

# The Development of Mathematical Model Consciousness in Junior Secondary Students: A Lesson Study of the Instruction of Congruent Triangles

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**Abstract:** *The Compulsory Education Course Standards for Mathematics 2022 have highlighted the educational objectives of junior secondary mathematics by emphasizing the development of mathematical competence and practical learning. Model consciousness, as one of the fundamental mathematical competencies to be developed at the junior secondary level, can facilitate students' comprehension of the universal application of mathematics. Teachers of mathematics in junior high school should construct effective classroom activities based on the cognitive qualities of their students in order to enhance their mathematical model consciousness and comprehension of the substance of mathematics knowledge. This paper is a lesson study of the education of Congruent Triangles, and its purpose is to investigate strategies for fostering mathematical model consciousness among junior high school pupils.*

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## **Background and Objectives**

**A**CCORDING to the *Compulsory Education Course Standards for Mathematics 2022* (henceforth referred to as the New Course Standards), the junior secondary level core mathematical competencies include abstract thinking, computing skills, geometric intuition, spatial thinking, reasoning ability, data consciousness, model consciousness, application awareness, and innovation disposition (Ministry of Education, 2022). Computing abilities, geometric intuition, spatial thinking, and data awareness serve as the basis for training abstract thought and reasoning ability, which are required for the development of model consciousness. The model consciousness of students contributes to their comprehension of the universality of mathematical application, hence enhancing their application consciousness and innovative disposition. Therefore, model awareness becomes an essential component of students' mathematical competence.

The New Course Standards define mathematical model consciousness as students' understanding of applying mathematical models to solve issues and connecting math knowledge to actualities through mathematical modelling. Junior high school students should develop an understanding of the fundamental process of mathematical modelling, the ability to abstract mathematical subjects from real-world or specific situations, and the ability to use mathematical symbols to establish equations, inequalities, functions, etc. to represent quantitative relationships and the reasoning process in mathematical inquiry. Mathematical modelling is also valuable for theme-based multidisciplinary education (Ministry of Education, 2022).

Shi (2016) argued that the purpose of mathematics education is to equip students with the skills necessary to view the real world from a mathematical viewpoint, investigate it using mathematical methods, and communicate it using mathematical terms. A mathematical model can also be referred to as mathematical language. We discover in our teaching practice that due to their incapacity to build mathematical models during the learning process, a sizable portion of students struggle when applying their knowledge of mathematics to evaluate and solve particular situations. Therefore, developing students' awareness of mathematical models is essential for the general growth of their fundamental mathematical skills.

This lesson study on the instruction of "Congruent Triangles" has the following goals: (i) to arouse teachers' interest in developing student mathematical model consciousness and to investigate how to promote it through effective design of classroom activities; (ii) to improve students' learning processes to maximize their engagement and to search for strategies to incorporate assessment of student mathematical modelling thought into classroom evaluation.

## **Teaching Design**

### ***Learning Contents***

Geometric measures, geometric characteristics, and geometric relations are the three parts of the study of geometry at the compulsory education level (which includes primary and junior secondary schools). The study of geometric relationships includes the section on congruent triangles. The unit on triangles comes before this part. A study of the relationship between two triangles takes the place of the study of a single triangle at this point. The most fundamental and prevalent congruent shapes in the physical universe are congruent triangles. Students have previously studied the ideas of a line segment, an angle, a parallel line, and a triangle. This lesson introduces the new topic and serves as the first lesson in the unit on the congruence of triangles. In order to prepare them for further study of this topic, students learn the definition and characteristics of congruent triangles through concrete examples in this lesson. They also create the mathematical model of congruent triangles.

### ***Learning Situations***

- *Students' Prior Relevant Experiences:* Students in the eighth grade have a basic concept of plane and space geometry as well as a beginning comprehension of the characteristics of polygons and circles. They have also mastered straight lines, rays, line segments, angles, and parallel lines. Students are capable of mathematical thinking such as analogies and have the fundamental skills of geometric analysis and proof.
- *Knowledge to Be Learned:* Congruent shapes and triangles.
- *Projected Learning Difficulty:* How to establish triangle congruence by rational and deductive reasoning.
- *Individual Disparities:* Student variations in symbol consciousness, observation and induction skills, and other skills.

