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“Optimization of wireless networks in 5G”

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ABSTRACT

To address the limitations of current Radio Access Networks (RANs), Cloud-RANs (C-RANs) have been proposed introducing increased transport bandwidth requirements and strict latency and synchronization constraints. To relax the stringent C-RAN requirements, flexible functional splits have been proposed relying on transferring some of the processing functions away from the Radio Units and locating these centrally at a central unit (CU). In this master thesis, the concept of flexible functional splits is addressed by combining servers with low processing power (cloudlets) and relatively large-scale DCs placed in the access and metro network domains respectively. The remote processing requirements of the functional split options, impose the need for a high bandwidth transport interconnecting radio units and the CU. The selection of the optimal split option is performed in a dynamic fashion through a novel mathematical model based on Evolutionary Game Theory (EGT). This model allows to dynamically identify the optimal split option with the objective to unilaterally minimize the infrastructure operational costs in terms of power consumption. The speed of convergence and the stability of the proposed scheme is theoretically analyzed for different scenarios and use cases. Finally, the impact of various parameters characterizing the 5G network on the split option selection is analytically determined.

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Victoria-Maria Alevizaki

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Chapter 1 Introduction

The demanding requirements of 5G and vertical operational and end-user services including unprecedented level of scalability (x1000) on the numbers of interconnected elements, high bandwidth, low latency and ubiquitous coverage, introduce the need for a common, flexible and open underlying network infrastructure involving a variety of advanced interconnected radio access network technologies. To address the limitations of current Radio Access Networks (RAN), Cloud-RANs (C-RANs) have been proposed introducing increased transport bandwidth requirements and imposing strict latency and synchronization constraints. To relax the stringent C-RAN requirements, taking advantage of its pooling and coordination gains the adoption of alternative architectures exploiting the option of flexible functional splits has been proposed. The concept of flexible splits relies on transferring some of the processing functions away from the RU and locating these centrally at a CU.

In this master thesis, the concept of flexible functional splits is addressed by appropriately combining servers with low processing power (cloudlets) and relatively large-scale DCs placed in the access and metro domains respectively. The remote processing requirements associated with some of the functional split options, impose the need for a high bandwidth transport interconnecting RUs and the CU. On the other hand, the variability of remote processing requirements across the various split options introduce the need for a transport network that offers finely granular and elastic resource allocation capabilities. The selection of the optimal split option is performed in a dynamic fashion through a novel mathematical model based on Evolutionary Game Theory (EGT). This model allows network operators to dynamically adjust their FH split option with the objective to unilaterally minimize their operational expenditures in terms of power consumption. The speed of convergence and the stability of the proposed scheme is theoretically analyzed for different scenarios and use cases. Finally, the impact of various parameters characterizing the 5G network (i.e. bandwidth, cost transmission, power consumption etc) on the split option selection is analytically determined.

In Chapter 2, the Fifth-Generation key enablers- the Densification and the Centralization of Baseband processing -are presented. Chapter 3 introduces the notion of Flexible Centralization of Wireless Networks. Five possible functional splits are analyzed and the required data rate for each split is calculated. In Chapter 4, a model for calculating the power consumption of the base station, as well as the network's power consumption is discussed. In Chapter 5, we examine the Evolutionary Game Theory, which we will use as a mathematical tool in order to approach this problem. Chapter 6 implements the replicator equation of evolutionary game theory, in order to find the optimal split. Lastly, in Chapter 7 we examine the optimal split's dependence on the system parameters, with the usage of numerical examples.

Chapter 2 Evolution of Mobile Wireless Communication Networks towards 5G

Over the years there has been a rapid development in mobile communications. The growing number of users, in combination with the limited bandwidth, has led to the research of new technologies to improve service quality and enable more users to the network. The evolution of mobile wireless communication networks is shown in Figure 1. In order for wireless communications to reach the current technological standards, they evolved through several stages, which are called generations. The first mobile phone networks emerged in the 1980s, GSM followed in the 1990s, 3G reached the turn of the century, and the LTE started unfolding in 2009. Each generation was meant to correct the mistakes of its predecessor. GSM has corrected the security weaknesses of analogue communications, 3G came to cover the lack of GSM mobile data and the 4G eventually was needed to make data usage more user-friendly. Now, the next generation of mobile technology (5G) is beginning to take shape.

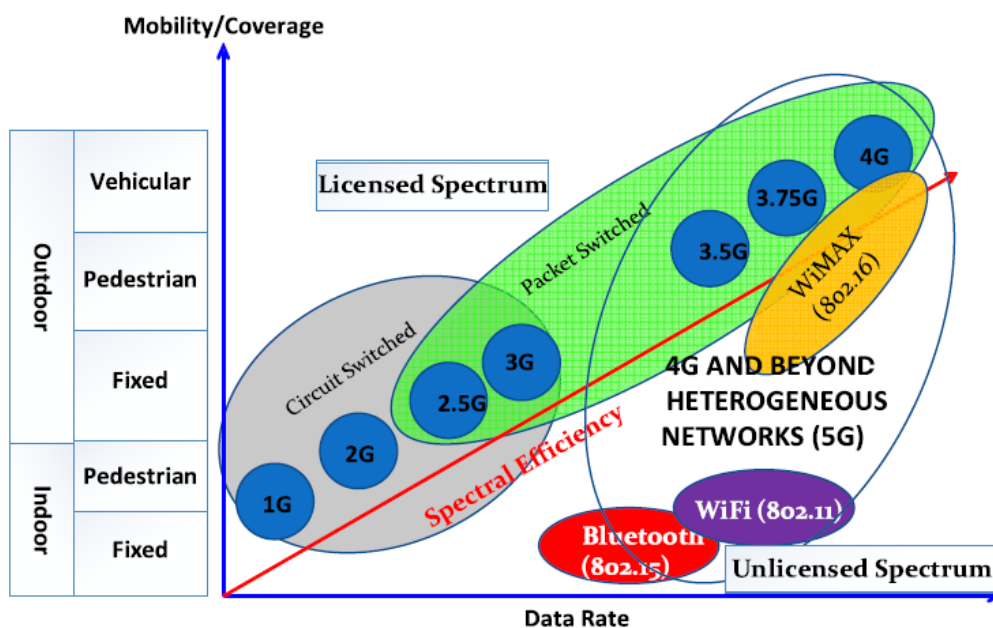


Figure 1: Evolution of wireless technologies. [1]

The evolution towards the fifth-generation networks is characterized by an enormous increase in data traffic. Future mobile networks shall provide a high amount of traffic with varying data rates from machine to machine (low data transfer) to high-speed 3D applications. This enormous growth is attributed to the rapidly increasing: a) number of network-connected end devices, b) Internet users with heavy usage patterns, c) broadband access speed, and d) popularity of applications such as cloud computing, video, gaming etc. [2] Thus, the need is emerged for the wireless networks to be able to handle changing traffic patterns, both spatially and temporally, as well as terminals with different quality requirements and services. [3] [4]

One of the key enablers of 5G implementation is densification of the network. 5G demands the movement towards a denser network (Densification) in order to increase the efficiency of the spectral range. This leads to a network of much smaller cells, reducing the distance between the terminals and the radio access points (RAPs). Small cells will not be homogeneous, but will form a flexible heterogeneous network where resources can be dynamically adapted, as the users' demand in space, time and spectral resources varies.

Nonetheless, the establishment of a denser network induces several problems. Initially, due to the numerous cells, there is an increase of intercell interference phenomena. Moreover, as the network density increases, the possibility of an access point (RAP) to transfer a small percentage - or even zero - of the total network traffic rises, because of the time-varying traffic fluctuations. [3]

Another key enabler of 5G implementation is the centralization of the Baseband Processing. The idea of centralized baseband processing emerged several years earlier, to facilitate the establishment of wireless base stations at large buildings or on a campus. The development of digital radio interfaces as well as Remote Radio Headers (RRHs) was crucial for the connection between the Base Unit (BU) and the Remote Unit (RU) through optical fiber. [5] With the invention of multipoint transmission and reception, this idea evolved on a larger scale, creating centralized wireless access networks (C-RANs). A classical C-RAN is shown in Figure 2. In C-RAN concept, RRHs are connected to a data center (DC) where all the digital

processing is executed. Radio signals are exchanged through dedicated transmission lines (called fronthaul) between the RRH and the data center (DC).

The problems of densification can be handled by the centralization of baseband processing. As far as the intercell interference is concerned, central processing will allow the employment of effective Radio Resource Management (RRM) algorithms that coordinate the resources in multiple cells. Furthermore, it enables the optimization of the performance at a signal-level, for example through the processing of multiple cells and the Intercell Interference Coordination (ICIC). RRM and ICIC enhance the wireless network's efficiency by avoiding, canceling or even exploiting the interference between neighboring cells. [3] [6]

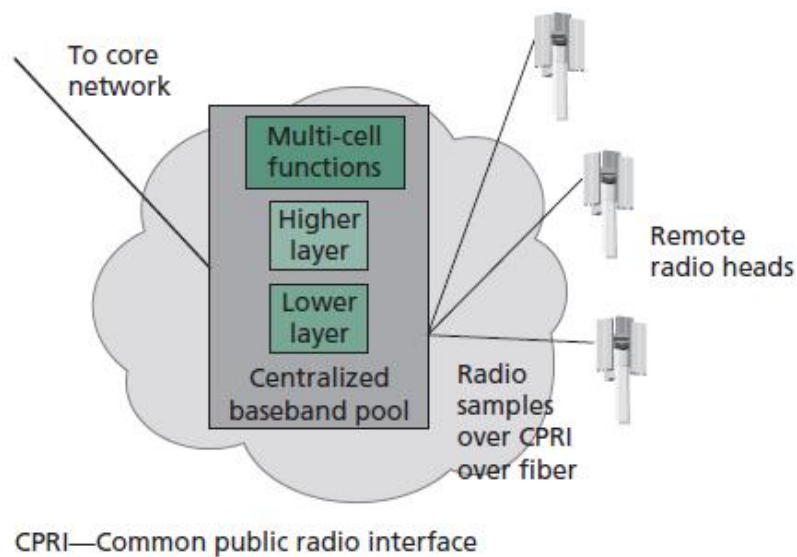


Figure 2: The classic centralized access network (C-RAN). [5]

Centralization of digital processing is also important at network-level, as it is required for the orchestration and optimization of overly dense networks. For example, it provides the ability to selectively activate and suppress RUs, thus addressing the problem of motion fluctuations. In addition, pooling resources may allow the development of a flexible software. Depending on the actual scenario, different algorithms can be used, that have been optimized for specific usage situations, e.g. based on traffic characteristics, cell-to-cell relations, or the allocation of wireless access networks. It also allows the operator to apply the latest algorithms on a large scale. [6]

On the other hand, at present, only optical fibers are capable of supporting the data rates that are required for the connection between the RRH and the data center, e.g. 10 Gbps for TD-LTE with bandwidth 20MHz and eight receiving antennas. [3] This need for a high-capacity fronthaul connection is the main drawback of C-RAN. Due to the necessary use of optical fiber, current C-RAN networks are characterized by low flexibility and scalability, as only points with existing fiber optic infrastructure can be selected. In addition, current C-RAN infrastructures are based on baseband processor pools that do not allow the development of flexible and adaptive software and therefore make it impossible to exploit the huge capabilities of cloud computing. [4] [6]

Therefore, there is a dispute over what is more efficient: the existence of centralized processing that requires large capacity links to connect the DCs with the RRHs or the old-fashioned decentralized processing using traditional backhaul to transfer data to / from RANs. To relax the stringent FH requirements of C-RAN architectures, while taking advantage of its pooling and coordination gains, solutions relying on alternative architectures adopting flexible functional splits have been proposed. [2] The introduction of flexible splits allows dividing the processing functions between the DC and the RU. Based on these solutions, a set of processing functions is performed at the RU and the remaining functions are performed centrally.

In the next chapters, a model for identifying the optimal split, depending the circumstances, is proposed. The overall objective of the model is twofold: The need to address the huge increase in data traffic combined with a significant reduction in energy consumption.

Chapter 3 Flexible Centralization of Wireless Networks

In this chapter, the idea of Radio Access Network as a Service (RANaaS) is introduced, an idea of a RAN that takes into consideration the actual needs and features of the network. Then, we examine possible split points to determine which functions will be executed locally on RU and which will be centralized. The required data rate for each split is also calculated.

3.1 Radio Access Network as a Service (RANaaS)

Centralized processing and management of 5G mobile networks should be flexible and adaptable to the real service requirements. Thus, a co-operation between full centralization and decentralization is of critical importance. This collaboration raises the new concept of the RAN as a Service (RANaaS), which partially centralizes the wireless access network functions according to actual needs and features of the network. This idea belongs to the more general «X as a Service» concept, according to which any kind of operation can be packaged and delivered as a form of a service that may be centralized within a cloud platform. [7] This enables increased storage and data processing capabilities that are provided by a cloud platform hosted on data centers. Thus, in the following, we refer to the concept of centralization toward commodity cloud-computing platforms as cloud-RAN. [3]

A major feature of RANaaS is the flexible assignment of functionality between RUs and the central processor. Cloud-computing platforms permit the necessary adaptivity for dealing with time and special traffic fluctuations in mobile networks. [6] This is imperative in order to improve the economic and ecological operation and use of mobile networks. An additional advantage of cloud-RAN technology is that it does not use specialized hardware and software, like currently proposed by C-RAN, but general-purpose processing technology (GPPs). [3]

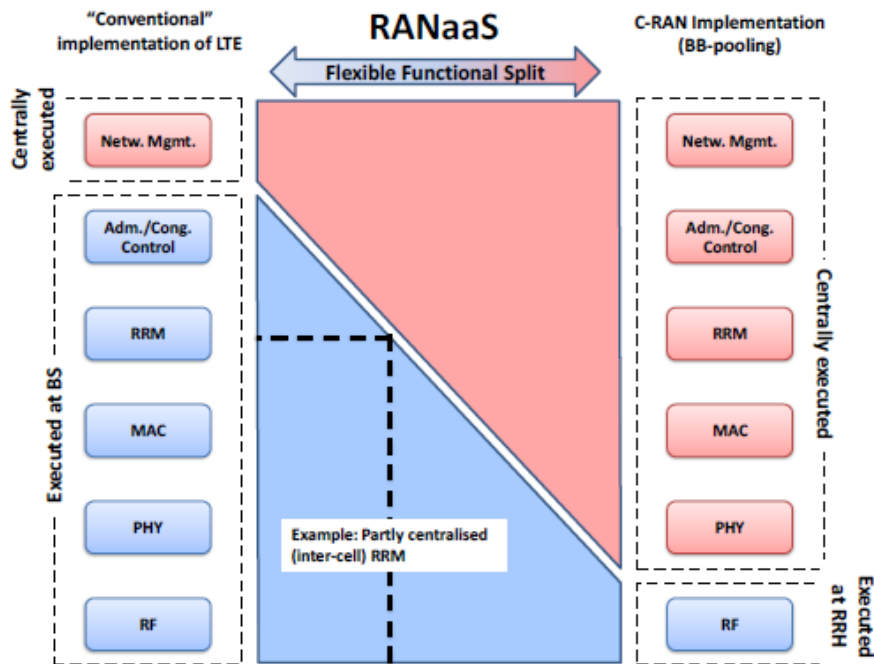


Figure 3: Flexible functional split. [6]

Figure 3 shows the functional split that is recommended by RANaaS. As it is evident, the split offers more alternatives in processing design and, therefore, flexibility in the actual execution of functions. The left side of the figure represents a traditional LTE implementation, where all functions up to Admission/Congestion Control are locally carried out at the RU, at the base station (BS). On the right side C-RAN is depicted, where only RF processing is executed locally and all other functionalities are centralized. RANaaS on the other hand doesn't centralize all RAN functions, but only a part of them. [6]

Cloud-RAN will further advance scalar algorithms, designed for cloud computing environments, and therefore exploit mass parallelism. This means that the algorithms will not simply be conveyed to cloud computing platforms but will be redesigned to gain from the available computing resources. [3] Cloud-RAN allows the development of algorithms that scale with the need for co-operation and coordination between individual cells, for example depending on traffic demand and user density, RUs can be grouped differently or use different algorithms.

