTUNITY

Business drivers of the last decades like heavily increasing mobile data usage, smartphone growth and mobile applications led inevitably to data-driven network modernization projects, more towards packet-switched solutions. The LTE, the successor of the GSM and UMTS systems, can be told that it is the fastest developing mobile system technology ever. Based purely on the IP protocol, it broke the barriers of the spectrum capacity and contributed to the excessive cellular data usage.

As a core service the voice had to adapt during this IP transformation and a more cost-effective solution was specified. Launching VoLTE requires an IMS domain, a mature but complex technology which includes all the elements and the interfaces presented in the previous Chapter 2. The IMS system should be integrated with the EPC core network as well as with many IT and provisioning systems in order to be fully operational. The EPC is the domain that can provide the registration path to the subscribers toward the IMS, through different 3GPP and non-3GPP access networks.

VoLTE represents a large transformation in the mobile industry. It lays the foundation for Operators to move toward the next generation of voice telephony. It can be regarded as just one more data application for the LTE network, but more demanding with specific requirements for real-time traffic, QoS and interoperability with existing circuit-switched voice core systems in order to provide voice, video and SMS on the LTE Network.

In this chapter, some indicative market potentials and considerations about VoLTE launch will be presented, as well as the advantages and the drawbacks of the VoLTE deployment from technical and effort points of view.

3.1 General Concerns about VoLTE Deployment

Carrying years of legacy voice experience, many Operators decided to allocate budget and their expert staff to modernize the network and activate the VoLTE service. However, Operators should examine except for the benefits all the challenges that may occur when deploying the VoLTE solution. Of particular importance it should be the evaluation of the market conditions, the competition and the product trends that can change the demand at the time of the activation. Below are some key points that should be considered before deciding to implement VoLTE.

3.1.1 Industry Nature and Competition

The Telecommunication industry has a capital intensive and network-oriented nature. Lim et al. (2007) [32] pointed out that investments in Telecommunications are generally high-risk projects due to the uncertain market environments surroundings, such as the demand and the technological changes. However, optimizing the network and launch new services are essential to the future of firms' business model. According to Taylor and Baker (1994) [33], achieving and maintaining customer-perceived service quality is considered as crucial for the successful provision of overall customer satisfaction. Thus, Operators should implement enhanced and richer communication services like VoLTE to compete with the other players and increase their market share.

Several strategies and marketing tactics are being built to create a stronger brand name and encourage loyalty. Gerpott et al. (2001) [34] recognized customer retention as a critical factor in the success of the telecommunication providers. In the race for customer engagement, technology leadership is proved to be a useful tool. VoLTE is the state of the art for the mobile voice services, which can improve user experience and bring new subscribers. Additionally, “first to launch” may positively impact the brand and reputation, especially on the innovator customers.

3.1.2 VoLTE-Capable Devices Availability

Operators should estimate the dynamic users and the potential penetration of the VoLTE service into the market before launching. The rate of adoption of the VoLTE solution will be dictated by the maturity of the LTE coverage and the availability of VoLTE-capable devices. The cost of the handsets is an important factor as well, especially in emerging markets with low income per capita. Nowadays, there is a wide variety from plenty of vendors, and the prices are steadily declining. According to Ericsson, over 3,200 VoLTE-enabled 4G device types are estimated to be in the market, such as mobile phones, indoor customer premises equipment (CPE), fixed wireless phones, tablet PCs, and smart watches [35].

3.1.3 Voice Growth vs. Data Growth

Even though the voice was the core component of the Telecom industry for many years, the market focus has shifted towards data. With the advent of smartphones and mobile broadband services, data volumes continue to skyrocket, while fixed and mobile voice traffic has continued to slide down for many carriers. Lee & Lee (2009) [36] claimed the development of the Internet as the reason for the rapid shift from voice-based to data-based services, exploring the substitutive and complementary relationships between fixed, mobile and Internet services among different national and economic environments.

A VoLTE investment under these circumstances sounds costly. However, it gives the opportunity of decommissioning the legacy 2G/3G systems while it reduces the costs by maintaining only one network. Also, it provides Operators with the option to deploy 5G and take advantage of innovations in terms of IP-based and cloud computing solutions, which enables a more cost-efficient network implementation and operation.

3.1.4 OTT Threat for Operators

Data utilization is not the only reason for the lost revenues of cellular voice services. Few new Mobile Network Operators (MNOs) players can enter the telecommunication market, as there are still many entry barriers, like the infrastructure costs and the limited spectrum resources. However, Fritz et al. (2011) [37] saw that due to the widespread adoption of mobile internet access, Over the Top (OTT) service providers and applications like Viber, Skype and WhatsApp have entered the market and utilize the existing EPC network of the MNOs to provide voice, messaging and video call services without incurring any cost. Krüssel (2018) [38] deduced that such OTT applications are increasingly preferred by subscribers because they are benefited from the low latency and the improved speed of the LTE for an almost free service while creating network data congestion for the Operators.

OTT applications have gradually substituted the traditional MNOs’ services and decreased their main revenue streams. Operators have become the data pipes for such applications, and the only income occurring is from the data subscriptions. Awwad (2021) [39] stated that the OTTs` services massive penetration into telecom industry is driving the MNOs to reconsider their strategies and revenue sources. Czarnecki and Dietze (2017) [40] mentioned that in order to accommodate the increase in traffic volume while maintaining a good level of customer experience, the Operators' investment decisions focused mainly on expanding the network capacity. Continuing to invest in their network and adopt new technologies is one of the few ways they have to protect the existing revenues and potentially drive further growth. VoLTE is an option to make them more competitive with OTT applications and get more users connected.

3.2 Technical Benefits from VoLTE launch

Until today, many mobile Operators across the world decided to deploy a VoLTE capable network. Deploying VoLTE can be a complex and demanding project, maybe more challenging than GSM deployment 30 years before. However, by utilizing an all-IP cloud-based IMS network to provide VoLTE, new voice-related use cases and functionalities can be developed while the network becomes more cost-efficient and modernized to support future technologies.

3.2.1 Network Modernization and Enhancements

Like any modernization project, the VoLTE solution offers plenty of enhancements to the Operators. VoLTE supports the transmission of voice signalling and user plane traffic as data packets over the EPC, transforming the legacy networks into a full IP-centric solution. VoLTE succeeds at least three times faster call setup time than the 2G/3G networks while offering QoS with a guaranteed bit rate. It allows data streaming in 4G rates concurrently with ongoing voice calls while supporting High Definition (HD) voice quality, using the next-generation codecs, like the Adaptive Multi-Rate Wideband (AMR-WB) and the Enhanced Voice Services (EVS) codec. Without an end to end VoLTE functionality, Operators have to maintain their legacy 2G/3G systems and utilize the CSFB feature, to support originating and terminating calls for all users. This technique reduces the voice quality and doesn't offer a guaranteed bit rate [41].

Moreover, VoLTE is considered the base for enhancing and developing new voice-related services. HD voice calls and collaboration across different types of smart devices and wearables can improve voice services and create a high impact on customer experience. Voice over Internet of Things (VIoT) is another breakthrough feature available for IoT devices. VIoT can provide new solutions to the market and support several innovative use cases in key areas, like industrial automation, smart home, robotics, transport etc. The same functionalities can also be deployed from an OTT provider by developing mobile applications and services running on top of the Operator's data network. However, the advantage of the traditional MNOs over the OTTs is the seamless services across all access networks [41].

3.2.2 Shutting down 2G/3G and Costs Reduction

One of the major reasons to enable VoLTE is for Operators to be able to transform their circuit-switched core infrastructures to a full IP-centric domain and to shut down their legacy systems. Decommissioning of GSM and GPRS legacy nodes could offer significant cost savings. From power consumption point of view, only one subsystem will be up and running, so fewer elements on service. Fewer elements active mean that less space is required for installations, reducing the costs of deploying data centers and operating several technologies. Redundancy schemes need to be more robust due to the fewer signalling points, however, the network becomes simpler to monitor and maintain.

Migrating to all-IP networks allows utilizing server-based infrastructure and cloud computing. This type of technology is more efficient than bare-metal legacy systems, as it can host several different services, using the same computes nodes, as optimal as possible. Also, it offers higher computing performance per unit for data processing and dynamic allocation of resources for reliability purposes. Cloud architecture offers scalability and automation benefits as well. For example, Operators can scale up or down based on their traffic needs; increase or decrease resources such as processing power, data storage and networking capacity without any service disruption. Through automated lifecycle management and software-defined network tools, monitoring, management and service operation become easier.

3.2.3 Spectrum Refarming Capability

Another critical factor is the spectrum, which is limited and the most expensive resource for Telecom Operators to acquire. Available frequencies are shared between the different radio technologies that operators are utilizing. Migrating the services and shutting down the legacy systems allows the firm to liberate the corresponding radio frequencies and allocate them for the newer generations of mobile systems. This process is called Spectrum Refarming. Many Operators have already decided to offload and power off their 2G or 3G networks or both of them in order to reuse the spectrum in new technologies, like LTE and 5G. Governments and regulators contribute to this process of radio network evolution and expedite the renewals of the licenses to ensure service continuity. In the EU, the licensing framework is now technology-neutral, meaning that refarming can be operator-driven and technology agnostic, as long as a non-interfering technology is deployed in a given spectrum [42].

The spectrum bands, which have been allocated to the LTE by the providers, according to the [42], are:

- 850 MHz,
- 900 MHz,
- 1800 MHz,
- 1900/2100 MHz,
- 2600 MHz.

Reinforcement of LTE increases the total capacity of the radio network. LTE supports more efficient spectrum utilization and facilitates an increased transmission speed, using the Orthogonal Frequency-Division Multiple Access (OFDMA) and the Multiple Input Multiple Output (MIMO) schemes. These schemes allow the eNodeB to transmit simultaneously several data streams over the same carrier. VoLTE inherits these benefits and excels in 2G and 3G voice techniques, offering better quality and improving voice delivery. Consequently, the roll-out of LTE/VoLTE allows the Operators to offload the congested legacy systems and maximize revenues by exploiting the spectrum to its fullest potential [43].

3.2.4 Fixed Mobile Convergence Initiative

What mainly implies the launch of the VoLTE service is the operation and maintenance of two different and complicated domains; the LTE as the main access network and the IMS as the voice core network. However, as stated before, IMS is an access-agnostic network. It can establish SIP sessions and deliver multimedia services, mainly voice, for different applications, regardless of the access network technology. Yi et al. (2012) [44] recognized IMS as the 3GPP integration point between the IP-based multimedia services and mobile access, while it can support integration with the wired network.

This feature brought the idea of using fixed and mobile assets more consistently and reviewing the possibility of synergies between wireless and wireline access networks. In the VoLTE case, the so-called Fixed Mobile Convergence (FMC) initiative allows the Operators to reduce the costs and create larger scale markets by leveraging the investment and sharing the same IMS core between the Mobile and the Fixed VoIP services. Eido (2017) [45] analyzed the main benefits behind the implementation of FMC network which are:

- Minimization of the number of network elements and their locations,
- Optimization of the number of caches and the cache locations,
- Simplification of network control and route management,
- Improved service delivery due to the availability of different paths over the different access technologies,
- Improved network performance (higher throughput, reduced latency, higher services quality e.g., 3D video services or video streaming/conferencing with High Definition (HD) quality),
- Lower complexity by simplifying the network structure,
- Reduced cost and energy consumption by sharing control functions and improving network structure,
- Improved usage of available network resources,
- Unifying authentication for users regardless of the access network's type,
- Unifying IP edges of all networks (fixed and mobile gateways) within a single entity, and thus allowing flexible use of common functions,
- Enabling efficient network level load balancing schemes,
- Potential separation between control and data plane by using Software Defined Networking (SDN) technology.
- Enhancing mobile segment of FMC network architecture, which will be able to support enormous traffic growth generated by future HD services.

3.2.5 Seamless Service Continuity with Wi-Fi networks

Apart from combining mobile and fixed services into a common core, the IMS can be integrated with non-3GPP untrusted and trusted access networks, like the wireless Wi-Fi infrastructures, through the EPS. In this case, the UE can be connected to the IMS and perform voice calls, experiencing seamless service continuity between LTE and Wi-Fi, and seamless voice handover from Wi-Fi to LTE. The voice service over Wi-Fi, known as VoWiFi, is an innovative trend in mobile networks, which can offer a substantial return on minimum investment cost for the Operators. The overall solution can be seen as an extension of the EPC architecture, which requires the deployment of two new elements, the enhanced Packet Data Gateway (ePDG) and the Authentication Authorization and Accounting (AAA) [46].

The UE, under Wi-Fi coverage, sets up an IPsec tunnel and connects to the ePDG, gets authenticated and authorized by the AAA and the HSS, and attaches to the P-GW, establishing the IMS APN default bearer. Then, it can perform an IMS registration following the same process as through LTE, which was described in chapter 2. Due to VoWiFi functionality, subscribers can have indoor “cellular” calls using their WLAN, in areas with poor LTE coverage, without the need for Operators to invest anything in radio access technology. Initiatives like this allow them to augment their carriers with new sites, extend the reach of VoLTE to Wi-Fi and increase service penetration of their networks by utilizing different backhaul infrastructures [47].

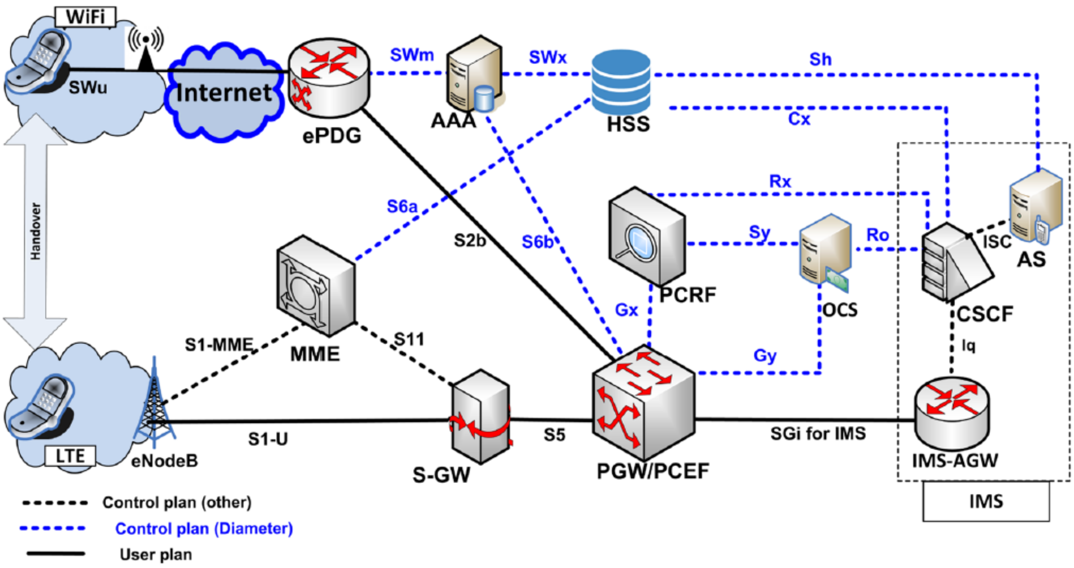

Figure 17: Current VoLTE and VoWiFi Architecture [46]

3.2.6 Migration to Fifth Generation Technology

According to the latest specifications, the IMS network will be reused in the upcoming 5G, non-standalone and standalone architecture as the core network for voice services. For the non-standalone option, Operators, who have already deployed IMS and voice over LTE solutions, can simply deploy the 5G New Radio (NR) access network in order to launch 5G. The standalone option introduces a new 5G Core network (5GC), which will be integrated with the IMS and it will support all existing voice, video and messaging services of LTE. Based on the Ericsson Mobility Report for Voice and Communications Services trends of November 2021, 90 per cent of all combined LTE and 5G subscriptions will be IMS voice capable by the end of 2027, meaning 6.9 billion subscriptions [35].

