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ABSTRACT

Many decisions require not a single solution alternative, but a set or portfolio of alternatives. For example, a business generally produces not a single product, but a portfolio of products, and a project is tackled not by a single individual, but by a team of individuals. Unfortunately, the portfolio selection problem has been largely neglected in the literature. Though many specific problem environments have been discussed using heuristic, ad hoc methods, no general methodology or theory has been developed. This paper proposes an object-oriented framework to provide a theoretical foundation for future research on the portfolio selection problem.

INTRODUCTION

Most decisions involve multiple objectives. A decision alternative is chosen based on a multitude of often conflicting decision criteria. A solution is sought that provides the best compromise with respect to optimizing these various desired objectives. Multi-criteria decision making, in the last twenty years, has become an established field of research, with extensive theory, a wide choice of solution methods, and many available computer-based decision support packages (see for example Weistroffer and Narula, 1997; and Figueira, Greco, and Ehrgott, 2005). However, there are many decision settings that require a set of alternatives to be selected, rather than a single alternative. Examples include deciding on an investment portfolio, selecting projects for research and development, choosing colleges to apply to for graduating high school seniors, selecting members on a team, planning the meal composition in a student cafeteria, etc. Rather than selecting a best single investment option, a portfolio that balances security with expected payoff is desired. Budget constraints usually prevent pursuing all requested projects for research and development, and a limited size portfolio that balances requirements and resources must be selected. College applications should include schools that are most desirable and schools that are most likely to accept. Selecting members on a team requires a balance of players with various skills, as well as considerations of players' personalities to ensure proper teamwork among members. Planning cafeteria meals requires selecting multiple food items that complement each other in taste and nutrition, while providing variety and observing budget restraints. Thus in each of these situations, one cannot simply find the top *n* choices to constitute the portfolio or team, but rather the dependencies between the various selected individual alternatives must be taken into account. The utility attached to the alternatives is not additive, that is, the utility of the portfolio cannot, in general, be expressed as a sum of individual alternative utilities. The synergistic effects of the portfolio must be considered.

Literature on financial investment portfolio selection (e.g. Markowitz, 1952; Elton and Gruber, 1995; Ballestero and Romero, 1996), as well as on research and development (R&D) project selection (e.g., Golabi, 1985; Stewart, 1991; Henig and Katz, 1996) is abundant. However, most of these publications do not address generalized mappings of solution methodologies to portfolio problem types. Rather, most published papers discuss very specific decision situations and suggest very specific solution methods, usually based on heuristics, and cannot easily be generalized. In addition, many specific techniques that have been proposed in the literature are not widely used because they tend to be too complex, do not address all relevant issues associated with portfolio selection, require too much input data, or may be too difficult to understand for decision makers to use (Ghasemzadeh and Archer, 2000). According to Hess (1993), "management science has failed altogether to implement project selection models".

In this paper we look at theoretical concepts to construct a comprehensive, object-oriented model of the multicriteria portfolio selection problem. This model does not present a general solution algorithm for solving portfolio problems, but rather, the objective is to provide a framework for identifying portfolio problem types in real world decision environments, and for mapping possible solution techniques to these problems. Liu and Stewart (2004) write: "The two main factors resulting in low productivity are the nature of decision analysis as an art instead of a science, and the difficulties in reusing the past experiences and relevant knowledge when making a decision and developing a DSS." The proposed framework is intended to give structure to the area of multicriteria portfolio problems and their solutions in order to facilitate the reuse of past experiences and previously tried out methods. An object-oriented approach is taken because of the inherent reusability and extensibility of object orientation (see for example Graham, 1994). The model presented here is meant to furnish a basic groundwork. It is anticipated that in the future this model will be further developed and extended by adding and improving components for a dynamically evolving, wide-ranging framework.