As mentioned before, cloud-RAN permits partial (or full) unloading of the classic features, that are typically processed locally, on a cloud-computing platform, in order to take

advantage not only of computing power but also of centralized processing. In theory, such a separation can occur at each protocol level or at the interface between each level. However, the 3GPP LTE implies some timing limitations and feedback loops between the individual protocol layers. [6] Generally, the lower the separation of functions is imposed, the higher is the overhead and the more stringent is the transport capacity.

It is obvious that this new pattern depends to a large extent on the availability of resources on the RANaaS platform and also on the transport capacity. Hence, a common design of both is crucial.

3.2 Functional Split

As described in the previous section, the first attempts to implement a centralized wireless access network were based on the complete centralization of digital processing. However, this process is extremely costly due to the requirement of large bandwidth and short-delay of the network that connects the Central Units, such as a BBU, to the local RF equipment, such as a RRH. In this section, in order to reduce the required bandwidth, the digital processing chain is analyzed, and possible split points are defined to determine which

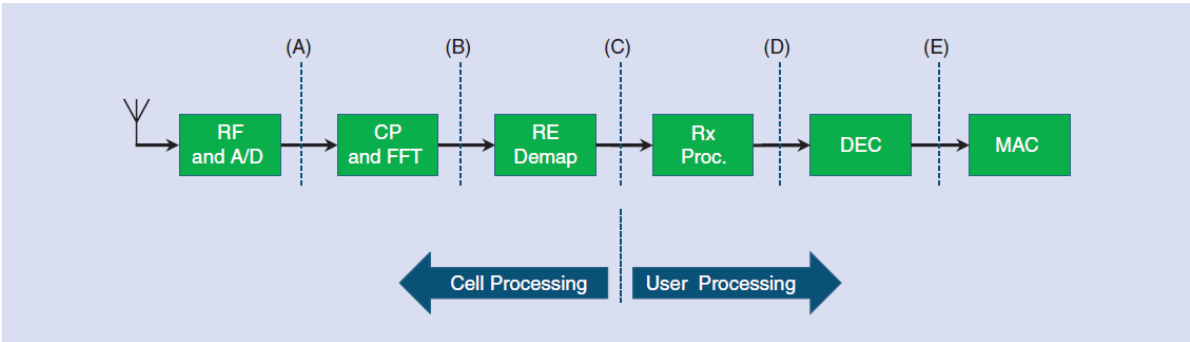


Figure 4: The functional splits between RUs and the cloud-platform for the uplink transmission. [3]

functions will be executed locally on RU and which centrally on the cloud platform. The influence of each split on the required connection rate is also calculated.

The analysis will focus on the uplink as the processing load in this case is higher than in the case of downlink. In this case, the received signals undergo RF processing, are transferred

to the baseband and converted from analog to digital (A/D conversion). Then the cyclic prefix (CP) is removed, and the signal is transformed to the frequency domain. Next, the subframes are disassembled (resource demapping). At this point the per cell processing comes to an end. Functions like equalization, IDFT, QAM, multi-antenna processing are performed on a per-user basis. The processing of the first layer ends with the Forward Error Correction (FEC). Lastly, the packets are forwarded to higher-level operations (MAC, RLC, PDCP). [5]

Figure 4 shows the digital uplink processing chain and possible points where the split of functions can be placed. In the following, each split point is analyzed and the required transport rate is calculated.

I. SPLIT A: IQ Forwarding

In this case, the received signals, after being transferred to the baseband and converted from analog to digital, they are forwarded to the cloud platform. Therefore, the whole frame, including the circular prefix, must be transmitted through the transport connection. This approach is used in the common public radio interface (CPRI). [3] The major advantage of this split is that almost no digital processing equipment is needed in RUs, making them smaller and cheaper. However, for the implementation of a flexible function split, digital processing equipment at the access points is needed, in which case the above advantage is canceled. The required transport capacity is: [3]

$$R_A = N_o \cdot f_s \cdot 2 \cdot N_Q \cdot N_R \quad (3.2.1)$$

Where N_o is the oversampling factor, f_s the sampling frequency, N_Q are the quantization bits per I/Q and N_R the number of the receiving antennas.

II. SPLIT B: Subframe forwarding

Here we place the split after the removal of the cyclic prefix (CP) and the Fast Fourier Transformation (FFT). Thus, guard subcarriers can be removed. As FFT can be highly efficient with dedicated equipment, it is better if it is executed

locally at access points. Thus, if these functions are done locally, the signal to be forwarded to the platform requires a lower transport rate than before. This rate is calculated according to the formula: [3]

$$R_B = N_{sc} \cdot T_s^{-1} \cdot 2 \cdot N_Q \cdot N_R \quad (3.2.2)$$

Where N_{sc} is the number of subcarriers that are being used.

III. SPLIT C: RX data forwarding

Another split of the functions of digital processing is after the disassembly of the subframes (resource demapping). If only part of the resource elements (Res) is used by the user's equipment, then only they will remain after the disassembly and therefore should be forwarded to the cloud platform. The required transport capacity is the fraction of the REs that are used: [3]

$$R_C = R_B \cdot \eta \quad (3.2.3)$$

Where η is the percentage of used REs. The main advantage of this split is that the rate depends on the actual wireless network load over the theoretically maximum.

IV. SPLIT D: Soft-bit forwarding

One of the per-user functions is the MIMO receive processing. In a MIMO system that uses the diversity of receivers, multiple antenna signals are combined during channel equalization, thus removing the transport rate's dependence on the number of receive antennas. The transport capacity decreases and is calculated based on the formula: [3]

$$R_D = R_C / N_R \quad (3.2.4)$$

On the other hand, in the case of spatial multiplexing with N_S levels per user equipment, the rate is given by the formula: [3]

$$R_D = R_C \cdot N_S / N_R \quad (3.2.5)$$

V. *SPLIT C: MAC-PHY*

The last functional split that we will assume is between the two network layers. In this case, all generating and tracing functions are performed locally on the remote unit (RU), while the higher-level functions, such as timing, are performed centrally. [5] The resulting transport rate is highly dependent on the modulation and the coding scheme, which are expressed by a factor that is called spectral efficiency S : [3]

$$R_E = N_{sc} \cdot T_s^{-1} \cdot \eta \cdot S \quad (3.2.6)$$

Chapter 4 Power Efficiency for Radio Networks

One of the major problems that the fifth-generation network needs to solve is energy consumption. This is particularly important for base stations, which consume most of the total power in cellular networks. In this chapter, a model for calculating the power consumption of the base station, as well as the network’s power consumption, is analyzed.

4.1 Power Modeling of Base Stations

To optimize the energy consumption of new generation networks, we must first calculate the energy efficiency of today's mobile communications. An Energy Efficiency Evaluation Framework (E³F), which allows quantitative assessment of the performance in different traffic and load scenarios, has been developed by the EARTH (Energy Aware Radio and NeTwork technologies) program. [8] This framework describes a detailed base station power model, and focuses on how power varies in different scenarios.

The base stations are divided according to the geographical area they serve. Thus, there are four main categories: macro, micro, pico and femto. [8] As it is expected, each category has different energy consumption. This depends on the components used for each station, for example macro and micro stations, which require more resetting, use less dedicated hardware and more Field Programmable Gate Arrays (FPGAs), which leads to higher power consumption.

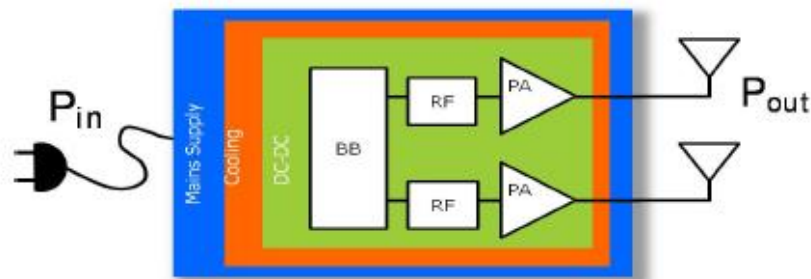


Figure 5: Base station components included in the power model. [8]

In order to study the energy efficiency in this model, power versus energy was used, since the input and output of electronic systems are usually measured in terms of power. The base station is divided into the parts shown in Figure 5. Those parts relate to the functions that occur in the base station. The BB unit refers to digital processing, RF to analog, PA to signal amplification and finally there is also the unit that is responsible for the power conversion and the cooling of the system (Overhead).

The division of the base station helps in understanding the power consumption of each system's operation and how it varies with the system parameters. These parameters are the bandwidth, the number of the antennas, the modulation, the coding rate, the fraction of the bandwidth that is being used (frequency-domain duty cycle - df) and the fraction of time at which the base station is operating (Time-domain duty cycle - dt). The rest of the time, the base station is considered to be sleeping. The number of the antennas is considered to be the same in both uplink and downlink [8]. Table 1 shows the typical values in which the above parameters range.

The total power at the base station is measured after calculating the power consumption of each element.

Table 1: Parameters affecting scaling of baseband and RF power consumption. [8]

<i>Symbol</i>	<i>Description</i>	<i>Range</i>
<i>BW</i>	Bandwidth (MHz)	1.4-20
<i>Ant</i>	Number of antennas	1, 2, 4
<i>M</i>	Modulation (bits per symbol)	1, 2, 4, 6
<i>R</i>	Coding rate	1/3 – 1
<i>dt</i>	Time-domain duty cycling	0 - 1
<i>df</i>	Frequency-domain duty cycling	0 - 1

4.1.1 Baseband Processing

Digital baseband processing includes all the functions that need to be done after the signal has been transferred to the baseband and has been digitized. Taking into consideration the leakage power, the total power consumption of the digital processing unit is [8]:

$$P = P_{Dynamic} + P_{Leak} \Rightarrow$$

$$P = P_{DPD} + P_{Filter} + P_{CPRI} + P_{OFDM} + \quad (4.1.1)$$

$$+ P_{FD,lin} + P_{FD,nl} + P_{FEC} + P_{CPU} + P_{Leak}$$

$P_{Dynamic}$ in the above formula is the total power that is consumed by the distinct operations of signal processing and P_{Leak} is the leakage power.

Table 2 shows the reference values of the baseband processing operations of the four base station types for downlink and uplink, respectively. Those values were calculated for a system with 20MHz bandwidth, a single antenna, 64 – QAM, coding rate equal to 1 and a load of 100%. The term load refers to the fraction of time and frequency resources that the system uses ($load = dt \times df$). The chosen CMOS technology is 65 nm.

Table 2: Complexity of baseband operations in downlink and uplink (GOPS). [8]

GOPS per operation type	Macro		Micro		Pico		Femto	
	D	U	D	U	D	U	D	U
DPD	160	-	160	-	0	-	0	-
Filter	200	200	160	160	120	160	100	150
CPRI/SERDES	360	360	300	300	0	0	0	0
OFDM	80	80	80	80	70	80	60	60
FD (linear)	30	60	30	60	20	40	20	30
FD (non-linear)	10	20	10	20	5	10	5	10
FEC	20	120	20	120	20	120	20	110
CPU	200	200	200	200	30	30	20	20

For the computations, the factor GOPS (Giga Operation Per Second) was used. GOPS expresses power models that depend on the internal performance of the selected technology

(GOPS/W). For the 65 nm General Purpose CMOS technology, this factor is 40 GOPS/W for macro and micro stations, and 120 GOPS/W for pico and femto stations. Furthermore, the reference value for the leakage power is defined as [8]:

$$P_{Leak,ref} = \eta_{Leak} \cdot P_{Dynamic,ref} \quad (4.1.2)$$

The coefficient η is 0.1 for the 65 nm technology. It is worth noting that power consumption is highly dependent on technology. Specifically, for each new generation CMOS the dynamic power is halved, but the leakage power is tripled.

As shown in Table 2, the deformation reduction (DPD) does not apply to small base stations (pico and femto) as they have different type of power amplifiers and power levels. Moreover, in small base stations there is no specific backbone network. [8] On the other hand, filtering and signal accuracy are more stringent at large base stations, which affects the power that is consumed during filtering and MIMO-OFDM processing. Last but not least, comparing the downlink and uplink results, it is obvious that MIMO processing and decoding are much more complex for the uplink.

Finally, the change in power for the various operations of baseband processing with each system parameter in downlink is shown in Table 3. In uplink, all exponents are the same except in the case of non-linear processing in the frequency domain (there is cubic dependence instead of square).

Table 3: Scaling exponents for the baseband sub-components in downlink. [8]

Downlink							
Digital scaling exponents	BW	M	R	Ant.	dt	df	
DPD, Filter and OFDM	1	0	0	1	1	0	
CPRI/SERDES	1	1	1	1	1	1	
FD (linear)	1	0	0	1	1	1	
FD (non-linear)	1	0	0	2	1	1	
FEC	1	1	1	1	1	1	
CPU	0	0	0	1	0	0	
Leakage	1	0	0	1	0	0	

4.1.2 Analog Processing

The RF processing unit consists of elements like clock/carrier generator, modulators, multiplexers, filters, buffers, low-noise amplifiers and analog/digital converters. Table 4 and Table 5 demonstrate the power consumption of each element, considering the same case as in that of digital processing. Some elements do not exist in small base stations, so they have a zero value in the tables. Additionally, a downscaling factor reduces the power of small base stations due to less constraining specs and different hardware implementation of small cells. [8]

All analog elements have a scaling exponential of 1 in relation to the number of antennas (Ant) and the operating time (dt). The clock generator has zero all the other exponential coefficients. For the rest of the elements, the exponential coefficient is 1 in relation to the bandwidth (BW) as well as the bandwidth that is being used (df) for small base stations. [8]

Table 4: RF analog component power (mW) in the downlink. [8]

Downlink				
Power per analog component (mW)	Macro	Micro	Pico	Femto
IQ Modulator	1000	1000	1000	1000
Variable attenuator	10	10	0	0
Buffer	300	300	0	0
Forward Voltage-Contr. Osc. (VCO1)	170	170	170	170
Feedback Voltage-Contr. Osc. (VCO2)	170	170	0	0
Feedback mixer	1000	1000	0	0
Clock generation and buffering	990	990	990	990
Digital-to-Analog Converter (DAC)	1370	1370	200	200
Analog-to-Digital Converter (ADC)	730	730	140	140
TOTAL	5740	5740	2500	2500
Downscaling factor	1	2	7	12
Total after downscaling (W)	5,7	2,9	0,4	0,2

Table 5: RF analog component power (mW), in the uplink. [8]

Uplink				
Power per analog component (mW)	Macro	Micro	Pico	Femto
First Low-Noise Ampl. (LNA1)	300	300	300	300
Main variable attenuator	10	10	10	10
Second Low-Noise Ampl. (LNA2)	1000	1000	0	0
Dual mixer	1000	1000	1000	1000
Dual IF Variable Gain Ampl. (VGA)	650	650	0	0
Clock generation and buffering	990	990	990	990
Analog-to-Digital Converter (ADC)	1190	1190	290	290
TOTAL	5140	5140	2590	2590
Downscaling factor	1	2	7	12
Total after downscaling (W)	5,1	2,6	0,4	0,2

The type of technology affects power consumption in this case too. An empirical formula for finding power consumption is: [8]

$$Power(tech) = Power(65nm) \cdot \left(1 + \frac{tech/65 - 1}{2} \right) \quad (4.1.3)$$

$Power(65\text{ nm})$ is the power reference for the 65 nm CMOS technology.

4.1.3 Power Amplifier

The power amplifier cannot be modeled by a single power reference value and specific scaling rules. Therefore, the PA model is represented by a table containing output power measurements in relation to power consumption. The power model selects the point with the minimum power consumption that satisfies the output power's and linearity's limitations.