Additionally, in 5G standalone, there is no CSFB functionality defined, meaning that the only option to deliver voice is through IMS. 5G voice services will be deployed using several solutions, combined or stepwise, based on the 5G coverage build-out; LTE-New Radio (NR) dual connectivity, Evolved Packet System (EPS) fallback and finally, Voice over New Radio (VoNR), for full 5G standalone solution [35]. Several techniques like Cloud-Native Network Function (CNF) and SDN technology are recommended to be deployed by Operators to meet the 5G requirements, along with the new design adaptations, like Control/User Plane Separation (CUPS) and Network Slicing [48]. Eido (2017) [45] insisted that SDN and Network Functions Virtualization (NFV) would allow operators to deploy software network functions instead of physical network elements, which can reduce the CAPEX and OPEX costs and provide more flexibility and better network controlling.

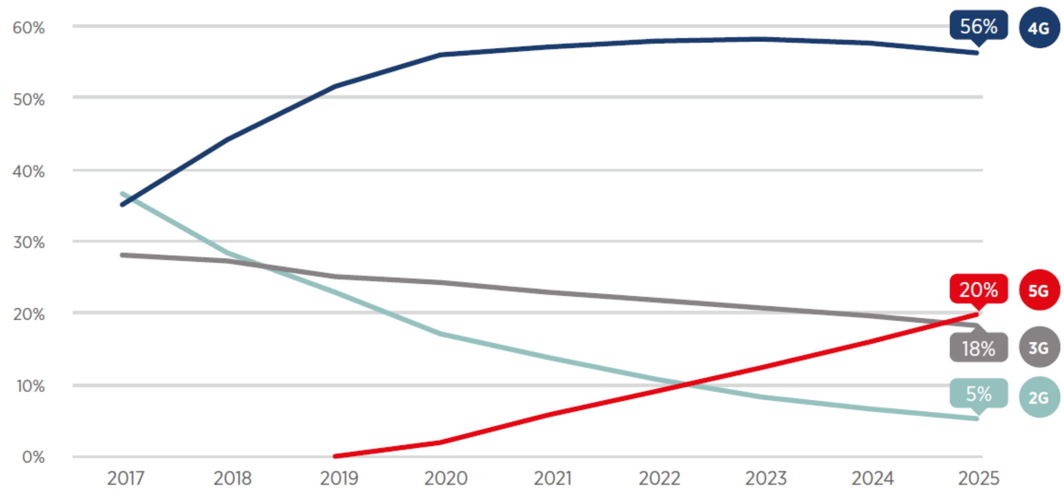

Figure 18: Percentages of global connections (excluding licensed cellular IoT) [2]

3.3 Technical challenges of VoLTE deployment

VoLTE service requires the deployment of the IMS domain, which is a complicated system with many different elements. IMS deployment can bring many difficulties to the surface, functional and non-functional. Investment costs, time and effort to deploy and optimize such a complicated system as the IMS domain should be considered.

3.3.1 Deployment and Feature Parity

The network features need time, effort and plenty of engineers to be implemented or transferred from the legacy domain. Moreover, using elements from different vendors is feasible, but it can be a challenge, as specifications can be interpreted slightly different, causing interworking problems, extensive testing, and possibly extra costs Building the VoLTE service should be seen as a good opportunity for network planners to reconsider old obsolete capabilities and assess utilization and costs before migrating them. Decisions about the feature parity between legacy and IMS systems should also be taken based on the service's regulatory requirements.

3.3.2 Obsolete Services Migration

Full migration to the IMS usually cannot be planned at once; some services may have to be executed in the legacy systems for a long time. The reason can be that the legacy platforms don’t support the new IP-based protocols or the replacement of them will take time; or the migration is not worth it because the service is obsolete. Whatever the reason, as long as these services exist and cannot be transferred soon, the Operators should maintain both networks, which add extra costs. Moreover, this hybrid situation generates extra signalling load into both domains, and more complicated call flows for the designers. From the quality point of view, it can create disruptions or even bigger latency, as signalling is rooted through more domains and is executed on many platforms.

3.3.3 Peripheral Supporting Systems

Integration and interoperability with critical IT systems should also be considered in the project plan and the necessary FTEs (Full Time Equivalent). For example, the Information Technology (IT) systems for provisioning and billing purposes, like the Charging Data Records (CDR) collection; the Intelligent Networks (IN) platforms for serving intelligent functionalities, like the service execution; the Charging systems for the Online and Offline charging functions. These are just some of the basic and necessary components to be integrated into the IMS before the go-live; transparent for the subscribers but vital for the Operators and the services though.

3.3.4 LTE Coverage

Kapoor (2019) [49] expressed that the service Operators should focus on increasing their coverage not only to increase the customer base of the company but to provide the VoLTE service to more customers. Coverage is a key challenge for VoLTE activation indeed. Operators should avoid activating the service while still having poor coverage areas. Increased speech disruptions and drop calls could lead to a frustrating user experience.

As long as the LTE coverage is not high and the legacy systems are not phased out, voice service will perform an inter-radio handover every time the UE has an active VoLTE call and loses the 4G coverage. As mentioned before the feature which provides voice call handover capability is the SRVCC. Seamless handover is challenging to be achieved but improves user experience and service performance when reached. The number of SRVCC attempts is higher when coverage of 4G is low, and there are many gaps in the field. However, due to mobility, it should decrease along with the LTE coverage growth, turning the functionality obsolete and performing only intra-radio handovers. According to Poikselkä et al. (2012) [50], LTE coverage growth is also a good reason for providers to activate VoLTE service if not done yet.

3.3.5 Network Validation

Prerequisite for customers to embrace VoLTE technology is the emergence of VoLTE-capable devices. VoLTE functionality is hard-coded to the devices by the vendors. For mobile Operators to ensure that IMS parameters configuration is aligned with the settings of the handsets, they invite UE suppliers to run performance tests and certify their networks. This process is time-consuming and may add delays to the project. However, until all subscribers migrate to smartphones with VoLTE capability settings enabled, Operators have to maintain a 2G/3G network and route all the calls of the non-VoLTE devices through the legacy network.

3.3.6 Reskilling and Transformation

To enhance LTE functionalities and introduce the era of 5G, terms like cloudification, data centers and automation concepts should become familiar to the Operators. Telecom companies should develop software skills, as the networks are transformed from standalone bare-metal platforms with coupled proprietary releases to cloud-based infrastructures with open source software and automation tools. Krüssel (2018) [38] established that the new business challenges for programmability, agility and fast delivery of applications demand, except for system modernization and new technical skills, enterprise reformation. Project management, traditionally waterfall structured, should adopt a more agile and service-oriented approach in order to adequate the new service delivery requirements.

4. REAL OPTIONS APPROACH FOR INVESTMENTS UNDER UNCERTAINTY

Telecommunications projects are usually capital-intensive and technically demanding tasks, while they are subject to a high degree of risk. Moreover, once activated it is relatively hard to recover the invested capital. Thus, Operators should build their strategy based on all the parameters that can affect the future of the revenue flows before activating any project. These parameters should be acknowledged and analyzed in the context of project management and incorporated into the decision making tools in order to introduce a risk-free project into the market.

The risk is commonly defined as the probability of damage, loss or any other undesirable event that may occur. For the investments, risk could be the loss of a percentage of the initial capital and it is measured as the variance over the expected outcome. Caballero and Pindyck (1996) [51] distinguished risk as industry-wide and firm-specific. Industry-wide risk affects the evolution of the total market, the general willingness of the firms to proceed with new projects and possibly the entry of new players. A firm-specific refers to the inherent risks that affect the firm exclusively. Adding both risks, the total asset risk can be found. Of course the future is unpredictable and unforeseen events can change the status of a project. The higher the risk is, the higher the expectation over future prices.

Previous Chapter 3 covered some of the most important factors that may create uncertainty and add risks to a telecommunication project like VoLTE. Intense competition, changing market conditions, the short-lived life cycle of high-tech products and consumer preferences are indicative of an even longer list. This chapter will present the basic characteristics of the capital investments as well as an alternative approach to treating them as options, exploiting a much more dynamic framework based on modern research. Moreover, the Geometric Brownian motion will be introduced as the proposed process to model uncertainty.

4.1 Characteristics of the Investments

According to Dixit and Pindyck (1994) [52], investment decisions in real world have three main characteristics, which are:

- Uncertainty,
- Irreversibility,
- Flexibility.

4.1.1 Uncertainty

Uncertainty has a determinant role in investment decisions as it may impact the price of both financial and real assets. In the real world, uncertainty can be derived from any political, financial, technological, social or regulative event, creating an unfriendly and unstable environment for investors. Changing and uncertain market conditions as well as a competitive environment can affect except for the motivation of the investors, the project values and increase the riskiness of the future cash flows. Dixit and Pindyck (1994) [52] suggest decision-makers assess the probabilities of the alternative outcomes that can mean greater or smaller profit for the venture to deal with uncertainty.

4.1.2 Irreversibility

Irreversible investment means that at least part of the initial cost cannot be recovered when the project gets active. In other words, if the economic conditions turn unfavorably, it is costly to reverse the investment decision. Conversely, complete reversibility means that physical and financial resources are fully recoverable at any time after the activation, which is clearly unrealistic for the majority of the investments for Holland, Ott and Riddiough (2000) [53]. Caballero and Pindyck (1996) [51] described that uncertainty affects irreversible investments in two ways: first through the effect on the expected marginal profitability of capital and second through the impact of competitors' investment on this marginal profitability. Thus, when an investment is irreversible, the firms' willingness to invest is decreasing while the demanded rate of return is increasing.

By analogy with the options, when a firm makes an irreversible investment expenditure, technically it exercises its option to invest. Based on Dixit and Pindyck (1994) [52], this lost option value is the opportunity cost, which is considered part of the sunk cost. Opportunity cost is an economics term that refers to the profit lost when one alternative is selected over another. In other words, it is the actual value of something given up to obtain something else. When a firm chooses one investment over another, understanding the potential missed opportunities allows for better decision-making.

4.1.3 Flexibility

The option of delaying is also important when an irreversible investment under uncertainty is about to be exercised. The **flexibility** of the investment decision gives the firm, or the option holder, the opportunity to delay until the acquired benefits are adequate or immediately perceptible. Other reasons for postponing the investment can be to collect new better information or expect some sort of funding. Dixit and Pindyck (1994) [52] considered that the new information that may arrive might affect the desirability or timing of the expenditure. Based on the market conditions, the firm elaborates these parameters and builds a strategy in order to choose the most suitable time to maximize the benefits. Of course, postponing the activation of a project or the exercise of an option can incorporate risks, especially in competitive markets.

4.2 Real Options Approach as an Investment Evaluation Tool

One of the most common techniques in analyzing investment projects is the Net Present Value (NPV). NPV criterion calculates the difference between the discounted expected present value of the revenues and the discounted expected present value of the costs, considering that the investment is sensitive to the discount rate more than anything else. Applying the NPV rule, a specific lifetime should be chosen, assuming that the investment starts now. If the result of the difference is greater than zero, the rule will consider the investment attractive and will urge positively. Otherwise, the investment will be declined.

However, NPV and other traditional investment valuation tools like the Internal Rate of Return (IRR) are broadly considered inappropriate for assessing projects with high risk since they don't capture the impact of the uncertainty, the cost's irreversibility, or the investment's flexibility. As Pindyck (1990) [54] claimed, the NPV rule is not valid because the decision to invest should be taken when the value of a unit of capital exceeds the purchase and installation cost by an amount equal to the value of keeping the option to invest these resources elsewhere alive. Neglecting the opportunity cost of investing leads to poor investment decisions and undervalued investment opportunities. Except for denying the characteristics above, those traditional valuing frameworks consider the investment reversible at any time.

For the markets with high volatility over demand and intense competition like the Telecommunications, where delays are possible and expenditures are partially irreversible, the traditional tools fail to capture fluctuation adequately and provide reliable and flexible results. Based on modern literature, one of the most appropriate tools to incorporate these features and support capital investment decisions under conditions of uncertainty is the Real Options Approach (ROA). ROA overcomes the limitation of the traditional theories as it combines uncertainties with flexibility in timing, which characterizes the majority of the investment decisions.

Options are financial derivatives that give their holder the right but not the obligation to buy or sell an asset, financial or real, for a certain price, on a specific future date or within a time period. Trigeorgis (1999) [55] defined an option as the right but not the obligation to benefit from uncertain future conditions. Investment opportunities on real assets are called **Real Options**. Real Options are investment opportunities, where the underlying asset is real. A firm holds a real option when management has for example the right to update an existing strategy with a new project or service. According to Myers (1977) [56], a firm may or may not exercise an option depending on the size of payments that have been promised to the firm's creditors.

ROA is proposed as an analytical tool which can value flexibility for real assets under conditions of uncertainty. The capital investments by analogy with the Real Options theory permit an enriched study under common characteristics. Myers (1977) [56] first referred that "growth opportunities" can be regarded as call options on real assets. According to this approach, investment opportunities should be treated as Real Options, and the methods used for pricing financial options can be applied to valuate non-financial or "real" assets. Avoiding the go-no-go decisions of the traditional tools, Agusdinata (2008) [57] explained that the benefits of applying options' methodology relies on maximizing the upside potential while minimizing the downside effects.

Real Options analysis allows decision-makers to evaluate investment opportunities and adjust a firm's strategy using tools developed for financial products. According to Charalampopoulos, Katsianis & Varoutas (2011) [58], the total strategic value of an investment opportunity is formed from the future stream of payments plus the flexibility parameter. The profitability streams can be calculated from the NPV rule. The flexibility parameter refers to the different types of options related to the investment opportunity, which belong to the following option categories:

- the option to defer,
- the option to alter the operating scale,
- the option to abandon,
- the option to switch,
- the compound option.

4.3 Geometric Brownian Motion to Model Uncertainty

In financial economics, stochastic processes play a significant role in building statistical models. Many financial problems, especially in the stock market, were solved by modelling them as continuous time-continuous space stochastic processes. One simple and widely adopted model is the Geometric Brownian motion (GBM), which is a non-negative variation of the Brownian motion (BM) with drift. Tsekrekos (2010) [59] indicated that the advantage of using the geometric Brownian motion is that it leads to tractable solutions and closed-form intuitive investment decision rules.

The process is named after the Scottish botanist Robert Brown, who in 1827 noticed that small particles of pollen suspended in fluids perform a peculiarly erratic movement. By heating the water, the movement became more intense; while cooling it, it returned to its original rhythm. Brown couldn't explain the phenomenon; however, he published his observations in the Edinburgh Philosophical Magazine [60]. It was not until 1905 that Albert Einstein published his dissertation "A New Determination of Molecular Dimensions" in Annalen der Physik journal. He explained that the movements were due to the collisions between the liquid's molecules and the relatively light pollen.