PROBLEM DESCRIPTION

Decision Analysis Context

The general multi-criteria decision making (MCDM) problem (see for example Steuer, 1986) involves a set of feasible decisions or possible actions, and a set of corresponding solutions or outcomes. The complexity arises from the reality that the outcomes are measured with respect to multiple criteria or objectives, and usually none of the outcomes will be optimum with respect to all of the criteria. Thus compromise solutions that are acceptable to the decision makers are sought. Mathematically, this problem can be expressed as follows:

Maximize $\mathbf{F}(\mathbf{x})\colon D \rightarrow S$,

where $D \subset \Re^k$ and $S \subset \Re^p$, and $\Re$ represents the real numbers, $\Re^k$ and $\Re^p$ are vector spaces. $\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_p(\mathbf{x})\}$ represents the p objectives, $\mathbf{x} = \{x_1, x_2, \ldots, x_k\}$ represents the k decision variables, D is the decision domain, and S is the solution domain. Since in general, the p objectives conflict with each other and cannot be maximized simultaneously, a compromise solution is sought that maximizes the overall utility, i.e.

Maximize $U\colon \mathbf{F}(D) \rightarrow \Re$.

An example of a MCDM problem might be designing a new auto engine. The decision domain would include all possible combinations of, say, quality of material used (represented by the value of x_1), engine capacity (represented by the value of x_2), etc. The solution domain would include corresponding decision outcomes with respect to fuel efficiency, perhaps represented by $f_1(\mathbf{x})$, engine power, represented by $f_2(\mathbf{x})$, construction cost, represented by $f_3(\mathbf{x})$, durability, represented by $f_4(\mathbf{x})$, etc. Ideally, we would like an engine to be fuel efficient (i.e., minimize fuel consumption), be powerful (i.e., maximize horse power), be reliable (i.e., maximize durability), and cheap to build (i.e., minimize cost). However, maximizing horse power and durability would likely result in less than optimum fuel consumption and cost.

An important special case of the MCDM problem is the multi-attribute decision making (MADM) problem, where the decision domain is a discrete set of solution alternatives. An example of a MADM problem would be selecting a candidate from a pool of new faculty applicants. The alternatives would be the set of applicants, each of whom would be evaluated based on multiple attributes, such as which courses the person would be able to teach, potential scholarship output, etc. The likelihood that one candidate would be optimum with respect to all attributes is slim.

The MADM problem can be described as follows:

Maximize $U(a)\colon A \rightarrow \Re$,

where A is the set of all solution alternatives. Each possible alternative is assessed based on its associated values for the multiple decision attributes. In the example above, each $a \in A$ represents one of the faculty candidates, whose utility is assessed based on attributes such as teaching ability, scholarship output, etc.

The portfolio selection problem adds another dimension of complexity. Instead of seeking a single alternative $a \in A$ that maximizes the overall utility, a set of alternatives, i.e., a portfolio $P = \{a_1, a_2, \ldots, a_n\}$ is required, where $P \subset A$.

Thus the problem becomes:

Maximize U: $\{P \mid P \subset A\} \rightarrow \Re$,

i.e., find the subset P of A that provides maximum utility (usually subject to some constraints).

Figure 1: General Portfolio.

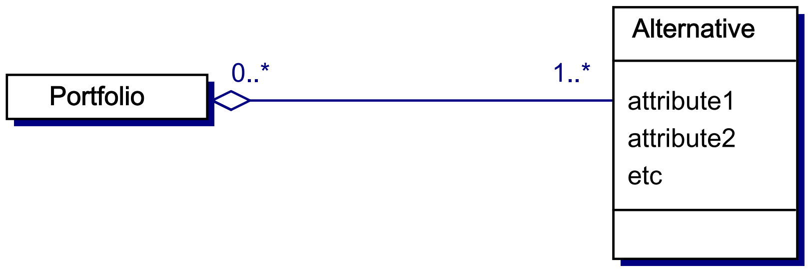


Figure 1 illustrates a general portfolio using unified modeling language (UML) notation. The rectangular boxes represent object classes, and the connecting line represents an *association*. The hollow diamond indicates that the association is an *aggregation* or "part of" relationship. Numbers or asterisks next to the association are *multiplicities*; they indicate how many objects in one class are related to how many objects in the other class. Thus a portfolio has one or more alternatives, and each alternative can be part of zero, one, or more portfolios. Object classes have attributes that can be shown in the middle compartment of the rectangle. The bottom compartment is to show object behaviors, if desired.