The maximum total output power is 46 dBm for macro cell, 41 dBm for micro cell, 24 dBm for pico and 20 dBm for femto cells. [8]

4.1.4 Overhead

In this category belong all the elements that are related to the system power supply, such as AC / DC or DC / DC conversion as well as the cooling of the system. The power in this case is assumed to depend linearly on the total power of the rest of the base station: [8]

$$P_{Overhead} = (P_{BB} + P_{RF} + P_{PA}) \times ((1 + \eta_{cool})(1 + \eta_{acdc})(1 + \eta_{acdc}) - 1) \quad (4.1.4)$$

The factor η represents the loss coefficient of each function in this category. Typical values for the loss coefficients are $\eta_{acdc} = 5\%$, $\eta_{acdc} = 10\%$ and $\eta_{cool} = 10\%$ for the big base stations, since small ones don't use a cooling system.

4.1.5 Power Consumption of the Base Station

The total power of the base station shall be calculated according to the formula: [8]

$$P_{Total} = P_{BB} + P_{RF} + P_{PA} + P_{Overhead} \Rightarrow$$

$$P_{Total} = \sum_{i \in I_{BB}} P_{i,ref} \prod_{x \in X} \left(\frac{x_{act}}{x_{ref}} \right)^{s_{i,x}} +$$

$$+ \sum_{i \in I_{RF}} P_{i,ref} \prod_{x \in X} \left(\frac{x_{act}}{x_{ref}} \right)^{s_{i,x}} + P_{PA} + P_{Overhead} \quad (4.1.5)$$

In the above formula, I_{BB} and I_{RF} represent the set of elements of the baseband and the analog processing, respectively, whereas $X = \{BW, Ant, M, R, dt, df\}$ is the set of the system parameters. The s in the above formula symbolizes the exponential scaling factor that relates each operation of the base station with the system parameters. For instance, if an element's power doesn't depend on the number of the antennas, then $s = 0$, that is a change in the number of the antennas won't affect the power consumption of that element.

In the interest of understanding the calculation of the power consumption, lets compute the power of baseband processing for the scenario of $X =$

$\{10\text{MHz}, 2, 4 (16\text{QAM}), \frac{3}{4}, 100\%, 30\%\}$ and 65nm CMOS technology. Beginning from the reference scenario ($X = \{20\text{MHz}, 1, 6 (64\text{QAM}), 1, 100\%, 100\%\}$), we convert the GOPS value of table $\alpha\delta\epsilon$ σε W :

$$P_{Filter,ref} = \frac{200 \text{ GOPS}}{40 \text{ GOPS}/W} = 5W, \quad P_{CPRI,ref} = \frac{360 \text{ GOPS}}{40 \text{ GOPS}/W} = 9W$$

$$P_{OFDM,ref} = \frac{80 \text{ GOPS}}{40 \text{ GOPS}/W} = 2W, \quad P_{FD,lin,ref} = \frac{60 \text{ GOPS}}{40 \text{ GOPS}/W} = 1.5W$$

$$P_{FD,nl,ref} = \frac{20 \text{ GOPS}}{40 \text{ GOPS}/W} = 0.5W, \quad P_{FEC,ref} = \frac{120 \text{ GOPS}}{40 \text{ GOPS}/W} = 3W$$

$$P_{CPU,ref} = \frac{200 \text{ GOPS}}{40 \text{ GOPS}/W} = 5W, \quad P_{Leak,ref} = 0.1 \cdot 26W = 2.6W$$

The scaling vector $[s_1, \dots, s_6]$ is shown in Table 3. Thus, the power consumption of each baseband element is:

$$P_{sub} = P_{sub,ref} \cdot \left(\frac{BW_{act}}{BW_{ref}}\right)^{s_1} \cdot \left(\frac{M_{act}}{M_{ref}}\right)^{s_2} \cdot \left(\frac{R_{act}}{R_{ref}}\right)^{s_3} \cdot \left(\frac{Ant_{act}}{Ant_{ref}}\right)^{s_4} \cdot \left(\frac{dt_{act}}{dt_{ref}}\right)^{s_5} \cdot \left(\frac{df_{act}}{df_{ref}}\right)^{s_6}$$

Thus, the results for each element are:

$$P_{Filter} = 5 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^0 \cdot \left(\frac{3/4}{1}\right)^0 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^1 \cdot \left(\frac{0.3}{1}\right)^0 = 5W$$

$$P_{CPRI} = 9 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^1 \cdot \left(\frac{3/4}{1}\right)^1 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^1 \cdot \left(\frac{0.3}{1}\right)^1 = 1.35W$$

$$P_{OFDM} = 2 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^0 \cdot \left(\frac{3/4}{1}\right)^0 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^1 \cdot \left(\frac{0.3}{1}\right)^0 = 2W$$

$$P_{FD,lin} = 1.5 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^0 \cdot \left(\frac{3/4}{1}\right)^0 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^1 \cdot \left(\frac{0.3}{1}\right)^1 = 0.45W$$

$$P_{FD,nl} = 0.5 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^0 \cdot \left(\frac{3/4}{1}\right)^0 \cdot \left(\frac{2}{1}\right)^3 \cdot \left(\frac{1}{1}\right)^1 \cdot \left(\frac{0.3}{1}\right)^1 = 0.6W$$

$$P_{FEC} = 3 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^1 \cdot \left(\frac{3/4}{1}\right)^1 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^1 \cdot \left(\frac{0.3}{1}\right)^1 = 0.45W$$

$$P_{CPU} = 5 \cdot \left(\frac{10}{20}\right)^0 \cdot \left(\frac{4}{6}\right)^0 \cdot \left(\frac{3/4}{1}\right)^0 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^0 \cdot \left(\frac{0.3}{1}\right)^0 = 10W$$

$$P_{Leak} = 2.6 \cdot \left(\frac{10}{20}\right)^1 \cdot \left(\frac{4}{6}\right)^0 \cdot \left(\frac{3/4}{1}\right)^0 \cdot \left(\frac{2}{1}\right)^1 \cdot \left(\frac{1}{1}\right)^0 \cdot \left(\frac{0.3}{1}\right)^0 = 2.6W$$

Therefore, the total power consumed by the digital baseband processing unit in this scenario is:

$$P_{Total} = P_{Filter} + P_{CPRI} + P_{OFDM} + P_{FD,lin} + P_{FD,nl} + P_{FEC} + P_{CPU} + P_{Leak} = 22.45 W$$

4.2 Power Consumption of IP Networks

A traditional IP network is constructed by four main parts, that are shown in Figure 6. The access network connects the end user with the edge switches in the provider's network. The link between the access network and the core network is called metro and edge network. The core network is composed by core routers that perform all the necessary routing and also serve as the gateway to neighboring core nodes. Finally, the VDN is an additional infrastructure in order to deliver video program content to the access node. [9]

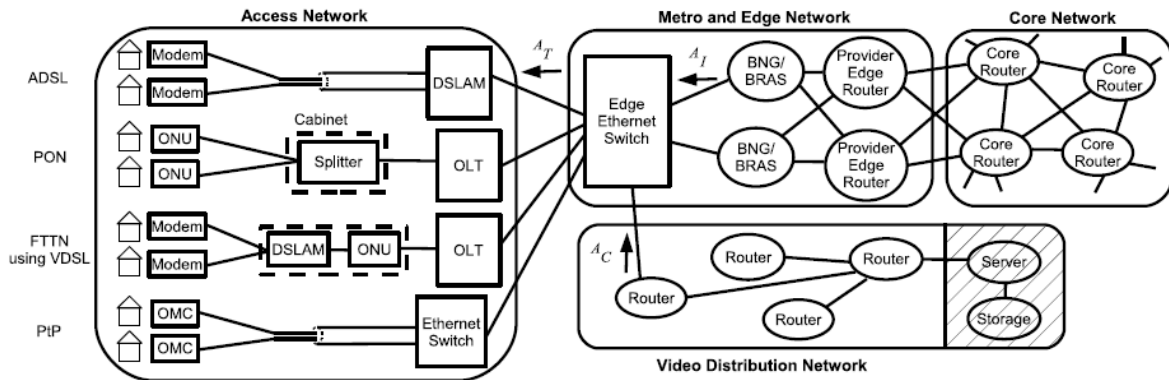


Figure 6: The structure of a traditional IP Network. [9]

There are quite a few types of access technologies that are in use or in development, however, our analysis is constrained to four popular access technologies. Asymmetric Digital Subscriber Line (ADSL) is the most commonly used technology. The broadband service is delivered by the coppered pairs that are installed to deliver a fixed-line telephone service. Due to the fact that ADSL uses copper pair, it has limited capacity. For example, ADSL2+, gives theoretical speeds of 24 Mb/s downstream and 1 Mb/s upstream. [9] Fiber based technologies give a desirable solution in order to overcome the limit in the capacity. In Passive shared Optical Networks (PON) groups of end users are connected to the provider's network by one single fiber, through a passive splitter. The capacity of PON networks is significantly higher than that of ADSL (G-PON has theoretical speeds of 2.4 Gb/s downstream and 1.2 Gb/s upstream). [9]

Another viable solution, in order to exploit the copper pair infrastructure is the technology Fiber To The Node (FTTN). These networks use a fiber to connect the network switch with a cabinet close to a cluster of customers. Then high speed DSL technologies (e.g. VDSL) are used to connect the cabinet with the end user. Lastly, the highest access speed network today is the Point to Point Network (PtP) that employs a dedicated fiber between the customer premises and the network access node. [9]

In order to see the dependence of the optimal split with the network technology, we calculate the power consumption of each part by using two power models, a linear and a nonlinear that takes into consideration the technology improvement rate.

4.2.1 Linear Model

The total power consumption of the network consists of the power that individual parts of the network consume, that is:

$$P_{NET,tot} = P_a + P_m + P_c + P_{vdn} + P_{TS}, \quad (4.2.1)$$

where P_a is the power consumption of the access network, P_m the metro and edge network's power consumption, P_c the core network's power consumption, P_{vdn} the power consumption of VDN and P_{TS} the power consumption of the transport systems.

Each access network technology absorbs different amounts of power per customer (P_a), which is [9]

$$P_a = P_{CPE} + \frac{P_{RN}}{N_{RN}} + \frac{2P_{TU}}{N_{TU}} \quad (4.2.2)$$

where P_{CPE} , P_{RN} , P_{TU} are the power consumption of the customer premises equipment, the remote node and the terminal unit, respectively.

The per customer consumption of the metro network can be expressed as: [9]

$$P_m = 2 \left(P_{ES} + 2A_I \left(\frac{\tilde{P}_{Gateway}}{C_{Gateway}} + \frac{\tilde{P}_{PEdge}}{C_{PEdge}} \right) \right) \quad (4.2.3)$$

In the above equation $\tilde{P}_{Gateway}/C_{Gateway}$ and $\tilde{P}_{PEdge}/C_{PEdge}$ are the total power consumption/ the capacity of the gateway and provider edge routers, respectively. P_{ES} is the power consumed by the edge Ethernet switches. The factor A_I relates to the peak access rate A_p . Peak access rate is the highest rate that can be provided to the customers. Nevertheless, network operators provide a lower worst case minimum rate to every customer, in order to manage the data traffic. The connection of those two rates is through a factor that is called oversubscription rate (M): [9]

$$A_I = \frac{A_P}{M} \quad (4.2.4)$$

The power consumption of the VDN is: [9]

$$P_{VDN} = 4 \times \frac{3A_C}{C} \times P \quad (4.2.5)$$

where P, C are the power consumption and the capacity of the VDN routers, while A_C is the per-customer capacity in VDN.

The per customer power consumption of the core network is given by [9]

$$P_c = \frac{8A_I(H + 1)}{C_{r,c}} \times P_{r,c} \quad (4.2.6)$$

where H symbolizes the number of core node hops. $P_{r,c}, C_{r,c}$ are the power consumption and the capacity of the core routers.

Last but not least, it is crucial to calculate the power consumption of the WDM transport systems, that are important in order to connect longer links between the metro edge routers sites and core router sites. These systems can be either terrestrial or undersea. Their power consumption is expressed by the formula: [9]

$$P_{TS} = \begin{cases} 4 \left(\frac{A_I(1 - U)}{C_{channel}} \right) \times \frac{H}{2} \times P_{channel}, & \text{terrestrial} \\ 4 \left(\frac{A_I U}{C_{channel}} \right) \times H \times P_{channel}, & \text{undersea} \end{cases} \quad (4.2.7)$$

where U is the proportion of traffic going to neighboring nodes through undersea WDM systems and $P_{channel}, C_{channel}$ are the cumulative power per channel and the rate of each channel, respectively.

4.2.2 Nonlinear Model

In recent years there have been exponential developments in the efficiency of routers and switches. A relation between the access rate and the technology improvement can be expressed as: [9]

$$\frac{P_R}{C_R} = \frac{P_0}{C_0} \times (1 - \alpha)^{\frac{\ln(A/A_0)}{\ln\beta}} \quad (4.2.8)$$

where α is the technology improvement rate and β the per year traffic growth rate. This model will be applied to the core, VDN and metro and edge networks, but not to the access network, because this equipment is replaced less frequently. The WDM transport systems obey to the formula: [9]

$$\frac{P_{WDM}}{C_{WDM}} = \frac{P_{0,WDM}}{C_{0,WDM}} \times \left(0.1 + 0.9(1 - \alpha)^{\frac{\ln(A/A_0)}{\ln\beta}} \right) \quad (4.2.9)$$

shows the dependence of the power per customer on the peak access rate, for the cases of four different access network technologies with no VDN. In this case there was assumed a technology improvement rate of 10%. It is evident that, as the access rate grows, the technologies that are copper pair-based consume a considerably large amount of power, in comparison with fiber-based technologies.

Chapter 5 Game Theory

As it was pointed out, the new generation of mobile communication networks must confront the constant increase of data traffic. This increment will lead to a heavy load on the base stations, which will cause an augmentation in power consumption. A viable solution would be the centralization of baseband processing that was discussed in Chapter 3. However, a fully centralized solution indicates high data rates for the connection of the remote and the centralized unit. Thus, a compromise must be found, between high data rate connections and high power consumption. In this chapter, we examine the necessary mathematical tools in order to approach this problem.

5.1 Classic Game Theory

Classic theory game origins back to 1930, when the economist Oskar Morgenstern realized the difficulty of economic provisions. The problem lays to the fact that the provision itself may affect the economic result. Morgenstern described this phenomenon by using the strife between Sherlock Holmes and Professor Moriarty. These two enemies will never mutually outguess each in order to end up to a final solution. [10]

As an introduction, we begin with the two-players game. Player *I* can choose from a finite set of n choices, which are called strategies. We represent this set by $\{e_1, e_2, \dots, e_n\}$. Similarly, player *II* chooses from another set of m strategies: $\{f_1, f_2, \dots, f_m\}$. If player *I* picks the strategy e_i and player *II* the strategy f_j , then both players receive payoffs a_{ij} and b_{ij} respectively. Hence, the game is described by two matrices $n \times m$, or alternatively we can describe the game by one matrix $n \times m$, whose ij -th element represents the pair (a_{ij}, b_{ij}) . The payoffs are measured on a utility scale that is consistent with the preferences of both players. [10]

Table 6: "Rock, Scissors, Paper", a classic example of a two players game. [10]

PLAYER I / PLAYER II	ROCK	SCISSORS	PAPER
ROCK	(0,0)	(1,-1)	(-1,1)
SCISSORS	(-1,1)	(0,0)	(1,-1)
PAPER	(1,-1)	(-1,1)	(0,0)

A classic example of a two players game is the game "Rock, Scissors, Paper". [10] [11] Each player has to choose between three strategies. The matrix that describes the game is described by Table 6. According to this game, if both players choose the same strategy, then it's a tie and the payoffs are zero. If the result is either (0,0) or (-1,1), then player I would have done better if he changes his strategy. The same applies for player II if the result is (0,0) or (1,-1). If one prediction becomes public, at least one of the players will have the incentive to deviate. This deviation would be predicted by his adversary, how consequently will also deviate. Thus, the players will result in an endless loop, both trying to outguess each other.

