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Optimal Inspection under Moral Hazard and Limited Liability of Polluter

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Abstract.
We have considered an environmental pollution that catastrophically destroys the environment once it occurs. While this kind of pollution could be avoided to some extent through precautionary activity, efforts to prevent pollution could not be observed by a government without inspection. In addition, the polluter might not be able to afford to compensate for the damage. The first best has not been achieved in the literature when moral hazard and limited liability are considered at the same time. By generalizing other policies, including the strict liability rule and the negligence rule, we derive an optimal inspection policy under moral hazard and limited liability. The optimal policy is composed of advance payment and ex-post payment after inspection. In other words, we can consider the optimal policy as a deposit/refund system. We derive the second-best policy by taking account of inspection cost.

Keywords: Moral hazard, Limited liability, Inspection, Environmental accident

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1. Introduction

Traditional monitoring and enforcement as an environmental policy is becoming controversial. Instead, voluntary programs and information policies like eco-labelling are increasingly advocated. However, recent survey evidence suggests that a large portion of the improvement in environmental performance being observed in many areas can be attributed to traditional economic incentives resulting from monitoring and enforcement, including liability (Gray and Shimshack 2011).

In this paper, we consider an environmental pollution that catastrophically destroys the environment once it occurs, such as accidents in nuclear power plants and oil tanker accidents. Even though such an environmental accident could be avoided through precautionary activity, it is quite difficult for a government to observe the actual effort to prevent an accident. In other words, the government will face a *moral hazard* problem. Shavell (2011) advocates that the liability rule has great advantages over other policies like tax in controlling such harmful externalities. The strict liability rule, which is the simplest version of liability rules, requires the polluter to compensate for the damage caused by the pollution. The social optimum or the first best can be achieved under that rule even if the government cannot observe precautionary activity or efforts to prevent pollution – in other words, even in the presence of *moral hazard*. Another advantage of liability over other policies is that the administrative cost can be saved under a liability rule since investigation and enforcement are necessary only in the case of accidental pollution.

However, the polluter might not be able to afford to compensate for such serious damage. *Limited liability*, which is known as a judgement proof problem in the literature of Law and Economics, is another source that can prevent the first best from being achieved. Since the seminal papers were published by Summers (1983) and Shavell (1986), it is well known that it is difficult to attain the first best, if *moral hazard* and *limited liability* should be considered at the same time.

There are many policies to alleviate this problem. Extending liability to the third party is one of them (see, for example, Pitchford (1995), Boyer and Laffont (1997), Lewis and Sappington (1999), Balkenborg (2001), Lewis and Sappington (2001), Dionne and Spaeter (2003), Hutchinson and van't Veld (2005), Hiriart and Martimort (2006), and Che and Spier (2008)). The extended liability is a rule where parties such as lenders, who have the contractual relationship with insolvent polluters, have to be liable for the remaining liability. The literature shows that partial liability rather than full liability to the lender may encourage polluters to make the second-best level of effort when effort is not observable (see Boyer and Laffont (1997) and Dionne and Spaeter (2003), for example).

Financial responsibility is another remedy to mitigate the inefficiency caused by limited liability and moral hazard. The most common instrument of financial responsibility is compulsory liability insurance. But as it is well known, compulsory liability insurance induces the efficient level of prevention only when the insurer is able to observe the precaution level performed by the firm (Shavell (1986) and Polborn (1998)). On the other hand, minimum asset requirement as a remedy for the limited-liability problem is investigated by Shavell (2005). While such a requirement can mitigate the problem, banning the operation of firms that do not meet the requirement might decrease social welfare. It is shown that compulsory liability insurance may be inferior to asset requirement when insurers cannot observe levels of effort.

Some researchers relax the assumption of *moral hazard*. They assume that the effort to prevent accidents is observable only if inspection is conducted. The negligence rule is advocated by many researchers, including Shavell (1986), Miceli and Segerson (2003), Ganuza and Gomez (2008), Shavell (2011), and Shavell (2013). This rule suggests inspection only in the case of an accident.

If insufficient effort compared with the first best is detected, then the firm will be required to pay the maximum amount of money that it can afford to pay. Otherwise, any penalty will never be imposed on the firm. The first best will not always be attained under this rule, although it can mitigate the inefficiency.

In this paper, we will hold the same assumption that the effort to prevent accidental pollution is observable only if inspection is conducted. We will derive the optimal inspection policy under moral hazard and limited liability. Jost (1996) also introduced monitoring and showed that the first best can be restored by combining a compulsory liability insurance with a liability rule. The inspection rule considered in this paper is more general than that in the literature, since the probability of inspection depends on occurrence of pollution.

We are supposed to go through a two-step procedure to derive the optimal policy. As the first step, we will derive the optimal policy that can encourage a firm to choose any target effort while minimizing the cost of inspection. The optimal policy is composed of advance payment and ex-post payment after inspection. Total payment of a firm must be set lower than the liability imposed on the firm. We show that the optimal policy is considered as a deposit/refund system. A firm is required to pay a fixed amount of money as a deposit in advance but a refund will be returned to the firm if it is found through inspection to be compliant with the standard of effort. The optimal policy works well so that any target effort including the first best can be achieved under the policy. However, the social cost will not be minimized at the first-best level of effort when we take inspection cost into account. We derive the second-best target of effort as the second step. We show that the second-best effort is smaller than the first best.

