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ABSTRACT

In recent years, managerial applications of artificial intelligence, especially in the area of financial services, has received considerable attention. In this paper, a fuzzy logic expert system is developed for approvals of small business loans. Previous studies have used non-fuzzy expert systems. But the fuzzification of the variables used in a business loan approval decision-making promises more efficient results. Furthermore, another distinct feature of this paper is its focus on small business loans. CubiCalc fuzzy expert system shell is used to develop the expert system. Knowledge acquisition is made using the resources and expertise of a Small Business Development Center. Tests to establish generalizability and validity of the results are conducted.

INTRODUCTION

In the domain of commercial lending, it is a loan officer's job to evaluate a company's financial position. As part of this evaluation, loan officers must gain a precise comprehension of the company's credit, collateral, capital, capacity, and character. The primary objective of a loan officer is to gain an in-depth understanding of the degree of risk a loan would entail. A subset of the commercial lending is the processing of the small business loans as part of the U. S. Small Business Administration's Low Documentation Loan Program. This Program focuses on the strength of character and credit of the applicants, and loans up to \$100,000 may be granted. Bank/Small Business Administration (SBA) reviews the application for repayment ability, management ability, owner investment, credit history, and business eligibility. If all factors are satisfactory, the loan is approved. A business qualifies as small based on the average number of employees for the preceding 12 months or on sales volume averaged over the last three fiscal years. There are more than 18 million small businesses in the United States and theses businesses are responsible for approximately half the U. S. gross domestic product (Ryans, 1995). Therefore, it is no surprise that small business and commercial mortgages were identified as two of the fastest-growing loan segments in a 1996 survey of 395 banks and thrifts by KPMG Peat Marwick (Milligan, 1996).

Historically, the complex, ill-structured nature of this loan processing problem prevented the financial institutions from applying any rigorous mathematical or analytical methods; usually banks and other lending agencies make loan decisions by utilizing experienced lending officers to perform the requisite tasks and evaluations. However, with the rapid exploitation of the expert system technology in managerial decision making, many banks and financial institutions have made investments in implementing rule-based systems to help automate the commercial loan processing.

In their evaluation process, loan officers have to manipulate vaguely defined and ambiguous linguistic variables like "above average," "below," "somewhat strong," and "very low." For example, a seasoned loan officer would evaluate a company's credit position as "strong" if efficiency and profitability positions are strong and liquidity position is normal (Levy et al., 1991).

Expert systems (ESs) have become important tools in managerial decision-making. An expert system (ES) is a computer program which summarizes knowledge of human experts and is used for decision-making and/or problem-solving. ESs are a branch of applied artificial intelligence (AI). Financial services sector is one area where expert systems, fuzzy logic and neural networks have found highly beneficial applications. This is due to the complexity and repetitiveness of decisions in the finance area. Business loan evaluation is one domain in the financial decision-making where artificial intelligence is employed extensively. An expert system that evaluates loan applications reduces the time and improves the quality of the evaluation.

In this paper, we first present a literature review of artificial intelligence applications in the financial decision making. This review of the literature is not exhaustive. The following section explains briefly the fuzzy logic. After that a detailed discussion of the system development follows where we include the nature of small business loans and explain the rules and membership functions of the fuzzy expert system developed. Following the development, we discuss the validation issues for expert systems and focus on the validation of the system developed. Finally, we make conclusions and give directions for future research.

ARTIFICIAL INTELLIGENCE APPLICATIONS IN THE FINANCIAL DECISION MAKING

Financial services sector is one area where expert systems have found highly beneficial applications. This is due to the complexity and repetitiveness of decisions in the finance area. A lot of financial institutions are investing in the development of expert systems for daily operations (Yiu & Kong, 1992).

The domain of finance is extremely broad and diverse. The system characteristics and requirements vary for different application areas. However, there are some common attributes in these applications such as the dynamic problem environments, availability of numerous models to approach the financial problems, and the need to create very large knowledge bases. Yiu and Kong (1992) compared rules, frames, and semantic networks as knowledge representation alternatives and recommended that the rule-based approach is the most appropriate for expert systems in financial decision-making. The authors also concluded that the need for constant updating and maintenance of the knowledge bases and creating powerful graphical user interface are the most important characteristics and requirements of ES applications in finance area.