Nevertheless, in an earlier paper, mathematician John von Neumann had found a way to overcome Morgenstern's dead end. The solution lays in randomness, that is to say let the chance decide. It's obvious that if players choose their strategies with equal probability, then no one will have the motive to deviate. In the next decade, the collaboration of these two led to the birth of game theory. Landmark of their work was the publication of "Theory of Games and Economic Behavior" in 1944. Few years later, John Nash introduced the concept of equilibrium that stands for an even wider context and became the corner stone of game theory. [10] [12]

Classic game theory separates the games into three categories: matrix games, continuous static game games and differential games. [12] Matrix game, like the "Rock, Scissor, Paper" game, have a finite number of options. When a player chooses his strategy, he receives a payoff that is defined by an element of the matrix. Player I's strategy equals to the

selection of one row of the matrix, while player II's strategy equals to the selection of one column. Hence, all payoffs are given by the elements of the matrix. The payoff of each player is specified by the combination of their strategies. Continuous static games, strategies and payoffs are associated by a continuous manner. The use of the word "static" indicates that the strategy of each player is stable. Differential games are characterized by strategies and payoffs that constantly change with time, and by a dynamic system of differential equations.

One should not confuse game theory with the theory of optimization. In a conventional game, the goal of every player is the choice of the strategy that will maximize his payoff. When the player's payoff is related only to his own strategy, we refer to it as a problem of game theory. The game consists of players, strategies, payoffs and rules that determine how these strategies lead to the corresponding payoffs. In the contrary, when each payoff is related to the strategies of other players, then this is an optimization problem. The main component of optimization theory is the concept of maximum, while game theory varies in the potential solutions to predict the outcome of the game and the best choices of the players. [12]

5.1.1 Nash Equilibrium

In order to fully understand the concepts of game theory, here we analyze a very simple model of strategic interaction. Let's assume that player I chooses to play a strategy e_i with probability x_i . This mixed strategy is given by a vector $x = (x_1, \dots, x_n)$, where $x_i \geq 0$ and $x_1 + \dots + x_n = 1$. [10] [11]. The set of all the mixed strategies is represented by Δ_n and it is a simplex in R_n . The unit vectors e_i of the standard base that construct this simplex symbolize the original strategies. We refer to them as "pure strategies". Respectively player II chooses a mixed strategy y from the simplex Δ_m , that is constructed by the unit vectors f_j .

If player I chooses the pure strategy e_i , while player II a mixed strategy y , then the expected payoff of player I is: [10]

$$(Ay)_i = \sum_{j=1}^m \alpha_{ij} y_j, \quad (5.1.1)$$

where A is his payoff matrix. If player I also picks a mixed strategy x , then his expected payoff becomes: [10] [11]

$$x \cdot Ay = \sum_{i=1}^n x_i (Ay)_i = \sum_{i=1}^n \sum_{j=1}^m \alpha_{ij} x_i y_j \quad (5.1.2)$$

Equally, player II receives a payoff:

$$x \cdot By = \sum_{i=1}^n x_i (By)_i = \sum_{i=1}^n \sum_{j=1}^m b_{ij} x_i y_j, \quad (5.1.3)$$

where A is his payoff matrix.

Assuming that player I knows the strategy y of his opponent, he must use the strategy that is the best response to y . The set of “best responses” is defined as: [10]

$$BR(y) = \underset{x}{argmax} x \cdot Ay \quad (5.1.4)$$

That is the set of all $x \in \Delta_n$ for which $z \cdot Ay \leq x \cdot Ay$ for all $z \in \Delta_n$. The set of best replies is never void, because the function $z \mapsto z \cdot Ay$ is continuous and Δ_n is compact. Additionally, if $x \in BR(y)$, then all the “pure strategies” e_i with $x_i \geq 0$ belong to the set: [10]

$$(Ay)_i = e_i \cdot Ay \leq x \cdot Ay \quad (5.1.5)$$

When player I has found the best response to strategy y of his adversary, he has no reason to deviate, as long as player II doesn't change his strategy. Player II will remain to his strategy only if he has no reason to deviate, that is if he has found the best response to the strategy of player I. Two strategies form a “Nash equilibrium” if mutually they are the best response to each other, i.e. $x \in BR(y)$ and $y \in BR(x)$. [10] A Nash equilibrium states that no player has the incentive to deviate from their strategies. Nash equilibrium exists in every strategic game. This is true for even larger game categories, for every number of players, every set of strategies and ever payoff function.

A symmetric Nash equilibrium pair is of most importance, that is a pair of a Nash equilibrium in which $x = y$. A symmetric Nash equilibrium point is therefore determined by a x strategy that has the property of being the optimal response to itself: [10]

$$z \cdot Ax \leq x \cdot Ax, \quad (5.1.6)$$

for all $z \in \Delta_n$, i.e no player can improve his payoff by changing unilaterally. In every symmetrical game there exists a symmetric Nash equilibrium.

Mixed strategies are the corner stone of Nash equilibrium. Their importance is evident in the game "Rock, Scissors, Paper". In this case the mixed strategy $x = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, that consists of the choice of equal probability between the three pure strategies, leads to a Nash equilibrium pair. There is no reason for the players to deviate. Instead, if player I chooses another strategy but strategy x of his opponent, then he will still receive a zero payoff, but player II will have the motive to deviate, which lead to the deviation of player I etc.

5.2 Evolutionary Game Theory

Up to now, the discussion was about games of two players who try to guess each other's strategy in order to find the best response. These games, which belong to the classic game theory, were addressed to mathematical and economical problems. Beginning from the sixties and seventies, theory as well as its applications were motivated by problems of evolutionary biology, like the sex ratio or the investigation of fight behavior in the animal kingdom. Consequently, the necessity of an essential change of scene was emerged, especially the introduction of thinking in terms of population in game theory.

Evolutionary game theory studies the behavior of large populations, whose members repeatedly participate in strategic interactions [11]. The mathematician and biologist Maynard Smith was the first who studied the evolution from the angle of game theory, in his book "Evolution and Theory of Games". He adjusted the methods that were created to explain the economic behavior, in the biologic natural selection. Evolution through natural selection belongs to the evolutionary game theory in the sense that it also has players, strategies, set

of strategies and payoffs. According to Maynard, evolution is about the survival of a distinct strategy among a population of individuals who probably use many different strategies.

There are several points in which evolutionary game theory separates itself from the classic. To start with, evolutionary game theory doesn't belong to any of the three categories of classical games. One could say that it is a hybrid between continuous static games and differential ones. Furthermore, the attention now focuses on the strategies that will remain as time goes by, instead of classical game theory, where the focus was on the players who tried to choose the proper strategies in order to optimize their payoff. In classical game theory, each player may have different sets of strategies and different payoffs that correspond to them. On the other hand, in evolutionary game theory there are groups of individuals that are identical in the sense that they choose from the same set of strategies and have the same payoffs that correspond to the same strategies. In classical game theory, individuals try to optimize their payoffs, based on their personal interest, while in an evolutionary game natural selection is the optimization factor. In conclusion, one can say that classic game theory focuses on the winners, whereas evolutionary game theory focuses on the survivors. This doesn't mean that winners can't be survivors rather that survivors don't have to be the winners. [12]

A significant part of the importance of the evolutionary game theory is its appliance in many different problems, like language evolution, social dilemmas or animal behavior. Its methods are becoming more and more popular in computer science, in engineering and automation, since they assist the design and the control of multi user systems (like driving drivers through motorway networks, or routing packets over the Internet).

5.2.1 Evolutionary Stability

In his effort to explain evolution in terms of game theory, Maynard introduced a basic principle: The evolutionary success of a given behavioral characteristic depends on the dominance of all behavioral characteristics. [11] As a consequence, natural selection among the traits can be modeled as a random match of animals to play Normal Form Games. A

Normal Form Game is a strategic interaction in which each of the n players chooses a strategy and receives a payoff that depends on the strategic choices that were made by all players. [11]

The main problem that emerges from this assumption is finding the proper solution for the games. Every evolutionary system produces trajectories of changing strategic values. [12] The question that arises is if these trajectories end up in a stable point, and if those stable points have anything in common. Specifically do these points have the property of producing the best strategy given the circumstances? One may consider that the best strategy is the one that maximizes the total payoff, however such a solution may not be appropriate given the nature of evolutionary game theory.

Working on this direction, in 1973, Maynard Smith with Price proposed a concept of stability for populations of animals who share a common trait. That is, in order for a strategy to be evolutionary stable, it must resist the invasion of other strategies. Particularly, let's assume that an animal population, that is programmed to play the mixed strategy x , is invaded by a group of mutants who play the mixed strategy y . The mixed strategy x forms an Evolutionary Stable Strategy (ESS), if despite the choice of y , the expected payoff of a domestic animal at a random match after the invasion is bigger than that of a mutant's. This is true as long as the size of the mutants' group is sufficiently small. [11] [12]

Based on the above definition, the resistance in an invasion is strong only when the majority of the population is using the ESS. ESS should be the best strategy from all the alternatives when it is common to the entire population. Soon it became clear that the concept of ESS had many similarities with the Nash equilibrium from a classic game. Both concepts refer to strategies which when they are played by the entire population, then no one will benefit by changing strategy unilaterally. They differ, though, to the fact that Nash equilibrium is associated with the payoffs without taking into consideration the dynamic of the population, whereas an ESS must also refer to the population dynamics, i.e. how the population is affected by the strategies that exist inside and between the populations. [12]

The definition of an Evolutionary Stable Strategy can be expressed by the assistance of two conditions: [11]

$$x \cdot Ax \geq y \cdot Ax, \text{ for all } y \in \Delta_n \quad (5.2.1)$$

for all $y \neq x$, $[x \cdot Ax = y \cdot Ax]$ implies that $[x \cdot Ax > y \cdot Ay]$

First condition is the description of Nash Equilibrium. Native strategy x must be the best response to itself. Second condition states that if a mutant's strategy y , is the best response to x , then the native earns a higher payoff against the mutant, than the mutant against himself.

In conclusion, the concept of ESS of Maynard Smith attempts to explain the dynamic evolution of natural selection with the usage of a static definition. This can be extended in order to cover a wide range of strategic sets, and has been generalized in many directions.

5.2.2 Population Games

Population game provide a simple and general frame for studying strategic interactions in large populations, whose members play pure strategies. Consider a population, where each of the players has a given strategy. Randomly in time, two random players meet and play a game by using their strategies. These strategies are analyzed as behavioral programs, in the sense that they can be taught, inherited or imprinted by every other way. In order to simplify the analysis, all the individuals are assumed to be identical. They differ only in their strategies. Such games are called symmetric. This assumption isn't always true, like the case of sellers and consumers or the case of parents and offsprings. [10]

In symmetric games of two players, there is no distinguish among player I and II. Both players have the same set of strategies, i.e. $n = m$ and $f_j = e_j$ for all j . If a player chooses the strategy e_i over someone who chose the strategy e_j , then he receives the same payoff, regardless if he is player I or II. For the payoff matrices, this means that $a_{ij} = b_{ji}$ or $B = A^T$. The behavior of the entire population is described by a population state $x \in \Delta_n$, where x_j represents the percentage of players who choose the pure strategy e_j . It must be noticed that the same symbol x that was used before to state a mixed strategy of a certain player, is now used to denote a population state. [10] [11]

As population game is a continuous payoff function $F : \Delta_n \rightarrow R_n$. Scalar $F_i(\mathbf{x})$ expresses the payoff of strategy e_i when the population state is \mathbf{x} . That is, a player that plays the strategy e_i has expected payoff $F_i(\mathbf{x})$, while the mean payoff of the population is $\bar{F}(\mathbf{x}) = \sum_i x_i F_i(\mathbf{x})$. The expected payoff $F_i(\mathbf{x})$ and the mean payoff $F(\mathbf{x})$ are given by the expressions: [11]

$$F_i(\mathbf{x}) = (A\mathbf{x})_i = \sum_j \alpha_{ij} x_j, \quad (5.2.2)$$

$$\bar{F}(\mathbf{x}) = \mathbf{x}A\mathbf{x} \quad (5.2.3)$$

5.2.3 Revision Protocols

Before tackling the evolution dynamics it's necessary to introduce a basis for the evolutionary model. This foundation is the revision protocol, which describes the timing as well as the results of the players' myopic decisions about their behavior in strategic interactions. A revision protocol is a map $\rho: R_n \times \Delta_n \rightarrow R_+^{n \times n}$ that accepts as inputs the vectors of payoffs $\boldsymbol{\pi}$ and the population states \mathbf{x} and returns non negative matrices as output. The scalar $\rho_{ij}(\boldsymbol{\pi}, \mathbf{x})$ is called the conditional switch rate from strategy e_i to strategy e_j . [11]

A revision protocol ρ , a population game F and a population size N form a continuous evolutionary process- a Markov process. According to Markov process, each player in the population is supplied with a stochastic alarm clock. The times between the ringings of each players alarm are independent and they are dominated by an exponential distribution with rate R . When the clock rings, it creates a revision opportunity for the alarm's owner. If a player who uses strategy e_i gets a revision opportunity, he changes his strategy from e_i to e_j with a probability ρ_{ij}/R . If a change takes place, then the population state changes from state \mathbf{x} to state \mathbf{y} that correspond to the change of the player's strategy.

There are many types of revision protocols. One of them is the imitative protocol, that has the form: [11]

$$\rho_{ij}(\boldsymbol{\pi}, \boldsymbol{x}) = x_j \hat{\rho}_{ij}(\boldsymbol{\pi}, \boldsymbol{x}) \quad (5.2.4)$$

In accordance with this protocol, a player of e_i strategy, with the arrival of a revision opportunity, chooses a random opponent and observes his strategy e_j . The player changes to the strategy of his opponent with probability $\hat{\rho}_{ij}$. Take into consideration that the player does not need to be aware of the value of the population state x_j . This term in the above equation simply declares the observation of the opponent's strategy.

In imitative protocols only strategies that already exist in the population may be chosen from the players that have a revision opportunity. In other protocols, the behavior of the players doesn't depend directly on the current behavior of the population, but vicariously through its effect on the payoffs. These protocols are called evaluative protocols and require from the players to directly calculate the payoff of every strategy, unlike the indirect calculation that is offered by the imitative protocols. [11]

5.2.4 Mean Dynamic

As it is described above, a revision protocol ρ , a population game F and a population size N form a Markov process. The next step of the analysis is the assumption that the population evolves as time passes, in the sense that the percentages of the players who use specific strategies may change.

Each of the N players receives revision opportunities with rate R of exponential distribution. Thus for dt time span, there are expected Rdt revision opportunities. The expected number of revision opportunities that are received by the players who use strategy e_i is approximately $Nx_i Rdt$, where \boldsymbol{x} is current state of the population. When a player with strategy e_i receives an opportunity, he can switch to the strategy e_j with probability ρ_{ij}/R . Hence the expected number of those switches for the next dt time span is $Nx_i \rho_{ij} dt$. This leads to the mathematical expression of the expected change in the number of players who use the strategy e_i in dt time units: [11]

$$(x_i)_{t+dt} - (x_i)_t = N \left(\sum_j x_j \rho_{ji}(F(\mathbf{x}), \mathbf{x}) - x_i \sum_j \rho_{ji}(F(\mathbf{x}), \mathbf{x}) \right) dt \quad (5.2.5)$$

By dividing the above equation with the number of the players in the population (N) and eliminating the time differential dt , we end up with a differential equation that describes the rate of change in the percentage of players using strategy e_i : [11]

$$\dot{x}_i = \sum_j x_j \rho_{ji}(F(\mathbf{x}), \mathbf{x}) - x_i \sum_j \rho_{ji}(F(\mathbf{x}), \mathbf{x}) \quad (5.2.6)$$

The differential equation that arose is called “Mean Dynamics”. The first term describes the inflow of players of other strategies who change to strategy e_i , while the second term refers to the outflow of players who use strategy e_i , to other strategies.

5.2.5 Replicator Equation

Replicator Equation is the most notable dynamic of evolutionary game theory. According to the replicator equation the rate of change in the percentage of players using one strategy is: [11]

$$\dot{x}_i = x_i (F_i(\mathbf{x}) - \bar{F}(\mathbf{x})) \quad (5.2.7)$$

As stated in ((6.1.6) the percentage growth rate \dot{x}_i/x_i of the strategies that are currently used is equal to the excess of the current payoff versus the average population’s payoff. This means that strategies employed at present will be spread or eliminated depending on whether their payoff is better or worse than the average. On the other hand, unused strategies will remain unused. Notice that $\sum \dot{x}_i = 0$.