We show that there is a trade-off between the magnitude of liability and frequency of inspection. As liability imposed on a firm increases, the inspection frequency can be reduced. In addition, we show that if liability is larger than twice as much as the target effort, inspection is *almost unnecessary* to attain the first-best effort. It is also shown that the optimal policy is superior to the strict liability rule and the negligence rule.

The rest of the paper is organized as follows. The next section provides a theoretical background to the problem that we address in this paper. Our scheme is proposed in Section 3. Section 4 gives an interpretation of the optimal policy. We derive the second-best target effort by taking inspection cost into account in Section 5. Section 6 provides the conclusion.

2. Theoretical Background

We consider an environmental pollution that catastrophically destroys the environment once it occurs. We assume that such a pollution might be avoided to some extent by precautionary activity or effort. We consider a simple model. Let x stand for pecuniary effort to prevent environmental accidents. Probability of pollution is represented by a decreasing and convex function $p(x)$. We assume that pollution incurs a fixed amount of damage represented by D. The first-best effort will minimize the expected social cost $x + p(x)D$. The first-best effort represented by x_{fb} satisfies the following first-order condition:

$$1 + p'(x)D = 0, \qquad (1)$$

where the subscript *fb* represents first best.

Suppose that the effort to prevent pollution is not observable while a firm can afford to compensate for the damage. In this case, the strict liability rule will encourage the firm to make the first-best effort. To see this, note that the firm will choose x to minimize the total cost represented by $x + p(x)D$ under the strict liability rule.

By contrast, suppose that the firm cannot afford to compensate for the damage while in turn the effort is observable. It is still possible to attain the first best by a combination of tax and subsidy as shown by Proposition 1.

Proposition 1

Suppose that the effort to prevent pollution is observable. Let us consider a subsidy combined with tax. A subsidy represented by τ will be provided to the firm per unit of effort that exceeds the first-best level or x_{fb}. In addition, the firm will be required to pay a fixed amount of tax represented by *T* in the case of pollution. The first best can be attained if the subsidy rate τ is set at $\tau = 1 - \frac{T}{D}$ for any *T*.

Proof.

Under this policy, the firm will choose effort *x* to minimize the cost represented by $x - \tau(x - x_{fb}) + p(x)T$. The first-order condition for this problem will be written as

$$1 - \tau + p'(x)T = 0 \qquad (2)$$

By comparing (1) and (2), we can see that the first best can be attained if we set the subsidy rate at $\tau = 1 - \frac{T}{D}$. Note that we can choose any positive value for *T*. Therefore, by letting *T* be infinitesimal, the minimized cost will converge to x_{fb}. It means that any limited-liability constraint will not bind as long as the firm can afford to pay the first-best effort or x_{fb}.

However, it is quite difficult to attain the first best if we have to address *moral hazard* and *limited liability* at the same time.

3. Optimal policy with advance and ex-post payment and probabilistic inspection

3.1. The model

We relax the assumption of moral hazard following Shavell (1986), Jost (1996) and Miceli and Segerson (2003). It is assumed that the effort to prevent pollution is observable only if inspection is conducted. We derive the optimal inspection policy to achieve the target level of effort while minimizing the inspection cost under several conditions, including limited liability and the budget constraint for the government.

We consider a more general inspection rule compared with the strict liability rule and the negligence rule, which require inspection with probability one only when pollution occurs. In our model, inspection is a probabilistic event depending on whether pollution occurs. The probability of inspection in the case of pollution is represented by s, while that in the no-pollution case is represented by q. Similar probabilistic monitoring is considered by Jost (1996), but the probability of monitoring is fixed irrespective of occurrence of pollution.

The policy considered in this paper contains two-part payments, or advance payment and ex-post payment, which are represented by K_0 and K_1, respectively. The firm is required to pay K_0 prior to deciding on their level of effort. Note that it cannot be dependent on the actual effort x, and it must be set at a fixed level. In addition to advance payment K_0, ex-post payment K_1 will be imposed on the firm after a level of effort is chosen and when inspection is conducted. Note that it can depend on the actual effort x, since the effort level is revealed through inspection. We assume that the effort can be represented by expenditure on precautionary activity – that is, it is pecuniary[d]. Total payment by the firm, including effort, advance payment, and ex-post payment should not exceed a fixed amount of money, represented by L. This setting of the policy reflects limited liability of the firm. We assume that liability L is exogenously given. Timing of events under the policy is described in Figure 1.

We should admit that it is not difficult to achieve the first best in our setting, since we assume that inspection perfectly reveals the actual effort. In other words, effort to prevent pollution becomes observable, if inspection is always conducted with probability one irrespective of occurrence of pollution. However, inspection might be costly in some circumstances. We have to derive the optimal policy that can achieve the second-best effort by taking inspection cost into account. We are supposed to meet the goal in two steps. Firstly, in this section we will seek the minimum frequency of inspection that will make a target level of effort represented by x_t be realized on the equilibrium. The target level of effort x_t can be different from the first-best effort x_{fb}, which is defined by (1), as long as it is less than the first-best effort. Secondly, the optimal target of effort, or the second-best effort, represented by x_{sb} will be derived by taking inspection cost into account in Section 5, where the subscription *sb* represents second best.

