INNOVARE JOURNAL OF ENGINEERING & TECHNOLOGY

Vol 10, 2022 ISSN - 2347-1573

Research Article

INTEGRAL TRANSFORM AND FRACTIONAL KINETIC EQUATION

AARTI PATHAK[1], RAJSHREEMISHRA[2], D.K. JAIN[3], FAROOQ AHMAD*[4], PEER JAVAID AHMAD[5]

[1]Research Scholar Department of SOMAAS Jiwaji University Gwalior, Madhya Pradesh, India. [2]Department of Mathematics Government Model Science College Gwalior, Madhya Pradesh India. [3]Department of Engineering Mathematics Computing Madhav Institute of Technology and Science. Gwalior Madhya Pradesh India. [4]Department of Mathematics Government College for Women Nawakadal. Srinagar, Jammu & Kashmir. [5]Department of Statistics Government College for Women Nawakadal. Srinagar, Jammu & Kashmir. Email: sheikhfarooq85@gmail.com

Received: 23 November 2022, Revised and Accepted: 16 December 2022

ABSTRACT

With the help of the Laplace and Fourier transforms, we arrive at the fractional kinetic equation's solution in this paper. Their respective solutions are given in terms of the Fox's H-function and the Mittag-Leffler function, which are also known as the generalisations and the Saigo-Maeda operator-based solution of the generalised fractional kinetic equation. The paper's findings have applications in a variety of engineering, astronomy, and physical scientific fields.

Keywords: Fractional kinetic equation, Saigo-Maeda operator & Mittag-Leffler function.

 DOI: http://dx.doi.org/10.22159/ijet.2022v10i1.46890. Journal homepage: https://innovareacademics.in/journals/index.php/ijet

2020 Mathematical Sciences Classification: Primary: 26A33, 33C99, Secondary: 33E12, 33E99.

INTRODUCTION

Astronomers and physicists are now paying more attention to the available mathematical techniques that may be used to effectively solve a variety of physics and astrophysics problems as a result of the tremendous importance of mathematical physics in distinct astrophysical.

A symmetric gas sphere in thermal and hydrostatic equilibrium with negligible rotational and magnetic fields can be thought of as a star (much like the Sun). The mass, brightness, effective surface temperature, radius, core density, and temperature of the star are its defining characteristics. Based on the aforementioned traits as well as some additional data regarding the equation of state, nuclear energy generation rate, and opacity, the mathematical models of stellar structures and their properties are examined. These stellar models explain how the star's mass, pressure, temperature, and luminosity change as it moves away from its centre. The assumptions of thermal equilibrium and hydrostatic equilibrium show that the mathematical model, which uses equations to describe the star's interior structure, does not depend on time. Kourganoff (1973) [141 &25].

In a recent study, Ferro et al. (2004) [48] demonstrated that a very small deviation from the Maxwell-Boltzmann particle distribution and the use of simple statistical mechanics can be applied to describe the modified nuclear reaction rates in stellar plasmas. This is consistent with the need for modification to the nuclear reaction rates of stellar plasmas and their chemical makeup. Nonlinear reaction-type (kinetic) equations have few exact solutions that are known. For further information, read Kourganoff (1973) [91] and Haubold & Mathai (2000) [67], which illustrate how a linear kinetic equation's solution describes minor variations from the nonlinear kinetic equation's equilibrium solution. It is possible to determine the rate of change of by for any response that has a time-dependent characteristic.

If an arbitrary reaction is characterized by a time dependent quantity $N = N(t)$ then it is possible to calculate the rate of change of ${dN}/{dt}$ by mathematical equation

$$\frac{dN}{dt} = -d + p$$

where d is the destruction rate and p is the production rate of N. In general, through feedback or other interaction mechanisms, destruction and production depend on the quantity N itself: $d = d(N)$ or $p = p(N)$ which is a complicated dependence since the destruction or production at time t depends not only on $N(t)$ but also on the past history $N(\tau), \tau < t$ of the variable N. This may be formally given by the following equation (Haubold & Mathai 2000)[67].