Turban (1988), identified the following successful applications in the financial services sector: credit authorization, advice on buying shares, capital investment decisions and foreign exchange advise. In addition, Chorafas and Steinmann (1991) and Doherty and Pond (1993), explored the wide range of expert system applications within banking, including credit analysis, risk analysis, securities analysis, financial planning, insurance claims processing, identification of stolen credit cards and auditing. Chase Manhattan Bank conducted a pilot project on the use of expert systems in approvals of personal loans, tax loans, and credit cards in 1991. The system was successful and is currently operating on a mainframe environment (Yiu & Kong, 1992).

Investment management is a very difficult decision-making area of finance. Siriopoulus, Perantonis, and Karakoulus (1994) describe an expert system for technical analysis (ESTA). They list the benefits provided to users by artificial intelligence models. They also suggest that a combination of neural networks and expert systems can provide additional benefits.

Another problem domain in the finance area where expert systems are used is personal financial planning. Phillips, Brown, and Nielson (1990) introduced two types of financial expert systems: (1) integrated systems and (2) specialized systems that focus on a smaller area. Personal planning expert systems that are currently available are PlanPower from Applied Expert Systems, PFPS from Chase Lincoln First Bank, and Personal Financial Analysis from Price Waterhouse (Brown, Nielson, Phillips, 1990). An expert system that evaluates loan applications reduces the time and improves the quality of the evaluation. MARBLE (managing and recommending business loan evaluation) is a knowledge-based decision support system designed to help credit analysts and loan review committees in decision-making on loan applications (Shaw & Gentry, 1988).

FUZZY LOGIC OVERVIEW

This section presents an overview of fuzzy set theory and gives a review of the literature. Most of the time decisions are made in an environment where facts and rules contain various shades of vagueness, imprecision, and errors. Since uncertainty is prevailing in the most of application domains, it is essential that expert systems should be equipped with the uncertainty-handling mechanisms to improve their performance. Fuzzy logic and fuzzy rule-based systems are a means of dealing with imprecision and a method of modeling human behavior. In a fuzzy expert system, linguistic variables are used in place of, or in addition to, numerical variables and simple relations between these variables are defined by conditional statements with fuzzy logic operators.

Fuzzy Set Theory is a mathematical tool for describing impreciseness, vagueness and uncertainty. The term "fuzzy" refers to the situation in which there are no well-defined boundaries for the set of activities or observations to which the descriptions apply. Zadeh (1965) proposed Fuzzy Set Theory. A fuzzy set is a class of objects with a continuum of membership grades. A membership function, which assigns to each object a grade of membership, is associated with each fuzzy set. Usually, the membership grades are in the interval [0,1]. When the grade of membership for an object in a set is one, this object absolutely belongs to that set; when the grade of membership is zero, the object absolutely does not belong to that set. Borderline cases are assigned numbers between zero and one. Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth, truth values between completely true and completely false. Fuzzification is a methodology to generalize any specific theory from a crisp (discrete) to a continuous (fuzzy) form.

A fuzzy subset F of a set S can be defined as a set of ordered pairs, each with the first element from S, and the second element from the interval [0,1], with exactly one ordered pair present for each element of S. This defines a mapping between elements of the set S and values in the interval [0,1]. The value zero is used to represent complete non-membership, the value one is used to represent complete membership, and values in between are used to represent intermediate degrees of membership.

A fuzzy expert system is an expert system that uses a collection of fuzzy membership functions and rules, instead of Boolean logic, to reason about data. The general inference process proceeds in four steps:

1. Under *fuzzification,* the membership functions defined on the input variables are applied to their actual values, to determine the degree of truth for each rule premise.
2. Under *inference,* the truth value for the premise of each rule is computed, and applied to the conclusion part of each rule. This results in one fuzzy subset to be assigned to each output variable for each rule.
3. Under *composition,* all of the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable.
4. Finally is the *defuzzification,* which is used to convert the fuzzy output set to a crisp number.

This section introduced fuzzy logic and its importance. A review of past studies was presented, showing that fuzzy expert systems have effectively been applied to a variety of domains in financial decision making. The next section will discuss the design and development of a fuzzy expert system for approval of small business loans.

FESLAP DEVELOPMENT METHODOLOGY

This section provides a description of the design and development of the fuzzy expert system for small business loan application processing (FESLAP). First, the nature of the small business loans is discussed. Second, the architecture of the system is presented. Finally, design and development of different components of the system architecture is detailed.

Small Business Loans

Successful small business expansions and new formations lead the way in creating new markets, innovations and jobs that fuel economic growth and prosperity. One key to successful business start-up and expansion is the ability to obtain and secure appropriate financing. However, as many budding entrepreneurs quickly discover, raising capital can be a complex and frustrating process.