The problem to be solved in the first step is written as follows:

$$\min_{s,q} \pi = sp(x_t) + q(1 - p(x_t))$$

subject to

$$\operatorname*{argmin}_x \; C(x, s, q) = x_t \qquad (3)$$

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[d] Some authors assume that precautionary effort is non-monetary (Summers (1983), Shavell (1986), and Jost (1996)), while others assume a pecuniary effort (Beard (1990) and Dari-Mattiacci and Geest (2006)). The main results in this paper do not depend on this assumption, even though the analysis becomes simpler in the case of non-monetary effort.

$$x_t + K_0 \leq L \quad (4)$$

$$x + K_0 + K_1 \leq L \quad (5)$$

$$K_0 + \left(1 - p(x_t)\right)qK_1 + p(x_t)sK_1 \geq 0, \quad (6)$$

where $C(x, s, q)$ represents the cost of the firm and it is composed of effort, advance payment, and the expected ex-post payment; in other words,

$$C(x, s, q) = x + K_0 + \left(1 - p(x)\right)qK_1 + p(x)sK_1. \quad (7)$$

The inspection policy should encourage the firm to make the target level of effort on the equilibrium. Constraint (3) means that the cost is minimized when the firm complies with the standard or the target effort. Constraints (4) and (5) represent limited liability. Constraint (4) means that advance payment K_0 plus the target level of effort should not exceed the liability L. In other words, the government is allowed to request the firm to pay up to $L - x_t$ in advance before the level of effort is chosen. Note that the standard of effort (x_t), advance payment (K_0), ex-post payment (K_1), and inspection frequency (s and q) are announced in the beginning. On the other hand, if inspection is conducted, the government is also allowed to impose ex-post payment K_1 on the firm depending on the actual effort level revealed by inspection. The constraint (5) requires that total expenditure, including effort, advance and ex-post payments, should not exceed the liability represented by L. The constraint (6) is the budget constraint of the government, which means that the expected payment of the firm or the expected revenue of the government must be non-negative. Without this constraint, limited-liability restriction can be mitigated substantially by giving a big reward or subsidy to the firm if it is found to be compliant with the standard through inspection. We will later show that inspection is *almost unnecessary* to attain the first best if the budget constraint of the government does not bind (see Lemma 4).

There is a possibility that the firm has an incentive to violate the standard by choosing an insufficient effort. Therefore, there must be a mechanism to encourage the firm to comply with the standard on the equilibrium. Our strategy is to increase the cost of the firm for non-compliance on one hand, and on the other hand to give a reward to the firm if compliance is proved by inspection. As a result, the government can reduce inspection frequency until the firm becomes indifferent between compliance and non-compliance.

3.2. Optimal payment rule

We will characterize the optimal policy as Proposition 2, which consists of the following eight lemmas. Prior to proving them, we will pose the following assumptions:

Assumption 1

Liability must be large enough to cover the first-best effort but it cannot cover the damage. In other words, we assume that $x_{fb} \leq L < D$.

Assumption 2

We assume that $p'(x) < 0$ and $p''(x) > 0$.

The first lemma shows that ex-post payment should be set at the maximum if an insufficient effort is detected.

Lemma 1

In order to increase the cost for non-compliance, K_1 should be set at the maximum if an insufficient effort is detected through inspection. In other words, it must be that

$$K_1 = L - K_0 - x > 0 \text{ for } x < x_t. \quad (8)$$

Proof.

See Appendix A. □

The rationality of raising ex-post payment to the maximum is that it can encourage the firm to choose compliance on the equilibrium by increasing the cost of the firm for non-compliance. As a result, the government can decrease the frequency of inspection until the firm becomes indifferent between compliance and non-compliance. To see this, note that the envelope theorem applied to the cost minimization yields

$$\frac{\partial C(x,s,q)}{\partial s} = p(x)K_1 > 0 \quad (9)$$

$$\frac{\partial C(x,s,q)}{\partial q} = \big(1 - p(x)\big)K_1 > 0. \quad (10)$$

Note also that K_1 is positive for $x < x_t$, since the following result follows from (8) and (4): $K_1 = L - K_0 - x > L - K_0 - x_t \geq 0$. Note that the last inequality follows the constraint (5). The signs of the derivatives of the cost with respect to inspection frequency represented by (9) and (10) imply that the government can reduce the inspection frequency until the increment of the cost induced by raising ex-post payment up to the limit will be cancelled out.

The same logic can be applied to the optimal advance payment. It is also optimal to set advance payment at the maximum, or $K_0 = L - x_t$. To see this, note that the envelope theorem yields $\frac{\partial C(x,s,q)}{\partial K_0} = 1$. Therefore, we can obtain the next lemma.

Lemma 2

The advance payment should also be set at the maximum; in other words,

$K_0 = L - x_t$. (11)

3.3. Optimal policy with inspection

Now we move on to seeking the optimal frequency of inspection. Suppose that the firm does not comply with the standard. In general, the less frequently inspections are conducted, the less compliant the firm becomes off the equilibrium. It implies that the government can reduce inspection frequency by encouraging the firm to choose zero-effort if it decides to become non-compliant with the standard. Since our policy is aimed at compliance with the target on the equilibrium, inspections that encourage more effort at non-compliance or off the equilibrium are not worth it. It should be stressed that the firm must be indifferent between zero effort and the standard on the equilibrium, so that it can be expected to be compliant with the standard.