### ***Teaching Focuses***

- *Key Points:* The concepts of congruent triangles and congruent shapes; reasoning and calculation based on congruent triangle properties
- *Teaching Challenges:* The study of corresponding relations in congruent triangles; the development of a mathematical model for congruent triangles.
- *Instruction Strategies:* Students' capacity to recognize geometric figures will be improved. Students will also develop a mathematical model consciousness in the process of analyzing and solving problems. Finally,

students will be made aware of the reciprocal translation between figures and symbols.

## **Lesson Implementation**

### ***The First Experiment in Class A: Focusing on the Holistic Mastery of Knowledge***

#### *The Objectives of the Lesson Design*

To help students build the mathematical model of congruent triangles, it is important to give them the tools to abstract the idea of congruent triangles from everyday life, recognize corresponding relationships in congruent triangles, represent their properties with symbols, and calculate their sides and angles using geometric analysis.

#### *Classroom Processes*

##### *i. The Introductory Situation*

- Students were instructed to study the following four pairs of images and record their characteristics.



Students can recognize that the two pictures in each pair correspond.

- This technique aimed to prompt students' consideration on geometric congruence by using familiar imagery from daily life.

##### *ii. Interpretation of Learning Objectives*

- To comprehend the concept of congruent triangles and their corresponding relationships through the observation of actual examples; to investigate the properties of congruent triangles through analogies with general congruent figures and use the properties to calculate the degree of angles and the length of line segments; to hypothesize the methods for determining the congruence of two triangles with the aid of teaching aids.

- The objective of this procedure is to assist students in comprehending the inquiry procedure and fundamental concepts of this course.

### iii. *Conceptual Comprehension*

- The interaction between the teacher and students:

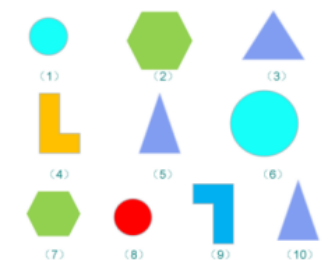
*The teacher: "How should geometric congruency be defined in mathematics?"*

*Students: "Identical in size and shape."*

*The teacher: "Give some examples from your personal surroundings."*

*Students: "Classroom windows, doors, etc."*

Instant assessment: Identify congruent figures among the following shapes,

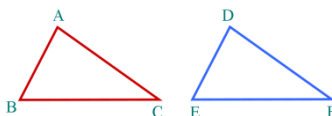


- The purpose of this method is to allow students to abstract geometric congruence from physical examples and recognize the application of mathematics to real-life situations; and to assess students' understanding of the notion of geometric congruence.

### iv. *The Inquiry of the Properties of Congruent Triangles*

- Step One

The teacher stacked two sheets of different colored cardboard and cut two congruent triangles from them.



*The teacher: After a sequence of movements of the two models, ask students to discuss the relationships between the two figures.*

*Students: "They are congruent because they can coincide."*

*The teacher: demonstrate the symbol “ $\cong$ ” that denotes geometric congruence (“ $\simeq$ ” means the same shape, “ $=$ ” means the same size).*

- Step Two

*The teacher: Make the two models coincide to direct students to explore the traits of sides and angles in congruent triangles and their correspondent relations.*

*Students: Identify the vertices, sides, and angles that correspond in congruent triangles by aligning the two models.*

*The teacher: Instruct students to consider the properties of congruent triangles.*

*Students: After group discussion, identify the relationships between the two sets of sides and angles and use symbols to express their attributes.*

- The purpose of this process is to help students obtain the concept of congruent triangles through the analogy of general congruent shapes and to perceive the mathematical research process of transitioning from generality to particularity; to guide students in discovering the corresponding relationships in congruent triangles through overlapping, observation, and other activities; and to express the properties of congruent triangles in symbolic language.