One of the revision protocol that produces the replicator equation is the pairwise proportional imitation protocol: [11]

$$\rho_{ij}(\boldsymbol{\pi}, \mathbf{x}) = x_j [\pi_j - \pi_i]_+ \quad (5.2.8)$$

This protocol is an imitative protocol. When one player receives a revision opportunity, he chooses an opponent and imitates his strategy only when the payoff of the opponent's strategy is higher than his, with probability proportional to the difference of the payoffs.

One interesting property of the replicator equation is that it remains unchanged even by adding an arbitrary function $b(x)$ to all payoffs $F_i(x) = (Ax)_i$. The factors that are added to the payoffs are also added to the average payoff $\bar{F}(x) = x \cdot Ax$, cancelling the difference. This means that a constant c_j can be added in the j -th column of the matrix A , without changing the dynamic of the replicator. Another significant property is described by the formula below: [10]

$$\left(\frac{\dot{x}_i}{x_j}\right) = \frac{x_i}{x_j} [(Ax)_i - (Ax)_j], \quad x_j > 0, \quad (5.2.9)$$

i.e. the relative sizes of two strategies change according to the difference of their payoffs. Generally, it is true that: [10]

$$\dot{V} = V \left[p \cdot Ax - \left(\sum p_i \right) x \cdot Ax \right], \quad V = \prod x_i^{p_i} \quad (5.2.10)$$

The points where all the payoffs values $(Az)_i$, for all $z_i > 0$, are equal, are called resting points of the replicator equation. The typical value of those payoffs is the average payoff $z \cdot Az$. [10] All vectors e_i of Δ_n simplex are resting points. This is obvious since if all players use the same strategy, then the adoption of the opponents' strategy won't lead to any difference. In the interior Δ_n , the resting point exists as the solution of the linear equation:

$$(Ax)_1 = \dots = (Ax)_n \quad (5.2.11)$$

If there is no resting point inside of Δ_n , then all the trajectories in Δ_n convert on the limits for $t \rightarrow \pm\infty$. On the contrary, if resting point exists in the interior of Δ_n , then there is a trajectory that is bounded away from the borders.

As an example, let's assume a symmetric game $n \times n$ with a payoff matrix A and a symmetric Nash equilibrium point z . Based on (5.1.6) we conclude that:

$$(Az)_i \leq \mathbf{z} \cdot Az \quad (5.2.12)$$

For all $i = 1, \dots, n$. Equality should apply to all i for which $z_i > 0$. The \mathbf{z} -point is the resting point of replicator equation. It is also a saturation point, which means that for $z_i = 0$:

$$(Az)_i - \mathbf{z} \cdot Az \leq 0 \quad (5.2.13)$$

Every saturation point is also a point of symmetrical Nash equilibrium, but the opposite is not true. In the interior of Δ_n each resting point is saturated. In the borders, though, there may be resting points that are not saturated. In this case there are strategies that although they are not present in the state population \mathbf{z} , have a better payoff than the average. [10]

5.2.6 Lyapunov stability

Nonlinear differential equations, like the replicator equation, cannot be resolved in any reasonably convenient way. One of the approaches that is being used for these cases, is the concept of stability, which leads to a qualitative understanding of the behavior of the solutions, rather than detailed quantitative information.

Firstly, it is important to become familiar with the stability concept. We assume an ODE system of the form:

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}) \quad (5.2.14)$$

Notice that the right part of the above formula, doesn't contain the independent variable t . These ODE systems are called autonomous. [13] The aim is to characterize the stability of the equilibrium points. These points are found by solving the equation:

$$f(\mathbf{x}) = 0 \quad (5.2.15)$$

This also means that for an equilibrium point it is true that $\frac{d\mathbf{x}}{dt}=0$, thus it constitutes a stable solution, that is an equilibrium.

Let's say that an equilibrium point is the point \mathbf{x}^0 . The equilibrium point is said to be [14]:

- stable if, for each $\varepsilon > 0$, there is a $\delta = \delta(\varepsilon) > 0$ such that

$$\|\mathbf{x}(0) - \mathbf{x}^0\| < \delta \Rightarrow \|\mathbf{x}(t) - \mathbf{x}^0\| < \varepsilon, \text{ for every } t \geq 0.$$

According to the above definition every solution that begins near \mathbf{x}^0 -that is the distance δ - remain close to \mathbf{x}^0 -that is the distance ε .

- unstable if it is not stable.
- asymptotically stable if it stable and δ can be chosen such that

$$\|\mathbf{x}(0) - \mathbf{x}^0\| < \delta \Rightarrow \lim_{t \rightarrow \infty} \mathbf{x}(t) = \mathbf{x}^0.$$

This means that the trajectories that begin near \mathbf{x}^0 , need not only to stay close to it, but also to approach \mathbf{x}^0 as $t \rightarrow \infty$.

Asymptotically stability is a stronger property than the property of simple stability. An equilibrium point must already be stable in order to examine if it is asymptotically stable. On the other hand, the limit of asymptotically stability's definition, does not imply by itself the simple stability. There can be examples where all trajectories approach \mathbf{x}^0 as $t \rightarrow \infty$, but for which \mathbf{x}^0 isn't a stable equilibrium point. [14]

Having discussed the concept of stability, it is time to introduce the Lyapunov's Indirect Method for finding the stability of a nonlinear system's equilibrium points. Let us go back to the nonlinear autonomous system 5.2.14:

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x})$$

By expanding the nonlinear function f as a Taylor series about the equilibrium $\mathbf{x} = \mathbf{x}^0$ we get:

$$f(\mathbf{x}) = f(\mathbf{x}^0) + \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}^0)\mathbf{x} + O(\mathbf{x}^2) = \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}^0)\mathbf{x} + O(\mathbf{x}^2) \quad (5.2.16)$$

If the initial condition $\mathbf{x}(0) = \mathbf{x}_0$ is chosen close enough to \mathbf{x}^0 , then \mathbf{x} will be 'small' for some time interval extending from zero. Thus, we should be able to neglect the higher-order terms, and approximate our nonlinear system by the linear system:

$$\dot{\mathbf{x}} = A\mathbf{x}, \text{ where } A = \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}^0), \text{ the Jacobian matrix at } \mathbf{x}^0. \quad (5.2.17)$$

The stability of the equilibrium point \mathbf{x}^0 is determined by the eigenvalues λ_i of the Jacobian matrix A . Thus [15]:

- \mathbf{x}^0 is asymptotically stable if $\Re(\lambda_i) < 0$ for all eigenvalues λ_i of A ,
- \mathbf{x}^0 is unstable if $\Re(\lambda_i) > 0$ for some eigenvalues λ_i of A .

However, if some eigenvalues have $\Re(\lambda_i) = 0$ then the linearization method of Lyapunov cannot determine the stability of the equilibrium point because the higher order terms of the Taylor Series become significant.

Chapter 6 Optimization of functional splits

Having already discussed the functional splits of digital processing, in this chapter our interest focuses in finding the optimal split, depending on the circumstances. Each split has different demands as far as network and baseband processing is concerned. The lower the functional split is placed within the protocol stack, the higher is the demand on the network rate and, consequently, the network power. This is obvious since more data need to be transferred through the network. On the other hand, the baseband processing resources, that is the processing power, are reduced, since the functions that were typically performed within the base station, are now performed on the cloud. In this chapter, we use evolutionary game theory in order to extract the optimal functional split.

6.1 Functional splits and game theory

Let's assume a population of N players. Each player chooses a strategy from the finite set S , that consists of 5 strategies- the 5 possible functional splits that were analyzed in the previous chapter. In order to simplify the problem, we will assume the case of the uplink. Table 7 sums up the required transfer rate and required total processing power.

Table 7: Network and processing demands of each functional split.

Strategies (Uplink)	Required data rate	Total Processing Power
SPLIT 1	R_1	$\alpha \cdot P_{RF} + \beta \cdot (P_{OFDM} + P_{FD,l} + P_{FD,nl} + P_{FEC})$
SPLIT 2	R_2	$\alpha \cdot (P_{RF} + P_{OFDM}) + \beta \cdot (P_{FD,l} + P_{FD,nl} + P_{FEC})$
SPLIT 3	R_3	$\alpha \cdot (P_{RF} + P_{OFDM} + P_{FD,l}) + \beta \cdot (P_{FD,nl} + P_{FEC})$
SPLIT 4	R_4	$\alpha \cdot (P_{RF} + P_{OFDM} + P_{FD,l} + P_{FD,nl}) + \beta \cdot P_{FEC}$
SPLIT 5	R_5	$\alpha \cdot (P_{RF} + P_{OFDM} + P_{FD,l} + P_{FD,nl} + P_{FEC})$

The total processing power is the sum of the required processing power on the antenna and the processing power on the cloud. It is easily concluded that the cost of local processing is higher than the cost of the remote one. For simplification purposes, we will assume the same power model as in the case of a macro base station, discussed in Chapter 4, for the case of the cloud. Taking this into consideration, the differentiation between the required local and remote processing power will be inserted by multiplying with a factor α and β the two terms, respectively, with $\alpha > \beta$. The required rate depends on the type of the network. Both network and CPU resources will be depicted in the payoff matrix of the game. In order to be able to do elementary operations between those two parameters we assume a linear (with no improvement rate) and a nonlinear dependence of the network power and the network rate:

$$P_{Net} = f(R) \quad (6.1.1)$$

Consequently, for each split j corresponds a total network power consumption P_{N_j} and a total baseband processing power consumption P_j . Random in time, two players of the population meet and play a game. In order to find the split with the minimum power consumption we describe the game with a 5×5 matrix whose elements are calculated by the formula:

$$a_{ij} = -P_i(\text{Network} + \text{CPU}) - P_j(\text{Network} + \text{CPU}), \quad i, j = 1, \dots, 5. \quad (6.1.2)$$

Therefore, the matrix is:

$$A = \begin{bmatrix} a_{11} & \frac{a_{11} + a_{22}}{2} & \frac{a_{11} + a_{33}}{2} & \frac{a_{11} + a_{44}}{2} & \frac{a_{11} + a_{55}}{2} \\ \frac{a_{11} + a_{22}}{2} & a_{22} & \frac{a_{22} + a_{33}}{2} & \frac{a_{22} + a_{44}}{2} & \frac{a_{22} + a_{55}}{2} \\ \frac{a_{11} + a_{33}}{2} & \frac{a_{22} + a_{33}}{2} & a_{33} & \frac{a_{33} + a_{44}}{2} & \frac{a_{33} + a_{55}}{2} \\ \frac{a_{11} + a_{44}}{2} & \frac{a_{22} + a_{44}}{2} & \frac{a_{33} + a_{44}}{2} & a_{44} & \frac{a_{44} + a_{55}}{2} \\ \frac{a_{11} + a_{55}}{2} & \frac{a_{22} + a_{55}}{2} & \frac{a_{33} + a_{55}}{2} & \frac{a_{44} + a_{55}}{2} & a_{55} \end{bmatrix} \quad (6.1.3)$$

Where:

$$a_{ii} = -2 \cdot (P_{Net_i} + P_{CPU_i}), \quad i = 1, \dots, 5. \quad (6.1.4)$$

The aggregate behavior of the players is described by a vector $\mathbf{x} \in \mathbf{X}$, where x_j , $j = 1, \dots, 5$, represents the proportion of the population that plays strategy j :

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad x_1 + x_2 + x_3 + x_4 + x_5 = 1 \quad (6.1.5)$$

So as to calculate the rate of change in the proportion of players choosing strategy i , the replicator equation is deployed:

$$\begin{aligned} \dot{x}_i &= x_i(F_i(\mathbf{x}) - \bar{F}(\mathbf{x})) \Rightarrow \\ \dot{x}_i &= x_i((A\mathbf{x})_i - \mathbf{x}A\mathbf{x}). \end{aligned} \quad (6.1.6)$$

The term $(A\mathbf{x})_i$ represents the payoff of strategy i , while the term $\mathbf{x}A\mathbf{x}$ represents the mean payoff of the population. In order to calculate these terms, we will use the property of the replicator equation of remaining unchanged even by adding a constant c_j in the j -th column of the matrix A . Thus, we add the term $\frac{a_{ii}}{2}$ to the i -th column of the matrix A , which leads to the matrix:

$$A' = \begin{bmatrix} \frac{a_{11}}{2} & \frac{a_{11}}{2} & \frac{a_{11}}{2} & \frac{a_{11}}{2} & \frac{a_{11}}{2} \\ \frac{a_{22}}{2} & \frac{a_{22}}{2} & \frac{a_{22}}{2} & \frac{a_{22}}{2} & \frac{a_{22}}{2} \\ \frac{a_{33}}{2} & \frac{a_{33}}{2} & \frac{a_{33}}{2} & \frac{a_{33}}{2} & \frac{a_{33}}{2} \\ \frac{a_{44}}{2} & \frac{a_{44}}{2} & \frac{a_{44}}{2} & \frac{a_{44}}{2} & \frac{a_{44}}{2} \\ \frac{a_{55}}{2} & \frac{a_{55}}{2} & \frac{a_{55}}{2} & \frac{a_{55}}{2} & \frac{a_{55}}{2} \end{bmatrix} \quad (6.1.7)$$

First one shall calculate the matrix $A'\mathbf{x}$:

$$A'x = \begin{bmatrix} \frac{a_{11}}{2} \\ \frac{a_{22}}{2} \\ \frac{a_{33}}{2} \\ \frac{a_{44}}{2} \\ \frac{a_{55}}{2} \end{bmatrix} \quad (6.1.8)$$

The mean payoff follows:

$$x \cdot (A'x) = \frac{a_{11} - a_{55}}{2} x_1 + \frac{a_{22} - a_{55}}{2} x_2 + \frac{a_{33} - a_{55}}{2} x_3 + \frac{a_{44} - a_{55}}{2} x_4 + \frac{a_{55}}{2} \quad (6.1.9)$$

Hence the system of differential equations that emerges after substituting equations 6.1.8 and 6.1.9 in equation (6.1.6) is:

$$\begin{cases} \dot{x}_1 = \frac{x_1}{2} \cdot [(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + a_{11} - a_{55}] \\ \dot{x}_2 = \frac{x_2}{2} \cdot [(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + a_{22} - a_{55}] \\ \dot{x}_3 = \frac{x_3}{2} \cdot [(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + a_{33} - a_{55}] \\ \dot{x}_4 = \frac{x_4}{2} \cdot [(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + a_{44} - a_{55}] \\ x_5 = 1 - x_1 - x_2 - x_3 - x_4 \end{cases} \quad (6.1.10)$$

6.2 Linearization and stability

The system of differential equations that emerged in the previous section is a nonlinear dynamical system. Since this system cannot be easily solved by analytical methods it is important to examine its qualitative behavior without actually solving it. Our attention concentrates in finding the stability of a solution. The guidebook for this direction is the Lyapunov stability theorem that has been discussed in Chapter 5.