By substituting (8) and (11) into (7), the cost function can be rewritten as

$$C(x, s, q) = x + L - x_t + \left(\left(1 - p(x)\right)q + p(x)s\right)(x_t - x) \quad \text{for } x < x_t. \tag{12}$$

It immediately follows from (12) that

$$L - x_t < C(0, s, q) = L - \left(1 - \left(1 - p(0)\right)q - p(0)s\right)x_t < \lim_{x \to x_t} C(x, s, q) = L. \tag{13}$$

It can also be shown that the cost function is increasing at $x = x_t$, since the derivative of the function with respect to x yields

$$\left.\frac{\partial C(x,s,q)}{\partial x}\right|_{x=x_t} = 1 - \left(\left(1 - p(x_t)\right)q + p(x_t)s\right) > 0. \tag{14}$$

Therefore, assuming convexity of the cost function[e], the firm will choose zero-effort if and only if

$$\left.\frac{\partial C(x,s,q)}{\partial x}\right|_{x=0} = 1 - \left(\left(1 - p(0)\right)q + p(0)s\right) + p'(0)x_t(s - q) \geq 0. \qquad (15)$$

Figure 2 illustrates the cost function without reward for compliance.

The next lemma defines the limit for frequency of inspection in the case of pollution or *s*, above which the firm will choose a positive effort even if it becomes non-compliant off the equilibrium.

Lemma 3

Suppose that the firm becomes non-compliant or $x < x_t$ off the equilibrium. The frequency of inspection represented by (q, s) that encourages the firm to choose $x = 0$ should satisfy the following inequality:

$$s \leq \frac{1}{p(0) - p'(0)x_t} + \frac{p(0) - p'(0)x_t - 1}{p(0) - p'(0)x_t} q. \qquad (16)$$

In addition, the firm will always choose zero effort irrespective of inspection frequency, if $p(0) - p'(0)x_t < 1$.

Proof.

The firm will choose $x = 0$ off the equilibrium when it decides to deviate from the standard, if and only if (15) is satisfied. This inequality can be rewritten as (16). If $p(0) - p'(0)x_t < 1$, it can be shown that the whole rectangle $[0,1] \times [0,1]$ is included in the area represented by (16), as illustrated by Figure 3. □

The case corresponding to $p(0) - p'(0)x_t > 1$ is illustrated by Figure 4. Let us call (16) the

[e] We will check the convexity of the cost function under the optimal policy by (24).

zero-effort constraint from here on. Frequent inspection is not necessary to encourage the firm to choose zero effort off the equilibrium, which means that the government can reduce the inspection frequency. We can consider the zero-effort constraint represented by (16) as a necessary condition for an efficient inspection frequency.

Note that inspection is *almost unnecessary* if we are allowed to ignore the budget constraint for the government that is to be considered later, since (q,s)=(0,0) satisfies zero-effort constraint represented by (16). This result is summarized in the following lemma.

Lemma 4

If the budget constraint for the government represented by (6) does not bind, inspection is *almost unnecessary* to encourage the firm to choose zero effort in the case of non-compliance.

Lemma 4 seems obvious, since the firm is expected to choose zero effort with less frequent inspection if it is not compliant with the standard. However, when we say that inspection is *almost unnecessary* to attain any target effort, it does not mean that $(q, s) = (0,0)$. A positive frequency of inspection is still necessary even though it is infinitesimal. Our strategy is to motivate the firm to comply with the standard on the equilibrium by giving it a reward if it is found to be compliant through inspection.

The next lemma will give us another piece for the optimal policy, or the subsidy to the firm that is found to be compliant with the standard through inspection.

Lemma 5

Under the optimal policy, K_1 should be set at a certain negative fixed level represented by *r* if the firm is found to be compliant with the standard. In other words, it must be that

$$K_1 = r^* < 0 \quad \text{for} \quad x \geq x_t, \tag{17}$$

where r^* is given by

$$r^* = -\left(\frac{1-(1-p(0))q-p(0)s}{p(x_t)s+(1-p(x_t))q}\right)x_t < 0. \tag{18}$$

Proof.

See Appendix B. □

The idea of the optimal policy is illustrated by Figure 5. The cost of the firm drops discontinuously at $x = x_t$ due to the subsidy.

The budget constraint for the government represented by (6) has not been considered yet. The constraint requires that the expected payment of the government must be non-negative on the equilibrium. To realize the reason why the budget constraint is necessary, let us consider a situation in which inspection is *almost unnecessary*. The reward for compliance goes to infinity in the absolute value as frequency of inspection becomes infinitesimal (see Lemma 5). The government can finance such an extremely big reward if it does not confront the budget constraint.

By substituting (11), (17), and (18) into (6), the budget constraint can be rewritten as

$$s \geq \frac{2x_t-L}{p(0)x_t} - \frac{1-p(0)}{p(0)}q. \tag{19}$$

The next lemma shows that inspection is still *almost unnecessary* if a sufficient liability can be imposed on the firm.

Lemma 6

If $2x_t \leq L$, inspection is *almost unnecessary* to encourage the firm to choose zero-effort in the case of non-compliance or off the equilibrium.

Proof.

According to Lemma 4, inspection is *almost unnecessary* as long as the budget constraint does not bind. We can easily see that the area represented by (19) includes the origin, or (0,0) in (q,s) plane if $2x_t \leq L$. □

However, inspection with positive frequency is necessary, when we suppose that $x_t \leq L < 2x_t$. The next two lemmas show that frequency of inspection will increase as liability decreases.