Brownian motion is a continuous stochastic process used to model the uncertainty of particles' motion in different media. A stochastic process S_t is said to follow a GBM process if any relative change within a small time interval *[t, t+d_t]* is given by the following equation:

$$\frac{dS_t}{S_t} = a\, d_t + \sigma\, dz_t \quad (1),$$

where

$$S_t = e^{X_{(t)}} \quad (2),$$

and

$$X_t = a\, t + \sigma\, z_t \quad (3).$$

The process X_t is a Brownian motion with drift, the a and σ are the constants for growth and volatility respectively, and z_t is a Wiener process (Brownian Motion). The dz_t is the increment of the Wiener process $z(t)$, which for an infinitesimally small change in z is given by:

$$dz_t = e_t * \sqrt{dt} \quad (4).$$

The random variable e_t is normally distributed, with mean equals zero and unit standard deviation. Therefore, the Wiener component dz_t is also normally distributed, with zero mean and variance:

$$V(dz_t) = (dz_t)^2 = dt \quad (5).$$

To find the solution of the differential equation (1), a function $F(S_t)$ will be defined with [61]:

$$F(S_t) = \ln(S_t) = X_t \quad (6),$$

so that

$$\frac{\theta F}{\theta t} = 0 \quad (7),$$

$$\frac{\theta F}{\theta S} = \frac{1}{S} \quad (8),$$

$$\frac{\theta^2 F}{\theta S^2} = -\frac{1}{S^2} \quad (9).$$

The Ito's Lemma for the differential *dF* is:

$$dF = \frac{\theta F}{\theta t} dt + \frac{\theta F}{\theta S_t} dS_t + \frac{1}{2} \frac{\theta^2 F}{\theta S_t^2} dS_t^2 \ (10).$$

From the (7), (8) and (9), the equation (10) becomes:

$$dF = \frac{1}{S_t} dS_t - \frac{1}{2S_t^2} (dS_t)^2 \ (11).$$

Replacing the equation (1) into the (11):

$$dF = \frac{1}{S_t} (a \ S_t \ dt + \sigma \ S_t \ dz_t) - \frac{1}{2S_t^2} (a \ S_t \ dt + \sigma \ S_t \ dz_t)^2$$

$$dF = a \ dt + \sigma \ dz_t - \frac{1}{2} a^2 dt^2 - a \ \sigma \ dt \ dz_t - \frac{1}{2} \sigma^2 \ dz_t^2 (12).$$

From the (5), the equation (12) becomes:

$$dF = a \ dt + \sigma \ dz_t - \frac{1}{2} a^2 dt^2 - a \ \sigma \ dz_t^3 - \frac{1}{2} \sigma^2 \ dt \ (13).$$

The terms $(dt)^2$ and $(dz_t)^3$ can be omitted, as they go faster to zero than d_t and dz_t respectively. Thus, the equation (13) can be written as:

$$dF = (a - \frac{1}{2} \sigma^2) \ dt \ + \sigma \ dz_t \ (14).$$

From the equation (6), *dF* is replaced with $ln(S_t)$ into the equation (14) and taking the integration of both sides, knowing that $z_0 = 0$:

$$\int_0^t d \ [ln(S_t)] = \int_0^t (a - \frac{1}{2} \sigma^2) \ dt \ + \int_0^t \sigma \ dz_t$$

$$ln(S_t) - ln(S_0) = (a - \frac{1}{2} \sigma^2) \ t + \sigma \ z_t$$

$$ln \frac{(S_t)}{(S_0)} = (a - \frac{1}{2} \sigma^2) \ t \ + \sigma \ z_t$$

$$S_t = S_0 * e^{[(a - \frac{1}{2} \sigma^2) \ t \ + \sigma \ z_t]} \ (15).$$

The equation (15) proves that S_t is a "geometric" stochastic process following an exponential Brownian motion with drift rate equals $a - \frac{1}{2} \sigma^2$. Finally, the GBM solution can be represented in the form:

$$S_t = S_0 * e^{X_t} \ (16),$$

where

$$X_t = (a - \frac{1}{2} \sigma^2) \ t + \sigma \ z_t \ (17).$$

GBM has some important properties; first it is a Markov process, which means that the past values don't impact the probability distribution of the future values. The value, which is important for the investment decision, is the present value. Utilizing the equation (16), it will be proved that the future state $S_{(t+h)}$ (*h* time units after time *t*) is independent of the past state *S(u)*, with $0 \leq u < t$ [62]:

$$S_{(t + h)} = S_0 * e^{X_{(t+h)}}$$

$$S_{(t+h)} = S_0 * e^{X_{(t)} + X_{(t+h)} - X_{(t)}}$$

$$S_{(t+h)} = S_0 * e^{X_{(t)}} e^{X_{(t+h)} - X_{(t)}}$$

$$S_{(t+h)} = S_t * e^{X_{(t+h)} - X_{(t)}} \text{ (18).}$$

Based on equation (18), the future states $S_{(t+h)}$ depend on the present state S_t multiplied by the future increment of the Brownian Motion process, namely $X_{(t+h)} - X_{(t)}$. The BM has independent increments, so the $S_{(t+h)}$ does not depend on the past values before the time t.

Since BM has independent increments, the GBM also has independent increments. This means that between any non-overlapping time intervals, the probability distribution of the GBM process is discrete. Additionally, since these are changes in the natural logarithm, the absolute log-changes should be normally distributed over any finite time since BM changes are normally distributed.

5. INVESTMENT STRATEGY DECISIONS BY DYNAMIC PROGRAMMING

According to the traditional valuing theories, businesses should invest as long as the difference between the present expected return value and the present expected costs value is greater than zero, based on the classic NPV criterion. However, as it was described in the previous chapter, this rule ignores critical parameters of the market, like uncertainty and irreversibility.

Uncertainty is an important factor to consider in the development of a business strategy, as it can negatively influence the expected profit stream. Due to the costly irreversible activation and shutdown of many projects, the decision to proceed with any investment may incorporate risks for the future of the business. Moreover, the activation time cannot be a “now or never” decision. The businesses should evaluate the option to wait for better information, better market conditions or higher profitability level, for example.

Based on these observations, many questions may appear; how many times should the exercised price be over the expected costs to allow risk aversion? When is it the right time to activate the project? Before abandoning an investment, how tolerate the firm should be in the losses? Our contributions aims at answering all these questions by proposing a Real Options valuation framework based on the method described by Dixit and Pindyck (1994) [63] and providing an alternative approach in the treatment of the investment opportunities.

The dynamic programming will be deployed hereafter to build the optimal investment rules and estimate the opportunity costs, adopting the work of Dixit and Pindyck (1994) [63]. The inherent uncertainty is applied in the demand, which will be considered a stochastic variable following a Geometric Brownian Motion (GBM) process. Uncertainty will be incorporated into the stochastic model as the constant volatility coefficient. Growth and volatility can be derived from the market's historical data, provided that actual changes are lognormally distributed over time.

5.1 Presentation of the Entry and Exit Decision Option

Supposing an Operator has the resources and the opportunity to proceed into an investment, like holding a real call option. In oligopolistic industries, the competition reduces the flexibility to reconsider undertaking a project [64]. In fact, there is almost a zero value of waiting and an upper barrier on the price process [65]. However, for simplicity, the Operator would be considered a monopolist, with no entry threat and with the flexibility to postpone the project for better demand or more information. The project utilizes a sunk cost I, operates with an operating cost C and generates a market price of value P, according to the stochastic demand variable D. Let’s consider the operation side of the firm to be very simple and the cost to be constant for each unit. The inverse demand function of the firm’s cash flow would be:

$$P = Y\, D(Q) \quad (19),$$

in which Y is the stochastic shift variable over the demand and Q is the quantity or, in this case, the number of firms that entered the market. As long as more firms are entering and offering the same service, the Q increases and the P decreases, along with the inverse demand function curve of the market. For the amount of time that the market remains a monopoly and there is no new entry, Q is fixed, and there is an immediate correspondence between the demand D and the price P. Without further loss of generality, the price value P will be used as the stochastic variable in this solution instead of the demand D.

Considering an exogenous stochastic market price *P*, each moment t_i, the Operator must decide whether to invest or not or, in the world of Real Options, whether to execute the call option or not. The firm's objective is to maximize the benefits at the time zero when the sunk cost is incurred and the project gets activated. As mentioned before, the magnitudes *I* and *C* will be considered non-stochastic constants, unlike the market price *P* and the demand *D*, from which uncertainty arises. Upon investment and during the operation period, the project will generate a profit flow of *P – C*, which can be positive or negative. Another question occurs here: how long the project should last when the cash flow is negative. Or, more precisely, when it is the right time to execute the put option and abandon the project.

A prerequisite to deciding on the activation and abandonment of the project is to estimate the profitability thresholds. By being a Markov process, all it needs to know is the current price value P_0 at the time zero and be at least as large as the optimal investment rule *P**, which will be calculated in the next steps. P_0 includes all the information and it is the only value needed for the forecasting of the future values. Except for the optimal *value* $P_0 = P^*$ to enter the market and exercise the call option, the firm should also decide the price *P*, in which the operation should be suspended and abandoned in case of losses.

Based on the above, the firm's strategy should consist of two discrete but interlinked decisions; the optimal investment decision or *P High* (P_H), and the optimal abandonment decision or *P Low* (P_L). When the demand increases and the price value *P* reaches a sufficiently profitable level, let's say P_H, the Operator should proceed with the investment and activate the project. At that time $t_{i=0}$, the *P* should be:

$$P = P_0 \geq P_H.$$

The project shall stay active as long as the *P* remains above a minimum threshold, let's say P_L, with:

$$P_L < P_H.$$

The P_L should be less than P_H, which is reasonable in real life, as the decision to suspend incurs a lump sum exit cost *E*; and, again, an entry cost *I*, or at least a portion of it, in case of reactivation in the future. So, an early decision to abandon can create extra payments for the firms. However, if *P* falls below P_L, the investment should become inactive.

To summarize, there should be an idle project for the price range $(0, P_H)$, with the Operator to hold a call option, which contributes to the firm's asset value, and to wait for better information and market conditions; and an active project for (P_L, ∞), with a put option to be activated, if revenues fall below threshold P_L.

5.2 Framework build on Dynamic Programming methodology

Since a GBM is used, which is a special case of the Wiener process, before deploying the dynamic methodology, the related historical data of the market should be collected and assessed for the goodness of fit to the lognormal distribution model. After the validation, the demand *D* and, consequently, the market prize *P* can be considered a stochastic variable, evolving exogenously, following a Geometric Brownian Motion process, with:

$$d P = a P dt + \sigma P dz \quad (20),$$

in which $a > 0$ and $\sigma > 0$. The constant *a* is the growth rate for *P* and the constant *σ* is the random uncertainty variable, which implies the standard deviation parameter. As it was explained, the *dz* is the increment of a Wiener process *z(t)*, which for an infinitesimally small change in *z* it is given by:

$$dz = e_t * \sqrt{dt} \quad (21).$$

The random variable e_t is normally distributed, with mean equals zero and unit standard deviation, so the Wiener component *dz* is also normally distributed, with zero mean and variance:

$$V(dz) = (dz)^2 = dt \, (22).$$

From the formulas above, it is extracted that the expected value for *dz* would be:

$$E[dz] = 0 \quad (23),$$

and the expected value for the differential *P* will follow the trend, which is:

$$E[d\,P] = a\,P\,dt \quad (24).$$

The method, which will be utilized to calculate the opportunity cost *F*, the price thresholds P_H, P_L, and the expected net present value *V* of the future growth, is the dynamic programming, by breaking the time into shorter decision periods.

The assumption here is that there is a risk-neutral firm which expects a rate of return ρ equals:

$$\rho = a + \delta \quad (25),$$

with $\rho > 0$. The constant *a* is the expected rate of capital gain and the constant δ is the dividend rate, which should be positive and different from zero.

Regarding the expected net present value of the project, two decision periods will be built, the current and the whole upcoming, by forming a suitable Bellman equation. The equation will be expressed as the sum of the immediate benefits at the time *t,* during a short time interval d_t, augmented by the expected value of the asset beyond the interval d_t, discounted by the factor $e^{-\rho d_t}$:

$$V(P, t) = \pi(P, t)\,dt + E\left[V(P + dP) * e^{-\rho d_t}\right] \quad (26).$$

The Ito's Lemma equation for the differential *dV* is:

$$dV = \frac{\theta V}{\theta t} dt + \frac{\theta V}{\theta P}\,dP + \frac{1}{2}\,\frac{\theta^2 V}{\theta P^2} dP^2 \; (27).$$

Replacing the (27) into the (26) and expanding the right-hand side of the equation (26), by replacing the $e^{-\rho d_t} = 1 - \rho\, d_t$ and the $\theta V/\theta t = 0$ for a small d_t, yields the equation (28):

$$V(P) = \pi(P)\,dt + (1 - pdt)\,[V(P) + V^{'}(P)\,dP + \tfrac{1}{2}\,V^{''}(P)\,dP^2] \quad (28).$$

Since the price variable *P* takes the form of a Geometric Brownian motion, equation (20) can be substituted into the equation (28):

$$V(P) = \pi(P)\,dt + (1 - p\,dt)[V(P) + V^{'}(P)\,a\,P\,dt + V^{'}(P)\,\sigma\,P\,dz +$$

$$+ \tfrac{1}{2}\,V^{''}(P)\,(a^2\,P^2 dt^2 + 2\,a\,P\,dt\,\sigma\,P\,dz + \sigma^2\,P^2 dz^2)] \quad (29).$$

The term $(d_t)^2$ can be omitted, as it goes faster to zero than d_t. Using the equations (22) and (23) into the equation (29):

$$V(P) = \pi(P)\,dt + V(P) - \rho\,dt\,V(P) + V^{'}(P)\,a\,P\,dt - p\,dt\,V^{'}(P)\,a\,P\,dt +$$

$$+\frac{1}{2} V''(P)\, \sigma^2 P^2\, dt - p\, dt\, \frac{1}{2} V''(P)\, \sigma^2 P^2\, dt \quad (30).$$

Respectively, the term $(d_t)^2$ into the equation (30) can be ignored again, as it goes faster to zero than d_t. The equation (30) can be divided throughout by d_t and rearranged; then for simplicity the drift *a* is assumed equivalent to the capital gain *a*, so it is replaced with $\rho - \delta$ from equation (25). Therefore, the result is the following second-order non-homogenous linear differential equation (31):

$$\frac{1}{2} \sigma^2 P^2 V''(P) + (\rho - \delta)\, P\, V'(P) - \rho V(P) + \pi(P) = 0 \ (31).$$

5.3 Calculation of the Optimal Investment Rules

Based on the entry and exit options, two discrete periods can be identified for the investment project, the idle and the active. At the time zero, when the project is still idle and there is no operating profit yet, the price *P* is $P \leq P_H$ and the profit π is $\pi(P)=0$. The value of the project $V_0(P)$ can be considered as the value of a call option and the equation (31) will become:

$$\frac{1}{2} \sigma^2 P^2 V_0''(P) + (\rho - \delta)\, P\, V_0'(P) - \rho\, V_0(P) = 0 \ (32).$$

For this idle period, the equation (31) is transformed into the second-order homogeneous linear differential equation (32). These types of differential equations are called Euler equations, where all possible solutions are of the general form:

$$V_0(P) = P^{\beta} \ (33).$$

By replacing the general solution (33) into the differential equation (32) and simplifying, yields the equation:

$$\frac{1}{2} \sigma^2 P^2 \beta\, (\beta - 1)\, P^{\beta-2} + (\rho - \delta)\, P\, \beta\, P^{\beta-1} - \rho\, P^{\beta} =$$

$$\frac{1}{2} \sigma^2 \beta\, (\beta - 1) + (\rho - \delta)\, \beta - \rho = 0 \quad (34),$$

The equation (34) is giving a quadratic equation in β. So the solution of equation (32) should be a linear combination of the two independent solutions of the above quadratic equation (34):

$$V_0(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} \quad (35),$$

where A_1 and A_2 are two arbitrary constants to be found, and β_1 and β_2 are the roots of the fundamental quadratic equation (34). Calculating the solution of the quadratic equation (34) for the roots β_1 and β_2:

$$\beta_{12} = \frac{\frac{1}{2} \sigma^2 - (\rho - \delta) \pm \sqrt{[(\rho - \delta) - \frac{1}{2} \sigma^2]^2 + 2\, \sigma^2 \rho}}{2 * \frac{1}{2} \sigma^2} \Leftrightarrow$$

$$\beta_1 = \frac{1}{2} - (\rho - \delta) / \sigma^2 + \sqrt{[(\rho - \delta) / \sigma^2 - \frac{1}{2}]^2 + \frac{2\rho}{\sigma^2}} > 1 \quad (36)$$

and

$$\beta_2 = \frac{1}{2} - (\rho - \delta) / \sigma^2 - \sqrt{[(\rho - \delta)/ \sigma^2 - \frac{1}{2}]^2 + \frac{2\rho}{\sigma^2}} < 0 \quad (37).$$

When the price P starts to increase, the investment will become more attractive. Since root β_2 is negative, the chance to raise the price P over an optimal threshold and make the project profitable, in the near future, is quite remote. In order to acquire the optimal positive value V_0, when P is increasing, and make sure that, if $V(P)$ goes to zero, P will also go to zero, coefficient A_2 of the negative root should be considered equal to zero and $A_1 \geq 0$. So, the general solution function (35) of the equation (32) is finally equal to:

$$V_0(P) = A_1 P^{\beta_1} \quad (38),$$

in which P receives values between the range $0 < P \leq P_H$ and the constant A_1 should be $A_1 \geq 0$.