A loan officer's primary concern when reviewing a loan request is whether or not the loan will be repaid. To help answer this question, many loan officers will order a copy of the applicant's business credit report from a business credit reporting agency. Using the credit report, and the information the applicant provides, the lending officer will consider the following issues:

- Has the applicant invested savings or personal equity in his/her business totaling at least 25% - 50% of the loan he/she is requesting? A lender or investor will not finance 100% of the applicant's business.
- Does the applicant have a sound record of credit worthiness as indicated by his/her credit report, work history and letters of recommendations?
- Does the applicant have sufficient experience and training to operate a successful business?
- Has the applicant prepared a loan proposal and business plan which demonstrates his/her understanding of the business and his/her commitment to the success of the business?
- Does the business have sufficient cash flow to make the monthly payments on the loan request?

The U. S. Small Business Administration also has a Low Documentation Loan Program (LowDoc) for business loans up to $100,000. The Low Documentation Loan Program focuses on the strength of character and credit of the applicants. No predetermined percentage of equity is required. A lack of adequate collateral is not a determining factor. Primary considerations are:

1. Willingness to pay debts, as indicated by credit history. Co-signers may be considered if applicants have no credit history.
2. Historical or expected earnings evidencing repayment ability.
3. The requested financing provides the business a good chance of achieving success.

Design Considerations

Previous research (De at al., 1993; Date, 1990) have identified the drawbacks associated with traditional, uniprocessor problem solving such as reduced levels of systems modularity, adaptability, responsiveness and availability, accompanied by a concurrent increase in processing bottlenecks and conflict levels. In order to contain such limitation, the distributed technique recommends the use of multiple processors, each of which works on some facet of the solution process. Therefore, we utilized a distributed approach in developing FESLAP. In the context of expert problem solving, the distributed system is composed of multiple, interconnected expert systems. Each knowledge-base in the system possesses knowledge that is potentially useful in the partial resolution of a complex decision problem faced by the entire system and, consequently, could be made responsible, for some portion of the total problem-processing effort. Since the complete domain of small business loan approval decision making is complex (i.e., 11 decision variables), we decided to divide the knowledge-base into smaller components and to use a distributed approach in the system architecture.

The system was developed on an Intel Pentium Computer with 64 MB RAM. CubiCale® Version 2 environment is used for the development of different expert system modules and fuzzy processing. CubiCale® is designed as a true information modeling environment (Cox & Schwarts, 1993). This is because it supports ideas like hedges, the fuzzy operators, arithmetic, logical and trigonometric functions in rules, and explicit rule weights. The ability to weight rules means that the designer can specify added weights to certain rules or set the weight low or to zero when attempting to perform rule-set optimization.

System Architecture

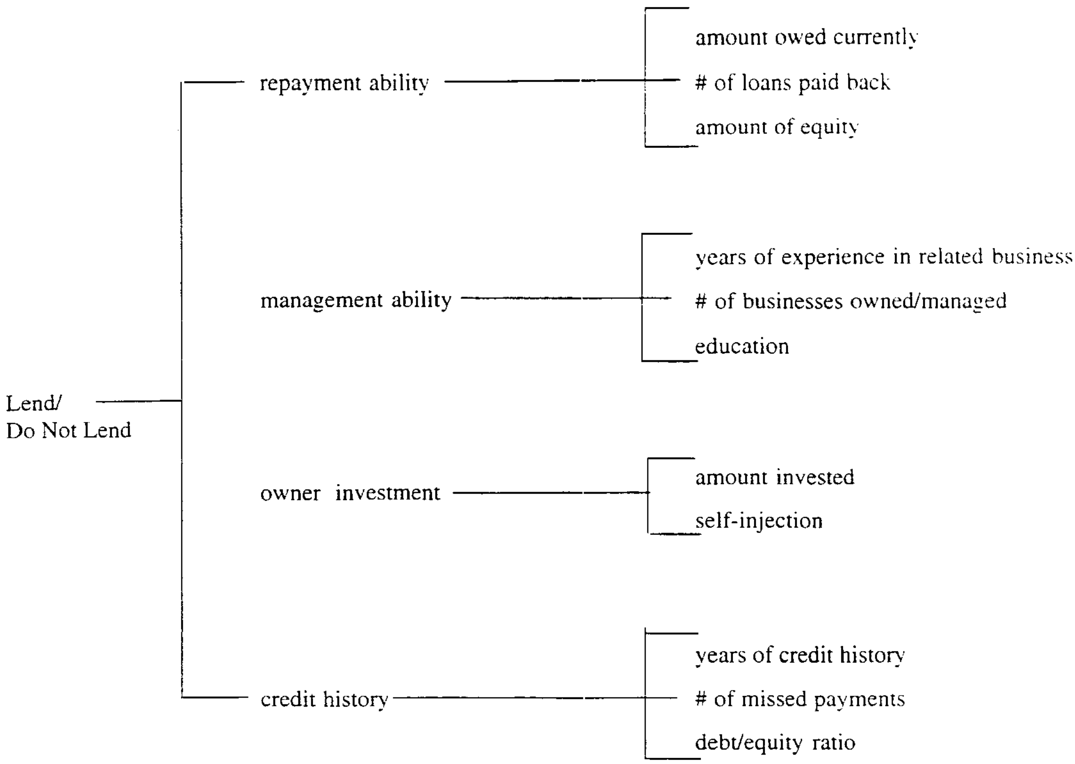
The design of the fuzzy expert system required several steps. First, the human expert was chosen. The director of the Small Business Development Center (SBDC) at the University of Texas-Pan American is the human expert. This person has an MBA and five years experience as business counselor at the SBDC office for small business entrepreneurs. The expert is highly computer-literate and familiar with artificial intelligence and expert systems.