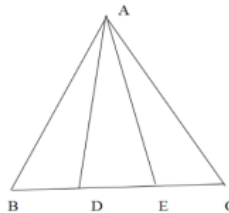
v. *The Application of the Properties of congruent triangles*

- The explanation of an example question:

*As shown in the figure,  $\triangle ABE \cong \triangle ACD$  is known,*

*(1). If  $BE = 6$ ,  $DE = 2$ , find the length of  $BC$ .*

*(2). If  $\angle BAC = 75^\circ$ ,  $\angle BAD = 30^\circ$ , calculate the degree of  $\angle DAE$ .*



- The goal of this procedure is to explain how to apply the properties of congruent triangles to problem solving using the example question,

based on students' prior understanding of the sum and difference of line segments, and to emphasize the usual steps for addressing the question.

### vi. *Knowledge Transfer*

- Speculating on the criteria for determining if two triangles are congruent.

*The teacher: "What to consider while determining triangle congruence?"*

*Students: "Equal sides and angles."*

*The teacher: "What are the necessary conditions?"*

*Students: Make personal guesses freely.*

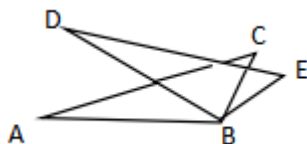
- The goal of this procedure is to provide students the chance to visualize the material covered across the entire unit and make connections between concepts.

### vii. *In-class Assessment*

- The exercise.

*As shown in the figure,  $\triangle ABC$  and  $\triangle DBE$  are congruent triangles, so there are ( ) pairs of equal angles in the figure.*

*A.1, B. 2, C. 3, D. 4*



- The goal of this procedure is to assess the lesson's effectiveness by asking a question that is appropriate for students' average cognitive abilities.

## *An Evaluation of the Instruction Results*

- i. Using concrete examples, students were able to grasp the idea of geometric congruence, although the initial situation did not spark their interest in acquiring additional information.
- ii. With the aid of teaching aids, students could use characteristics of generic congruent figures to comprehend the idea of congruent triangles and explain the corresponding relationships between congruent



triangles. The idea of a mathematical model of congruent triangles has not been formed, yet pupils were not proficient enough in using the properties of congruent triangles to solve problems.

- iii. The knowledge transfer approach helped students understand the subject as a whole, but it has to be enhanced to better raise students' awareness of mathematical models.

## ***The Second Experiment in Class B: Constructing a New Framework by Analogy Based on Students' Prior Knowledge***

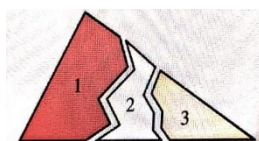
### *The Objectives of the Lesson Design*

To modify classroom activities in response to problems encountered in the first experiment; to plan a more intriguing introduction to pique students' curiosity; to associate the corresponding relations between general congruent figures with those between the elements (vertexes, sides, and angles) in congruent triangles; to hypothesize the process of studying geometric figures, including congruent triangles, in analogy with that of studying parallel lines and triangles; and to construct the mathematical model of congruent triangles.

### *Classroom Processes*

#### *(i) The Introductory Situation*

- Student X shatters a triangular piece of glass into three pieces, and he now needs to go to the glass shop to obtain an equivalent piece.



- The objective of this procedure is to prompt students to consider the question, “What does ‘an identical piece of glass’ mean?” and “How can it be expressed mathematically?”

#### *(ii) Interpretation of Learning Objectives*

- To abstract congruent shapes from physical examples and identify the corresponding relationships between them; to apply knowledge about general congruent shapes to the study of the relationships between the basic elements of congruent triangles; to use prior knowl-

edge of parallel line analysis to project the process of analyzing the congruence of triangles.

- The objective of this procedure is to assist students in comprehending the inquiry procedure and fundamental concepts of this course.

### (iii) *Conceptual Comprehension*

- To abstract the mathematical concept of geometric congruence, students needed to see the following images in pairs and identify those that could coincide.