Similarly, at the time $t_i = 1$, when the project is up and running, the price P should be always $P \geq P_L$ and the profit is equal to:

$$\pi(P) = P - C \quad (39).$$

The general solution (35) of the second-order linear differential non-homogenous equation (31) for project value $V_1(P)$ is formed as the combination of the solution of the homogeneous part, which is called the complementary solution, and the solution of the non-homogenous part, which is called the particular solution. The general solution (40) is created, in which the particular solution will be referred to as $v(P)$:

$$V_1(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} + v(P) \quad (40).$$

The method, which will be used to find the $v(P)$, is called the Undetermined Coefficients. As a first step, the equation (31) should be splitted into the homogenous and the non-homogenous part; then substituting the profit equation (39) into it, the equation (41) is created:

$$\frac{1}{2} \sigma^2 P^2 V''(P) + (\rho - \delta) P V'(P) - \rho V(P) = - P + C \quad (41).$$

The non-homogeneous part, on the right side, is the first-degree polynomial $- P + C$, with constant coefficients -1 and 1. Then, another first-degree polynomial (42) should be created with a similar degree and equal to the non-homogeneous polynomial part:

$$v'(P) = - A_3 P + A_4 C \quad (42).$$

The first derivative of the $v'(P)$ would be:

$$(v'(P))' = - A_3,$$

and the second derivative would be:

$$(v'(P))'' = 0.$$

Replacing the term $V(P)$ with the polynomial equation (42) into the equation (41):

$$(\rho - \delta) P (- A_3) - \rho(- A_3 P + A_4) = - P + C \Leftrightarrow$$

$$A_3 \delta P - A_4 \rho = - P + C \quad (43).$$

The corresponding terms on both sides of the equation (43) should have the same coefficients. Therefore, equating the coefficients of like terms:

$$A_3 \delta = -1 \leftrightarrow A_3 = - \frac{1}{\delta}$$

$$- A_4 \rho = 1 \leftrightarrow A_4 = - \frac{1}{\rho}$$

Replacing the values of the coefficients A_3 and A_4 into the equation (42), the polynomial would become:

$$v'(P) = P / \delta - C / \rho \quad (44),$$

which is the solution of the non-homogenous part of the equation (41). Thus, the general solution (40) yields the equation (45):

$$V_1(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{\rho} \quad (45).$$

It is important to highlight here that the particular solution equation (44*)* gives a very useful portion. This value defines the expected net present value of the project running forever, with a profit flow *P - C* and an initial price P_0, assuming, for simplicity, that the operating costs have a constant value *C* [66]:

$$E\int_0^\infty (P_t - C) * e^{-\rho t} d_t =$$

$$E\int_0^\infty (P e^{at} - C) * e^{-\rho t} d_t =$$

$$P\int_0^\infty e^{-(\rho - \alpha) t} d_t - C\int_0^\infty e^{-\rho t} d_t =$$

$$\frac{P}{-(\rho - \alpha)} * (-1) + \frac{C}{\rho} (-1) =$$

$$P / \delta - C / \rho \ (46).$$

Covering the time that the project is active, every instant, the firm should decide either to continue the project or to stop it and receive a termination payoff. In order to cease the operation, the optimal suspension value P_L should be estimated as part of the firm's strategy. This value is expected to be found in the general solution equation (45). In fact, the lower threshold P_L can arise only from the complementary part of the equation (45), where the negative root β_2 is the term that could give the price variable *P* a lower value, provided that the coefficient B_1 of the positive root β_1 is considered equal to zero [67]. So finally, the general solution becomes:

$$V_1(P) = B_2 P^{\beta_2} + P / \delta - C / \rho \quad (47),$$

in which price variable *P* receives values between the range $P_L < P < \infty$.

Having found the general equations (38) and (47) of the idle and the operational period, boundary conditions should be introduced in order to contribute to the calculations about the unknown terms. The following conditions (48), (49) and (50) should be met and satisfied by the price *P**:

$$V(0) = 0 \quad (48),$$

$$V(P^*) = P^* - I \quad (49),$$

$$V'(P^*) = 1 \quad (50).$$

As mentioned before, *P** is the optimal price value to invest. Condition (48) defines that the project will be of no value if the profit *P* goes to zero. Condition (49) is called the "value-matching", as it matches the value of the project with the payoffs upon investment. The last condition (50) originates from the economic field and remarks the continuity, with the requirement for the equation slope of both the project's value and the payoffs function to meet tangentially. This condition is called the "smooth-pasting" and occurs from the derivative of the condition (49), where the values and the derivatives have to match at the boundary.

Due to the different project status at times 0 and 1, two different profit thresholds exist, the P_H and the P_L, respectively, as pointed out before. As a result, the boundaries (49) and (50) can be modified as follow. At the time 0, when the optimal price value P^* is equal to $P^* = P_H$:

$$V_0(P_H) = V_1(P_H) - I \quad (51),$$

$$V_0'(P_H) = V_1'(P_H) \quad (52).$$

At the time 1, the price value would be equal to $P = P_L$:

$$V_1(P_L) = V_0(P_L) - E \quad (53),$$

$$V_1'(P_L) = V_0'(P_L) \quad (54).$$

with E to be the lump-sum abandonment cost, which can be positive or negative but less than I. Replacing the equations (38) and (47) into the boundary equations creates a system of four non-linear equations. The system (55) has four unknowns, the thresholds P_H and P_L, and the coefficients A_1 and B_2:

$$\begin{cases} -A_1 P_H^{\beta_1} + B_2 P_H^{\beta_2} + P_H / \delta - C / \rho = I \\ -\beta_1 A_1 P_H^{\beta_1 - 1} + \beta_2 B_2 P_H^{\beta_2 - 1} + 1 / \delta = 0 \\ -A_1 P_L^{\beta_1} + B_2 P_L^{\beta_2} + P_L / \delta - C / \rho = -E \\ -\beta_1 A_1 P_L^{\beta_1 - 1} + \beta_2 B_2 P_L^{\beta_2 - 1} + 1 / \delta = 0 \end{cases} \quad (55).$$

5.4 Calculation of the Opportunity Cost

Apart from the value of the project, the same methodology of dynamic programming can be applied to estimate the value F of the option, which constitutes the opportunity cost of the investment project. The investment opportunity has no cash flow since the project is not active yet. The only return from holding the option could be the capital appreciation. The value can be acquired before the call option is exercised. When the project starts, the firm receives the value of the project but gives up the opportunity to invest. Thus, the opportunity cost should be calculated in the total sunk cost of investing.

Building the Bellman equation during a continuous-time, when the value V of the project is not optimal enough to invest and until the time it will be, the expected rate of growth for the option value will be equal to the expected return of the investment opportunity, for a time interval d_t :

$$E(dF) = \rho F d_t \quad (56).$$

The Ito's Lemma equation for the differential dF is:

$$dF = \frac{\theta F}{\theta t} d_t + \frac{\theta F}{\theta V} d_V + \frac{1}{2} \frac{\theta^2 F}{\theta V^2} d_V^2 \quad (57).$$

Using the equation (57) in equation (56):

$$\frac{1}{2} F''(V) (dV)^2 + F'(V) dV - \rho F(V) d_t = 0 \quad (58).$$

Considering the value of the project V follows a Geometric Brownian motion:

$$d V = a V d_t + \sigma V d_z \quad (59)$$

Then, substituting the equations (59) and (25) into the equation (58), simplifying using the equations (22) and (23), and dividing through by d_t, yields:

$$\frac{1}{2} \sigma^2 V^2 F''(V) + (\rho - \delta) V F'(V) - \rho F(V) = 0 \quad (60).$$

The second-order homogenous differential equation (60) is the same as the equation (32), but in *V*. Again, the solution of the equation (60) has the form of the general solution function (35). But the coefficient A_2 of the negative power of *V* needs to be set as zero in order for *V* to reach the optimal threshold in the near future:

$$F(V) = A V^{\beta_1} \quad (61).$$

The boundary conditions for the option value *F* between the range zero and *V**, which is the optimal value to invest, are:

$$F(0) = 0 \quad (62),$$

$$F(V^*) = V^* - I \quad (63),$$

$$F'(V^*) = 1 \quad (64).$$

Equation (63) confirms an important point here about investment opportunities. Each time a firm enters an investment, the full investment cost would be the sunk cost *I* plus the opportunity cost *F(V)*, which is given up, when the project gets active. The equation for the optimal exercised value *V** can be found, combining the general solution (61) with the boundary conditions (62), (63) and (64) as below:

$$\frac{V^*}{V^* - I} = \frac{V^* * F'(V^*)}{F(V^*)} = \frac{V^* * A * \beta_1 * V^{*(\beta_1 - 1)}}{A * V^{*(\beta_1)}} = \beta_1 \Leftrightarrow$$

$$V^* = \frac{\beta_1}{\beta_1 - 1} * I \quad (65),$$

and the coefficient *A* would be:

$$A = \frac{V^* - I}{V^{*(\beta_1)}} = \frac{(\beta_1 - 1)^{(\beta_1 - 1)}}{\beta_1^{\beta_1} * I^{(\beta_1 - 1)}} \quad (66).$$

The equation (65), even in this simplified example regarding market conditions and costs, presents how incorrect can be the NPV rule when uncertainty is not being considered. The magnitude β_1 is bigger than the unit, thus the optimal project value *V** will be bigger than the direct cost *I* by the factor β_1 (β_1-1).

Another approach is to find the value of the option as a function of the price *P*, rather than the value of the project *V*, by using again the same procedure of the dynamic programming. The Ito's Lemma equation for the differential *dF* would be:

$$dF = \frac{\theta F}{\theta t} d_t + \frac{\theta F}{\theta P} d_P + \frac{1}{2} \frac{\theta^2 F}{\theta P^2} d_P^{\,2} \quad (67).$$

Using equation (67) in equation (56):

$$\frac{1}{2} F''(P) (dP)^2 + F'(P) dP - \rho F(P) d_t = 0 \quad (68).$$

Then, substituting the equations (20) and (25) into the equation (68), simplifying using the equations (22) and (23) and dividing through by d_t:

$$\frac{1}{2}\,\sigma^2\, P^2\, F''(P) + \left(\rho - \delta\right) P\, F'(P) - \rho\, F(P) = 0 \quad (69).$$

The equation (69) has the following general solution and boundary conditions, for the price range zero and optimal *P**:

$$F(P) = A\,P^{\beta_1} \quad (70),$$

$$F(0) = 0 \quad (71),$$

$$F\left(P^*\right) = V(P^*) - I \quad (72),$$

$$F'\left(P^*\right) = V'(P^*) \quad (73).$$

While keeping it active forever, the expected present value of the project was expressed with the equation (46). Considering now that the project stays active forever, but with no operating costs, a more simplified portion of the project value *V(P)* can be found. This portion can also be interpreted as the value of the project before it gets activated, and it can be used in the calculations of the opportunity costs. The equation (74) is this fundamental expression of the revenue flow P_t, with $E(P_t) = P * e^{at}$, discounted by the interest rate ρ and using the equation (25) for simplicity [63]:

$$V(P) = \int_0^{\infty} P_t\, e^{-\rho t}\, d_t$$

$$V(P) = P \int_0^{\infty} e^{-(\rho-\alpha)t}\, d_t$$

$$V(P) = -\left(\frac{P}{\rho - \alpha}\right)\left[\lim_{t\to\infty} e^{-(\rho - \alpha)t} - \lim_{t\to 0} e^{-(\rho - \alpha)t}\right]$$

$$V(P) = -\left(\frac{P}{\rho - \alpha}\right) * [0 - 1]$$

$$V(P) = \frac{P}{\delta} \quad (74).$$

Using the fundamental equation (74) into the value - matching equation (72) for the optimal price value *P** yields:

$$F\left(P^*\right) = V(P^*) - I = \frac{P^*}{\delta} - I \quad (75).$$

Combining the general solution (70) with the boundary equations (71), (73) and (75), the equation (76) for the optimal price value *P** is:

$$F\left(P^*\right) = \frac{P}{\delta} - I \Leftrightarrow$$

$$\left(A\,P^{*\beta_1}\right)' = \left(\frac{P^*}{\delta} - I\right)' \Leftrightarrow$$

$$\beta_1\, A\, P^{*(\beta_1 - 1)} = \frac{1}{\delta} \Leftrightarrow$$

$$\delta\, \beta_1\, A\, P^{*(\beta_1 - 1)} = \frac{A\,P^{*\beta_1}}{\frac{P^*}{\delta} - I} \Leftrightarrow$$

$$P^*\left(\beta_1 - 1\right) = \delta\, \beta_1\, I \Leftrightarrow$$

$$P^{*} = \frac{\beta_1}{(\beta_1 - 1)} * \delta * I \ (76).$$

The equation of the optimal project value *V** arises, using the equation (74) and substituting the equation (76). The resulting equation (77) is the same as the equation (65):

$$V\left(P^{*}\right) = \frac{P^{*}}{\delta}$$

$$V\left(P^{*}\right) = \frac{\frac{\beta_1}{(\beta_1 - 1)} * \delta * I}{\delta}$$

$$V\left(P^{*}\right) = \frac{\beta_1}{(\beta_1 - 1)} I \ (77).$$

5.5 Investments' Decay

In the previous subchapters, the analysis of the optimal values of the project was performed, assuming that the project is running to infinity, following a non-stationary Wiener process with a constant drift rate. However, in real life, the projects don't last forever and don't run under the same conditions. Prolonged usage and physical sorts of decay make important the maintenance or even the replacement of the infrastructure. On the other hand, Dixit (1989) [66] believed that an investment rust rapidly when it's not used.

Moreover, technology is ever-growing, deploying new services and short-lived products at the fastest pace in the last decade. Such rapid evolution can drastically affect both the demand of a service due to the fast introduction of substitutes. The migration to a more cost-efficient solution and the shutdown of the legacy projects could be proven necessary. In addition, disruptors and OTT players can affect a project's life cycle by launching competitive and innovative applications to the market, driving projects to their death.

5.5.1 Sudden Death Option

The Operator has to take into consideration a lot of parameters, including the previous ones regarding decay and based on both the evolution of the technology and the new telecommunication trends, to set a useful lifetime for each investment.

Thus, following the technical milestones and the consumers' needs for new benefits, a fixed lifetime *T* should be defined forming the Sudden Death Option. During the period *T* the service will be active, but after this point, the operation of the project will suddenly stop. The fundamental expected present value of the project, during this period, without any operating costs, is rising at the exponential growth rate of *a*, discounted by the interest rate *ρ*:

$$V(P) = E \int_0^T P_t e^{-\rho t} d_t$$

$$V(P) = \int_0^T P e^{at} e^{-\rho t} d_t$$

$$V(P) = \frac{P}{-(\rho - \alpha)} \int_0^T e^{-(\rho-\alpha)t}$$

$$V(P) = -\left(\frac{P}{\delta}\right)\left[\lim_{t \to T} e^{-(\delta)t} - 1\right]$$

$$V(P) = \frac{P}{\delta}\left[1 - e^{-\delta T}\right] \quad (78).$$

Building again the value of the option as a function of the price value *P*, the Ito's Lemma equation for the differential *dF* will be used:

$$dF = \frac{\theta F}{\theta t} d_t + \frac{\theta F}{\theta P} d_P + \frac{1}{2}\frac{\theta^2 F}{\theta P^2} d_P^{\,2} \quad (79).$$

Using the equation (79) into the Bellman equation (56) yields:

$$\frac{1}{2} F''(P)\,(dP)^2 + F'(P)\,dP - \rho\,F(P) d_t = 0 \quad (80).$$

Substituting the equations (20) and (25) into the equation (80), simplifying using the equations (22) and (23) and dividing through by d_t:

$$\frac{1}{2}\sigma^2 P^2 F''(P) + (\rho - \delta) P F'(P) - \rho F(P) = 0 \quad (81).$$

The equation (81), for the price range *(0, P*]*, has the same general solution and boundary conditions as the equation (69):

$$F(P) = A P^{\beta_1} \quad (82),$$

$$F(0) = 0 \quad (83),$$

$$F(P^*) = V(P^*) - I \quad (84),$$

$$F'(P^*) = V'(P^*) \quad (85).$$

The smooth pasting condition will be applied for the optimal value *P**. Substituting the expression (78) into the condition (85), it gives a numerical result for the derivative of the value of the option:

$$F'(P^*) = V'(P^*) = \frac{1 - e^{-\delta T}}{\delta} \quad (86).$$

Replacing the equation (78) and the general solution (82) into the value matching condition (84), the expression (87) for the optimal value *P** is extracted, which indicates how many times should be over the investment cost *I*:

$$F(P^*) = V(P^*) - I \Leftrightarrow$$

$$(AP^{\beta_1})' = \left(\frac{P^*}{\delta}\left[1 - e^{-\delta T}\right] - I\right)' \Leftrightarrow$$

$$P^* = \frac{\beta_1}{(\beta_1 - 1)} * \delta * I * \frac{1}{[1 - e^{-\delta T}]} \quad (87).$$

An important note here is that if the equation (87) is replaced into the equation (78), the equation created is identical to the equations (65) and (77), confirming the validity of the methodology.

