Image enhancement in wavelet domain based on histogram equalization and median filter

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ABSTRACT

During the acquisition of a new digital image, noise may be introduced as a result of the production process. Image enhancement is used to alleviate problems caused by noise. In this work, the purpose is to propose, apply, and evaluate enhancement approaches to images by selecting suitable filters to produce improved quality and performance results. The new method proposed for image noise reduction as an enhancement process employs threshold and histogram equalization implemented in the wavelet domain. Different types of wavelet filters were tested to obtain the best results for the image noise reduction process. Also, the effect of canceling one or more of the high-frequency bands in the wavelet domain was tested. The mean square error and peak signal to noise ratio are used for measuring the improvement in image noise reduction. A comparison made with two related works shows the superiority of the methods proposed and implemented in this research. The proposed methods of applying the median filter before and after the histogram equalization methods produce improvement in performance and efficiency compared to the case of using discrete wavelet transform only, even with the cases of multiresolution discrete wavelet transform and the cancellation step.

Keywords: Histogram equalization; Mean square error and peak signal to noise ratio; Median filter; Multiresolution discrete wavelet transform; Threshold technique.

INTRODUCTION

In many systems dealing with images, the ability to get detailed information from received images, depends on the quality of the image. Changes occur in images, generally due to noise, therefore, reducing the quality of the image. A delicate operation called the image enhancement method can remove much of the noise from the image. Image enhancement methods can be considered a perfect part of image processing which is in demand in many fields such as weather forecasting, astrophysics, medical fields, and so on. Recovering the original image from the noisy image, thus reducing the loss of information, can be considered as actual gain (Bhargava, 2018; Koranga, 2018). Also, noise can be defined as the undesirable by-product of image capture that obscures the desired information. Many types of noise appear in images as a cause of multiple sources due to process or procedure, transmission, and reception. Besides, noise takes various forms. The noise used in this work is the Additive Gaussian Noise (AGN), for the possibility of this type of noise appearing in images is more significant than other types, and noisy image with AGN is more complicated for the image enhancement process (Sami, 2016). Equation 1 below shows the additive noise components (Ramadhan, 2017; Saxena, 2014):

$$In(i,j) = I(i,j) + n(i,j) \quad (1)$$

In Equation 1, I(i,j) is the original image, without noise, n(i,j) is the noise, In(i,j) is the noisy form of the image, and (i,j) is the location of the pixel.

Methods used for image enhancement include many different kinds such as filters and others for changing various properties of an entire image. Figure 1 shows the generic block diagram for image enhancement algorithms.

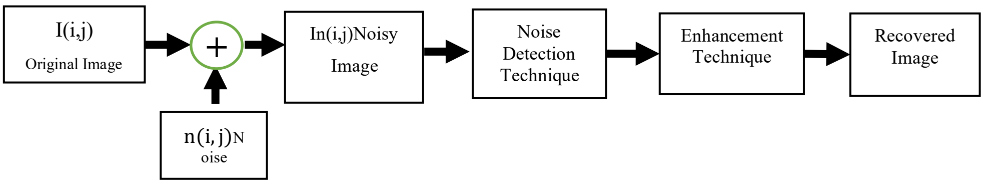

Figure 1. The block diagram of image enhancement algorithm.

The noise detection technique used in this work starts by implementing the estimation of the noise level inside images. The noise estimator is used by applying the Robust Median Estimator Equation (Ramadhan, 2017; Sami, 2016). Also, in this paper, we propose image enhancement approaches using suitable filters to produce better performance. The contribution of this work is a new method for image noise reduction using threshold and histogram equalization approaches applied in the wavelet domain. Different types of wavelet filters (Haar, Daubechies (dbN), Symlet (symN), Coiflet (coifN), Biorthogonal (biorNr.Nd), and Reverse Biorthogonal (rbioNr.Nd)) were tested aiming for the best results. The effect of canceling one or more of the high-frequency bands in the wavelet domain was also inquired. The rest of this paper covers the proposed image enhancement scheme, the next section displays the experimental results followed by a discussion of findings and conclusions.