$$\frac{dN}{dt} = -d(N_t) + p(N_t)$$

where the function N_t is defined by $N_t(t^*) = N(t - t^*), t^* > 0$
Haubold & Mathai (2000) [67] studied a special case of this equation, when spatial fluctuations or In homogeneities in the quantity $N(t)$ are neglected, given by the equation

$$\frac{dN_i}{dt} = -c_i N_i(t) \tag{3}$$

with the initial condition that $N_i(t = 0) = N_0$ is the number density of species i at time $t = 0$ and constant $c_i > 0$ known as the standard kinetic equation. A detailed discussion of the above equation is given in Kourganoff (1973) [91]. The solution of the above standard kinetic equation can be put into the following form:

Mathematical Prerequisites:

A generalization of the Mittage-Leffler function (Mittage-Leffler, 1903,1905)[124][125]

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha + 1)}, (\alpha \in C, Re(\alpha) > 0) \tag{4}$$

was introduced by wiman(1905) [189] in the general for

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(n\alpha+\beta)}\ ,(\alpha,\beta\in C,Re(\alpha)>0)\ \ (5)$$

The main result of these functions are available in the handbook of Erdelyi Magnus. Oberhettinger and Tricomi (1955,Section18.1)[42][43] and the monographs written by Dzherbashyas(1966,1993)[32][33], Prabhakar(1971)[143] introduced a generalization of (5) in the form

$$E^{\gamma}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n z^n}{\Gamma(n\alpha+\beta)n!}\ ,(\alpha,\beta,\gamma\in C,Re(\alpha)>0)\ (6)$$

Where

$$(\gamma)_0=1,\qquad (\gamma)_k=\gamma(\gamma+1)(\gamma+2)\dots\dots\dots(\gamma+k-1)\ (k=1,2\dots..)$$
$$\gamma\neq 0 \qquad (4.2.4)$$

For $\gamma=1$

$$E^{1}_{\alpha,\beta}(z)=E_{\alpha,\beta}(z),$$

For $\gamma=1,\beta=1$

$$E^{1}_{\alpha,1}(z)=E_{\alpha}(z)\qquad (7)$$

The Mellin-Barnas integral representation for this function follows from the integral

$$E^{\gamma}_{\alpha,\beta}(z)=\frac{1}{\Gamma(\gamma)}\frac{1}{2\pi\omega}\int_{\Omega}\frac{\Gamma(-\xi)\Gamma(\gamma+\xi)(-z)^{\xi}}{\Gamma(\beta+\xi\alpha)}d\xi\qquad (8)$$

where $\omega=(-1)^{1/2}$ The contour Ω is straight line parallel to the imaginary axis at a distance 'c' from the origin and separating the poles of $\Gamma(-\xi)$ at the point $\xi=\nu(\nu=0,1,2\dots.)$ from those of $\Gamma(\gamma+\xi)$ at the points $\xi=-\gamma-\nu(\nu=0,1,2\dots.)$. If we calculate the residues at the poles of $\Gamma(\gamma+\xi)$ at the points $\xi=-\gamma-\nu(\nu=0,1,2\dots.)$ then it gives the analytic continuation formula of this function in the form

$$E^{\gamma}_{\alpha,\beta}(z)=\frac{(-z)^{-\gamma}}{\Gamma(\gamma)}\sum_{\nu=0}^{\infty}\frac{\Gamma(\gamma+\nu)}{\Gamma(\beta-\alpha\gamma-\alpha\nu)}\frac{(-z)^{-\nu}}{\nu!},|z|>1\qquad (9)$$

From (2.7) it follows that for large z its behavior is given by

$$E^{\gamma}_{\alpha,\beta}(z)\sim O(|z|^{-\gamma}),|z|>1\qquad (10)$$

The H-function is defined by means of Mellin-Barnes type integral in the following manner (Mathai and Saxena,1978 p-2) [114]

$$H^{m,n}_{p,q}(z)=H^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right]=H^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_1,A_1)\dots.(a_p,A_p)\\(b_1,B_1)\dots.(b_q,B_q)\end{matrix}\right.\right]$$
$$=\frac{1}{2\pi i}\int\theta(s)z^{-\xi}\,d\xi\qquad (11)$$

$$\text{where }\theta(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j+B_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j-A_j\xi)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-B_j\xi)\prod_{j=n+1}^{p}\Gamma(a_j+A_j\xi)}\qquad (12)$$

$$m,n,p,q\in N_0\ with\ 1\le n\le p,1\le m\le q,A_j,B_j\in R_+\ a_j,b_j\in R$$
$$(i=1,2\dots..p,j=1,2\dots\dots q)\qquad \mathbf{(13)}$$
$$A_i(b_j+k)\neq B_j(a_i-l-1)\ \ (k,l\in N_0\,;i=1,2\dots.n,j=1,2\dots m)$$