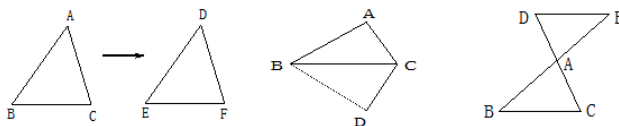


Students perceived the qualities of geometric congruence by comparing the size and shape of two congruent figures; to expand their grasp of the idea, they were asked to provide more instances of congruent shapes in everyday life.

- The objective of this procedure is to enable students to abstract the mathematical model of geometric congruence from real examples and get an initial understanding of the features of congruent figures.

### (iv) *The Inquiry of the Properties of Congruent Triangles*

- The teacher cut multiple pairs of congruent triangles from two pieces of different colored cardboard and performed the following actions: *translating  $\triangle ABC$  along line  $BC$  to obtain  $\triangle DEF$ ; flipping  $\triangle ABC$  along  $BC$  to obtain  $\triangle DBC$ ; rotating  $\triangle ABC$  by  $180^\circ$  around point  $A$  to obtain  $\triangle AED$ .*



With reference to the definition of congruent figures and the characteristics of the figures before and after the movements, students compared the description of congruent triangles with their own words; using symbols to represent the congruence of triangles and summarizing the properties of congruent triangles, they identified the corre-

sponding equal sides and angles between triangles after translation, flipping, and rotation; they then applied the properties of congruent triangles to analyze relations of sides and angles in more complex figures.

- The objective of this procedure is to help students understand the concept of congruent triangles by analogy with general congruent shapes; to understand that the resulting figures and the original figures are congruent after movements such as translation, rotation, and flipping; to understand the relationships between the corresponding vertexes, sides, and angles in congruent triangles with the aid of the aforementioned movements; and to represent the properties of congruent triangles.

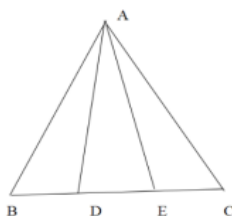
### (v) *The Application of the Properties of Congruent Triangles*

- An explanation of an example question:

*As shown in the figure,  $\triangle ABE \cong \triangle ACD$  is known,*

*(1). If  $BE=6$ ,  $DE=2$ , find the length of  $BC$ .*

*(2). If  $\angle BAC=75^\circ$ ,  $\angle BAD=30^\circ$ , calculate the degree of  $\angle DAE$ .*



- The objective of this procedure is to explain how to apply the properties of congruent triangles to problem solving using the example question, based on students' past understanding of the sum and difference of line segments, and to emphasize the usual problem-solving steps.

### (vi) *Knowledge Transfer*

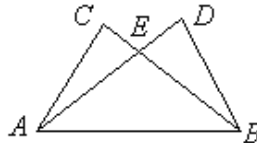
- Examine the process of learning parallel lines and compare it to the process of studying geometric shapes such as congruent triangles.
- The objective of this procedure is to teach students about the interconnectedness and integrity of mathematics, as well as to understand the process of geometrical study - from definition to property determination, deduction, and application - and to use the mathematical model to analyze and solve problems.

(vii) *In-class Assessment*

- The basic exercise:

As shown in the figure,  $\triangle ABC \cong \triangle BAD$ ,  $AC$  and  $BD$  are corresponding sides,  $AC=8\text{cm}$ ,  $AD=10\text{cm}$ ,  $DE=CE=2\text{cm}$ , then the length of  $BE$  is ( ).

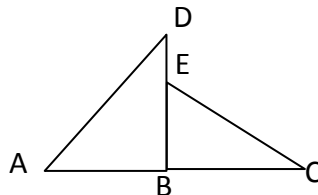
- A. 8cm      B. 10cm      C. 2cm      D. Undeterminable



- The exercise with a higher-level of difficulty:

As shown in the figure,  $\triangle ABD \cong \triangle EBC$ ,  $AB=3\text{cm}$ ,  $BC=4.5\text{cm}$

- (1). Find the length of  $DE$ .
- (2). Determine the relation between  $AC$  and  $BD$  and explain the reason.