Portfolio Cardinality

One example of a portfolio selection problem may arise when a university department is hiring not one new faculty member, but rather needs to fill a fixed number (*n*) of positions. Here, the set of solution alternatives A represents the total applicant pool, and the portfolio P is a subset of *n* applicants. The intuitive approach is to rank all candidates and pick the *n* top ranked ones. However, this may not be a good strategy. For example, in order to cover all the various courses taught in the department, picking candidate X may very well make some other otherwise highly ranked candidate become less desirable, if that person's course specialties overlap with candidate X's specialties. The value of each candidate is dependent on the other selected candidates. It may not even be best to include the highest ranked candidate out of the candidate pool. It is possible that two medium rated candidates have a higher combined value than picking the top candidate together with his/her best complementary candidate. This eliminates sequentially selecting the top candidates as a general strategy.

Figure 2: Faculty Selection Problem.

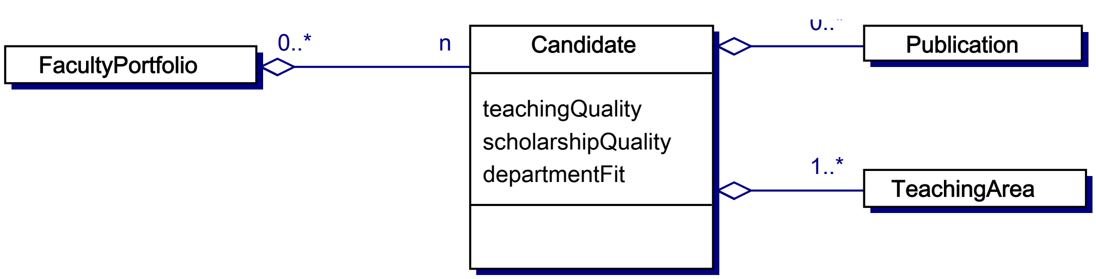
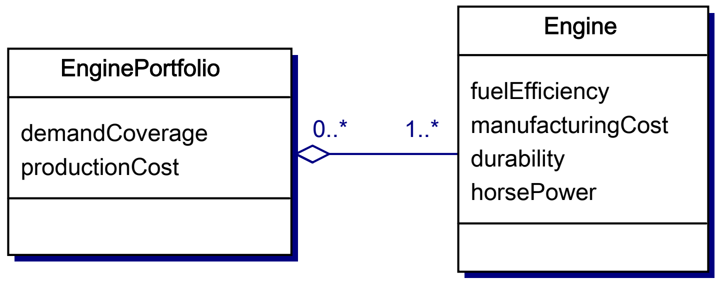


Figure 2 shows an example portfolio for the faculty selection problem. Each portfolio has *n* candidates, who are evaluated on teaching quality, scholarship quality, department fit. Also, each candidate has zero or more publications, and each candidate has one or more teaching areas (publication and teaching area are a "part of" each candidate).

Figure 3: Auto Engine Portfolio.