Where we employ the usual notations $N_0=(0,1,2\dots)$ $R=(-\infty,\infty)$ $R_+=(0,\infty)$ and C defines the complex number field. Ω is a suitable contour separating the poles of $\Gamma(b_j+B_j\xi)$ from those of $\Gamma(1-a_j-A_j\xi)$.

A detailed and comprehensive account of the H-function along with convergence condition is available from Mathai and Saxena (1978) [114]

It follows from (4.2.7) that the generalized Mittag-Leffler function

$$E^{\gamma}_{\alpha,\beta}(z)=\frac{1}{\Gamma(\gamma)}H^{1,1}_{1,2}\left[-z\left|\begin{matrix}(1-\gamma,1)\\(0,1)(1-\beta,\alpha)\end{matrix}\right.\right](\alpha,\beta,\gamma\in C,Re(\alpha)>0))$$
$$(14)$$

Putting $\gamma=1$ in(14)

$$E_{\alpha,\beta}(z)=H^{1,1}_{1,2}\left[-z\left|\begin{matrix}(0,1)\\(0,1)(1-\beta,\alpha)\end{matrix}\right.\right]\qquad (15)$$

If we further take $\beta=1$ in (15) we get

$$E_{\alpha}(z)=H^{1,1}_{1,2}\left[-z\left|\begin{matrix}(0,1)\\(0,1)(0,\alpha)\end{matrix}\right.\right]\qquad (16)$$

From Mathai and Saxena (1978,p.49) [11] it follows that the cosine transform of the H-function is given

$$\int_0^{\infty}t^{\rho-1}\cos kt\,H^{m,n}_{p,q}\left[at^{\mu}\left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right]dt$$
$$=\frac{\pi}{k^{\rho}}H^{n+1,m}_{q+1,p+2}\left[\frac{k^{\mu}}{a}\left|\begin{matrix}(1-b_q,B_q)\left(\frac{1}{2}+\frac{\rho}{2},\frac{\mu}{2}\right)\\(\rho,\mu)(1-a_p,A_p)\left(\frac{1}{2}+\frac{\rho}{2},\frac{\mu}{2}\right)\end{matrix}\right.\right]\qquad (17)$$

The Riemann-Liouvile fractional integral of order $\upsilon\in C$ is defined by Miller and Ross(1993,p.45;) [32] see also Srivastva and saxena,2001) [180].

$$_0D_t^{-\nu}f(t)=\frac{1}{\Gamma(\nu)}\int_0^t(t-u)^{\nu-1}f(u)du\qquad (18)$$

where $Re(\nu)>0$ following Samko, S.G., Kilbas, A. A. and Marichev, O.I. (1993,p.37) [159] we define the fractional derivative for $\alpha>0$ in the form

$$_0D_t^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_0^t\frac{f(u)}{(t-u)^{\alpha-n+1}}du,(n=[Re(\alpha)]+1)\qquad (19)$$

where $[Re(\alpha)]$means the integral part of $Re(\alpha)$.
In particular, if $0<\alpha<1$

$$_0D_t^{\alpha}f(t)=\frac{d}{dt}\int_0^t\frac{f(u)du}{(t-u)^{\alpha}}\qquad (20)$$

And in $\alpha=n\in N$ then

$$_0D_t^{\alpha}f(t)=D^nf(t)\qquad (21)$$

is the usual derivative of *n*.