We need to find the equilibrium points of system 6.1.10 by setting the right part of the equations equal to zero. After some calculation, we get four equilibrium points:

$$(x_1, x_2, x_3, x_4, x_5) = \begin{cases} (1,0,0,0,0) \\ (0,1,0,0,0) \\ (0,0,1,0,0) \\ (0,0,0,1,0) \\ (0,0,0,0,1) \end{cases} \quad (6.1.11)$$

The next step is to calculate the Jacobian matrix for each equilibrium point. For this direction, we shall differentiate the functions in the right part of the system with respect to each x_i parameter:

$$\frac{\partial F_1(x)}{\partial x_1} = \frac{1}{2} [2(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + (a_{11} - a_{55})]$$

$$\frac{\partial F_1(x)}{\partial x_2} = \frac{x_1}{2} (a_{55} - a_{22})$$

$$\frac{\partial F_1(x)}{\partial x_3} = \frac{x_1}{2} (a_{55} - a_{33})$$

$$\frac{\partial F_1(x)}{\partial x_4} = \frac{x_1}{2} (a_{55} - a_{44})$$

$$\frac{\partial F_2(x)}{\partial x_1} = \frac{x_2}{2} (a_{55} - a_{11})$$

$$\frac{\partial F_2(x)}{\partial x_2} = \frac{1}{2} [(a_{55} - a_{11})x_1 + 2(a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + (a_{22} - a_{55})]$$

$$\frac{\partial F_2(x)}{\partial x_3} = \frac{x_2}{2} (a_{55} - a_{33})$$

$$\frac{\partial F_2(x)}{\partial x_4} = \frac{x_2}{2} (a_{55} - a_{44})$$

$$\frac{\partial F_3(x)}{\partial x_1} = \frac{x_3}{2} (a_{55} - a_{11})$$

$$\frac{\partial F_3(x)}{\partial x_2} = \frac{x_3}{2} (a_{55} - a_{22})$$

$$\frac{\partial F_3(x)}{\partial x_3} = \frac{1}{2} [(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + 2(a_{55} - a_{33})x_3 + (a_{55} - a_{44})x_4 + (a_{33} - a_{55})]$$

$$\frac{\partial F_3(x)}{\partial x_4} = \frac{x_3}{2} (a_{55} - a_{44})$$

$$\frac{\partial F_4(x)}{\partial x_1} = \frac{x_4}{2} (a_{55} - a_{11})$$

$$\frac{\partial F_4(x)}{\partial x_2} = \frac{x_4}{2} (a_{55} - a_{22})$$

$$\frac{\partial F_4(x)}{\partial x_3} = \frac{x_4}{2} (a_{55} - a_{33})$$

$$\frac{\partial F_4(x)}{\partial x_4} = \frac{1}{2} [(a_{55} - a_{11})x_1 + (a_{55} - a_{22})x_2 + (a_{55} - a_{33})x_3 + 2(a_{55} - a_{44})x_4 + (a_{44} - a_{55})]$$

Thus, we write the linearized system and find the stability for each point, according to the Lyapunov First (indirect) Method.

1) Equilibrium point: (1,0,0,0,0)

The linear system near this equilibrium point is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{a_{55} - a_{11}}{2} & \frac{a_{55} - a_{22}}{2} & \frac{a_{55} - a_{33}}{2} & \frac{a_{55} - a_{44}}{2} \\ 0 & \frac{a_{22} - a_{11}}{2} & 0 & 0 \\ 0 & 0 & \frac{a_{33} - a_{11}}{2} & 0 \\ 0 & 0 & 0 & \frac{a_{44} - a_{11}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The eigenvalues of the jacobian matrix above are:

$$\lambda_1 = \frac{a_{55}-a_{11}}{2}, \lambda_2 = \frac{a_{22}-a_{11}}{2}, \lambda_3 = \frac{a_{33}-a_{11}}{2}, \lambda_4 = \frac{a_{44}-a_{11}}{2} \quad (6.2.1)$$

In accordance with Lyapunov theorem, the equilibrium is asymptotically stable if $\Re(\lambda_i) < 0$, for all eigenvalues λ_i of the jacobian matrix, and unstable if $\Re(\lambda_i) > 0$, for some eigenvalues. Thus, in order for the equilibrium to be stable, the conditions below must be valid:

$$\begin{cases} \alpha_{22} < a_{11} \\ \alpha_{33} < a_{11} \\ \alpha_{44} < a_{11} \\ \alpha_{55} < a_{11} \end{cases} \quad (6.2.2)$$

If the initial vector $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$ is close enough to the equilibrium point $(1,0,0,0,0)$ the evolution of strategies is shown in Figure 7.

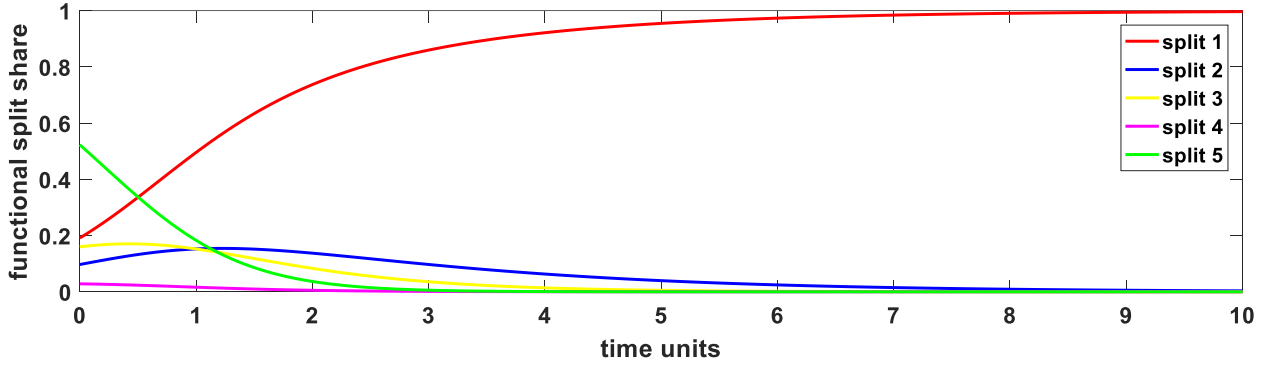


Figure 7: Evolution of strategies when a_{11} is higher than the other parameters.

2) Equilibrium point: $(0,1,0,0,0)$

The linear system near this equilibrium point is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{a_{11} - a_{22}}{2} & 0 & 0 & 0 \\ \frac{a_{55} - a_{11}}{2} & \frac{a_{55} - a_{22}}{2} & \frac{a_{55} - a_{33}}{2} & \frac{a_{55} - a_{44}}{2} \\ 0 & 0 & \frac{a_{33} - a_{22}}{2} & 0 \\ 0 & 0 & 0 & \frac{a_{44} - a_{22}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The eigenvalues are:

$$\lambda_1 = \frac{a_{11}-a_{22}}{2}, \lambda_2 = \frac{a_{55}-a_{22}}{2}, \lambda_3 = \frac{a_{33}-a_{22}}{2}, \lambda_4 = \frac{a_{44}-a_{22}}{2} \quad (6.2.3)$$

The equilibrium point is stable if:

$$\begin{cases} \alpha_{11} < a_{22} \\ \alpha_{33} < a_{22} \\ \alpha_{44} < a_{22} \\ \alpha_{55} < a_{22} \end{cases} \quad (6.2.4)$$

If the initial vector $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$ is close enough to the equilibrium point $(0,1,0,0,0)$ the evolution of strategies is shown in Figure 8.

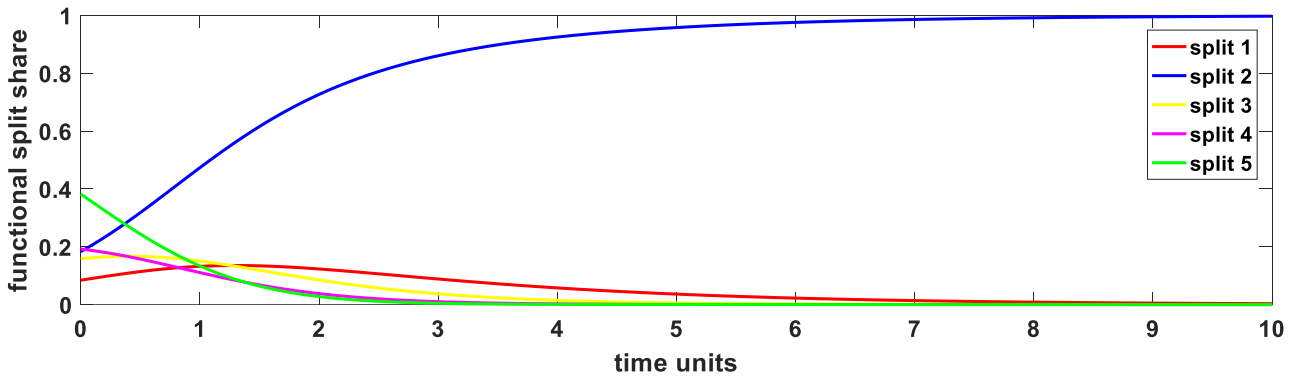


Figure 8: Evolution of strategies when a_{22} is higher than the other parameters.

3) Equilibrium point: $(0,0,1,0,0)$

The linear system near this equilibrium point is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{a_{11} - a_{33}}{2} & 0 & 0 & 0 \\ 0 & \frac{a_{22} - a_{33}}{2} & 0 & 0 \\ \frac{a_{55} - a_{11}}{2} & \frac{a_{55} - a_{22}}{2} & \frac{a_{55} - a_{33}}{2} & \frac{a_{55} - a_{44}}{2} \\ 0 & 0 & 0 & \frac{a_{44} - a_{33}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The eigenvalues are:

$$\lambda_1 = \frac{a_{11}-a_{33}}{2}, \lambda_2 = \frac{a_{22}-a_{33}}{2}, \lambda_3 = \frac{a_{55}-a_{33}}{2}, \lambda_4 = \frac{a_{44}-a_{33}}{2} \quad (6.2.5)$$

Therefore, this equilibrium point is stable if:

$$\begin{cases} \alpha_{11} < a_{33} \\ \alpha_{22} < a_{33} \\ \alpha_{44} < a_{33} \\ \alpha_{55} < a_{33} \end{cases} \quad (6.2.6)$$

If the initial vector $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$ is close enough to the equilibrium point $(0,0,1,0,0)$ the evolution of strategies is shown in Figure 9.

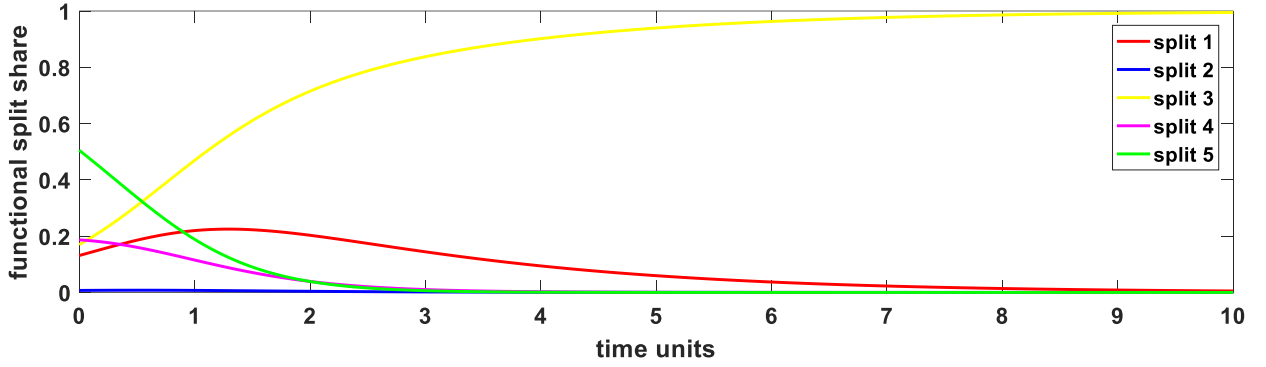


Figure 9: Evolution of strategies when a_{33} is higher than the other parameters.

4) Equilibrium point: $(0,0,0,1,0)$

The linear system near this equilibrium point is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{a_{11} - a_{44}}{2} & 0 & 0 & 0 \\ 0 & \frac{a_{22} - a_{44}}{2} & 0 & 0 \\ 0 & 0 & \frac{a_{33} - a_{44}}{2} & 0 \\ \frac{a_{55} - a_{11}}{2} & \frac{a_{55} - a_{22}}{2} & \frac{a_{55} - a_{33}}{2} & \frac{a_{55} - a_{44}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The eigenvalues of the jacobian matrix above are:

$$\lambda_1 = \frac{a_{11}-a_{44}}{2}, \lambda_2 = \frac{a_{22}-a_{44}}{2}, \lambda_3 = \frac{a_{33}-a_{44}}{2}, \lambda_4 = \frac{a_{55}-a_{44}}{2}, \quad (6.2.7)$$

The equilibrium point is stable if:

$$\begin{cases} \alpha_{11} < a_{44} \\ \alpha_{22} < a_{44} \\ \alpha_{33} < a_{44} \\ \alpha_{55} < a_{44} \end{cases} \quad (6.2.8)$$

If the initial vector $x = (x_1, x_2, x_3, x_4, x_5)$ is close enough to the equilibrium point $(0,0,0,1,0)$ the evolution of strategies is shown in Figure 10.

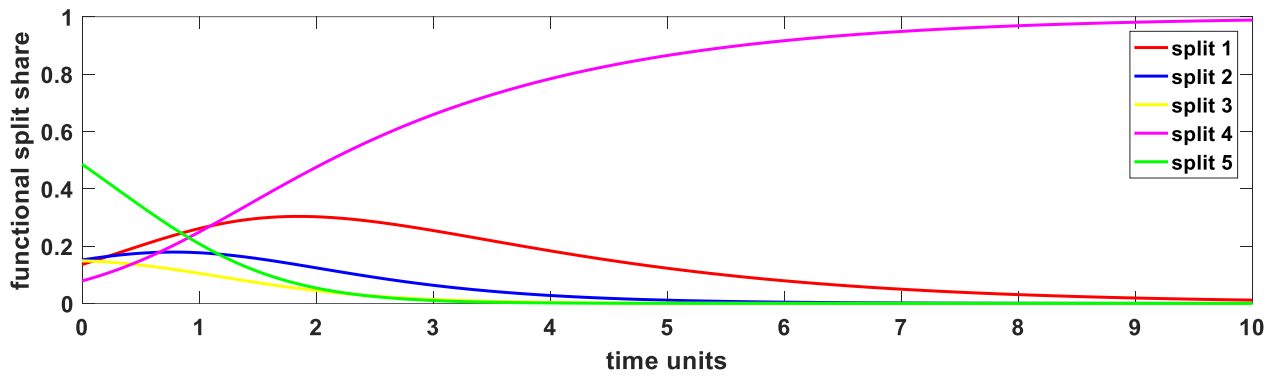


Figure 10: Evolution of strategies when a_{44} is higher than the other parameters.

5) Equilibrium point: $(0,0,0,0,1)$

The linear system near this equilibrium point is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{a_{11} - a_{55}}{2} & 0 & 0 & 0 \\ 0 & \frac{a_{22} - a_{55}}{2} & 0 & 0 \\ 0 & 0 & \frac{a_{33} - a_{55}}{2} & 0 \\ 0 & 0 & 0 & \frac{a_{44} - a_{55}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The eigenvalues are:

$$\lambda_1 = \frac{a_{11}-a_{55}}{2}, \lambda_2 = \frac{a_{22}-a_{55}}{2}, \lambda_3 = \frac{a_{33}-a_{55}}{2}, \lambda_4 = \frac{a_{44}-a_{55}}{2} \quad (6.2.9)$$

The equilibrium point is stable if:

$$\begin{cases} \alpha_{11} < a_{55} \\ \alpha_{22} < a_{55} \\ \alpha_{33} < a_{55} \\ \alpha_{44} < a_{55} \end{cases} \quad (6.2.10)$$

If the initial vector $x = (x_1, x_2, x_3, x_4, x_5)$ is close enough to the equilibrium point $(0,0,0,0,1)$ the evolution of strategies is shown in Figure 11:

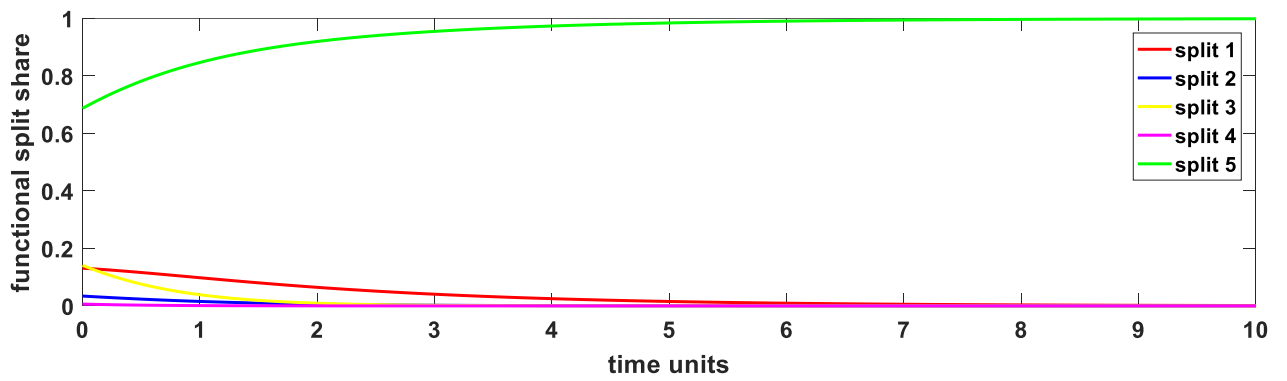


Figure 11: Evolution of strategies when a_{55} is higher than the other parameters.