To complicate matters further, *n*, the number of alternatives to be included in the portfolio may be variable; in fact, *n* set equal to a constant constitutes an important special case. Revisiting the engine design problem, let us suppose the company decides to build several new engines to meet the different demands of its customer base. The problem now becomes one of finding a portfolio of different engines that are not only measured on the criteria mentioned earlier, but also on the extent to which customer needs or desires are fulfilled. This may be a case where *n* is variable. On the one hand, a large number of different engines will likely satisfy a greater contingent of customers. But on the other hand, having fewer products reduces production costs. Figure 3 shows the auto engine portfolio example. Unlike in the faculty selection problem, where the number of members of the portfolio is fixed (the university authorized hiring *n* new faculty members), in the engine portfolio problem the number of members in the portfolio is open, and influences the values of the portfolio attributes "demand coverage" and "production cost". Note that these are true portfolio attributes that are distinct from individual alternative attributes. Portfolio attributes can be related to similar attributes of individual alternatives in complex ways.

Alternative Types

The above two examples of portfolio selection problems show one possible classification of such problems: fixed number of members in the portfolio versus variable number. Other factors that distinguish different types of portfolio problems include continuous decision variables (as in MCDM) resulting in possibly infinitely large decision alternative sets, versus a finite set of decision alternatives (as in MADM), resulting in a finite (though possibly very large) set of potential portfolios. The faculty selection problem discussed above clearly falls into the latter category, as there will always be a finite set of applicants. However, the alternative set in the engine design problem, as described earlier, may be represented by continuous functions, resulting in infinitely many alternatives, and thus allows for infinitely many different portfolios.

Dependency Types

Further, the dependencies between alternatives may have different consequences. For example, when building a team (whether a sports team or perhaps an IS development team), it is not only the individual capabilities of team members that should be taken into account, but also the synergistic effects derived from people being able to work together, i.e., the total being greater than the sum of the parts. On the other hand, when constructing a portfolio of investments, a primary portfolio goal may be reduction of risk. In general, the dependencies among alternatives could be classified into three types, which we label as indicated below.

<u>Additive</u>. All dependencies can be modeled as constraints, such as mutually exclusive. The portfolio objective functions are additive with respect to the corresponding alternative attributes.

<u>Multiplicative</u>. One or more dependencies can be modeled by multiplicative interaction terms in the portfolio objective functions. This is one way of representing synergistic effects existing among alternatives. The multiplicative dependency can also handle portfolio objectives such as probability of success.

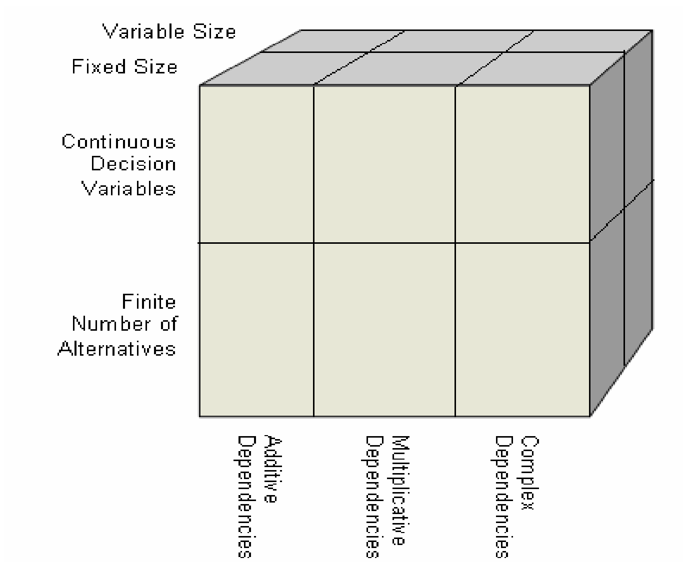
<u>Complex</u>. If a dependency exists that cannot be modeled as either additive or multiplicative, we call the dependency complex. Such dependencies can involve portfolio attributes that are not properties of individual alternatives. These

include ideas such as balance or variety in the portfolio. The portfolio objective functions may have mathematical expressions or exist on a subjective scale.

DECISION MODEL APPROACHES

The aspects described above can be represented by a three dimensional matrix resulting in twelve possible decision models, as depicted in Figure 4.