From Erdelyi, A., Magnus, W., Oberhettinger, F., and Tricomi, F.G (1954,p.182) [42] we have

$$L\{_0D_t^{-\nu}f(t)\}=s^{-\nu}F(s)\qquad (22)$$
$$F(s)=L\{f(t);s\}=\int_0^{\infty}e^{-st}\,f(t)dt\qquad (23)$$

where $Re(s)>0$

The Laplace transform of the fractional derivative is given by Oldham and spanier(1974,p.134,eq 8.1.3;) [133] see also (srivastva and saxena 2001) [180].

$$L\{_0D_t^{-\nu}f(t)\}=s^{\alpha}F(s)-\sum_{k=1}^{n}s^{k-1}\ {}_0D_t^{\alpha-k}f(t)|_{t=0}\qquad (24)$$

From among the numerous operators of fractional calculus studied in mathematical literature in one context or the other. We find it convenient to recall here the definition of the fractional calculus operator for a complex valued function $f(z)$.

MAIN RESULTS

In this section we present the solution of fractional kinetic equation with Laplace and Fourier transform in the form of following theorem

Theorem 1: Consider the fractional diffusion equation

$$N(x,t)-N_0t^{\mu-1}=-c^{\nu}{}_0D_t^{-\nu}\ {}_0D_x^{\nu}\,N(x,t)\ ,$$

with initial condition

$$_0D_t^{\nu-k}\,N(x,t)|_{t=0}=0\ \ and\ {}_0D_t^{-\nu-k}\,N(x,t)|_{x=0}=0,\qquad k=1,2\dots n$$
$$N(x,t)=\frac{N_0\Gamma(\mu)}{c^t}\,H^{1,0}_{1,1}\left[\frac{|x|^{\nu}}{(ct)^{\nu}}\left|\begin{matrix}(\mu+\nu,\nu)\\(1+\nu,\nu)\end{matrix}\right.\right]$$

Proof: $N(x,t)-N_0t^{\mu-1}=-c^{\nu}{}_0D_t^{-\nu}\ {}_0D_x^{\nu}\,N(x,t)$

Apply Laplace and Fourier transform with time variable and space variable respectively to (1) we get

$$N^*(k,s)-N_0\frac{\Gamma(\mu)}{s^{\mu}}=-c^{\nu}k^{\nu}s^{-\nu}N^*(k,s)$$

$$N^*(k,s)\{1+(s/c)^{-\nu}k^\nu\} = N_0 s^{-\mu}\Gamma(\mu)$$
$$N^*(k,s) = N_0 s^{-\mu}\Gamma(\mu)\left\{1+\left(s/_{kc}\right)^{-\nu}\right\}^{-1}$$
$$= N_0 s^{-\mu}\Gamma(\mu)\sum_{r=0}^{\infty}\frac{(1)_r\left[-\left(s/_{kc}\right)^{-\nu}\right]^r}{r!}$$
$$= N_0\Gamma(\mu)\sum_{r=0}^{\infty}\frac{(1)_r(kc)^{r\nu}(-1)^r}{r!}s^{-\nu r-\mu}$$

where $N^*(k,s)$ Laplace and Fourier transform of $N(x,t)$
Taking inverse Laplace transform

$$N(k,t) = N_0\Gamma(\mu)\sum_{r=0}^{\infty}(kc)^{r\nu}(-1)^r L^{-1}\{s^{-\nu r-\mu}\}$$
$$N(k,t) = N_0\Gamma(\mu)\sum_{r=0}^{\infty}(kc)^{r\nu}(-1)^r\frac{t^{\mu+r\nu-1}}{\Gamma(r\nu+\mu)}$$
$$= N_0\Gamma(\mu)t^{\mu-1}E_{\nu,\mu}(-c^\nu k^\nu t^\nu)$$

which can we expressed in terms of H-function

$$= N_0\Gamma(\mu)t^{\mu-1}H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(1-\mu,\nu)\end{matrix}\right.\right]$$

Now take inverse Fourier transformation

$$N(x,t) = \frac{1}{\pi}\int_0^\infty \cos kx\, t^{\mu-1}N_0\Gamma(\mu)H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(1-\mu,\nu)\end{matrix}\right.\right]dk$$
$$= \frac{t^{\mu-1}N_0\Gamma(\mu)}{\pi}\frac{\pi}{|x|}H_{3,3}^{2,1}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(1,1)(\mu,\nu)(1,\nu/_2)\\(1,1)(1,\nu)(1,\nu/_2)\end{matrix}\right.\right]$$