- The goal of this procedure is to stratify the in-class assessment according to students' academic levels.

*An Evaluation of Instruction Results*

- In this experiment, students were exposed to the topic through a real-world incident, which was more effective in sparking their interest in addressing problems with mathematical answers.
- Common figure movements such as translation, rotation, and inversion were incorporated into the study of the definition and properties of congruent triangles to aid students in establishing the mathematical model of congruent triangles and learning to determine the corresponding relations in more complex configurations via figure movements. However, this strategy was ineffective for kids with inadequate spatial vision skills. Therefore, it needs additional development.

- In the context of knowledge transfer, the similarity between the relationships between two straight lines and those between two figures was advantageous to students' perceptions of mathematics' interconnection and integrity, but had no influence on their awareness of mathematical models. It can be used as a supplement during the summary phase.

### ***The Third Experiment in Class C: Using Revised Learning Activities to Foster Mathematical Model Consciousness in Students***

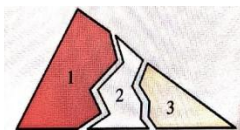
#### *The Objectives of the Lesson Design*

Revisions are made to classroom activities in response to obstacles encountered in the first two experiments. To achieve the goal of developing their mathematical model consciousness, students are required to make their own models of congruent triangles to better perceive the corresponding relations between them; to find common configurations of congruent triangles through movements of models; to understand the properties of congruent triangles by establishing mathematical models using physical models; and to apply the properties of congruent triangles to the calculus.

#### *Classroom Processes*

##### *i. The Introductory Situation*

- Student X shattered a triangular piece of glass into three pieces; he must now visit a glass shop to obtain a replacement.



- The objective of this procedure is to prompt students to consider the question, “What does ‘an identical piece of glass’ mean?” and “How can it be expressed mathematically?”

##### *ii. Interpretation of Learning Objectives*

- To be able to take the idea of geometric congruence from physical objects, relate the definitions and features of congruent figures and triangles, and figure out if two figures are congruent; to use self-made models to find the matching sides and angles in congruent tri-

angles and to learn how to use the matching relationships to find the unknown length of a side or degree of an angle; to figure out the sides and angles of congruent triangles by moving self-made models in ways like translation, rotation, and flipping, and to use the properties of congruent triangles to figure out the sides and angles.

- The goal of this process is to help students understand the inquiry procedure and what the main points of this lesson are.

### *iii. The Perception of Physical Examples*

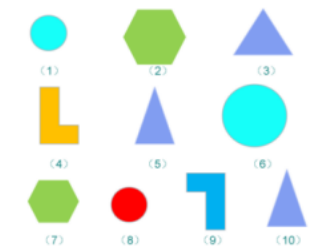
- Raising questions: What do the four pairs of images have in common? Can the images in each pair coincide if they are overlapped properly? What are the shape and size relationships between the two images in each pair?



How can the third set of images be made to coincide? The instructor utilized the software Easinote5 to illustrate that the two images can coincide after being rotated to convince pupils that figures can coincide through a variety of motions.

Students were invited to provide further instances of coincident plane geometric forms. In addition, students needed to construct models of two congruent triangles on their own, based on their knowledge of generic congruent figures, and to explain how they did so in order to strengthen their understanding of the idea.

Identify congruent forms among the figures shown below and summarize the criteria for determining congruent figures.

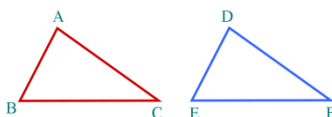


- The goal of this procedure is to enable students to abstract congruent geometric shapes from real-world objects and to recognize their ap-

plication in the actual world; to strengthen students' grasp of the congruence of figures, compare the fourth pair of images, which consists of two images of the same shape but different size, with the others; to construct models of congruent triangles to assess their knowledge of the idea and for the use in the succeeding classroom activities; to self-assess their ability to distinguish between congruent and non-congruent figures.