THE PROPOSED IMAGE ENHANCEMENT SCHEME

In this work, image enhancement approaches were designed and applied to digital images in the wavelet domain in order to enhance the quality and the contrast of image details. Performance measurement of the proposed techniques and all the work were done in the wavelet transform domain using filter families. In addition, we considered the effects of removing high-frequency bands in the multiresolution wavelet transformation. In evaluating the performance of the proposed image enhancement methods, values of peak signal-to-noise ratio (PSNR) and mean squared error (MSE) were calculated for noisy and original images. To compute the values of MSE and PSNR, the following equations were used:

$$MSE = \frac{1}{M.N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[F(i,j) - I(i,j)]^2 \quad (2)$$

$$PSNR = 10.\log_{10}\frac{(max)^2}{MSE} \quad (3)$$

In Equation 2, I (i, j) is the original image, F (i, j)is the enhanced image, *M and N: the* size of the original image, and *max*in Equation 3 is the maximum pixel value of grayscale image used in this work which is equal to 255.

The primary aim of digital image enhancement is to increase the quality of an image by removing any changes causing distortion and other unwanted effects; one of those adverse changes is noise. Therefore, noise reduction is considered as an image enhancement operation. There are many methods for noise reduction in digital images. Wavelet transformation is used as an excellent tool to convert the image from the spatial domain into the wavelet domain. Briefly, in the wavelet domain, a separation operation will distribute the frequencies inside the image into four bands for one analysis level of wavelet transformation. The bands in the wavelet domain can be divided into two types. The approximation band A, which contains the low frequencies; sometimes it is called LL band, and the three others called the high-frequency bands H, V, and D, also referred to as LH, HL, and HH, respectively. The bands H, V, and D contain the details and the edges of the objects in the image distributed inside in the form of the directions of the changes for the horizontal, vertical, and diagonal dimensions. In this work, Histogram Equalization Method and Threshold Technique are used in the wavelet domain for image enhancement. Additionally, another step that was implemented with each image enhancement method is the cancellation of one or more of the high frequency bands of the wavelet transformation. This latter step was devised to measure the effect of each group of high frequencies bands cancelled for the wavelet transformation.

HISTOGRAM EQUALIZATION METHOD

One of the image enhancement methods used in this work is the histogram equalization method applied to an image in the wavelet domain. Histogram equalization increases the dynamic range of grey-levels in a low-contrast image to cover the full range. It is employed in order to adjust the contrast of an image by modifying the intensity distribution of its histogram. Histogram equalization is achieved by having a transformation function T(r), which can be defined as the Cumulative Distribution Function (CDF) of a given probability density function (PDF) of grey-levels in a given image. Histogram equalization is an image processing technique which works on enhancing the visual appearance of an image by uniformly distributing the intensity levels (Arora, 2018). The histogram of an image can be considered as an approximation of its PDF; then the transformation function can be obtained. Processing of histogram equalization relies on CDF use. The CDF is a cumulative sum of all the probabilities lying in its domain (Bovik, 2009; Gonzalez, 2008). The discrete form for the CDF is shown in Equation 4:

$$cdf(x) = \sum_{k=-\infty}^{x} P(k) \tag{4}$$

Apart from equalization advantage, this method introduces a significant change in image brightness, that is, its mean grey-level. That is, due to the uniform distribution specification of the output histogram, the CDF method shifts the brightness of the output image to the median grey-level (Nithyananda, 2016).

Let f be a given image represented as mr by mc matrix of integer pixel intensities ranging from 0 to L−1, where L is the number of possible intensity values, often 256. Let P denote the normalized histogram of f with a bin for each possible intensity. So,

$$P_n = \frac{numberofpixelswithintensityn}{totalnumberofpixels} \quad , n = 0,1,..., L-1 . \tag{5}$$

The histogram-equalized image g will be defined by

$$g_{i,j} = floor\left((L-1)\sum_{n=0}^{f_{i,j}} P_n\right), \tag{6}$$

where floor () rounds down to the nearest integer.