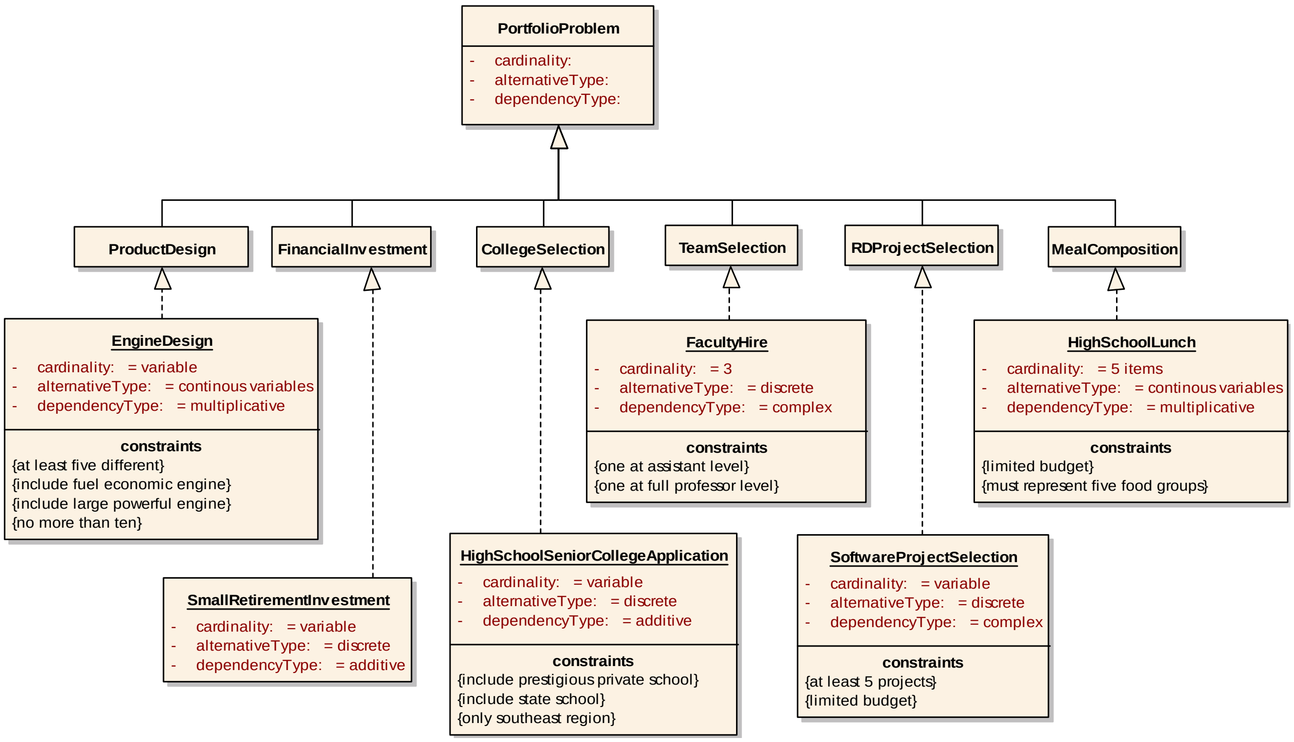
Figure 4: Types of Portfolio Decision Models.



Many special cases of the portfolio problem exist, often represented through constraints. For example, when hiring multiple new faculty members, constraints might be that one new hire must be at the assistant professor level and one at a senior level. Such constraints may limit the solution space of feasible portfolios considerably and thus make the selection easier. As mentioned earlier in this paper, many proposed solution techniques were designed for such special cases. Looking at all possible theoretical situations based on constraints is not really feasible; however, a classification with respect to such constraints might be useful if one can determine the most common types of real world decision situations. In other words, develop a number of representative model situations that cover many of the real world problems, such as the financial investment portfolio problem, the college selection problem, the team selection problem, the meal composition problem, the R&D project selection problem, etc. Many of these, though, will have multiple versions, complicating matters further.