#### iv. *The Inquiry of the Properties of Congruent Triangles*

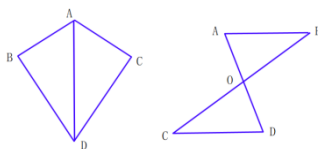
- Raising the question: What relationships exist between the sides and angles of the two triangles that are congruent?



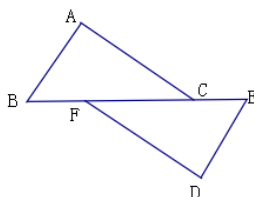
Students aligned the models in their hands and identified the vertices, sides, and angles in the two triangles that corresponded.

What are the relationships between the sides and angles of congruent triangles, as defined by the definition? Use mathematical symbols to represent congruent triangle qualities.

Students were instructed to place the two congruent triangle models in their hands according to the graph below and determine the relevant sides and angles.



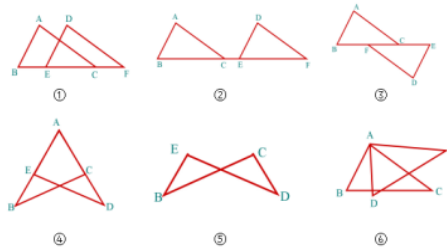
- Self-assessment: in the following figure,  $\triangle ABC \cong \triangle DEF$ , write the equal sides and angles of the two triangles,



- Further consideration: identify the equal line segments and angles in this picture and explain why; how many moves are required to align the two triangles?
- The objective of this procedure is to help students understand the idea of congruent triangles by comparing them to other shapes that are also congruent; to find the matching relationships in congruent triangles, describe their properties, and show them with mathematical symbols; to learn that two figures can be the same by moving in the same way, using models they made themselves; to use physical models to figure out the mathematical model of congruent triangles; to improve the ability to think by answering questions that get progressively harder.

### v. *The Construction of Mathematical Model of Congruent Triangles*

- First, put the two models of congruent triangles next to each other. Then, use translation, rotation, and flipping to make new shapes. Try to draw the patterns that have changed.



Observe the configurations of group members' models and practice identifying corresponding sides and angles.

Display all student-created patterns and identify their relevant relationships without the use of physical models.

- The objective of this procedure is to increase students' understanding of mathematical models by having them arrange physical models and hone their geometric intuition through the manipulation of figures.

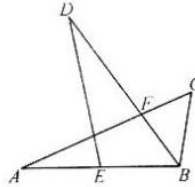
### vi. *The Application of the Properties of Congruent Triangles*

- An explanation of the example question:



As shown in the figure, it is known that  $\triangle ABC \cong \triangle DEB$ ; point  $E$  is on  $AB$ ;  $AC$  and  $BD$  intersect at point  $F$ ;  $AB = 6$ ,  $BC = 3$ ,  $\angle C = 55^\circ$ ,  $\angle D = 25^\circ$ .

- (1). Find the length of  $AE$ .
- (2). Calculate the degree of  $\angle AED$ .



- This approach is intended to inform students of the standard symbols for reasoning, facilitate their transition from physical model-based analysis to spatial representation without actual models, and cultivate an awareness of mathematical models.

### vii. *The Lesson Summary*

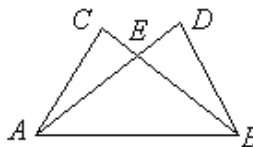
- Analyze the procedure for learning parallel lines and that for studying geometric forms, especially congruent triangles.
- The objective of this procedure is to make an analogy between the relationship of two straight lines and that of two figures to perceive the interconnectedness and integrity of mathematics; to comprehend the path of geometrical figure study – a process from definition to property determination, deduction, and application, incorporating the mathematical model into problem analysis and solution.

### viii. *Homework*

- The basic exercise:

As shown in the figure,  $\triangle ABC \cong \triangle BAD$ ,  $AC$  and  $BD$  are corresponding sides,  $AC = 8\text{cm}$ ,  $AD = 10\text{cm}$ ,  $DE = CE = 2\text{cm}$ , then the length of  $BE$  is ( ).