After the detailed investigation of the problem at hand, the human expert identified the repayment ability, management ability, owner investment, and credit history as important evaluation criteria for the loan lending decision process of the SBA. Figure 1 illustrates the decision framework for loan approvals.

Repayment ability summarizes the SBA's perception about the capability of the borrower to pay the loan back. The amount that the client currently owes to other lenders decreases his/her ability to pay the loan back. If the number of loans (both personal and business) paid back to satisfactory and the amount of equity of the applicant is high then the ability to pay back the loan is high.

Management ability evaluates the managerial skills of the applicant. Criteria to look at to assess the management ability of the applicant are experience, number of businesses owned or managed previously, and education. If number of years of experience in related businesses is high, number of businesses owned/managed is high, and years of education is high then the management ability is high.

Figure 1. An Illustrative Decision Framework for Small Business Loan Approvals



Owner investment summarizes the amount successfully invested by the applicant before and the amount that will be injected by the applicant as part of the loan. If amount invested before and amount of self-injection are high then owner investment is high.

The final category is credit history. This category encapsulates the years of credit history, number of missed payments, and the debt/equity ratio of the applicant. If years of credit history is high, number of missed payments is low, and the debt/equity ratio is low then credit history is high.

The evaluation process illustrated in Figure 1 is modeled with rules that are written in the CubiCalc® fuzzy expert system shell. Some variables in the model may have different numerical scales. But all variables are recoded to an interval of [0,60] and inputs are fuzzified using triangles and trapezoids. The ranges for very low, low, medium, high, and very high for each variable are determined together with the expert. The final variable in the model is Lend/Do Not Lend. It is used to approve or disapprove the loan amount of the applicant. The variables (see Table 1) are derived from the decision framework illustrated in Figure 1.

Table 1. List of Variables

First Level			
	OWE	=	amount currently owed
	PAID	=	number of loans paid back
	EQUITY	=	amount of equity
	EXPER	=	years in related businesses
	BUS	=	number of businesses owned/managed
	EDU	=	years of education
	INV	=	amount invested before this loan
	INJECT	=	amount of self-injection
	HISTORY	=	years of credit history
	MISSED	=	number of payments missed
	D/E	=	debt/equity ratio
Second Level			
	REPAY	=	repayment ability
	MGMT	=	management ability
	OWNER	=	owner investment
	CREDIT	=	credit history
Third Level			
	LEND	=	lend/do not lend

The next step involved describing the membership functions on these variables. The human expert and relevant literature were again consulted in the design of membership functions. A membership function is a function that maps one or more variables to a degree of membership (zero to one) in a fuzzy set. Membership functions are made up of function names (fuzzy labels) and membership values. Two decisions were necessary in defining the membership functions for the fuzzy variables, including identification of (1) the universe of discourse and (2) the number and description of membership functions related to each variable. The universe of discourse represents a range of measurement value to the user related to a specific variable. Kickert (1979) demonstrated that, for linguistic semantic values, no significant computational differences exist between selection of a wide versus narrow universe of discourse. Therefore, the universe of discourse value range of [0,60] was used for all fuzzy variables. After identification of variables and their universe of discourse, it was necessary to develop membership functions for each variable. The selection of appropriate membership functions is a means to incorporate knowledge into a fuzzy system. Identification as to the number, size, and shape of the membership functions representative of a given variable is often subjective and arbitrary. Zimmerman (1987) indicated that definitive criteria regarding membership function development is still lacking.