This is equivalent to transforming the pixel intensities, k, of f by the function below:

$$T(k) = floor\left((L-1)\sum_{n=0}^{k} P_n\right). \tag{7}$$

The motivation for this transformation comes from thinking of the intensities of f and g as continuous random variables X, Y on [0, L−1] with Y defined by Arora ,2018, and Rajput ,2013:

$$Y = T(X) = (L-1)\int_0^X P_X(x)dx \tag{8}$$

In Equation 8, PX is the probability density function of f, and T is the cumulative distributive function (CDF) of X multiplied by (L−1).

THRESHOLDING TECHNIQUE

Thresholding technique is implemented in the wavelet domain because the wavelet transform provides a useful tool for frequencies separation, which is helpful for reducing noise of digital images. Significant enhancements can be achieved through computing a suitable threshold value (Boyat, 2013; Om, 2012; Ramadhan, 2017; Toufik, 2012). Equation 9 shows the mechanism of applying the threshold value to the pixels for subbands in the wavelet domain:

$$Pnew(x,y) = \begin{cases} P(x,y), & whenP \geq Thx \\ 0, & otherwise \end{cases} \tag{9}$$

In Equation 9,$P(x,y)$is the pixel value of the subband x for the noisy image, and $Pnew(x,y)$is the pixel value of the subband x for the enhanced image. Equation 10 is used to obtain the threshold value as follows:

$$Thx = \frac{\sigma^2}{\sigma_x} \quad . \tag{10}$$

In Equation 10, σ^2 is the noisy image noise value, which is computed by using Robust Median Estimator Equation, and σ_x is the standard deviation of the specific subband x (Tai-sheng, 2015).

The multiresolution 2D-DWT means that the image can be subjected to more than one analysis level. In this work, the multiresolution approach is applied with two analyses. In order to obtain less distortion in the image, a suitable threshold value must be used (Koranga, 2018; Ergen, 2012; Qian, 2015).

STEPS OF THE IMAGE ENHANCEMENT METHOD

In this paper, the term ‘proposed algorithm’ is used to refer to each one of the two methods of the digital image enhancement. The enhancement method is composed of the following steps affected on the digital image.It should be noted that 2D-DWT is applied to the noisy image. For the 1-level decomposition in 2D-DWT the result is four subbands (A1, H1, V1, and D1), and for applying the 2 levels of 2D-DWT decomposition, the band A1 is the input for the wavelet transformation.

Apply the main step for the enhancement method as follows. If the enhancement method is the histogram equalization, then the histogram equalizer is applied to all of the detail bands for all the decomposition levels in 2D-DWT. While if the enhancement method is thresholding technique, then a threshold value is computed for all of the detail bands. After that, a comparison is applied to compare pixels in the subbands against the specific threshold value for that band. The value of the pixel is cleared (zeroed), if the value of the threshold of that band is higher than the pixel value. Otherwise, skip to test the next pixel.

Apply the synthesis operation with the same number of levels as in the decomposition operation. The synthesis can be applied as an inverse of the 2D-DWT; the new subbands are generated using the enhancement step to obtain a new image called recovered image.

By using Equations 3, the PSNR for both the newly recovered and the original images to evaluate the proposed image enhancement method can be computed.

In addition to the new switch to both mentioned image enhancement methods, another step appended to test each method after canceling one or more of the first wavelet analysis level detail bands. After applying all the above algorithms, along with the suggestion of bands cancellation, another novel idea was implemented with all the above methods. In addition to the algorithms applied in the classic case, the median filter is applied before and after the algorithm of this work. The results of computations for all cases are given in the Experimental Results section.