- A.  $8\text{cm}$       B.  $10\text{cm}$       C.  $2\text{cm}$       D. *Undeterminable*

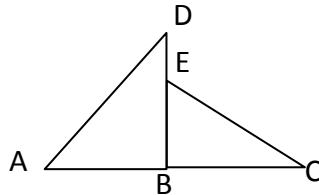


- The exercise with a higher level of difficulty:

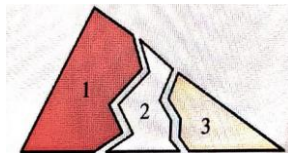
As shown in the figure,  $\triangle ABD \cong \triangle EBC$ ,  $AB=3\text{cm}$ ,  $BC=4.5\text{cm}$

(1). Find the length of  $DE$ .

(2). Determine the relation between  $AC$  and  $BD$  and explain the reason.



- The exercise for mathematical competence training:  
Student X shattered a triangle of glass into three pieces. He must now go to the glass shop to obtain an identical one. Which of the three bits should he keep? Try it with cardboard and scissors, and then give him some suggestions. Give your reasoning.



- The objective of this procedure is to differentiate homework assignments based on students' academic levels and to foster the development of their mathematical skills.

### *An Evaluation of the Instruction Results*

- i. The design of an introductory circumstance can effectively stimulate students' initiative in learning and encourage them to combine math studies with real-world experience. It was also reiterated in the final homework assignment.
- ii. The formation of mathematical model consciousness was explicitly stressed in the classroom processes of "Inquiry of the Properties of Congruent Triangles" and "Construction of Mathematical Model of Congruent Triangles". To meet the cognitive qualities of pupils, progressive modes of inquiry have been devised, and questions with more openness have been prepared.
- iii. Students were extremely engaged in model creation, model organization, and spatial visualization. Their past knowledge of figure move-

- ments (translation, rotation, and flipping) aided in the analysis of typical configurations of congruent triangles.
- iv. The findings of the self-assessment revealed that the majority of students had increased their awareness of mathematical models, allowing them to precisely identify the corresponding linkages between congruent triangles and use their properties to solve problems.
  - v. All classroom activities were concentrated on the students. Students could freely share their thoughts and confidently present the results of group research as significant actors in the session.

## ***A Comparison of the Three Experiments***

**Figures 1 and 2** show students' engagement in class and how many students used mathematical models in each of the three experiments. They show that optimizing learning activities can make a big difference in how much students participate in class and how many of them can use the mathematical model of congruent triangles on their own in class.

## **Teachers' Reflections**

The design of classroom activities is essential to the development of mathematical model consciousness among students. The growth of students' understanding of mathematical models is contingent upon their involvement in learning activities on a voluntary basis. Consideration must be given to students' past knowledge, cognitive qualities, and life experiences while planning classroom activities. Students are more willing to comprehend, discover, and investigate mathematical subjects when their dynamic thinking is fully activated. Second, a well-designed learning exercise can inspire students' inquisitiveness. Exploratory classroom activities can dramatically increase students' mathematical model awareness. In the third experiment, students engaged in a comprehensive investigation of the process of constructing the mathematical model of congruent triangles through model creation, model layout, and group cooperation, among other activities. Students became the discoverers and explorers of knowledge through this process. Thirdly, a learning task followed by self-reflection promotes model consciousness among students. The summary following each exercise, the exchange of results, and group conversations all help to the formation of knowledge linkages.