Applying a result of Mathai and Saxena (1978,p.4.eq1.2.1)[114] the above expression becomes

$$N(x,t) = \frac{N_0\Gamma(\mu)}{|x|}H_{2,2}^{2,0}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(\mu,\nu)(1,\nu/_2)\\(1,\nu)(1,\nu/_2)\end{matrix}\right.\right]$$

If we employ the formula Mathai and Saxena (1978) [114]

$$x^\sigma H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right] = H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p+\sigma A_p,A_p)\\(b_q+\sigma B_q,B_q)\end{matrix}\right.\right]$$
$$N(x,t) = \frac{N_0\Gamma(\mu)}{ct}H_{2,2}^{2,0}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(\mu+\nu,\nu)(1,\nu/_2)\\(1+\nu,\nu)(1,\nu/_2)\end{matrix}\right.\right]$$
$$N(x,t) = \frac{N_0\Gamma(\mu)}{ct}H_{1,1}^{1,0}\left[\frac{|x|^\nu}{(ct)^\nu}\left|\begin{matrix}(\mu+\nu,\nu)\\(1+\nu,\nu)\end{matrix}\right.\right]$$

Theorem 2: Consider the fractional diffusion equation

$${}_0D_t^\nu\, N(x,t) - E_\nu(-d^\nu t^\nu) = -c^\nu\frac{\partial^2}{\partial x^2}N(x,t)$$

with initial condition

$${}_0D_t^{\nu-k}\, N(x,t)|_{t=0} = 0\ \ k = 1,2..n$$

Where $n = [Re(\nu)]+1$; c^ν is diffusion constant.

Then for the solution of (2) is given by

$$\frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)*\frac{1}{(ct)^\nu}H_{1,1}^{1,0}\left[\frac{|x|^2}{(ct)^\nu}\left|\begin{matrix}(1-\nu/_2,\nu)\\(0,2)\end{matrix}\right.\right]$$
$$-\frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)H_{1,2}^{1,1}\left[d^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right]$$

Proof:

$${}_0D_t^\nu\, N(x,t) - E_\nu(-d^\nu t^\nu) = -c^\nu\frac{\partial^2}{\partial x^2}N(x,t)$$

Applying the Fourier transform with respect to the space variable *x* and the Laplace transform with respect to the time variable *t.* we get

$$s^\nu N^*(k,s) - \frac{s^{\nu-1}}{s^\nu+d^\nu} = -c^\nu k^2 N^*(k,s)$$
$$\{s^\nu+c^\nu k^2\}N^*(k,s) = \frac{s^{\nu-1}}{s^\nu+d^\nu}$$
$$N^*(k,s) = \frac{s^{\nu-1}}{\{s^\nu+d^\nu\}\{s^\nu+c^\nu k^2\}}$$
$$= \frac{s^{\nu-1}}{c^\nu k^2-d^\nu}\left[\frac{1}{s^\nu+d^\nu}-\frac{1}{s^\nu+c^\nu k^2}\right]$$

To invert equation (2).It is convenient to first invert the Laplace transformation and Fourier transform. Apply inverse Laplace transform we obtain

$$N(k,t) = \frac{1}{c^\nu k^2 t^\nu}[E_\nu(-d^\nu t^\nu) - E_\nu(-c^\nu k^2 t^\nu)]$$

Which can expressed in terms of H-function

$$N(k,t) = \frac{1}{c^\nu k^2-d^\nu}\left\{H_{1,2}^{1,1}\left[d^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right] - H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right]\right\}$$