FIRST CASE: IMAGE ENHANCEMENT BY HISTOGRAM EQUALIZATION IN WAVELET DOMAIN

In this case, 2D-DWT is used for preparing the work area for applying histogram equalization method to each band of the detail bands in wavelet domain. Figure 2 displays the block diagram for this case. This method is called as Proposed_1.

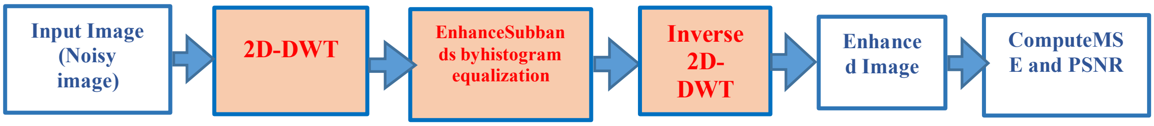

Figure 2. First case: Proposed_1 block diagram.

SECOND CASE: IMAGE ENHANCEMENT BY THRESHOLDING TECHNIQUE IN WAVELET DOMAIN

In this case, 2D-DWT is used for preparing the work area for applying the thresholding technique. Figure 3 shows the block diagram for this case. Image enhancement by thresholding technique in wavelet domain represents the steps that applied in both of Ramadhan (2017) and Sami (2016).

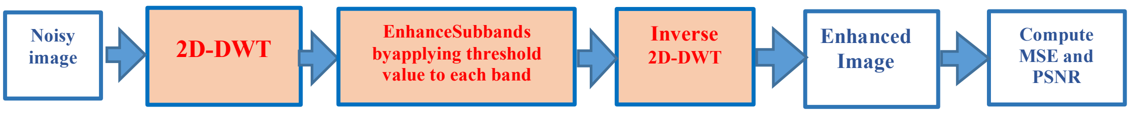

Figure 3. Block diagram of image enhancement by thresholding technique in wavelet domain.

THIRD CASE: IMAGE ENHANCEMENT BY MEDIAN FILTER ONLY

In this case, image enhancement was implemented by applying only a median filter to the noisy image in order to obtain an enhanced image. Figure 4 shows the block diagram for this case.

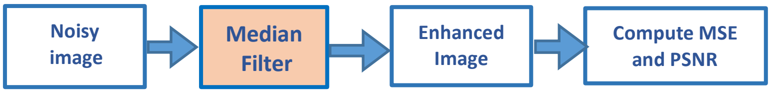

Figure 4. Block diagram of image enhancement by median filter only.

FOURTH CASE: IMAGE ENHANCEMENT BY MEDIAN FILTER BEFORE HISTOGRAM EQUALIZATION

In this case, image enhancement implemented in two steps: in the first step, median filter applied on the noisy image to obtain the first version of the enhanced image. Then in the second step the image further enhanced by histogram equalization. Figure 5shows the block diagram for this case which is the second proposed method; and will be called as Proposed_2.

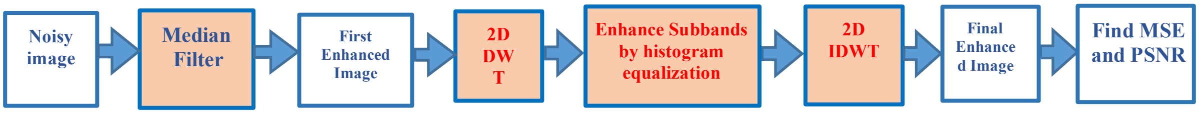

Figure 5. Proposed_2 block diagram.

FIFTH CASE: IMAGE ENHANCEMENT BY MEDIAN FILTER AFTER HISTOGRAM EQUALIZATION

In this case, the image enhancement method implemented in two steps. In the first step, histogram equalization performed in wavelet domain, then processed using the median filter to obtain the final enhanced image. Figure 6 shows the block diagram for this case which is the third proposed method; and will be called as Proposed_3.