Chapter 7 Numerical examples

In the previous chapter, we discussed a model for finding the optimal split, using the evolutionary game theory. The parameters a_{ii} that define the optimal split depend on the Base Station processing power along with the Network power. The Network power consumption depends on the required connection rate between the remote unit and the cloud platform, as well as the network technology. On the other hand, the Base Station processing power depends on the parameters that were described in Table 1. In this chapter, we examine these dependencies with the usage of numerical examples.

7.1 Effect of Network Technology on the Optimal Split

As it has been discussed in Chapter 4, different access network technologies translate into different relation between the access rate and the power consumption. Thus, parameters a_{ii} , which are related on power consumption of the network, depend on the access network technology in use. In order to see this dependence, let's assume the scenario of Table 8 with bandwidth $B = 20MHz$ for two types of access network technologies-PON and PtP.

Table 8: Exemplary system parameters for calculating the impact of bandwidth on the optimal split.

<i>Symbol</i>	<i>Value</i>	<i>Symbol</i>	<i>Value</i>
Ant	2	N_o	2
M	4	N_{sc}	1200
R	3/4	T_s	66.6 μ s
dt	100 %	N_Q	10
df	30 %	S	3 bit/cu
f_s	30.72 MHz	η	50 %

We will use two different models, in order to relate the access rate with the network power consumption. First, we will use a linear model of the form:

$$P_{Net} = \lambda \cdot R + \sigma \quad (7.1.1)$$

The parameter λ relates to the technical characteristics of the routers and the transport systems. The parameter σ depends on the technology of the access network and it is normalized to

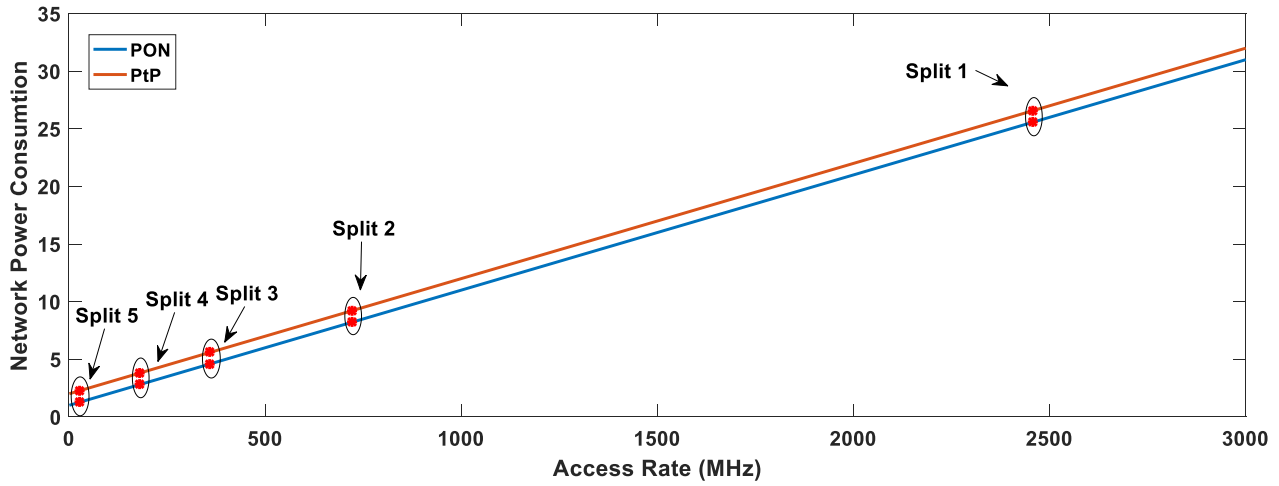
$$\sigma = \begin{cases} 1, & \text{for PON} \\ 2, & \text{for PtP} \end{cases} \quad (7.1.2)$$

Due to the recent improvements of router technologies and to the increment of data traffic, a more reliable model for power consumption is described by the non linear relation:

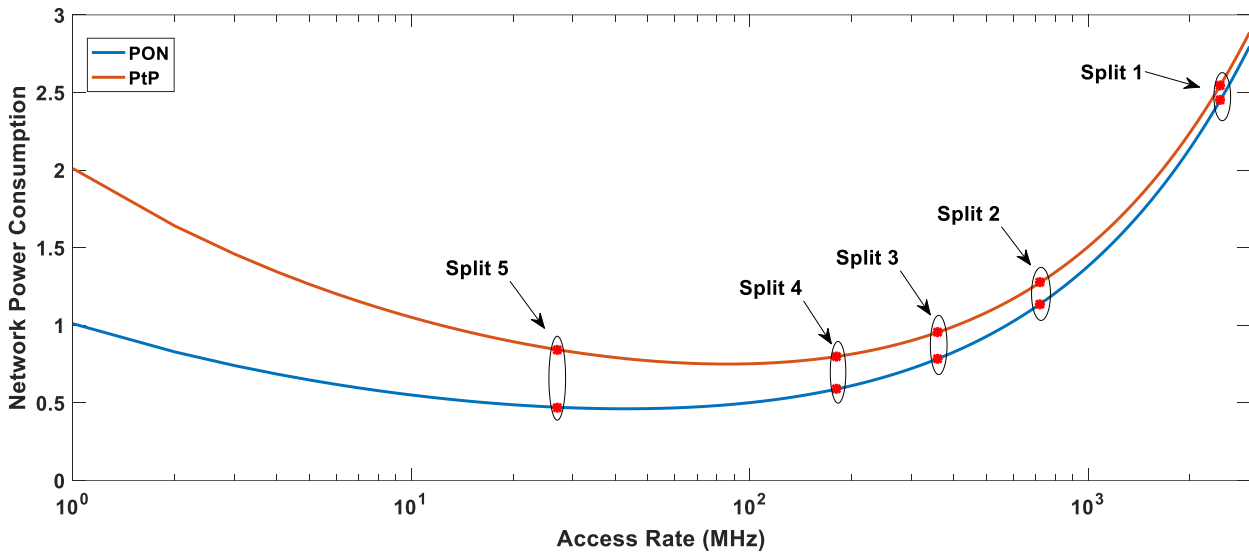
$$P_{Net} = (\lambda \cdot R + \sigma) \cdot (1 - a)^{\ln R / \ln b} \quad (7.1.3)$$

Where a is the router efficiency improvement per year and b is the per year traffic growth rate.

Figure 12 shows the scaling of the network power consumption with the access rate, using the two models. It is evident that, in the linear model the total power consumption is higher than that of the nonlinear model. According both models, for a certain access rate, PtP networks consume a larger amount of power. Furthermore, we notice that in the linear model, the difference between the power consumption of the two technologies remains the same as the access rate grows. On the contrary, in the nonlinear model, the variation of the power consumption between PtP and PON is growing with the increment of the access rate. Lastly, the optimal split in the case of PON is depicted in Figure 13. As we can see, the employment of the linear and the nonlinear model leads to different optimal splits.



(a)



(b)

Figure 12: Relation of network power consumption with access rate, based on (a) a linear model with $\lambda=0.01$ and (b) a non linear model with 10% router efficiency improvement per year and 42% per year traffic growth rate. Includes the access rates of the five functional splits, calculated for the scenario of Table 8.

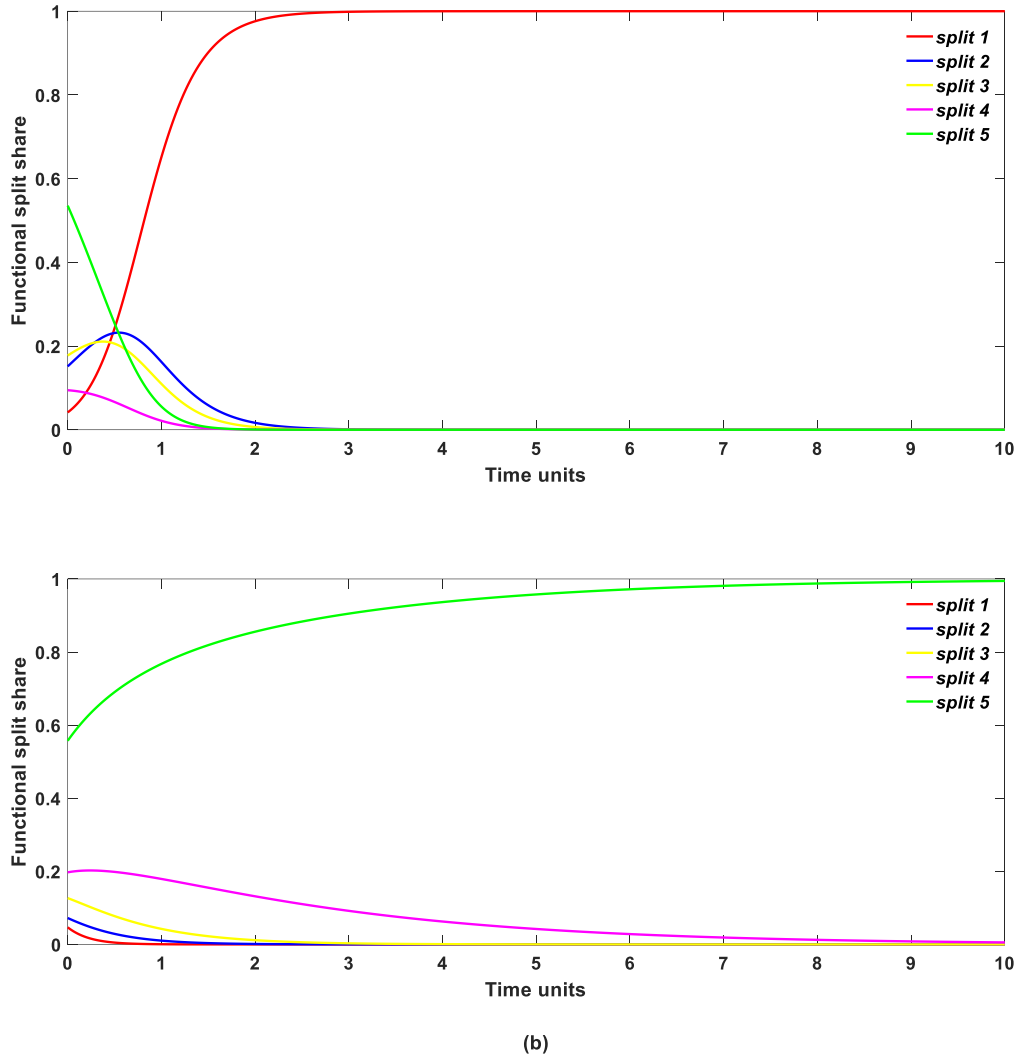
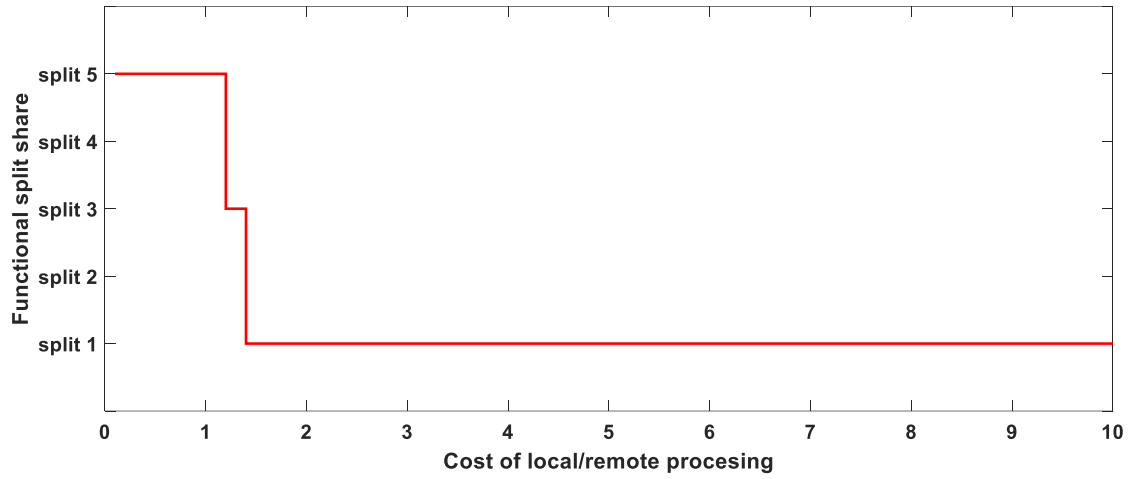


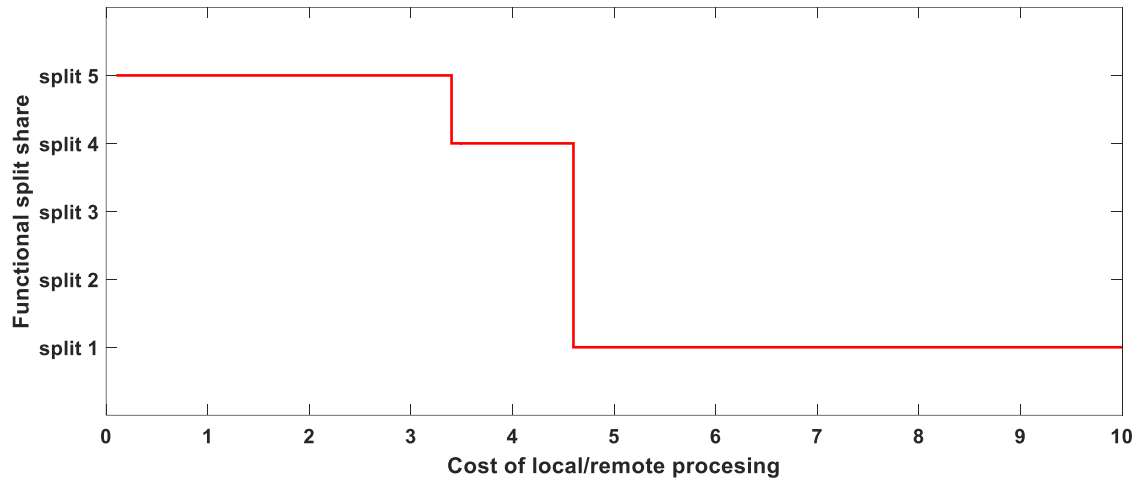
Figure 13: Optimal split of the scenario of Table 8 with $B = 20\text{MHz}$ for a PON technology access based on (a) the linear model with $\lambda=0.01$ and (b) the nonlinear model. For the calculations, the nonlinear model was employed with 10% router efficiency improvement per year and 42% per year traffic growth rate. A Macro base station was assumed. The ratio local/remote processing was assumed $\frac{a}{\beta} = \frac{2}{1}$.

7.2 Effect of Local Processing on Optimal Split

As it has been pointed out, the processing of the same function differs whether it is being executed locally or remotely. The relation of the power consumption between the antenna and the cloud is depicted in the ratio $\frac{a}{\beta}$ that was introduced in Table 7. It is obvious that for different ratio relations, the optimal split varies.



(a)



(b)

Figure 14: Optimal split with respect to the local/remote processing ratio $\frac{a}{\beta}$, for the scenario of Table 8 with $B = 20\text{MHz}$ and a PON technology access network, for (a) a Macro and (b) a Micro base station. For the calculations, the nonlinear model was employed with 10% router efficiency improvement per year and 42% per year traffic growth rate.

In order to see the dependence of the optimal split on the local/remote cost, we computed the optimal split for different scenarios of the ratio $\frac{a}{\beta}$, normalizing term β to 1 and investigating for different values of a . Figure 14 depicts the results for the case of Table 8 with $B = 20\text{MHz}$ and a PON technology access network. For the calculations, the nonlinear model was employed with 10% router efficiency improvement per year and 42% per year traffic

growth rate. Both cases of large and small base station were investigated. We can conclude that as the ratio gets higher, the optimal split converges to split 1. Furthermore, the small base stations converge to split 1 at higher values of the local/remote processing ratio, than in the case of the large base stations.