6. APPLICATION OF THE REAL OPTIONS IN THE GREEK TELECOMMUNICATION MARKET

To illustrate the use and the advantages of the Real Options, the framework of the "Sudden Death" is applied in the case of a VoLTE investment in the Greek Telecommunication market. The Optimal investment rule is calculated using the formulas that were derived based on the Sudden Death option. Then the result is analyzed and compared with the one that was found based on the NPV rule.

Moreover, in the context of the risk analysis, a Monte Carlo simulation is performed. A probability distribution is extracted that shows all the possible revenue outcomes and the likelihood that each outcome will occur. Identifying the risk, the Real Option methodology and the Traditional methodology are compared and evaluated for their accuracy and reliability to provide investment rules.

6.1 Characteristics of the Greek Mobile Telecommunication Market

In this chapter the proposed Real Options methodology is applied in a VoLTE Investment scenario for the Greek Mobile telecommunications market. A useful component of the methodology is the historical data. If data exists, it should be used to outline the variables such as the volatility and the growth rate. If not, similar projects or the competition can provide a good estimation of these variables.

For this example, the mobile registered subscriptions data were collected from the national regulatory authority, Hellenic Telecommunication & Post Commission (EETT) [68]. In Figure 19, the evolution of the subscriptions is presented since the deployment of the first GSM network in 1998.

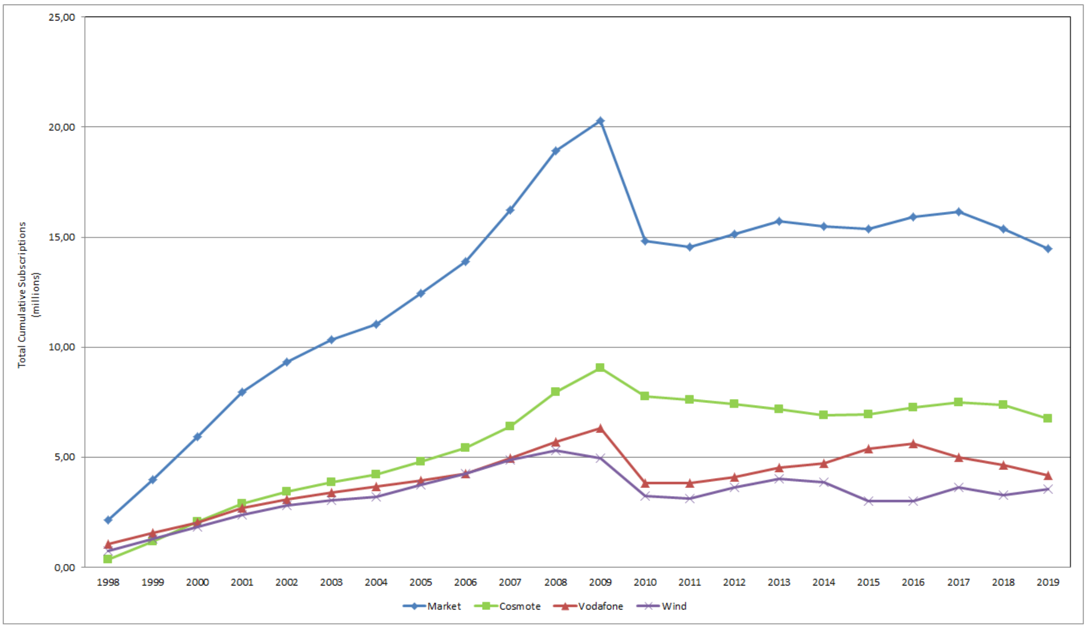

Figure 19: Total Mobile Register Subscriptions in Greek Telecommunication market.

For the majority of the time, the market consisted of three players, Cosmote, Vodafone & Wind. From 2002 to 2007, a fourth operator named Q-telecom entered the market. In 2007, Wind acquired Q-telecom. For simplicity, the related market share of Q-telecom will be considered part of Wind's share. Between 2014 and 2018, Cyta appeared in the market as an MVNO player. However, it will also not be mentioned because the firm didn’t manage to achieve more than 1% market share during the four years.

In 2017, Cosmote launched first the VoLTE service in the market. Three years later, at the beginning of 2020, Vodafone also activated VoLTE; followed by Wind at the end of the same year. Since then, all the VoLTE capable devices in Greece can register and perform VoLTE calls without the need to switch back to 2G or 3G network for telephony services.

6.2 Data Validation using Kolmogorov-Smirnov Normality test

The assessment for the log-normality of the stochastic variable is a prerequisite and must be confirmed in order to be able to apply the GBM diffusion process. For this purpose, the Kolmogorov-Smirnov test will be deployed in the historical data.

The goodness-of-fit Kolmogorov-Smirnov test was originally proposed in Andrei Nikolayevich Kolmogorov's paper in 1933 [69]. It is used to prove how well the distribution of any sample data conforms to some specific theoretical distribution functions $F(x)$. Given a random sample of a population, the test compares the observed versus the expected cumulative relative frequencies by calculating the maximal absolute difference between these values. This way, it examines to what extent the scores deviate from the initial hypothesis. In order to accept the initial hypothesis, this deviation should be very small with a high probability or p-value. Or reversely, a large deviation should have a low p-value.

In Annex I, the Kolmogorov-Smirnov normality test is deployed for the historical data of the Greek Market (Table 21), Cosmote (Table 22), Vodafone (Table 23) and Wind (Table 24) using the Microsoft Excel tool. In the first column, the absolute natural logarithm of the annual change $|ln(X_t/X_{t-1})|$ is calculated and sorted from smaller to bigger. According to the equation (15), these results should be normally distributed, following the BM properties.

To prove the initial hypothesis that the data comes from the normal distribution, the formula NORM.DIST is applied in the sorted results at the column CNDF (NORM.DIST). This function returns the cumulative normal distribution of the data given the mean and the standard deviation of the sample.

In the last column, the absolute values of the difference are calculated between the cumulative normal distribution of the sample and the expected values of the cumulative normal distribution function. The maximum value of the absolute values of the difference will be the largest deviation or the greatest vertical distance between the two distributions.

According to the Kolmogorov-Smirnov test criterion, for a sample of 21 observations and 95% confidence level, the maximum deviation or distance between the two distributions should be no more than 0,28724. The significance level is the accepted probability of making the wrong decision when the null hypothesis is true.

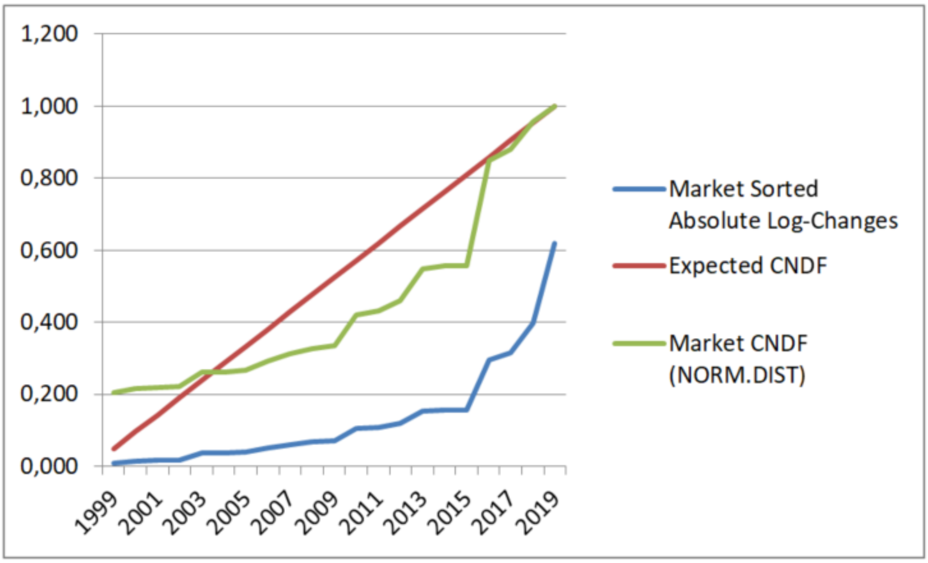

Figure 20: Log-Normality Test Results for Greek Market

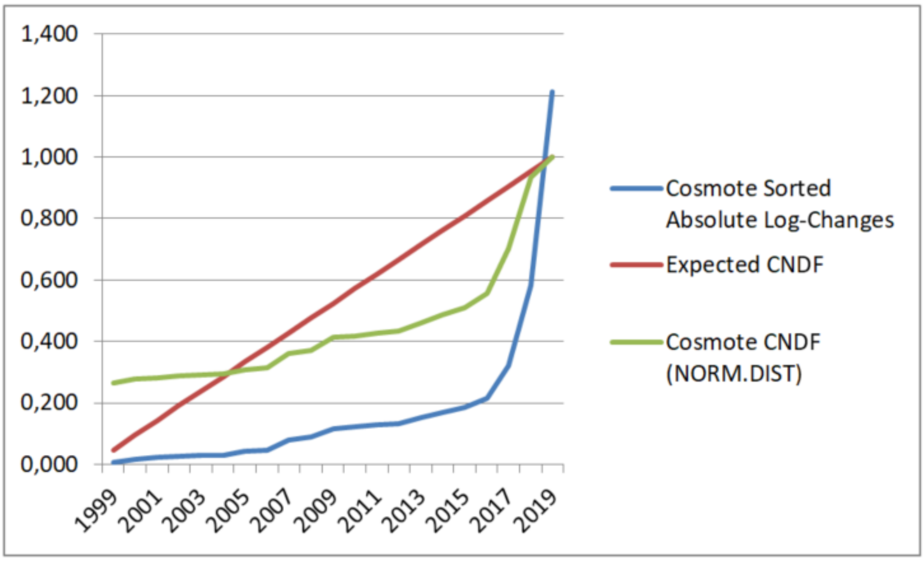

Figure 21: Log-Normality Test Results for Cosmote

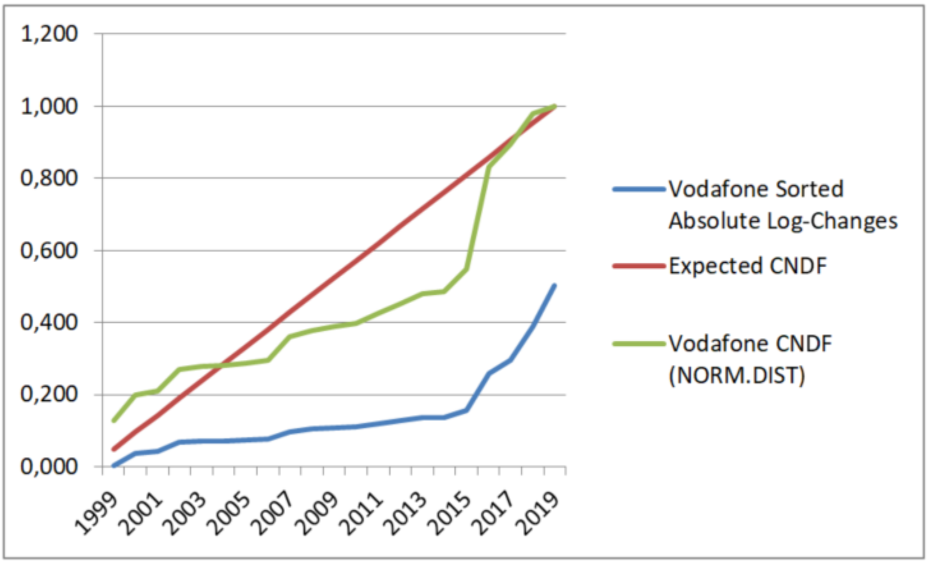

Figure 22: Log-Normality Test Results for Vodafone

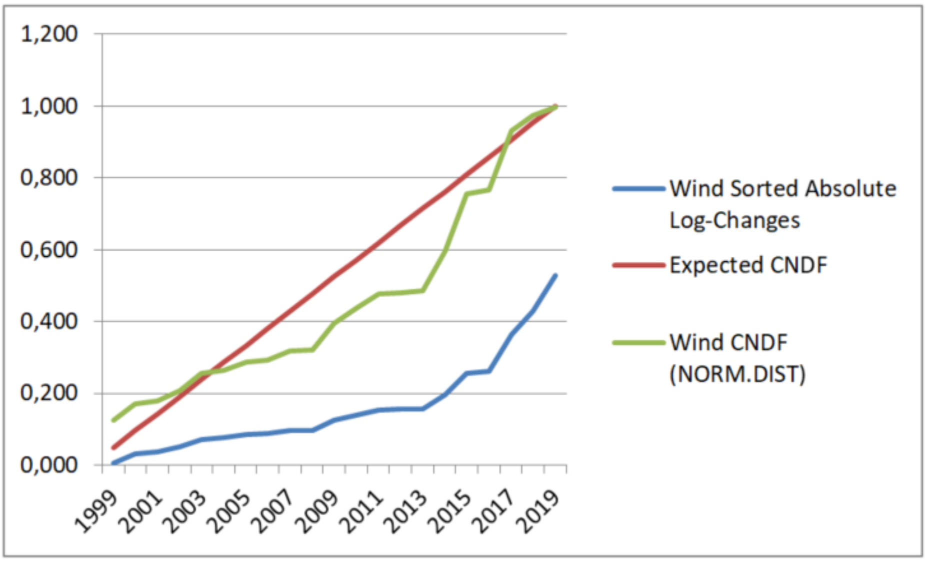

Figure 23: Log-Normality Test Results for Wind

Figures 20, 21, 22 and 23 depict the graphs of the sorted absolute log changes, the cumulative normal distribution, and the expected cumulative normal distribution function for the Greek Market and each Operator.

The Kolmogorov-Smirnov test was conducted with 95% confidence level and the results are summarized in Table 7. The normality test shows that the log-changes of the Market, Vodafone and Wind follow a normal distribution function since the maximum absolute values of the difference are below the critical threshold. On the other side, Cosmote log-changes are not normally distributed since the greatest distance is over the critical value.

Table 7: Kolmogorov-Smirnof Normality Test Results

	Max Deviation	Critical Value
Market	0,2542	0,28724
Cosmote	0,30243	
Vodafone	0,27793	
Wind	0,22941	

Consequently, the Real Options methodology will be applied only in the historical data of Vodafone and Wind.

6.3 Variables Extraction from the Historical Data

Since historical data of Vodafone and Wind are indeed log-normally distributed, according to Kolmogorov-Smirnov test, the next step is to perform a statistical analysis of the historical data and extract the parameters from the sample under study. In Table 25 of Annex II, the risk and the expected return rate are calculated.

As stated before, the risk can be distinguished either as industry-wide or as firm-specific. The industry-wide risk is derived from the uncertainty and competition of the market environment. Apart from the industry-wide, each player also owns a firm-specific or idiosyncratic risk related to its brand, firm's performance and more. The risk is applied as the constant volatility parameter σ into the methodology. It is found using the Microsoft Excel function STDEV.S for the actual annual log-changes *LN(i/i-1)* of the data set.

Regarding the expected return rate ρ, the mathematical type $\rho = a + \frac{1}{2}\sigma^2$ can be used, based on the solution yielded from the variance calculation of GBM [70].

According to the above equation, the volatility and the average growth rate should be calculated. Volatility σ was found previously while for the average growth rate a the Microsoft Excel function AVERAGE is used in the actual log-changes *LN(i/i-1)*. The results are presented in Table 8.