Figure 5 illustrates these decision model classifications using UML notation with representative example types. It shows three attributes that characterize the general portfolio problem, cardinality (fixed size or variable size portfolio), alternatives type (continuous variables or discrete alternatives), and dependency type. The connecting lines with the triangular arrow head indicate *subclass*, or "is a kind of" relationships. Product design, financial investment, college selection, team selection, research and development project selection, and meal composition all are types of portfolio problems. Other kinds of portfolio problems (subclasses) may be added to this model. Going back to our earlier examples of a faculty hiring portfolio and an auto engine portfolio, these can be viewed as examples of team selection and product design, respectively. The broken connecting lines with the triangular arrow heads indicate that these are realizations of the respective subclasses.

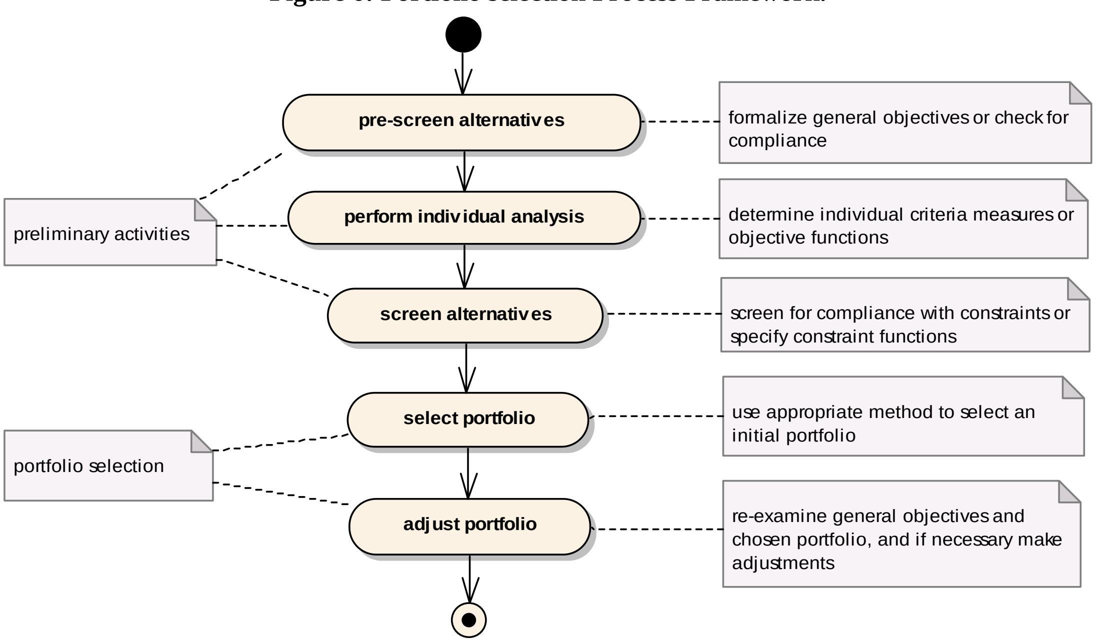
Figure 5: Characteristic Examples of Portfolio Problems.



PROCESS FRAMEWORK

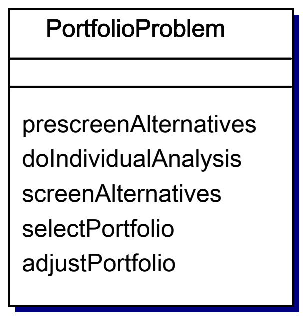
In the above we focused on modeling the static aspects of the decision problems. An alternate approach is to consider the portfolio selection problem as a process that includes specific steps or activities. The following general framework, shown as an UML activity diagram in Figure 6, is loosely based on one proposed by Ghasemzadeh and Archer (2000).

Figure 6: Portfolio selection Process Framework.