Lemma 7

Assume that $p(0) - p'(0)x_t < 1$. Then, the optimal inspection policy is characterized by

$s^* = \frac{2x_t - L}{p(0)x_t}$ and $q^*=0$, if $\left(2 - p(0)\right)x_t \leq L < 2x_t$, (20)

$s^*=1$ and $q^* = \frac{(2-p(0))x_t - L}{(1-p(0))x_t}$, if $x_t \leq L < \left(2 - p(0)\right)x_t$. (21)

Proof.

See Appendix C. □

Figure 6 illustrates the optimal path of inspection frequency when the liability decreases. The optimal path suggests that inspection in the case of pollution should be prioritized over one in the no-pollution case. The next lemma addresses the remaining case, which is represented by $p(0) - p'(0)x_t > 1$.

Lemma 8

Assume that $p(0) - p'(0)x_t > 1$.

The optimal inspection policy is characterized by

$$s^* = \frac{2x_t - L}{p(0)x_t} \quad \text{and} \quad q^*{=}0, \text{ if } 2x_t - \frac{p(0)x_t}{p(0) - p'(0)x_t} \leq L < 2x_t, \qquad (22)$$

$$s^* = \frac{(1 - p(0))(L - x_t) - p'(0)x_t(2x_t - L)}{-p'(0)x_t^2} \quad \text{and} \quad q^* = \frac{2x_t - L}{(1 - p(0))x_t} - \frac{p(0)}{1 - p(0)} s^*,$$

$$\text{if } x_t \leq L < 2x_t - \frac{p(0)x_t}{p(0) - p'(0)x_t}. \qquad (23)$$

Proof.

See Appendix D. □

By putting all lemmas together, we can summarize the optimal policy that can minimize the expected frequency of inspection or the inspection cost as the next proposition.

Proposition 2

The payment rule under the optimal policy will be characterized by

$K_0 = L - x_t$

$$K_1 \begin{cases} = r^* \leq 0 & \text{if } x \geq x_t \\ = x_t - x & \text{otherwise,} \end{cases}$$

where r^* is given by (18). The optimal inspection policy depends on the target effort (x_t) and the liability (L), as follows: The optimal frequency of inspection is given by (20) or (21) if $p(0) - p'(0)x_t < 1$, while it is characterized by (22) or (23) otherwise.

The optimal inspection problem remains to be examined in some respects. One of them is convexity of the cost minimization problem. In other words, we need to check whether the cost function remains convex with respect to x under the optimal policy. The second derivative of the cost function with respect to x can be calculated as

$$\frac{\partial^2 C}{\partial x^2} = (s^* - q^*)\big(-2p'(x) + p''(x)(x_t - x)\big) \quad \text{for } x < x_t. \qquad (24)$$

It follows from (24) that convexity is guaranteed if $s^* - q^* > 0$. We can see that this inequality is assured under the optimal frequency of inspection, which is given by Lemma 7 or Lemma 8.

We should also examine whether the optimal policy could prevent the firm from making an excessive effort. It can be shown that the cost function is an increasing function if $x \geq x_t$. To see this, the first derivative of the cost function can be calculated as

$$1 + (s^* - q^*)p'(x)r^* > 0 \quad \text{for } x \geq x_t. \qquad (25)$$

It follows that it is suboptimal for the firm to make an excessive effort. Beard (1990) shows that it is possible for the polluter (injurer in his model) to take too much care or effort compared with the first best under the strict liability rule. In our model, the firm will never do it, since excessive compliance is not compensated through a subsidy.

4. Interpretation of the optimal policy

The optimal policy derived in Proposition 2 can be considered as a deposit-refund system. The firm is requested to pay a deposit represented by K_0 in advance. Afterwards, a refund represented by r will be returned to the firm, if it is found to be compliant with the standard through inspection. Figure 5 illustrates how the optimal policy works to encourage the firm to comply with the standard on the equilibrium.

The optimal policy successfully breaks through the limited-liability restriction that has precluded the first best from being attained in the literature. Note that any target effort can be attained by the optimal policy as long as it is equal to the first-best effort or less. However, there is a trade-off between magnitude of liability and frequency of inspection. This is implied by the optimal path of inspection frequency, as illustrated by Figures 6 and 7. In other words, the less the liability becomes, the more frequently inspection should be conducted. Conversely, when the liability becomes large, the deposit-refund mechanism embedded in the policy will be enhanced. The government can take advantage of the enhanced mechanism to decrease inspection frequency, as shown by the next proposition.

Proposition 3

Assume that $L \leq 2x_t$. As the liability increases, the advance payment (deposit) and the subsidy for compliance (refund) will increase under the optimal policy while inspection frequency will decrease.

Proof.

According to Proposition 2, the advance payment K_0 is an increasing function of L. It follows from Lemma 7 and Lemma 8 that both s^* and q^* are decreasing in L for any case. If s^* and q^* decrease, it follows from Lemma 5 that the refund represented by r^* will increase. □

On the other hand, suppose that liability is more than twice as much as the target effort even though it cannot cover the damage (see Assumption 1). Limited-liability restriction usually emerges in this case. For example, the strict liability rule cannot attain the first best. Remarkably, however, inspection is *almost unnecessary* even in this case (see Lemma 6). In other words, the optimal policy derived in Proposition 2 can restore the first best without inspection as long as $L > 2x_t$. In addition, inspection is still *almost unnecessary* to attain the first best if the budget constraint does not bind (see Lemma 4).