Many authors (Masters, 1993; Viot, 1993; Kosko, 1994) proposed that simple membership function shapes, such as triangles and trapezoids, often improve maintainability and execution results, and are normally both effective and efficient. Although any shape membership functions can be used, most real world fuzzy expert systems make use of trapezoidal and triangular functions (Kosko, 1994). Another set of guidelines (Viot, 1993) regarding membership function development include the following: (1) a logical number of fuzzy sets as related to the specific domain of the system, (2) a sufficient number of fuzzy sets to provide coverage of the entire universe of discourse, and (3) overlapping membership functions. According to these guidelines, overlapping trapezoidal and triangular functions were used to describe variables in our application. The membership functions that are used to describe input variables are provided in Figures 2-4.

Figure 2. Membership Function for Variables Repay, Mgmt, Owner, Credit

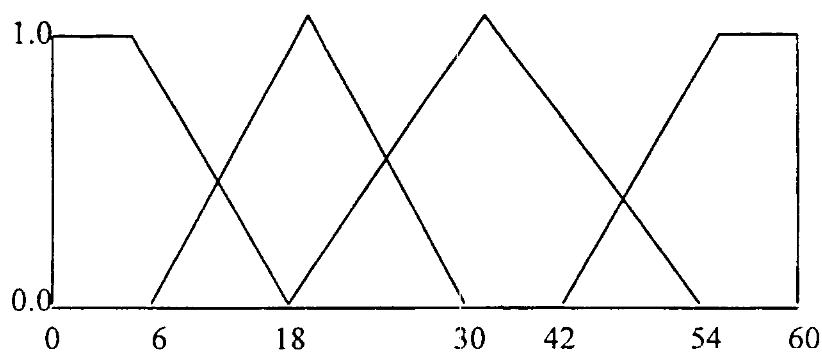


Figure 3. Membership Function for Variables Owe, Paid, Exper, Bus, Inv, Inject, History, Missed

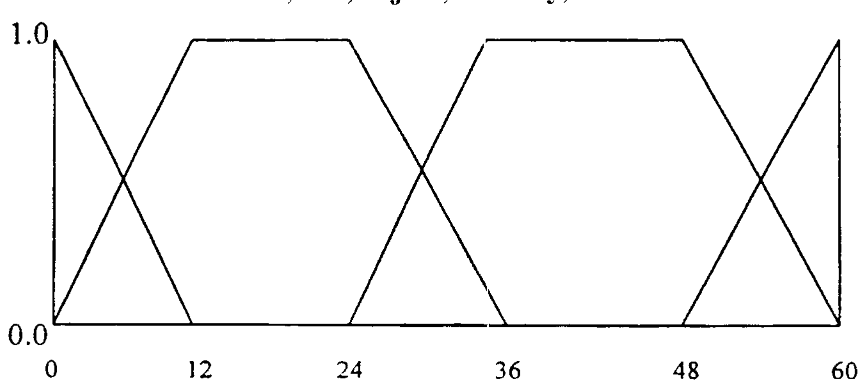
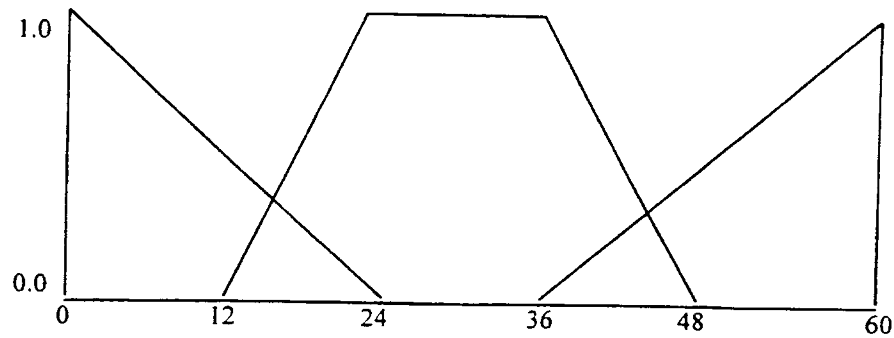