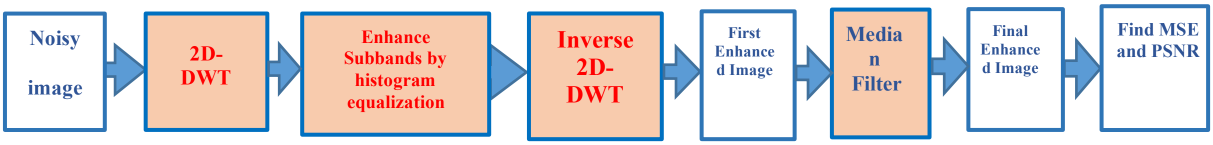

Figure 6. Proposed_3 block diagram.

SIXTH CASE: IMAGE ENHANCEMENT BY MEDIAN FILTER BEFORE THRESHOLDING TECHNIQUE

In this case, image enhancement was implemented in two steps: applying a median filter on the noisy image and then following with the thresholding technique in the wavelet domain. Figure 7 shows the block diagram for this case.

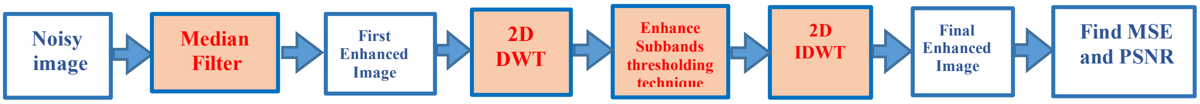

Figure 7. Block diagram of using median filter before thresholding technique.

SEVENTH CASE: IMAGE ENHANCEMENT BY MEDIAN FILTER AFTER THRESHOLDING TECHNIQUE

In this case, image enhancement was implemented in two steps: applying a thresholding technique in the wavelet domain followed by median filter processing. MSE and PSNR were then computed for both the final enhanced and the original images. Figure 8 shows the block diagram for this case of image enhancement.

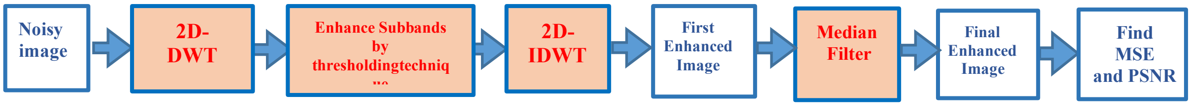

Figure 8. Block diagram of using median filter after thresholding technique.

COMPARING WITH RELATED WORKS

For the purpose of showing the contribution of the proposed methods of this research, namely, Proposed_1, Proposed_2, and Proposed_3, PSNR values of those and of the methods implemented by Ramadhan (2017) entitled Related_1 and Sami (2016) entitled Related_2 are displayed in Table 2 and Figure 10 in the next section. The proposed methods of applying the median filter before and after the histogram equalization methods produced better performance results and efficiency than using DWT only.

EXPERIMENTAL RESULTS

In this work, four grayscale images each of size 256 × 256 are shown in Figure9. The noise type used is the Additive White Gaussian Noise (AWGN) with zero mean (m = 0), and the values of the noise intensity used were σ =15, σ =20, and σ =25. The AWGN noise is added to an image to construct the noisy image. All the image enhancement algorithms in this work are implemented using MATLAB. As an addition to all the cases mentioned above, there is another step applied to test the same algorithms in each case but with canceling one or more of the first level analysis detail bands. The PSNR values are shown in Tables 1-2. In any table, the cells marked with a highlighted background represent the best results.

-a-: Hand -b-: Camera -c-: Lena -d-:Barbar

Figure 9. The four tested images used in this work.

The results in Table 1 and Table 2 displaying the summary of all the results outcomes and help show the following significant points:

1. For all of the images tested, the mix between using the image enhancement methods with the median filter produces results better than using each of the two methods solo.
2. The comparison results showed that the proposed methods produced higher PSNR than the enhancement methods used in Ramadhan (2017) and Sami (2016). The proposed methods produced better performance results than using DWT only with all cases.
3. The method of applying median filter first and then the enhancement method indicates that employing the threshold techniques with canceling the HH high-frequency band in the wavelet domain generates an improvement in the results.
4. Table 1 shows that applying the enhancement method to the first level of the wavelet analysis produces better results, and this means that most of the noise is concntrated inside the high-frequency wavele bands for the first level of the analysis.