7.3 Effect of Bandwidth on Optimal Split

In order to see the effect of bandwidth on optimal split we assume the scenario that is described in Table 8 for the unlink. The bandwidth affects the connection rates, thus, the Network power consumption, as well as the Processing power consumption.

First, we compute the connection data rates, using the formulas of Chapter 3. We should point out that these formulas have been calculated for the case of $B = 20MHz$. Thus, in order to see the effect of the scaling of bandwidth on optimal split, it is crucial to multiply the formulas with a factor $B/20MHz$.

The next step is to compute the power consumption of the RF components of the Base Station.

$$P_{RF} = P_{LNA1} + P_{Mva} + P_{LNA2} + P_{Dm} + P_{VGA} + P_{CGb} + P_{ADC} \quad (7.3.1)$$

Where P_{LNA1} , P_{Mva} , P_{LNA2} , P_{Dm} , P_{VGA} , P_{CGb} and P_{ADC} are the RF sub-components, shown in Table 5. The values that are depicted in Table 5 are for the reference scenario $X = \{20MHz, 1, 6 (64QAM), 1, 100\%, 100\%\}$. For obtaining the values of the RF sub-components for the exemplary scenario, we use the formula:

$$P_{RF \text{ sub-comp}} = P_{RF \text{ sub-comp,ref}} \cdot \prod_{x \in X} \left(\frac{x_{act}}{x_{ref}} \right)^{s_{i,x}}, \quad X = \{BW, Ant, M, R, dt, df\} \quad (7.3.2)$$

The scaling exponent s_i for the RF components is 1 as far as the number of the antennas (Ant) and the time domain duty cycling (dt) is concerned. For Clock generation and buffering all the other exponents are zero. The remaining exponents for the other RF sub-components have the value 1 with respect the frequency domain duty cycling (df) and the bandwidth (BW), except in the case of large Base Stations (Macro and Micro), where the exponent that

reflect the scaling of power with bandwidth is zero. The scaling exponents for the type of Modulation (M) and the Coding rate (R) are zero. Thus, we obtain:

$$P_{RF,sub} = \begin{cases} P_{RF,sub,ref} \cdot \left(\frac{Ant_{act}}{Ant_{ref}}\right)^1 \cdot \left(\frac{dt_{act}}{dt_{ref}}\right)^1 \cdot \left(\frac{df_{act}}{df_{ref}}\right)^1, & \text{Large BS} \\ P_{RF,sub,ref} \cdot \left(\frac{BW_{act}}{BW_{ref}}\right)^1 \cdot \left(\frac{Ant_{act}}{Ant_{ref}}\right)^1 \cdot \left(\frac{dt_{act}}{dt_{ref}}\right)^1 \cdot \left(\frac{df_{act}}{df_{ref}}\right)^1, & \text{Smal BS} \end{cases} \quad (7.3.3)$$

Where $P_{RF,sub}$ are all the RF subcomponents except P_{Cgb} . The power consumption of Clock generation and buffering (P_{Cgb}) is calculated by the formula:

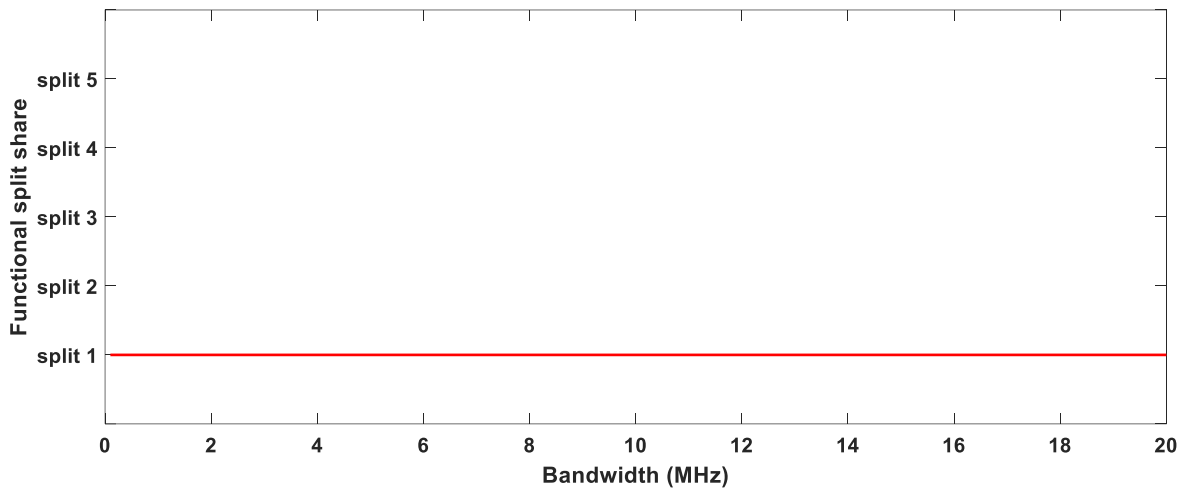
$$P_{Cgb} = P_{Cgb,ref} \cdot \left(\frac{Ant_{act}}{Ant_{ref}}\right)^1 \cdot \left(\frac{dt_{act}}{dt_{ref}}\right)^1, \quad \text{for all BS} \quad (7.3.4)$$

The calculation of the power consumption of the digital baseband processing is similar to the above. Hence for the digital baseband operations, the power consumption is given by the formula

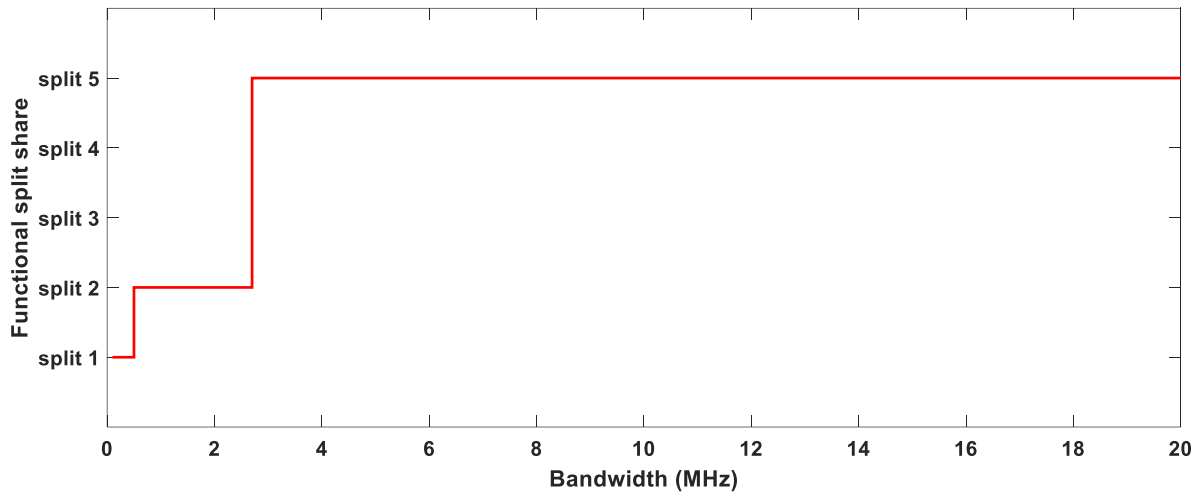
$$P_{BB,sub} = P_{BB,sub,ref} \cdot \left(\frac{BW_{act}}{BW_{ref}}\right)^{s_1} \cdot \left(\frac{M_{act}}{M_{ref}}\right)^{s_2} \cdot \left(\frac{R_{act}}{R_{ref}}\right)^{s_3} \cdot \left(\frac{Ant_{act}}{Ant_{ref}}\right)^{s_4} \cdot \left(\frac{dt_{act}}{dt_{ref}}\right)^{s_5} \cdot \left(\frac{df_{act}}{df_{ref}}\right)^{s_6} \quad (7.3.5)$$

The $P_{BB,sub,ref}$ for the uplink is given in Table 2. In order to convert the values of Table 2 from $GOPS$ to W , we divide the values with $40\ GOPS/W$ and $120\ GOPS/W$ for large and small base stations respectively. The scaling vector $[s_1, \dots, s_6]$ is given in Table 3.

Having the relations of the total power consumption, we calculate the parameters a_{ii} of the matrix A , using the exemplary scenario of Table 8. The formula that we will assume is the relation (6.1.4). The evolution of the game is depicted in Figure 15.



(a)

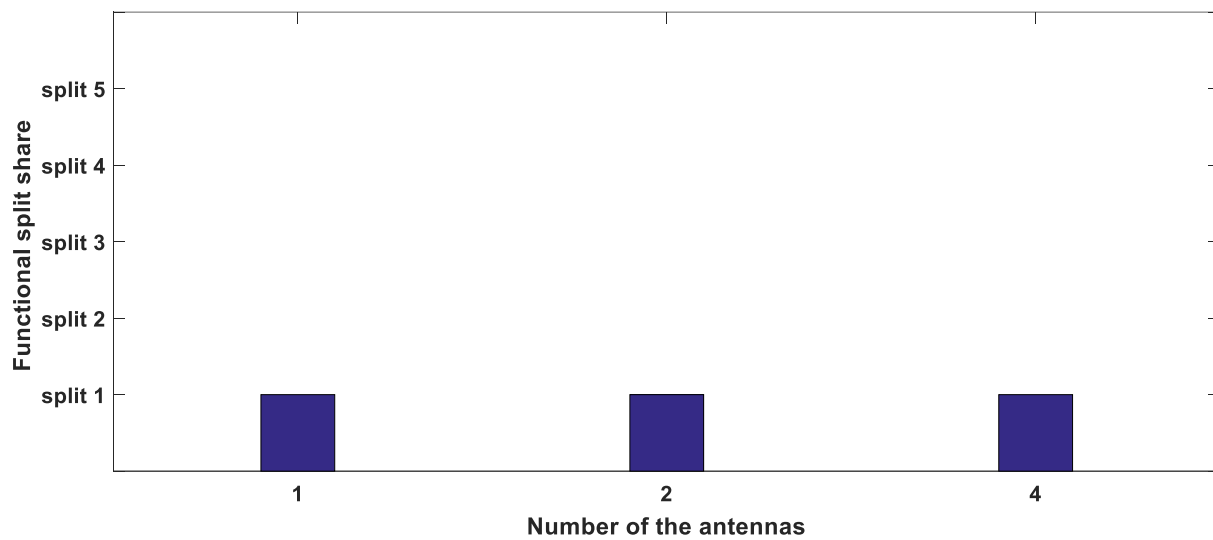


(b)

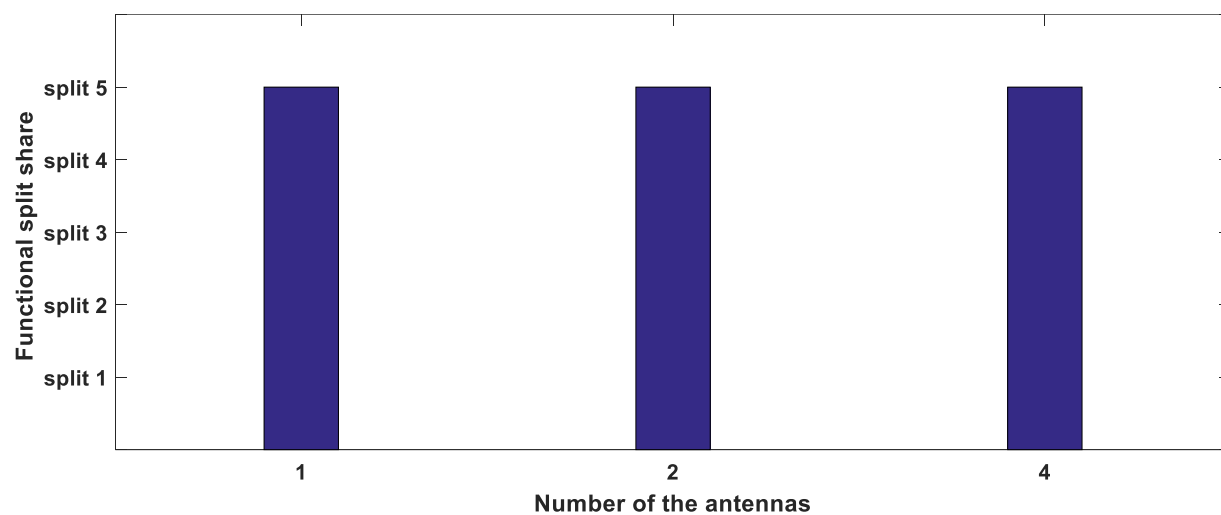
Figure 15: Scaling optimal splits with respect to bandwidth for the exemplary scenario of Table 8 for (a) a Macro Base Station and (b) a Pico Base Station using a non linear network power consumption with 10% router efficiency improvement per year and 42% per year traffic growth rate. The local/remote processing

$$\text{ratio } \frac{\alpha}{\beta} \text{ was assumed } \frac{2}{1}.$$

7.4 Effect of the Number of the Antennas on Optimal Split



(a)



(b)

Figure 16: Effect of the number of the antennas on the optimal split. Calculated for the scenario of Table 8 with $B = 20\text{MHz}$ and PON access network. The nonlinear model was employed, with 10% router efficiency improvement per year and 42% per year traffic growth rate. (a) Macro base station, (b) Pico base station.

The local/remote processing ratio $\frac{\alpha}{\beta}$ was assumed $\frac{2}{1}$.

The number of the antennas affects not only the processing power consumption, but also the required capacity for the data transportation, and thus, the network energy consumption. Hence, it is crucial to see the relation of the optimal split with the number of the antennas (Ant).

Once again, for the calculations we use the exemplary scenario of Table 8, only now we fix the bandwidth at $B = 20MHz$ and set the number of the antennas as the variable. We assume PON as the access network technology, and employ the nonlinear model for the calculations. In Figure 16, we can see change of the optimal split for the scenario of 1, 2, 4 received antennas. For the chosen scenario the number of the antennas doesn't affect the optimal split in both cases.

Chapter 8 Summary

In this master thesis, a model for optimizing the wireless networks in 5G was proposed. The overall objective of the model is twofold: The need to address the huge increase in data traffic combined with a significant reduction in energy consumption.

We initiate our analysis by an introduction focusing on the evolution of mobile wireless communication networks towards 5G. As we pointed out, in order to increase the efficiency of the spectral range, the densification of the network is necessary. The centralization of baseband processing offers useful solutions to the problems that arise due the densification. However, central processing demands high fronthaul data rates, that are supported only by optical fibers.

In order to reduce the required data rates, but also exploit the benefits of central processing, the option of introducing functional splits options has been proposed. In this architecture, the functions are performed either locally or centrally, or through a combination of local and central processing, depending on the circumstances. In our analysis, we used five different splits options and calculated the fronthaul capacity of each split option.

To optimize the energy consumption of new generation networks, one should first calculate the energy efficiency of today's mobile communications. Thus, a model for calculating the power consumption of the base station was analyzed. As it was described, the power consumption of base stations depends on system parameters, like bandwidth and number of received antennas. Then, the network power consumption was examined. We correlated the network power consumption and the access rate by using a linear model and a non linear that depends on the network technology as well as the technology improvement

Evolutionary game theory was employed in order to identify the optimal split option. We took into consideration both network and processing resources. The processing resources correspond to the sum of the required processing power on the antenna and the processing power on the cloud. For simplification purposes we assumed the same power model as in the

case of the macro base station, for the case of the cloud. Taking this into consideration, the differentiation between the required local and remote processing power was inserted by a ratio $\frac{\alpha}{\beta}$, where α depicts the cost of the local processing and β the cost of the processing on the cloud.

We used the replicator equation and extracted general rules for the prediction of the optimal split option. Finally, we used numerical examples to see the dependence of the optimal split on the network technology and on the cost of local/remote processing. The scaling of the optimal split option as a function of system parameters, such as bandwidth and number of the antennas, was investigated.

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