Thus, the process consists of preliminary screening activities as well as the actual portfolio selection. Preliminary activities start with pre-screening the alternatives for compliance with general decision objectives, or in the case of continuous variables, with formalizing these general decision objectives. Preliminary activities continue with determining the individual criteria measures, or in the case of continuous variables, with determining the objective functions. Finally, preliminary activities include checking for compliance with specific problem constraints, or in the case of continuous variables specifying constraint functions. The actual selection of the portfolio may include methods borrowed from multi-criteria decision making, and use these to find an initial portfolio, such as perhaps a portfolio consisting of the top *n* individual solutions. This portfolio would likely be sub-optimum for the reasons discussed earlier, thus requiring a further step of portfolio adjustment. This adjustment step may itself be an iterative and interactive procedure. The steps from Figure 6 can be incorporated into our UML model as behaviors of the portfolio problem class, i.e., things that are done in a portfolio problem. This is shown in Figure 7.

Figure 7: Portfolio Problem Behaviors.

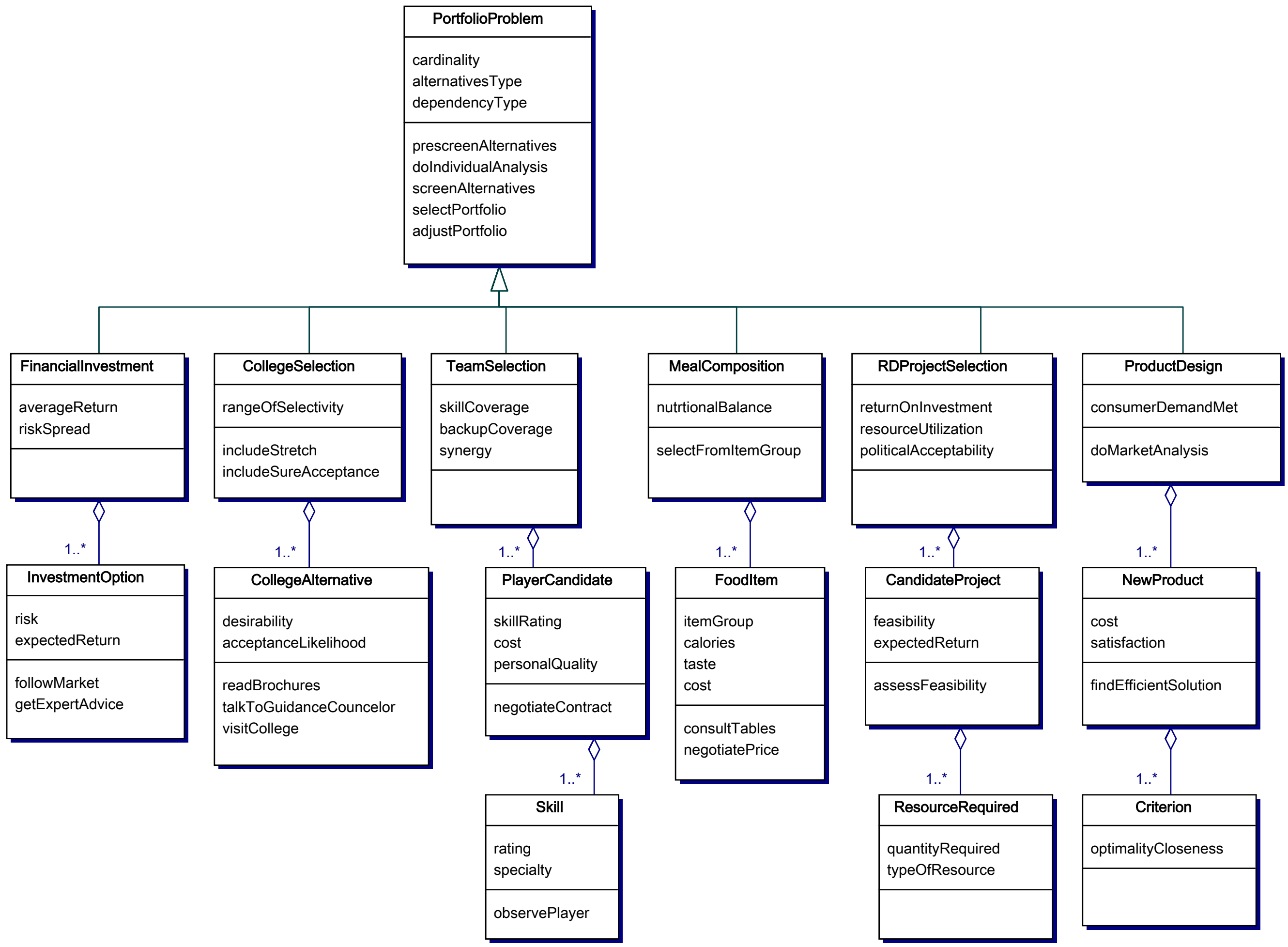


ADVANTAGE OF THE OBJECT-ORIENTED APPROACH

Liu and Stewart (2004) write: "Object orientation permits a system to be described in the concepts of the real world. Partitioning a problem domain into objects corresponding to a concept-oriented view of the real world counterparts is often more natural than a functional decomposition." Further, an advantage of the object-oriented approach in modeling the portfolio selection problem is that it allows us to develop each component (object class) separately in more detail, as well as add more components to the model, without disturbing the overall framework. Figure 8 shows what we have done above as a single diagram, with some possible additional aggregation classes, and with possible attributes and behaviors added to some of the classes.