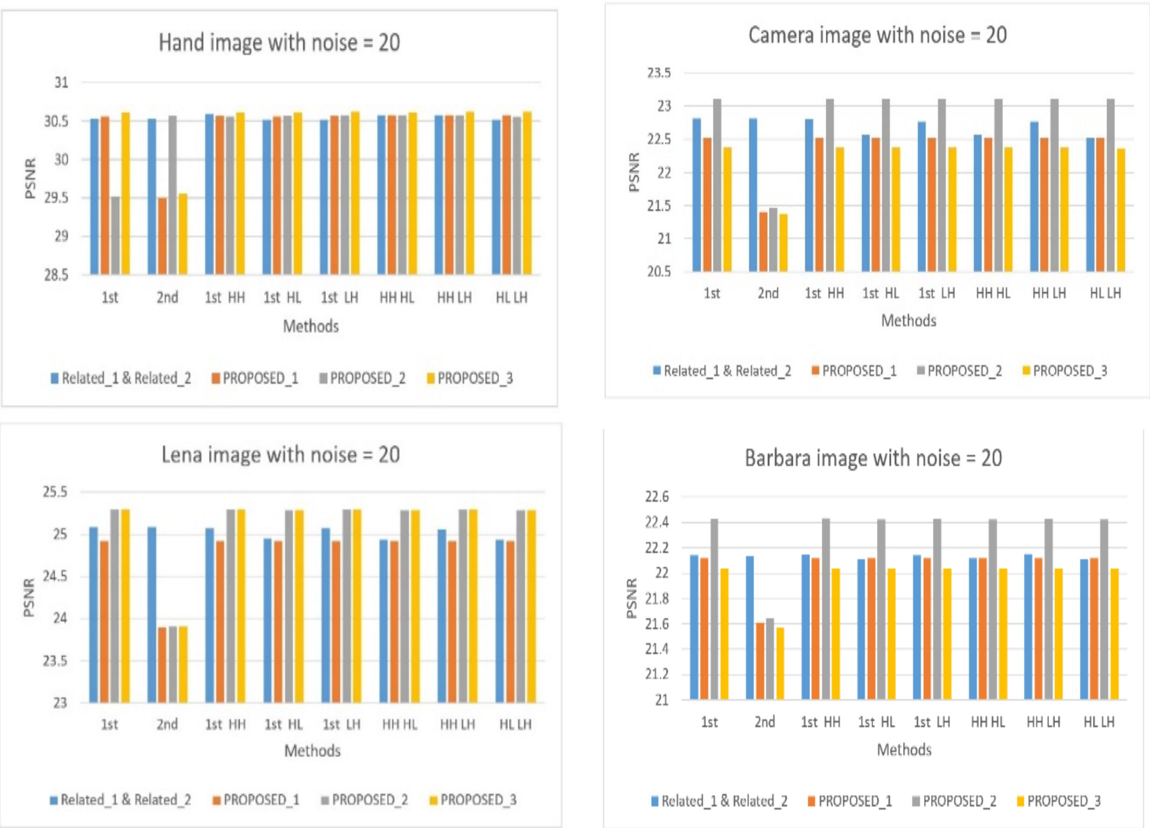

Figure 10. The comparison between the proposed methods and the related works; noise =20.

Table 1. Best case values of PSNR by image enhancement methods for all test images.