Table 8: Variables Volatility and Expected Rate of Return

Parameter	Vodafone	Wind
σ	0,178	0,201
a	0,066	0,074
ρ	0,081	0,094

Based on the official published financial reports of the Greek General Commercial Registry for the last 15 years, the dividend rate δ for Vodafone it was found to be equal to 0,005 per share and for Wind equal to 0,002 per share [71]. However, the effect of ρ, σ and I on the investment is of more interest for this analysis than the effect of δ. So, it will be considered equal to 0,005 for both operators in order to have the same impact on the results.

Regarding the investment costs, a draft cost estimate is provided in Table 26 of Annex II. It covers several cost areas of the VoLTE solution; the hardware purchase cost of the nodes, the hardware expansion, the software and the professional services. The aforementioned costs are related to the network deployment and are necessary to launch and run the service. For the project purposes, the actual hardware will be priced at 15.000 € per blade, and the expansions, upgrades and other services will be analogous to this price. The total estimated price to install and operate an IMS network of 168 blades and capacity for 5 million subscribers is 5.810.000 €. Based on the subscribers' volume, Vodafone has to deploy one time an IMS subsystem and Wind 0,8 times. On top of the network deployment cost, the software licenses are included, which are priced at 0,01 € per subscriber.

Last but not least is the spectrum cost. The 4G spectrum was auctioned to the three players by EETT in 2014 for 15 years [72]. In the latest report of 2019 [6], mobile data volume was estimated to be 225 million GB, while the voice volume at 28,7 billion minutes. The throughput of a VoLTE call using the AMR-WB 23.85 codec is 42 kbps [73]. Based on the volume ratio between voice and data traffic, 4% of the spectrum cost will be allocated to the VoLTE budget. With the growth of 5G and mobile data, maybe this ratio ought to decrease even more.

Thus, for each operator the total investment cost I is provided in the last column of Table 26. For exercise purposes, in the cost I it will be included the hardware cost, the software licenses and the spectrum cost plus the operating costs. The Operating cost includes the hardware expansions, the software updates and the professional services.

For the hardware cost, Vodafone has to pay a total of 2.520.000 € and Wind 0,8 times the 2.520.000, meaning 2.016.000 €. So, for Vodafone, the value *I* is equal to 7.541.040 + 3.290.000 € and for Wind 6.924.000 + 2.632.000 €.

6.4 Real Options Methodology Application

In order to find the optimal *P** and the value of the project *V(P*)*, the equation (78) and (87) will be used from the Sudden Death option methodology. The required parameters and their values are presented in Table 9. The roots β_1 and β_2 have been calculated from the equations (36) and (37) using a Microsoft Excel sheet. The lifetime of the VoLTE investment will be set to 15 years, considering the technology trends, the spectrum allocation time and the utilization of VoLTE service from both 4G and 5G technologies.

Table 9: Values of Coefficients of the Real Options solution

Parameter	Vodafone	Wind
σ	0,178	0,201
ρ	0,081	0,094
δ	0,005	0,005
β_1	1,054	1,045
β_2	-4,851	-4,451
T (years)	15	15
I (€)	10.831.040	9.556.000

In Table 10, the optimal investment rule *P** and the value of the project *V(P*)* (in millions) are presented for both Operators, Vodafone and Wind.

Table 10: Real Option Optimal Investment Values

Parameter	Vodafone	Wind
P* (€)	14.644.421	15.224.440
V(P*) (€)	211.630.959	220.012.992

Considering the Average Revenue per User (ARPU) to be 150€ per year, the minimum number of subscribers needed for each operator in order to enter risk-free and safe into the market and activate VoLTE is presented in Table 11.

Table 11: Real Option Optimal Subscribers Threshold

	Vodafone	Wind
Subscribers	97.630	101.497

6.5 NPV Methodology Application

As stated previously, traditional investment theories implement mainly the NPV rule to decide whether to invest or not in a new project. According to the NPV rule, if any project's net present value is a bit over zero, it will be positive evidence to proceed with the investment. Thus, putting the NPV equals zero, the minimum stream of profits can be found, assuming that the cost stream is well-known. Then, using the ARPU value, the profits can be transformed into the number of subscribers.

According to the NPV rule, the parameters that are needed to calculate the corresponding net present value are the discount rate which discounts the future cash flows to the present-day value, the lifetime of the project, the initial investment cost *I* and the future operating cost *C*. The discount rates were obtained from the financial reports for 2016 [74] [75]. The values for each parameter are presented in Table 12.

Table 12: Values of Variables of the NPV rule

	Vodafone	Wind
ρ	12,6%	13,47%
T (years)	15	15
ARPU (€)	150	150
I (€)	7.541.040	6.924.000
C (€)	3.290.000	2.632.000

As it was mentioned, in order to find the critical subscribers threshold, the NPV value will be set as zero and the number of subscribers will be considered the same for every year of the project. Moreover, the annual ARPU will be equal to 150 € and the number of subscribers cannot be more than the total subscribers database. Using a Microsoft Excel sheet and the Solver component, the NPV rule is deployed in Annex III, Table 27 and 28. The calculations give a constant minimum amount of subscribers needed per year in order to have a positive net present value. The results for both operators Vodafone and Wind are summarized in Table 13.

Table 13: NPV Results summary

	Vodafone	Wind
Subscribers per year	9.082	8.487
Total Subscribers of the project	136.223	127.303
Yearly Revenues (€)	1.362.300	1.273.050
Total Revenues of the project (€)	20.434.500	19.095.750

6.6 Monte Carlo Simulation for Risk Assessment

The Monte Carlo methodology is a probabilistic numerical technique used to estimate the outcome of stochastic process which cannot be modelled implicitly. It was invented in 1949 before the advent of modern computers by two mathematicians, Stanisław Ulam and John von Neumann, to simulate a chain reaction in highly enriched uranium, simply speaking an atomic explosion.

The model's contribution is to identify and quantify uncertainty related risk. It takes single-point estimates for inputs, considering that these inputs may vary due to uncertainty and variability, and generates thousands of scenarios in the context of the probabilistic modelling approach.

In this study, a Monte Carlo simulation was conducted for the risk assessment of the VoLTE project using the Crystal Ball spreadsheet-based application in Microsoft Excel 2016. The Crystal Ball offers Excel two important capabilities: the random simulation of the model and the replacement of single inputs that are subject to uncertainty with a probability distribution.

As initial value P_0, the NPV "Yearly Revenues" from Table 13 were used. The simulation derives a probability distribution allowing the estimation of the range of possible NPV values of the VoLTE project. The distribution fit obtained from 100,000 iterations was lognormal for both simulations.

For Vodafone, the results showed positive NPV values with a probability of 95,516% and negative NPV values with a probability of 4,484%. The mean value was found at 6.506.530,51 €, the standard deviation value at 4.953.886 € and the minimum NPV value at -4.620.328,22 €.

Table 14: Monte Carlo Simulation Statistics for Vodafone

100,000 Trials

Statistic	Fit: Lognormal	Forecast values
Trials	---	100,000
Base Case	---	$10,836,955.14
Mean	$6,504,959.84	$6,506,530.51
Median	$5,677,442.00	$5,681,193.54
Mode	$4,173,232.33	---
Standard Deviation	$4,931,036.08	$4,953,886.11
Variance	$24,315,116,773,485.10	$24,540,987,573,971.80
Skewness	1.16	1.23
Kurtosis	5.48	5.93
Coeff. of Variation	0.7580	0.7614
Minimum	($6,834,085.19)	($4,620,328.22)
Maximum	Infinity	$55,052,778.17
Mean Std. Error	---	$15,665.56

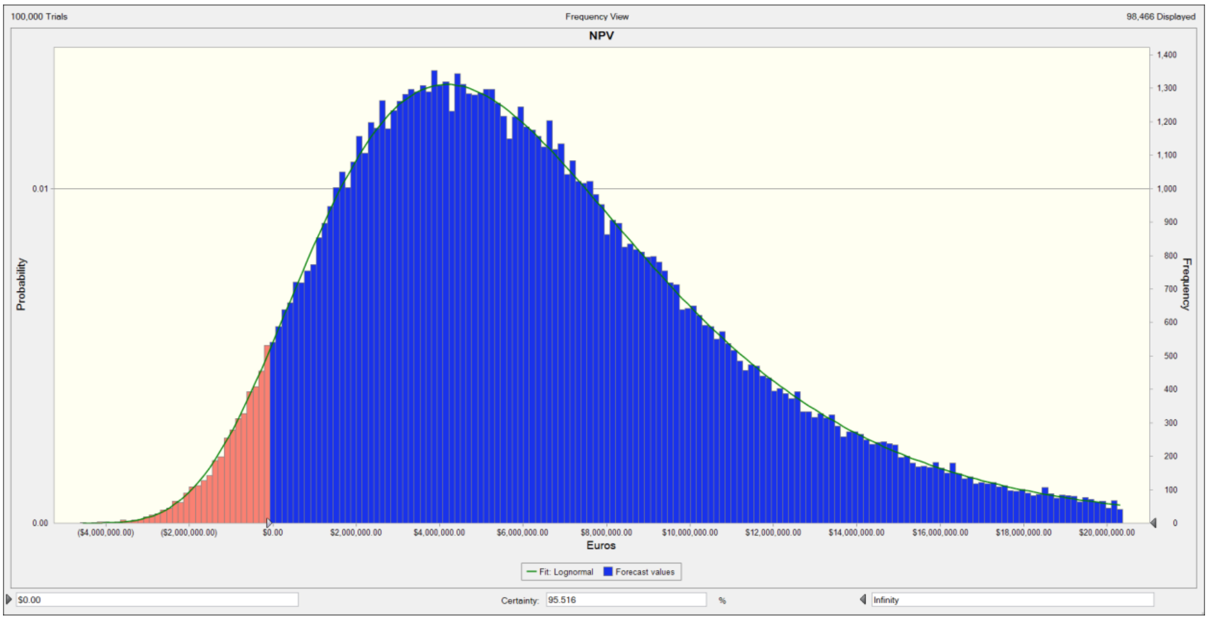

Figure 24: Monte Carlo Histogram for Vodafone – Probability Distribution and Positive Value range

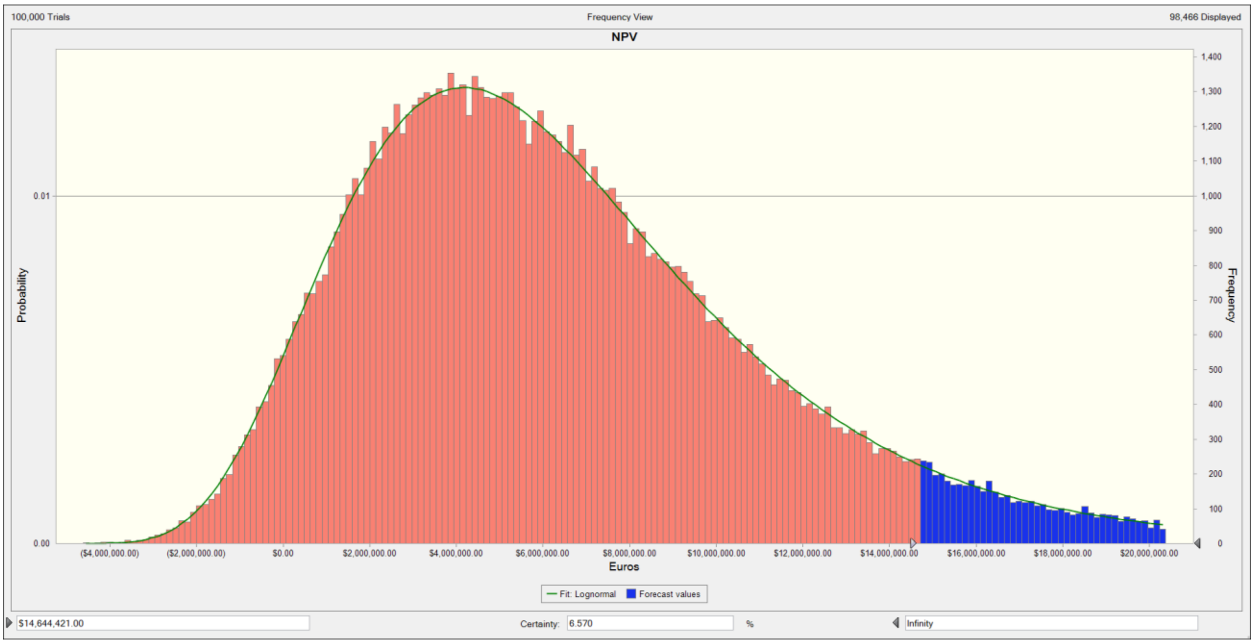

Figure 25: Monte Carlo Histogram for Vodafone – Optimal Threshold

For Wind, the results showed positive NPV values with a probability of 95,122% and negative NPV values with a probability of 4,878%. The mean value was found at 6.776.912,18 €, the standard deviation value at 5.467.962 € and the minimum NPV value at -4.377.666,42 €.

Table 15: Monte Carlo Simulation Statistics for Wind

Statistic	Fit: Lognormal	Forecast values
Trials	---	100,000
Base Case	---	$9,164,978.15
Mean	$6,774,321.14	$6,776,912.18
Median	$5,752,018.70	$5,734,224.35
Mode	$3,946,857.44	---
Standard Deviation	$5,437,306.58	$5,467,962.13
Variance	$29,564,302,877,418.80	$29,898,609,893,625.50
Skewness	1.36	1.41
Kurtosis	6.45	6.71
Coeff. of Variation	0.8026	0.8069
Minimum	($5,969,683.70)	($4,377,666.42)
Maximum	Infinity	$57,539,156.53
Mean Std. Error	---	$17,291.21

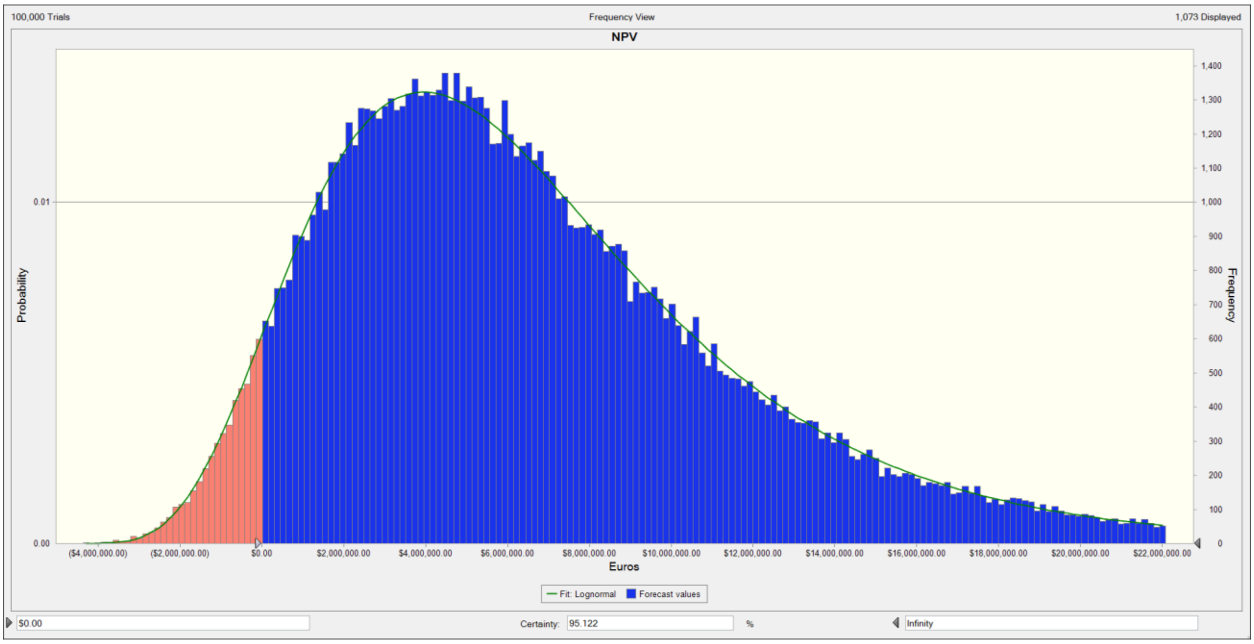

Figure 26: Monte Carlo Histogram for Wind – Probability Distribution and Positive Values range