Let us consider the optimal policy, under which inspection is *almost unnecessary.* According to Lemma 5, the reward for compliance represented by r^* goes to infinity in the absolute value as frequency of inspection becomes infinitesimal. Therefore, this policy could be interpreted as a kind of lottery. In other words, if the firm complies with the standard and if compliance is proved by much less frequent inspection, the firm will be given a big bonus as a reward for compliance.

The optimal paths of inspection frequency suggest that inspection should be prioritized in cases of pollution over those of non-pollution. Note that $s^* > q^*$ along the optimal path (see Figures 6 and 7). In order to understand the rationality of this, remember that the firm is encouraged to choose zero effort when it decides to be non-compliant. As a result, inspection in the case of pollution will be enhanced by the maximum probability of pollution, or $p(0)$. In other words, inspection in the case of pollution will effectively increase the cost in non-compliance.

Another remarkable feature of the policy is that the cost of the firm is fixed at x_t on the equilibrium, if liability is *less than* twice as much as the target effort, as stated by the next proposition. Note that the cost will usually increase in addition to effort under more prevalent policies. For example, Pigouvian tax will be an additional cost for firms and some of them might be encouraged to exit the market. In this respect, the optimal policy derived in this paper could avoid the distortion that would be generated by other policies.

Proposition 4

The expected cost of the firm on the equilibrium, which is represented by $C(x_t, s, q)$, is fixed at x_t, if $L \leq 2x_t$. Otherwise, it is fixed at $L - x_t$.

Proof.

Suppose that $L \leq 2x_t$. In this case the budget constraint for the government always binds (see Figures 6 and 7). It follows from (6) and (7) that the expected cost of the firm on the equilibrium can be calculated as

$$C(x_t, s, q) = x_t. \qquad (26)$$

On the other hand, if $L > 2x_t$, substituting (18) into (7) yields the following result:

$$\lim_{(s,q)\to(0,0)} C(x_t, s, q) = L - x_t. \qquad (27) \qquad \square$$

The expected cost of the firm on the equilibrium is illustrated by Figure 8.

The next two propositions deal with two other special cases. One is the strict liability rule and another is the negligence rule. Under the strict liability rule, inspection will always be conducted only when pollution occurs. Even though a firm that caused pollution is required to compensate for the damage, as a matter of fact, the firm will pay less than the damage because of limited liability. The next proposition shows that the strict liability rule is never optimal.

Proposition 5

The strict liability rule is not optimal.

Proof.

The strict liability rule is a special case of inspection policy. To see this, note that the same result under the strict liability rule can be replicated, if we set $(q, s) = (0,1)$, $K_0 = 0$, and $K_1 = L - x$. However, it follows from Proposition 2 that this set of policy variables is not optimal. □

Another special case is the negligence rule. Under this rule, inspection will also be conducted only when pollution occurs. When the polluter is found to be non-compliant with the standard, they will owe the maximum amount of payment that is equal to the liability.

Proposition 6

The negligence rule is not optimal either.

Proof.

The negligence rule is also a special case of inspection policy. The negligence rule can be replicated, if we set $(q, s) = (0,1)$, $K_0 = 0$, and

$$K_1 \begin{cases} = r = 0 & \text{if } x \geq x_t \\ = L - x & \text{otherwise.} \end{cases}$$

However, this set of policy variables is not contained in the optimal policy. □

5. The second-best policy

We will move to the second step to solve the original problem. The final question to be raised is how the target level of effort represented by x_t should be determined. It seems reasonable to think that the second-best effort will be determined so that the social cost, including inspection cost, can be minimized. We assume a fixed unit cost for inspection that is represented by *I*. Then the social cost represented by *SC* can be written as

$$SC = x_t + p(x_t)D + I\{p(x_t)s^* + (1 - p(x_t))q^*\}. \quad (28)$$

The terms in the bracket in (28) represent the expected inspection cost. The final proposition characterizes the second-best effort.

Proposition 7

The second-best target effort represented by x_{sb} that will minimize the social cost defined by (28) is smaller than the first-best effort; in other words, $x_{fb} > x_{sb}$.

Proof.

The second-best target effort represented by x_{sb} will be derived by the following first-order condition:

$$\frac{\partial SC}{\partial x_t} = \frac{\partial(x_t + p(x_t)D)}{\partial x_t} + I\left\{\left(\frac{\partial s^*}{\partial x_t} - \frac{\partial q^*}{\partial x_t}\right)p(x_t) + \frac{\partial q^*}{\partial x_t} + (s^* - q^*)p'(x_t)\right\} = 0. \quad (29)$$

Note that the first term is equal to zero at the first-best effort, or $x_t = x_{fb}$. Even though it looks complicated to calculate the terms in the bracket of (29), we can easily examine whether the inspection cost on the equilibrium will increase as the target effort increases. Suppose that $p(0) - p'(0)x_t < 1$. According to Lemma 3, the zero-effort constraint represented by (16) will not bind in this case. The feasible area for *s* and *q* is subject solely to the budget constraint, or (19), as illustrated by Figure 6. It is easy to see that the line represented by (19) will shift upward as x_t increases, which means that the feasible area will shrink. It follows that the minimized inspection cost will also increase as x_t increases. This means that the sum of the terms in the bracket in (29) is positive. Therefore, we can conclude that the second-best effort is smaller than the first-best effort, in other words, $x_{fb} > x_{sb}$ as illustrated by Figure 9.