Image name	Noise ratio	PSNR and method name	Cases			
			MF only	**Without MF**	**MF before the image enhancement methods in wavelet domain**	**MF after the image enhancement methods in wavelet domain**
Hand	**15**	**PSNR**	30.35	31.676	31.809	31.667
		Method		Th 1st HH	Th 1st HH	Th 1st HH
	20	**PSNR**	28.30	30.590	30.621	30.640
		Method		Th 1st HH	Th 2nd	Th 1st HH
	25	**PSNR**	26.87	29.849	29.849	29.962
		Method		Th All	Th All	His.E .1st LH
Camera	**15**	**PSNR**	24.677	22.875	24.689	22.875
		Method		His.E .1st LH	Th 1st	Th 2nd
	20	**PSNR**	24.068	22.812	24.097	22.813
		Method		Th 1st	Th 1st	Th 2nd
	25	**PSNR**	23.362	22.583	23.511	22.578
		Method		Th 1st HH	Th 1st	Th 1st
Lena	**15**	**PSNR**	26.55	25.450	26.595	25.445
		Method		Th 1st	Th 1st	Th 1st
	20	**PSNR**	25.48	25.090	25.864	25.086
		Method		Th 1st	Th 1st	Th 1st
	25	**PSNR**	24.623	24.765	25.243	24.766
		Method		Th 1st	Th 1st	Th 1st
Barbara	**15**	**PSNR**	22.327	22.316	23.044	22.300
		Method		Th 1st HH	Th 2nd	Th 2nd
	20	**PSNR**	21.95	22.150	22.742	22.129
		Method		Th 1st HH LH	Th 1st HH	Th 1st
	25	**PSNR**	21.52	21.933	22.423	21.904
		Method		Th 1st HH LH	Th 1st HH	Th 1st

Table 2. The PSNR values of the comparison between proposed methods and related works.

The method		The image											
		Hand			Camera			Lena			Barbara		
		15	20	25	15	20	25	15	20	25	15	20	25
Related_2	Related_1 1st	31.626	30.535	29.751	22.875	22.812	22.575	25.450	25.090	24.765	22.309	22.140	21.906
	2nd	31.624	30.534	29.749	22.870	22.812	22.575	25.448	25.086	24.762	22.307	22.136	21.905
	1st HH	31.676	30.590	29.818	22.597	22.811	22.583	25.423	25.076	24.762	22.316	22.146	21.923
	1st HL	31.549	30.521	29.748	22.825	22.562	22.366	25.263	24.948	24.650	22.263	22.111	21.895
	1st LH	31.562	30.518	29.764	22.590	22.768	22.538	25.416	25.074	24.763	22.306	22.140	21.915
	HH HL	31.609	30.580	29.815	22.817	22.559	22.374	25.236	24.936	24.644	22.267	22.118	21.909
	HH LH	31.621	30.579	29.831	22.548	22.766	22.544	25.390	25.063	24.759	22.314	22.150	21.933
	HL LH	31.500	30.521	29.778	22.541	22.516	22.329	25.231	24.935	24.648	22.259	22.112	21.902
	All	31.567	30.583	29.849	22.544	22.513	22.335	25.204	24.921	24.644	22.264	22.120	21.918
proposed_1	1st	31.537	30.562	29.829	21.416	22.515	22.337	25.209	24.924	24.645	22.264	22.118	21.916
	2nd	30.023	29.505	29.023	22.544	21.405	21.319	24.051	23.895	23.736	21.695	21.613	21.490
	1st HH	31.550	30.569	29.836	22.540	22.516	22.337	25.208	24.924	24.645	22.265	22.120	21.918
	1st HL	31.545	30.565	29.831	22.545	22.512	22.334	25.204	24.921	24.643	22.264	22.118	21.917
	1st LH	31.552	30.567	29.835	22.541	22.517	22.338	25.208	24.924	24.646	22.263	22.118	21.917
	HH HL	31.552	30.575	29.840	22.546	22.512	22.334	25.203	24.922	24.643	22.264	22.120	21.918
	HH LH	31.560	30.578	29.844	22.541	22.516	22.338	25.209	24.925	24.646	22.265	22.120	21.918
	HL LH	31.554	30.575	29.841	22.875	22.512	22.335	25.203	24.921	24.643	22.263	22.120	21.916