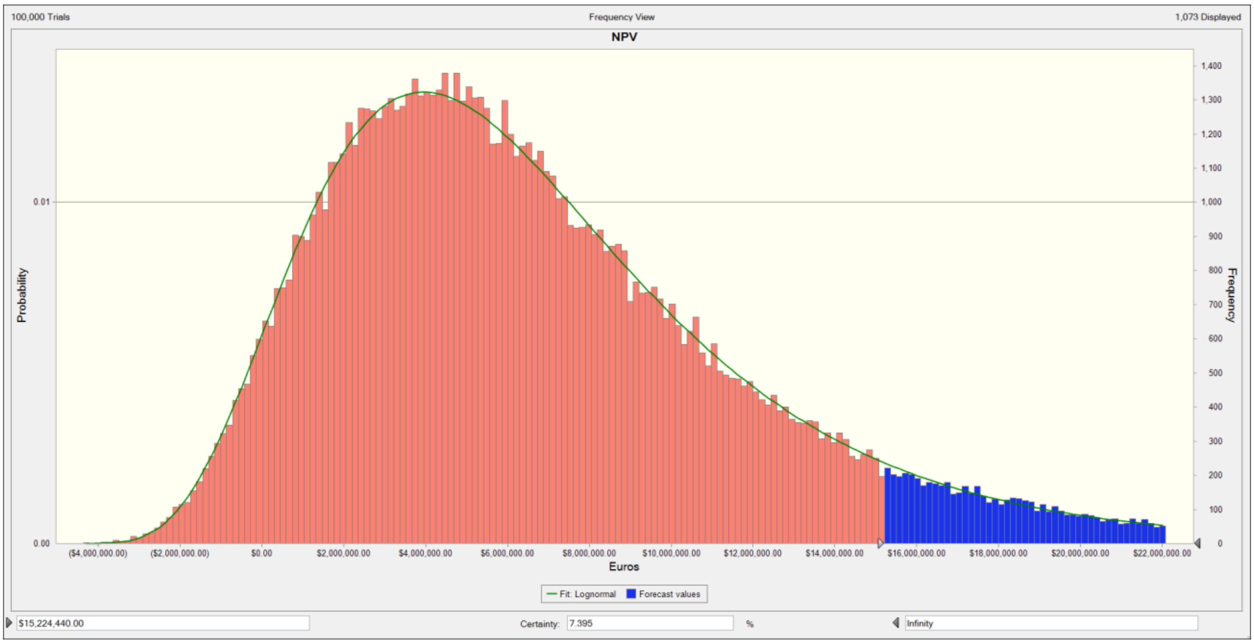

Figure 27: Monte Carlo Histogram for Wind – Optimal Threshold

6.7 Result Analysis

The proposed methodology was applied in a real world example assuming that the lognormal distributed variable of demand evolves linearly with constant drift rate *a* and standard deviation σ, following a GBM process. It can be concluded that the goal to provide the optimal investment rule regarding the VoLTE project using the solution derived from the Sudden Death option has been carried out successfully. Reviewing the Real Options results, comparing with the NPV results and analyzing the Monte Carlo simulation some interesting observations can be mentioned here.

Based on the Real Options results, the firm demands excessive project value to agree to join the irreversible investment, as it recognizes and inherits the uncertainty over the future rewards into the solution. This additional cash flow value is bigger than the sunk cost *I* by the fraction $\beta_1/(\beta_1-1)$, where $\beta_1 > 1$. Readers can verify this argument from the equations (65), (77) and (78). On the other side, the NPV solution consists of less parameter including the investment cost, the operating cost and most important the discount rate into the calculations to discount future cash flows back to their present value. However, this approach can be misleading and risky, especially for investments in competitive markets, as it ignores important factors like uncertainty and irreversibility. The results are presented in Table 16.

Table 16: Total Revenues Real Options vs. NPV

	Total Investment Cost (€)	Real Options Revenues (€)	Profit (%):	NPV Revenues (€)	Profit (%):
Vodafone	10.831.040	211.630.959	1.854%	20.434.500	88,6%
Wind	9.556.000	220.012.992	2.202%	19.095.750	99,8%

Regarding the optimal investment rule *P**, the results based on both the equation (87) for the Real Options solution and the NPV are summarized in Table 17. The value *P** gives the optimal revenues level to activate the VoLTE project. In this real-life example, the Real Options thresholds are 11.5 times higher on average than the NPV's. The Real Options methodology raises the thresholds to such a degree that the operator can enter the market and activate the service without any risks. One interesting finding here is that the Real Options threshold for Wind is higher than Vodafone's, while on the other hand NPV threshold for Vodafone is higher than Wind's. Even though the discount rate of Vodafone is less than Wind's, the costs for Vodafone are higher and affect the final result by increasing the revenue threshold.

Table 17: Optimal Investments Rules Real Options vs. NPV (€)

	Real Options (€)	NPV (€)
Vodafone	14.644.421	1.362.300
Wind	15.224.440	1.273.050

Furthermore, in Table 18, the optimal rule *P** is translated into the number of subscribers, after being divided by the ARPU value. In particular, the calculations for the Real Options resulted in 97.630 subscribers for Vodafone and 101.497 subscribers for Wind. The calculations for the NPV solution resulted in 9.082 subscribers for Vodafone and 8.487 subscribers for Wind. However, from Table 13 the total amount of subscribers needed for the entire lifetime of the project to consider it profitable based on the NPV solution is 136.223 subscribers for Vodafone and 127.303 subscribers for Wind. In this point the different logic of the two solutions is revealed.

The NPV rule takes it as granted that the project will be active the whole planned time and that these subscribers will undoubtedly join the project during its lifetime as expected. On the other hand, the Real Options solution requires a specific amount of subscribers from day one to cope with the uncertainty and seems indifferent to how long the project will be active after the activation day.

Table 18: Optimal Subscribers Entry Levels Real Options vs. NPV

(subscribers)	Real Options	NPV	Ratio
Vodafone	97.630	9.082	~11:1
Wind	101.497	8.487	~12:1

To facilitate further interpretation of the findings, Table 19 displays the distribution of the optimal subscribers' thresholds by incorporating a sensitivity analyses with respect to the different levels of demand volatility, expected interest rate and project useful lifetime. Up to volatility value equals 0.17 Vodafone requires less subscribers to enter the market comparing with Wind. On value 0.18 and onwards it's the opposite. Regarding the expected interest rate, it affects the optimal thresholds indirect through the fraction $\beta 1/(\beta 1-1)$, but the impact is proportional bigger than the volatility's. Regarding the lifetime parameter *T* the graph of the equation decreases faster in the interval between 2 and 5 years than at any other point along the curve.

Table 19: Sensitivity Analysis for Real Options optimal subscribers' thresholds

Subscribers	**Demand volatility (σ)**					
	0,16	**0,17**	**0,18**	**0,19**	**0,20**	**0,21**
Vodafone	94.451	96.176	98.004	99.932	101.961	104.091
Wind	94.714	96.230	97.835	99.530	101.313	103.186
	Expect Interest rate (ρ)					
	0,07	**0,08**	**0,09**	**0,1**	**0,11**	**0,12**
Vodafone	86.751	96.640	106.549	116.473	126.408	136.351
Wind	80.558	89.270	98.000	106.745	115.502	124.267
	Project Lifetime (T)					
(years)	**2**	**5**	**10**	**15**	**20**	**25**
Vodafone	708.970	285.717	144.644	97.630	74.130	60.036
Wind	737.050	297.033	150.373	101.497	77.066	62.414

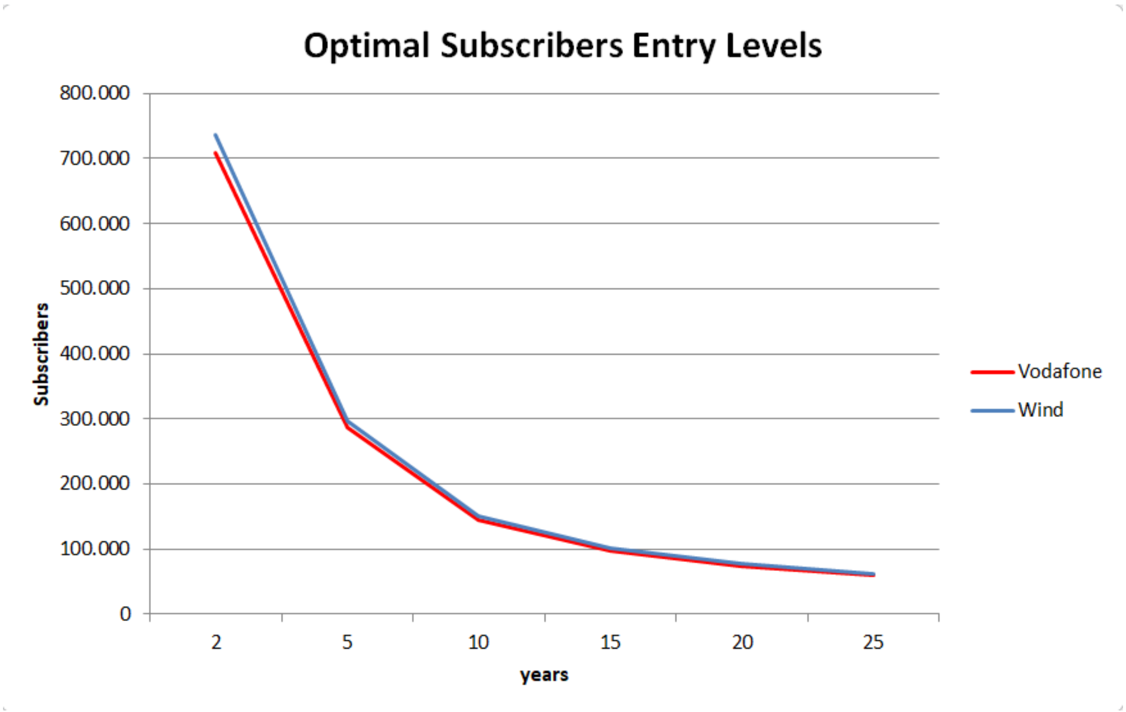

Figure 28: Optimal Subscriber-based Entry Levels

Moreover, the Monte Carlo Crystal Ball forecasted the entire range of possible results, providing a realistic picture of the risk inherent in the project and the like hood of any specific scenario. Table 20 summarizes the results for the different NPV value ranges running a one hundred thousand trials scenario. Under adverse conditions, negative NPV values may occur below 5 percentile. Conversely, the most likely scenario is a positive NPV value with a probability of ~95% for both operators. The standard deviation for Wind is higher than Vodafone's. Moreover, Vodafone's standard deviation equals about 76% of its mean and for Wind 80%, which also indicates higher uncertainty for Wind.

One more interesting insight is about the *P*.* The optimal value *P** is high in the upper tail, with a probability of ~ 7% of the value *P** or higher to be achieved. This means that in order to have a risk-free investment, the operator needs to postpone for long the activation of the project until the VoLTE-capable devices and the demand are increased and the profit flows reach that value. However, the competition and the fast evolution of technology may not allow that decision.

On the other hand, the traditional approach estimates an NPV equal to 20.434.500 € for Vodafone and 19.095.750 € for Wind. The probability of these values is even less than the probability of the optimal value *P**.

Table 20: Probabilities of different NPV value ranges

	Negative values	**Positive values**	**Optimal value P***
Vodafone	4,484%	95,516%	6,570%
Wind	4,878%	95,122%	7,395%

Looking at the histograms, the results are dispersed over a wide range. The difference between the higher and the lower value is 59.673.108 € for Vodafone and 61.919.824 € for Wind, meaning that there is much risk in the projects for both operators, especially for Wind.

7. CONCLUSION

In the presented study, the VoLTE solution was exposed from the point of view of technology and financial investment opportunity. The Introduction explored the evolution of mobile voice technology from 1G to 4G. Due to the ascertained increasing pace of technology evolution over the last years, telecommunication companies face many challenges in providing state of the art services to their customers and acquiring the desired skills and competencies. Their success depends a lot on their agility to predict, understand and adapt to the new trends of the quickly changing market.

The VoLTE is state of the art for mobile voice services. Based on the design principles and architecture presented, it is a challenging project which requires a high level of expertise and strict compliance with the specifications. Nevertheless, the advantages provided are significant for the mobile Operators, not only to maintain technical relevance but also to compete for a leading role in innovation. Among the benefits it may offer, the most essential are leveraging their IMS infrastructure, evolving to new generations and protecting their voice revenues from the rise of OTT solutions.

As an investment opportunity, it was remarked that Operators should evaluate mainly the cost of the investment, the service penetration based on the VoLTE capable devices and the coverage, and the potential for further network enhancements and cost-reduction. However, apart from the sunk costs, the investment incorporates risks due to the changing market conditions, which ultimately can affect the operators' demand and profitability. The related chapter summarized the limitations of the traditional NPV models to identify the stochasticity in the diffusion process and capture the market's inherent uncertainty while highlighting the need for more flexible valuation techniques.

The thesis proposed to model the stochastic diffusion of the VoLTE service as a Geometric Brownian Motion process and treat the VoLTE investment as a real option. This Real Options Approach relies on the option-like characteristics of the capital investments like uncertainty, irreversibility and time flexibility. In order to obtain the optimal investment rules for a risk-free investment and the opportunity cost, an option valuation framework was developed by using the dynamic programming methodology of the sequential decisions.

Dynamic programming is a useful tool, particularly in treating uncertainty. The whole logic behind the model is to evaluate a strategy plan by breaking it into decision components, starting from the initial one. The consequences of the initial decision incorporate risks, which are important to be modeled and calculated. Stochastic models like GBM can be very accurate in forecasting diffusion processes under uncertainty and are widely used in finance. However, the implementation didn’t avoid one critical point that requires further improvement in the methodology. Using the GBM model and considering the drift as a constant variable, it cannot accurately capture this S-shaped growth of most products and telecommunication services, like VoLTE, which may lead to positive growth over the saturation level. Further work in the context of the optimization of the stochastic models is needed.

To illustrate the proposed methodology, the Sudden Death framework was used for the evaluation of a real case scenario of a VoLTE investment in the Greek Telecommunications market. The historical data of the Greek market was crucial to extracting the solution’s parameters. Using the Real Options Approach under uncertainty in demand, the optimal investment rule was identified. The results were compared with the traditional methodology of the NPV rule, indicating the different logic and the much higher thresholds the Real Options approach is proposing.

A Monte Carlo simulation was conducted to further analyze the results of both methodologies by defining and quantifying the risks inherent in the investment. Performing probabilistic simulation analysis for future outcomes can help decision-makers to reduce the risks and improve the reliability of the decisions. The simulation provides interesting insights into both approaches. The optimal rule P^* proposed by the Real Options Approach indicates a "truly" risk-free condition with a high threshold level where the Operator has to delay the activation of the project to reach it. On the other hand, the NPV tool promises a positive outcome which has a very small probability to be achieved according to the simulation.