By contrast, suppose that $p(0) - p'(0)x_t > 1$. In this case, the feasible area is constrained by both (16) and (19). We can see that the line represented by (16) will rotate anticlockwise on the point represented by $(q, s) = (1,1)$ as the target effort increases, as illustrated by Figure 10. On the other hand, the line represented by (19) will shift upward as the target effort increases. Those observations suggest that the feasible area will shrink as the target effort increases (see Figure 10). It also means that the minimized inspection cost will increase. Therefore, we can derive the same conclusion as in the first case that the second-best target of effort is smaller than the first-best effort.

□

Finally, we consider a special case in which inspection does not cost at all; in other words $I = 0$. In this case, full inspection, $(q, s) = (1,1)$ is socially optimal. It is also optimal to set the target effort at the first best, in other words $x_t = x_{fb}$. Note that the minimum level of liability, or $x_t (= x_{fb})$ is sufficient to attain the target effort under full inspection. When the liability is set at the target effort, advance payment is unnecessary, or $K_0 = 0$ (see Proposition 2). Ex-post payment will also be reduced to a simple form, which is represented by

$$K_1 \begin{cases} = r^* = 0 & \text{if } x \geq x_{fb} \\ = x_{fb} - x & \text{otherwise.} \end{cases}$$

Note that this is equivalent to the optimal policy when effort is observable.

6. Conclusion

We examined an optimal policy based on inspection in a situation where the effort to prevent pollution is not observable and the damage caused by pollution cannot be compensated by the polluter due to asset constraint. We have addressed moral hazard and limited liability at the same time.

We have derived an optimal inspection policy that will achieve the target effort with the minimum frequency of inspection under moral hazard and limited liability. The policy is composed of the standard for effort, the advance payment, the ex-post payment imposed on non-compliance, and the reward for compliance aside from inspection frequency. The optimal policy is considered as a deposit-refund system. Our strategy is to increase the cost to the firm in non-compliance on one hand, and on the other hand to give a reward to the firm if compliance is proved by inspection. As a result, the government can reduce inspection frequency until the firm becomes indifferent between compliance and non-compliance. At the same time, the firm is encouraged to choose zero effort off the equilibrium. Zero effort increases the probability of pollution will go up to the maximum and makes it more effective to inspect when pollution occurs. This feature of the policy is consistent with prioritizing inspection in the case of pollution.

The optimal policy works to make any standard for effort be realized on the equilibrium. It will also break through the liability restriction. Liability can be set at any level as long as it covers the standard effort. However, if the government is allowed to raise the liability, the deposit-refund system embedded in the policy can be enhanced and the inspection frequency can be reduced. In particular, if the liability is more than twice as much as the target effort, the first best can be attained without inspection, even though the firm cannot compensate for the damage. We also showed that the optimal policy is superior to the well-known policies, in other words, the strict liability rule and the negligence rule. We also derived the second-best standard to minimize the social cost, including the inspection cost.

We have considered a situation in which an environmental pollution catastrophically destroys the environment once it occurs. However, it should be stressed that our contribution can be applied to other areas of economics. Analysis of the effect of limited liability was initiated in Law and Economics (Summers (1983) and Shavell (1986)), and pollution in the model could be replaced by accidents in general.

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Appendix

A. Proof of Lemma 1

We suppose that the firm will choose an insufficient level of effort, or $x < x_t$ to minimize the cost represented by (7). Then we can see that the minimized cost will increase as K_1 increases. To see this, we substitute $K_1 = l - x$ into (7), where l is a parameter and it is equal to $L - K_0$ or less (see (5)). We apply the envelope theorem to the cost minimization to obtain the following inequality:

$$\frac{\partial C(x,s,q,l)}{\partial l} = \left(1 - p(x)\right)q + p(x)s > 0.$$

It means that l should be set at the maximum, $L - K_0$, in order to increase the cost for non-compliance. In other words, it must be that

$K_1 = L - K_0 - x$ (8)

under the optimal policy, as claimed. □

B. Proof of Lemma 5

Suppose that the firm is not compliant with the standard. Under the policy proposed by previous lemmas, the firm will choose $x = 0$ and the expected cost is given by (13). On the other hand, if the firm decides to be compliant with the standard x_t, the expected cost will be calculated as

$C(x_t, s, q) = L + \big(p(x_t)s + \big(1 - p(x_t)\big)q\big)r.$

If the subsidy represented by *r* is determined so that the two expected costs are equivalent to each other, the firm is expected to comply with the standard on the equilibrium. Such a critical value for *r* can be calculated as claimed. □

C. Proof of Lemma 7

According to Lemma 3, the zero-effort constraint represented by (16) will not bind if $p(0) - p'(0)x_t < 1$. Only the budget constraint represented by (19) will bind. The area that satisfies (19) in the rectangle $[0,1] \times [0,1]$ is illustrated by Figure 6. It can be shown that the optimal frequency of inspection is given either by (0,*s*) or by (q,1). To see this, note that the slope of the iso-expected-inspection-frequency line is represented by $-\frac{1-p(x_t)}{p(x_t)}$ on the equilibrium and it is smaller than that of the line represented by the budget constraint, or $-\frac{1-p(0)}{p(0)}$. The condition that $(2 - p(0))x_t < L$ means that $s < 1$. As illustrated by Figure 6, if L decreases further less than $(2 - p(0))x_t$, inspection in the no-pollution case will be implemented or $q > 0$ while keeping $s = 1$ as claimed. □