Invert the Fourier transform

$$N(x,t)$$
$$= \frac{1}{\pi}\int_0^\infty coskx\frac{1}{c^\nu k^2-d^\nu}\left\{H_{1,2}^{1,1}\left[d^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right]dk\right.$$
$$\left.-\frac{1}{\pi}\int_0^\infty coskx\frac{1}{c^\nu k^2-d^\nu}H_{1,2}^{1,1}\left[c^\nu k^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right]dk\right\}$$
$$= -\frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)H_{1,2}^{1,1}\left[d^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right] + \frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)$$
$$*\frac{1}{|x|}H_{3,3}^{2,1}\left[\frac{|x|^2}{(ct)^\nu}\left|\begin{matrix}(1,1)(1,\nu)(1,1)\\(1,2)(1,1)(1,1)\end{matrix}\right.\right]$$
$$= -\frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)H_{1,2}^{1,1}\left[d^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right]$$
$$+\frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)*\frac{1}{(c^\nu t^\nu)^{1/_2}}H_{2,2}^{2,0}\left[\frac{|x|^2}{(ct)^\nu}\left|\begin{matrix}(1-\nu/_2,\nu)(1/_2,1)\\(0,2)(1/_2,1)\end{matrix}\right.\right]$$
$$= \frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)*\frac{1}{(ct)^\nu}H_{1,1}^{1,0}\left[\frac{|x|^2}{(ct)^\nu}\left|\begin{matrix}(1-\nu/_2,\nu)\\(0,2)\end{matrix}\right.\right]$$
$$-\frac{1}{2d^{\nu/_2}}\sin\left(d^{\nu/_2}x\right)H_{1,2}^{1,1}\left[d^\nu t^\nu\left|\begin{matrix}(0,1)\\(0,1)(0,\nu)\end{matrix}\right.\right]$$

The fractional kinetic equation has been extended to generalized fractional equation theorems (1) and (2). Their respective solutions are given in terms of Mittag-Leffler function and their generalization, which can also be represented as Fox's H-function.

CONCLUSION

In this paper we derive the solution of fractional kinetic equation with Laplace and Fourier transform. Their respective solutions are given in terms of Mittag-Leffler function and their generalization, which can also be represented as Fox's H-function. and the solution of generalized fractional kinetic equation involving Saigo-Maeda operator .The result proved in this chapter are application to wide range of engineering, astrophysics and physical science.

REFERENCES

1. Perdang, J, : Lecturer notes in steller stability, part I and II instito di Astronomia, Padov, (1976).
2. Clayton,D.D. : Principal of steller Evolution and Nucleosynthesis 2nd ed. City chicago: chicago Univ. chicago Press(1983).
3. Ferro, F. Lavogo.A., Quarati,P. : Temperature dependence of modified CNO nuclear reactions rates in dense stellar Plasmas (2003) arxiv:nuclth/0312106v1.
4. Kourganoff. V. : Introduction to the physics of steller Interiors D.Reidel Publishing company, Dordrecht(1973).
5. Haubold. H.J.,and Mathai, A.M., The fractional kinetic equation and thermonuclear functions, Astrophysics and space science Ap J., 327(2000) 53-63.
6. Mittag-Leffler function, M.G. : 1903 Sur la nouvelle function $\mathbf{E}_\propto(\mathbf{x})$, Comptes Rendus Acad. Sci, Paris (Ser.II) 137,554-558.
7. Mittag-Leffler function, M.G. : 1905 *Sur la representation analytique d'une branche uniforme d'une function monogene, Acta Math.*29. 101-18.
8. Erdelyi, A., Magnus,W., Oberhettinger, F and Tricomi, F.G. : Table of Integral Transform, McGraw-Hill, New York Vol 20, 1954.
9. Erdelyi, A., Magnus, W., Oberhettinger, F., and Tricomi, F.G. : 1955, *Higher Transcendental Functions, Vol.3, McGraw-Hill, New York-Toronto-London.*
10. Dzherbashyam,M.M.:1966,Integral transform and representation of function in complex domain(in Russian),Nauka, Moscow.
11. Dzherbashyam,M.M.: 1993, Harmonic Analysis and Boundary Value Problems in the Complex Domain, Birkhaauser Verlag,Basel.
12. Prabhakar, T. R. : A singular integral equation with a generalized Mittag-Leffler function in the kernel, Yokohama, Math., J., 19, 1971, 7-15.

13. Ahmed, H. M., On some fractional inegro-differential equations, Int. J. Contemp. Math. Science. Vol. 3, No. 2, 2008, 63-70.
14. Mathai,A.M and Saxena,R.K, : 1978, *The H-function with application in Statistics and other disciplines,Halsted Press[John Wiley and Sons*.
15. Srivastava, H. M. and Saigo, M. : Multiplication of fractional calculus operators and boundary value problems involving the Euler-Darboux equation, J. Math. Anal. Appl., 121, 1987, 325-369.