proposed_2	1st	31.651	29.515	29.829	23.327	23.115	22.831	25.736	25.293	24.898	22.645	22.431	22.196
	2nd	30.047	30.569	29.021	21.526	21.458	21.371	24.071	23.903	23.741	21.732	21.644	21.530
	1st HH	31.659	30.565	29.836	23.328	23.115	22.833	25.739	25.293	24.899	22.645	22.432	22.198
	1st HL	31.652	30.567	29.831	23.320	23.110	22.827	25.729	25.289	24.893	22.640	22.426	22.197
	1st LH	31.661	30.575	29.835	23.326	23.113	22.831	25.736	25.292	24.897	22.642	22.429	22.195
	HH HL	31.663	30.578	29.840	23.322	23.108	22.827	25.728	25.289	24.895	22.640	22.426	22.199
	HH LH	31.670	30.575	29.844	23.325	23.114	22.831	25.736	25.292	24.900	22.643	22.429	22.197
	HL LH	31.663	30.555	29.841	23.319	23.107	22.826	25.725	25.287	24.894	22.637	22.424	22.196
proposed_3	1st	31.497	30.610	29.953	22.400	22.374	22.207	25.054	25.293	24.543	22.172	22.039	21.853
	2nd	30.038	29.555	29.100	21.385	21.375	21.293	24.001	23.903	23.707	21.647	21.571	21.452
	1st HH	31.498	30.612	29.948	22.398	22.373	22.207	25.052	25.293	24.543	22.171	22.039	21.853
	1st HL	31.497	30.615	29.952	22.394	22.370	22.205	25.047	25.289	24.539	22.169	22.039	21.852
	1st LH	31.509	30.619	29.962	22.398	22.372	22.205	25.052	25.292	24.544	22.171	22.038	21.852
	HH HL	31.500	30.616	29.952	22.393	22.370	22.204	25.044	25.289	24.537	22.169	22.038	21.852
	HH LH	31.509	30.621	29.959	22.396	22.371	22.205	25.049	25.292	24.542	22.171	22.037	21.852
	HL LH	31.506	30.622	29.961	22.393	22.367	22.203	25.047	25.287	24.539	22.169	22.039	21.852

CONCLUSION

In this research, we proposed and applied image enhancement approaches using filters to produce high-quality results. A new method is devised for image noise reduction as an enhancement process using threshold and histogram equalization implemented in the wavelet domain. According to the results of the comparison, the contribution of the proposed methods of this research, namely, Proposed_1, Proposed_2, and Proposed_3, as PSNR values, is demonstrating superiority to the results presented by other methods implemented in the references, Ramadhan (2017) entitled Related_1 and Sami (2016) entitled Related_2.

For all the test images used in this work, MSE and PSNR are calculated as a performance indicator of different approaches to image enhancement. The values of PSNR are displayed in tables. A summary table is collection created to display the best results by image enhancement approach.

The tested methods of applying the median filter before and/or after threshold or histogram equalization methods, depending on the case, showed better performance results and efficiency than using DWT only. DWT only showed inferior results compared to other methods. However, using the mix of DWT and median filter together provided excellent results in relation to being able to recover much more details of the original images. Similarly, in all cases of canceling the detail bands in the wavelet domain, the hybrid filter produced clearer images and superior quality than using an enhancement method only.

It was noticed that the amount of noise that concentrates and gets spread into the detail bands for the first analysis level of 2D-DWT has a significant effect on the results of image enhancement. For this reason, it is safe to state that detail bands play a very critical role in the results and the performance of image enhancement systems. Also, another point has been noted which was that increasing the level of analysis in 2D-DWT, increased the quality of the image after enhancement. Such an increase in the analysis level is limited by the image size and by the situation that will have no improvement due to the saturation of the enhancement operation. In this work, two levels of DWT analysis were applied. The saturation of the enhancement operation means that the number of points in the data is very small and will not affect the result at all.

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