ABBREVIATIONS – ACRONYMS

3GPP	3rd Generation Partnership Project
5GC	5G Core
5GNR	5G New Radio
AAA	Authentication Authorization Accounting
AAR	Authorize Authenticate Request
AF	Application Function
AKA	Authentication and Key Agreement
AMR-WB	Adaptive Multi-Rate Wideband
APN	Access Point Name
ARPU	Average Revenue per User
AS	Application Server
AVP	Audio Video Profiles
BGCF	Border Gateway Control Function
BM	Brownian motion
CA	Carrier Aggregation
CCA	Credit Control Answer
CDMA	Code Division Multiple Access
CDR	Charging Data Records
CNF	Cloud-Native Network Function
CS	Circuit Switch
CSFB	Circuit-Switch Fallback
CUPS	Control/User Plane Separation
DNS	Domain Name System
ECGI	E-UTRAN Cell Global Identifier
EDGE	Enhanced Data Rates for GSM Evolution
ePDG	enhanced Packet Data Gateway
EPS	Evolved Packet System
ETSI	European Telecommunications Standards Institute
E-UTRAN	Evolved UMTS Terrestrial Radio Access Network
EVS	Enhanced Voice Services
FMC	Fixed Mobile Convergence
FTE	Full Time Equivalent

GBM	Geometric Brownian motion
GDP	Gross Domestic Product
GPRS	General Packet Radio Services
GSM	Global System for Mobile Communications
HD	High Definition
HSPA	High Speed Packet Access
HSS	Home Subscriber Server
I-CSCF	Interrogating Call Session Control Function
ICT	Information and Communication Technology
IFC	Initial Filter Criteria
IMS	IP Multimedia Subsystem
IMSI	International Mobile Subscriber Identity
IMT- 2000	International Mobile Telecommunications 2000
IMT Advanced	International Mobile Telecommunications Advanced
IN	Intelligent Networks
IoT	Internet of Things
IP	Internet Protocol
IP-CAN	IP connectivity Access Network
IPsec	Internet Protocol Security
IPSMGW	IP Short Message Gateway
IRR	Internal Rate of Return
IT	Information technology
ITU	International Telecommunication Union
LTE	Long Term Evolution
LTE-A	Long Term Evolution - Advanced
MAR	Multimedia Authentication Request
MGCF	Media Gateway Control Function
MIMO	Multiple Input Multiple Output
MME	Mobility Management Entity
MNO	Mobile Network Operator
MRB	Media Resource Broker
MRF	Media Resource Function
NAS	Non-Access Stratum
NE	Network Element

NFV	Network Functions Virtualization
NPV	Net Present Value
OCS	Online Charging System
OFDMA	Orthogonal Frequency-Division Multiple Access
PCC	Policy and Charging Control
PCC	Policy and Charging Control
PCEF	Policy and Charging Enforcement Function
PCRF	Policy and Charging Rules Function
P-CSCF	Proxy Call Session Control Function
PDCP	Packet Data Convergence Protocol
PDN	Packet Data Networks
P-GW	Packet Data Network gateway
PLMN	Public Land Mobile Network
PRACK	Provisional Response Acknowledgment
QCI	QoS Class Identifier
QoS	Quality of service
RES	user RESponse (used in IMS-AKA)
ROA	Real Option Approach
RRC	Radio Resource Control
RTP	Real Time Protocol
SAR	Server Assignment Request
S-CSCF	Serving Call Session Control Function
SDN	Software Defined Networking
SDP	Session Description Protocol
SGSN	Serving GPRS Support Node
S-GW	Serving Gateway
SIP	Session Initiation Protocol
SIP-UA	Session Initiation Protocol-User Agent
SIP-URI	Uniform Resource Identifier
SMS	Short Message Service
SRVCC	Single Radio Voice Continuity
TAC	Tracking Area code
T-ADS	Terminating Access Domain Selection
TAI	Tracking Area Identity

TAS	Telephony Application Server
TEID	Tunnel Endpoint Identifiers
TFT	Traffic Flow Template
UAR	User Authentication Request
UDR	User Data Request
UE	User Equipment
UICC	Universal Integrated Circuit Card
ULA	Update Location Answer
ULR	Update Location Request
VIoT	Voice over Internet of Things
VoIP	Voice over IP
VoLTE	Voice over LTE
VoWiFi	Voice over Wi-Fi
WCDMA	Wideband Code Division Multiple Access
WLAN	Wireless Local Area Network
XRES	eXpected user RESponse (used in IMS-AKA)

ANNEX I

Table 21: Kolmogorov-Smirnov Normality test for Greek Market

	Market - Kolmogorov-Smirnov Normality Test						
	Market Sorted Absolute Log-Changes	freq	cumul	Expected CNDF		Market CNDF (NORM.DIST)	Difference (ABS)
1999	0,008	1	1	0,048		0,203004795	0,155385748
2000	0,015	1	2	0,095		0,215600771	0,120362675
2001	0,016	1	3	0,143		0,218347833	0,07549069
2002	0,018	1	4	0,190		0,221470877	0,030994687
2003	0,037	1	5	0,238		0,260832561	0,022737323
2004	0,037	1	6	0,286		0,261152254	0,024562031
2005	0,040	1	7	0,333		0,267269727	0,066063607
2006	0,052	1	8	0,381		0,292682002	0,088270379
2007	0,060	1	9	0,429		0,312132027	0,116439402
2008	0,067	1	10	0,476		0,327722047	0,148468429
2009	0,070	1	11	0,524		0,336174865	0,187634659
2010	0,104	1	12	0,571		0,419068363	0,152360208
2011	0,109	1	13	0,619		0,430725384	0,188322235
2012	0,120	1	14	0,667		0,459413544	0,207253123
2013	0,153	1	15	0,714		0,547072394	0,16721332
2014	0,157	1	16	0,762		0,55507931	0,206825451
2015	0,157	1	17	0,810		0,555316844	0,254206966
2016	0,294	1	18	0,857		0,850057021	0,007085836
2017	0,315	1	19	0,905		0,878966394	0,025795511
2018	0,396	1	20	0,952		0,955454358	0,003073406
2019	0,618	1	21	1,000		0,999176342	0,000823658
	mean	0,14 €			max		0,254206966
	stdev	0,15 €			check	5%	0,28724
	count	21					

The normality criterio for 21 values and 5% probability stands for lower than 0,28724.

Table 22: Kolmogorov-Smirnov Normality test for Cosmote

	Cosmote - Kolmogorov-Smirnov Normality Test						
	Cosmote Sorted Absolute Log-Changes	freq	cumul	Expected CNDF		Cosmote CNDF (NORM.DIST)	Difference (ABS)
1999	0,008	1	1	0,048		0,265695267	0,218076219
2000	0,018	1	2	0,095		0,277681288	0,182443193
2001	0,021	1	3	0,143		0,282373439	0,139516296
2002	0,025	1	4	0,190		0,287272355	0,096796165
2003	0,030	1	5	0,238		0,29288023	0,054784992
2004	0,031	1	6	0,286		0,294090881	0,008376595
2005	0,043	1	7	0,333		0,309330717	0,024002617
2006	0,046	1	8	0,381		0,313734625	0,067217756
2007	0,080	1	9	0,429		0,359656084	0,068915345
2008	0,088	1	10	0,476		0,370159347	0,106031129
2009	0,117	1	11	0,524		0,412235005	0,111574519
2010	0,121	1	12	0,571		0,418052241	0,15337633
2011	0,128	1	13	0,619		0,427863686	0,191183933
2012	0,133	1	14	0,667		0,434409231	0,232257436
2013	0,151	1	15	0,714		0,461649162	0,252636552
2014	0,169	1	16	0,762		0,487904334	0,274000428
2015	0,184	1	17	0,810		0,509605567	0,299918243
2016	0,215	1	18	0,857		0,554707	0,302435857
2017	0,323	1	19	0,905		0,703643294	0,201118611
2018	0,584	1	20	0,952		0,933199685	0,019181267
2019	1,213	1	21	1,000		0,999932941	6,70593E-05
	mean	0,18 €			max		0,302435857
	stdev	0,27 €			check	5%	0,28724
	count	21					

The normality criterio for 21 values and 5% probability stands for lower than 0,28724.

Table 23: Kolmogorov-Smirnov Normality test for Vodafone

	Vodafone- Kolmogorov-Smirnov Normality Test						
	Vodafone Sorted Absolute Log-Changes	freq	cumul	Expected CNDF		Vodafone CNDF (NORM.DIST)	Difference (ABS)
1999	0,002	1	1	0,048		0,126819491	0,079200443
2000	0,038	1	2	0,095		0,198157055	0,10291896
2001	0,043	1	3	0,143		0,209011253	0,06615411
2002	0,067	1	4	0,190		0,269808257	0,079332067
2003	0,070	1	5	0,238		0,278325758	0,04023052
2004	0,071	1	6	0,286		0,281722623	0,003991663
2005	0,073	1	7	0,333		0,287148723	0,04618461
2006	0,076	1	8	0,381		0,295889077	0,085063304
2007	0,098	1	9	0,429		0,359404721	0,069166708
2008	0,104	1	10	0,476		0,3783578	0,097832677
2009	0,107	1	11	0,524		0,389244273	0,134565251
2010	0,110	1	12	0,571		0,396096363	0,175332208
2011	0,119	1	13	0,619		0,424647319	0,1944003
2012	0,127	1	14	0,667		0,451052652	0,215614015
2013	0,136	1	15	0,714		0,480809989	0,233475726
2014	0,137	1	16	0,762		0,483965285	0,277939477
2015	0,157	1	17	0,810		0,547897205	0,261626604
2016	0,259	1	18	0,857		0,831018177	0,02612468
2017	0,294	1	19	0,905		0,89388194	0,010879965
2018	0,390	1	20	0,952		0,978803916	0,026422964
2019	0,501	1	21	1,000		0,998342367	0,001657633
	mean	0,14 €			max		0,277939477
	stdev	0,12 €			check	5%	0,28724
	count	21					

The normality criterio for 21 values and 5% probability stands for lower than 0,28724.

Table 24: Kolmogorov-Smirnov Normality test for Wind

	Wind - Kolmogorov-Smirnov Normality Test						
	Wind Sorted Absolute Log-Changes	freq	cumul	Expected CNDF		Wind CNDF (NORM.DIST)	Difference (ABS)
1999	0,005	1	1	0,048		0,125385484	0,077766436
2000	0,031	1	2	0,095		0,170276646	0,075038551
2001	0,036	1	3	0,143		0,178287681	0,035430539
2002	0,050	1	4	0,190		0,206209267	0,015733076
2003	0,071	1	5	0,238		0,254025356	0,015930118
2004	0,075	1	6	0,286		0,263056128	0,022658157
2005	0,084	1	7	0,333		0,285787326	0,047546007
2006	0,087	1	8	0,381		0,292501827	0,088450554
2007	0,097	1	9	0,429		0,318379668	0,11019176
2008	0,098	1	10	0,476		0,31927415	0,156916326
2009	0,125	1	11	0,524		0,393972584	0,12983694
2010	0,140	1	12	0,571		0,436655874	0,134772697
2011	0,154	1	13	0,619		0,475882551	0,143165068
2012	0,155	1	14	0,667		0,479045252	0,187621415
2013	0,157	1	15	0,714		0,484872232	0,229413482
2014	0,195	1	16	0,762		0,595908318	0,165996444
2015	0,256	1	17	0,810		0,754977251	0,054546559
2016	0,262	1	18	0,857		0,767128645	0,090014212
2017	0,364	1	19	0,905		0,930766977	0,026005073
2018	0,428	1	20	0,952		0,974168694	0,021787742
2019	0,529	1	21	1,000		0,996343377	0,003656623
	mean	0,16 €			max		0,229413482
	stdev	0,14 €			check	5%	0,28724
	count	21					

The normality criterio for 21 values and 5% probability stands for lower than 0,28724.

ANNEX II

Table 25: Extraction of Volatility and Expected Return Rate from the Historical Data

Historical Data (1998 - 2019)						
Total Registered Subscribers				Annual Growth LN(i/i-1)		
Year	**Greek Market**	**Vodafone**	**Wind**	**Greek Market**	**Vodafone**	**Wind**
1998	2150000	1053500	752500			
1999	3990000	1556100	1276800	0,618323389	0,390064737	0,52871123
2000	5930000	2016200	1838300	0,396232982	0,259031861	0,364484284
2001	7960000	2706400	2388000	0,294404787	0,294404787	0,261614964
2002	9310000	3072300	2793000	0,156660091	0,126807128	0,156660091
2003	10330000	3408900	3047350	0,103963192	0,103963192	0,087156074
2004	11044000	3644520	3202760	0,066835011	0,066835011	0,049740578
2005	12448000	3921120	3734400	0,119672673	0,073152658	0,153574225
2006	13875000	4231875	4231875	0,108528692	0,07626783	0,125057994
2007	16226675	4949136	4868003	0,156567833	0,156567833	0,140038531
2008	18918092	5675428	5297066	0,15346222	0,136932918	0,084469348
2009	20298102	6333008	4932439	0,070408671	0,109629384	-0,071319489
2010	14815705	3837268	3215008	-0,314839617	-0,501014743	-0,428003707
2011	14557672	3828668	3115342	-0,017569627	-0,002243656	-0,031490965
2012	15151742	4106122	3636418	0,039997369	0,069962158	0,154660277
2013	15722476	4528073	4009231	0,036975772	0,097817431	0,097600394
2014	15473683	4704000	3868421	-0,015950571	0,03811665	-0,035753198
2015	15353553	5389097	2993943	-0,007793797	0,135964725	-0,256255156
2016	15934294	5624806	2979713	0,037126729	0,042808562	-0,004764213
2017	16167273	4995687	3621469	0,014515372	-0,118611408	0,195052807
2018	15354388	4652380	3285839	-0,051587717	-0,071196189	-0,097257754
2019	14458145	4178404	3542246	-0,060143373	-0,10744949	0,075138823
				Volatility Rate σ (STDEV.S)		
				0,18473848	0,177660995	0,201241749
				Average Growth a (AVERAGE)		
				0,09075191	0,065610066	0,073767387
				Expected Return Rate ρ (a + σ^2/2)		
				0,10781606	0,08139178	0,094016508

Table 26: Investment Costs for VoLTE Deployment

IMS Nodes	Cost area					
	# of blades	HW costs	HW expansions	SW (patches, upgrades etc.)	Professional Services	Summary for 5 mil. subscribers capacity
		Duration 15 years				
IMS (I/P/S-CSCF, BGCF):	80	1.200.000 €	480.000 €	450.000 €	510.000 €	2.640.000 €
TAS, MRFP:	53	795.000 €	345.000 €	315.000 €	375.000 €	1.830.000 €
IMS HSS:	35	525.000 €	270.000 €	245.000 €	300.000 €	1.340.000 €
Total	168	2.520.000 €	1.095.000 €	1.010.000 €	1.185.000 €	5.810.000 €
	# of subscribers (mil.)	**4G Spectrum (2015 - 2030)**	**4% of Spectrum cost**	**SW Licenses (0,01€ per sub)**	**Investment Cost (HW costs + Spectrum + SW Licenses)**	**Total**
Cosmote:	6,7	134.788.000 €	5.391.520 €	67.000 €	8.986.520 €	13.592.520 €
Vodafone:	4,1	124.501.000 €	4.980.040 €	41.000 €	7.541.040 €	10.831.040 €
Wind:	3,5	121.825.000 €	4.873.000 €	35.000 €	6.924.000 €	9.556.000 €

ANNEX III

Table 27: NPV for Vodafone's Volte Project

VODAFONE

Years:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Discount Rate:	1	1,126	1,267876	1,42762838	1,60750955	1,81005575	2,03812278	2,29492625	2,58408696	2,90968191	3,27630184	3,68911587	4,15394447	4,67734147	5,26668649	5,930288992
NPV:	-7541040	-6526038	-5624615,3	-4824062,3	-4113091,6	-3481678,9	-2920921,6	-2422913,4	-1980632,5	-1587843,1	-1239007,1	-929205,94	-654071,74	-409725,2	-192721,17	0,00

Project Lifetime:	15
p:	0,126
ARPU:	150
Cost I:	-7541040
Cost C/year:	-219333,33

Constraints:

Subscribers Database:		
9081,50	<=	4100000
NPV equals to zero:		
0,00	==	0

Results:

Minimum number of subscribers per year:	
Subscribers:	9081,50359
Total number of subscribers:	
Subscribers x 15 years:	136222,554

Table 28: NPV for Wind's Volte Project

WIND

Years:	0	1	2	3	4	5	6	7	8
Discount Rate:	1	1,1347	1,28754409	1,46097628	1,65776978	1,88107137	2,13445169	2,42196233	2,74820066
NPV:	-6924000	-5956731,4	-5104287,1	-4353036,3	-3690966,3	-3107490,5	-2593279	-2140109,4	-1740735,5

Years:	9	10	11	12	13	14	15
Discount Rate:	3,11838328	3,53842951	4,01505597	4,55588401	5,16956158	5,86590153	6,656038463
NPV:	-1388771,2	-1078588,5	-805227,55	-564317,19	-352005,26	-164896,83	0,00

Project Lifetime:	15
p:	0,1347
ARPU:	150
Cost I:	-6924000
Cost C/year:	-175466,67

Constraints:

Subscribers Database:		
8486,84	<=	3500000
NPV equals to zero:		
0,00	==	0

Results:

Minimum number of subscribers per year:	
Subscribers:	8486,84196
Total number of subscribers:	
Subscribers x 15 years:	127302,629

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