## **Findings of the Lesson Study**

### ***The Mathematical Modelling Process***

Mathematical modelling ability is the capability to abstract mathematical questions from real-world circumstances, to portray them using mathematic-

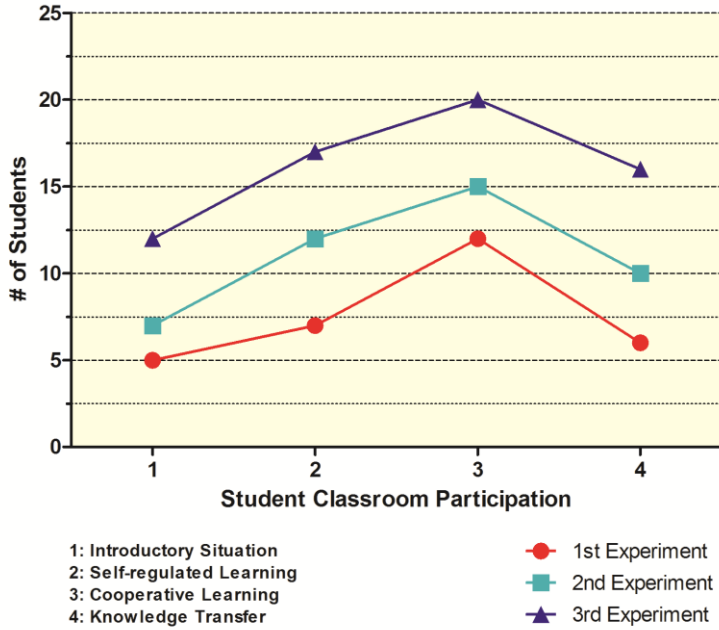


Figure 1. Student Classroom Participation.

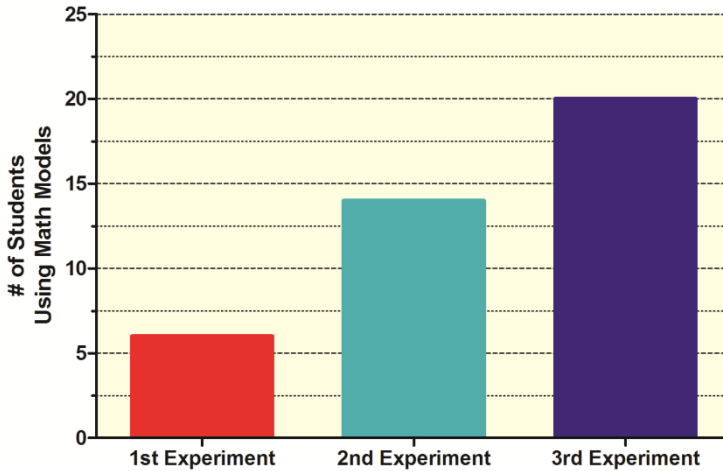


Figure 2. Numbers of Students Using Mathematical Models.

cal language, and to discover solutions through the building of mathematical models. Generally speaking, a mathematical modelling procedure consists of the following steps: discovering the problem, posing questions, evaluating the problem, developing the model, locating and confirming the solution, optimizing the model, and resolving the problem. Mathematical model consciousness, which is the mental disposition of solving real-world issues through the application of mathematical knowledge, has long been a driving factor in the evolution of mathematics.

## ***Strategies for Cultivating Mathematical Model Consciousness***

### *Comprehend Mathematical Models in Specific Situations*

The setting of an appropriate scenario is helpful to achieving educational goals. Even though mathematical knowledge is derived from real-world experience, real-world events must often be correctly understood prior to classroom application. The instructor should pay close attention to the creation of situations that correspond with textbook content, students' learning conditions, and teaching approaches. They do not need to be complex, but they must facilitate students' comprehension of mathematical models.

### *Emphasize the Application of Mathematical Models in Classroom Activities*

Mathematical models bridge the gap between mathematics and reality. Mathematical modelling training should not only teach students important concepts and principles, but also increase their understanding of how to apply applicable mathematical models to problem solving. Exploratory learning activities are extremely beneficial in promoting student mathematical competencies, particularly mathematical model consciousness (Yang, 2022).

### *Improve Mathematical Modelling Ability through Reflections*

Using mind-mapping, the teacher can encourage students to review the process of mathematical modelling and reinforce their model consciousness through associations, reflections, and knowledge transfer during the final step of class summary. As a result, students are able to develop effective learning strategies and gain expertise in mathematical modelling, so laying a solid foundation for the subsequent study of more advanced math concepts (Li, 2022).

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