Figure 4. Membership Function for Variables Equity, Edu, D/E



A set of fuzzy production rules were developed based on the discussions and interviews with the business counselors and the director of the SBDC office. Each rule consists of multiple antecedent variables but only one consequent variable. The format of the rule base is taken from Figure 1. There are two stages of production rules that link three levels of variables. Each of the four expert modules (Repayment Ability, Management Ability, Credit History) consists of 23 rules. The Owner Investment module consists of 17 rules. Therefore, the complete system consists of a total of 4 x 23 + 17 = 109 rules. Each rule has the form IF A, THEN B. As an example, Rule 16 of the management Ability module is given below:

IF	Exper if **High**
AND	Bus is **High**
AND	Edu is **Low**
THEN	Mgmt is **Medium.**

The fuzzified input values may fire several rules, though the degree to which each rule is fired is not fixed. The input data fires the A part of each rule to some degree. This becomes the activation of each rule. Centroid defuzzification method is then utilized by the system to calculate the crisp output number from each module.

SYSTEM VALIDATION

The widespread use and implementation of expert systems in a wide variety of organizations has increased the importance of Verification and Validation (V&V) process. In particular, systems where error in expert system advice or recommendation may lead to loss of life (Wang et al., 1994), loss of considerable amount of money, or damage to expensive physical equipment or facilities (Chen & Ishiko, 1990) have necessitated a need for careful V&V (Lee & O'Keefe, 1994).

According to O'Keefe et al. (1987), validation focuses on the needs of the user and the organization, and ensures that the 'right system' is built. O'Leary (1987), discussing the process of system validation, indicates that one objective of the process should be to determine what the system does know, what the system does not know, and what the system knows incorrectly. The objectives of the validation process were extended by O'Leary (1987) to include determining the decision-making level of the system, the theoretical base of the system, and the reliability of the system.

Hall et al. (1988) specifically addressed the issues and problems associated with the V&V of fuzzy expert systems. First, as fuzzy expert systems rely on ambiguity and uncertainty, the system evaluation itself may not be a precise process. Second, the authors reported that, in most fuzzy systems, it may not be feasible to test the system using all possible system inputs. Third, precise or exact solutions and answers cannot be expected from the evaluation of a fuzzy expert system. This study concluded that answers which are acceptable in terms of relations or ranges should be expected from the testing and evaluation of fuzzy expert systems, and that the testing and evaluation process of fuzzy expert system is not inherently different.

System validation refers to the system testing process which attempts to ensure that the system satisfies the needs of the users for which the system was designed (Geissman et al., 1988). In other words, validation helps ensure a realistic decision-making system (McEacharan, 1994). We focused on results for the validation of the fuzzy expert system model.

An input-output comparison test on actual data was performed to check if the system performance is acceptable. The input-output comparison stage is the ultimate test of validity. It substantiates that the system possesses a satisfactory accuracy range consistent with its intended application. The procedure used for this stage of validation was to provide the system with a set of inputs from real world cases and to check whether outputs provided by the system could be compared with the actual small business loan approval decision taken by the human expert. To test the validity of the developed fuzzy expert system, the historical test cases of the SBDC are used. The data consists of 90 business plans completed during the Fiscal Year 1996 by the SBDC.

Tables 2 and 3 indicate that the fuzzy expert system exactly matched the actual small business loan approval decision in 72 out of 90 cases (80%). In 7 cases, fuzzy expert system did not make a LEND recommendation although the loans were actually approved. Conversely, in 11 cases loans were not approved but fuzzy expert system made a LEND recommendation.

Table 2. Validation Test Case Results

	Actual Decision	System Recommendation	Common Cases
Lend	33	37	26
Do Not Lend	57	53	46

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Table 3. LEND vs. DO NOT LEND Decisions

ACTUAL	FUZZY SYSTEM	# OF CASES
LEND	LEND	26
LEND	DO NOT LEND	7
DO NOT LEND	LEND	11
DO NOT LEND	DO NOT LEND	46

CONCLUSION

In this paper, a decision framework for fuzzy logic to model the analysis of a small business loan problem has been developed. This fuzzy evaluation method is a systematic approach for analyzing a small business loan. The method consists of two stages of production rules and three levels of variables. The production rules has been developed based on specific input data given by the human expert.

This approach is easy to use and efficient. The fuzzy approach is a feasible technique to assist credit analysts on small business loan decisions. Although validation tests have been performed, further tests are necessary to conclude whether this approach become useful. Interviews with more experts can be made to validate the scale, membership functions, and production rules. Further data can be obtained from other SBDC offices to test the expert system and compare its predictions against actual funding decision. Also usability and usefulness of the system can be tested.

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