Each of the classes in Figure 8 can be analyzed and enhanced individually. Specific solution methods for portfolio problem types can be included as behaviors of those subclasses, and their underlying algorithms can be described individually using separate models, such as activity diagrams, which is part of UML notation. If more than one method has been suggested for a specific problem type, this type can be further subdivided, or the various solution techniques can be included in another aggregation class of that subclass, as shown in Figure 9.

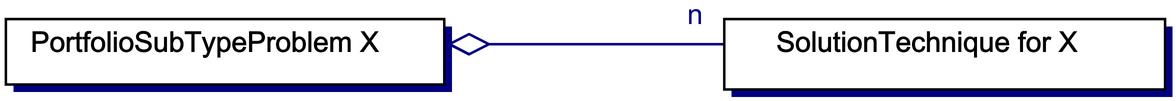
Figure 8: Partial Detailed Portfolio Problem Model.



IMPLICATIONS AND CONCLUSION

We have proposed a framework for the portfolio selection problem, using object-oriented notation. This framework does not restrict or limit the formulation and handling of specific portfolio problem types or applications, but it provides some basic structure towards the understanding of the problem area, and by classifying portfolio problems according to exemplifying characteristics, it may aid in the identification of appropriate solution approaches. The authors are not aware of any similar frameworks in the current literature.

Figure 9: Multiple Solution Techniques.



Further development of the framework is possible and desirable. The proposed sub-types of portfolio problems need to be examined as to how well they represent the most important problem types encountered in the real world.

Other subtypes may need to be added. Further, each subtype needs to be examined as to appropriate characteristics (attributes and aggregation classes) and already existing solution methods, as proposed in the literature.

The object-oriented approach has the advantage that it allows adding or modifying individual components (object classes) while keeping within the already developed framework. As the framework achieves a sufficient level of completeness (the whole idea of the object-oriented approach is that it will allow the framework to continue to grow and evolve), it should help direct the attention of future researchers to those areas in portfolio selection that are most in need of further development with respect to effective solution techniques and user friendly decision support systems. Further, it should allow practitioners that need to solve portfolio problems to identify the specific type of problem they are dealing with, and direct them to already published solution techniques that work for their type of problem.

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