D. Proof of Lemma 8

Not only the budget constraint represented by (19) but also the zero-effort constraint represented by (16) will bind in this case, as shown by Lemma 3. The feasible area is illustrated by Figure 7. We can see that the optimal point is the intersection point of the line represented by (19) and the *s* axis if the liability is sufficiently high. The optimal point or the intersection point will move upward from the origin as the liability decreases from $2x_t$ to $2x_t - \frac{p(0)x_t}{p(0)-p\prime(0)x_t}$. However, when the liability becomes even smaller than $2x_t - \frac{p(0)x_t}{p(0)-p\prime(0)x_t}$, the zero-effort constraint will also bind. The optimal point will move along the line represented by (16) toward the corner represented by $(q,s)=(1,1)$. Note that full inspection represented by $(q,s)=(1,1)$ is the optimal policy if the liability is equal to the standard, or $L = x_t$.

□

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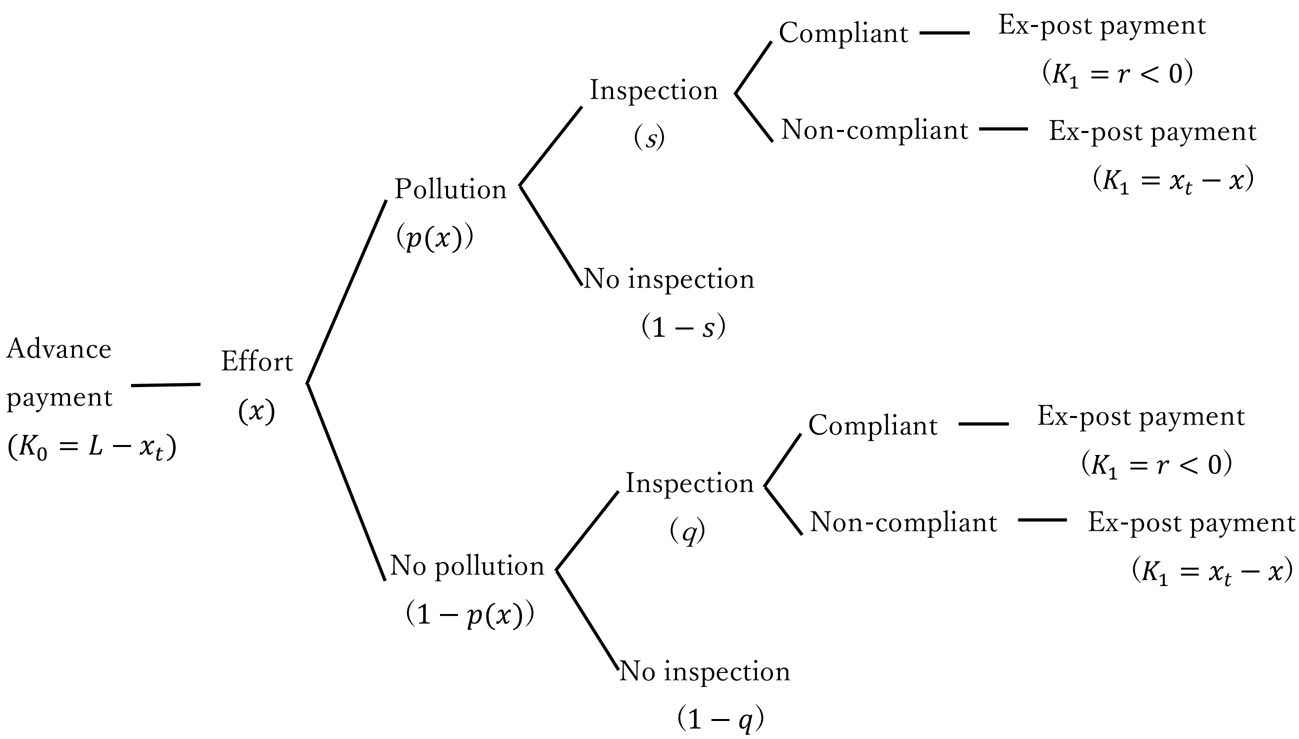

Fig. 1　Timing of events

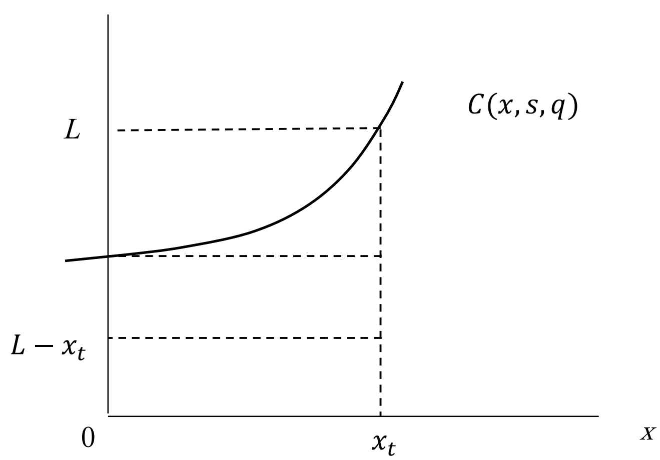

Fig. 2　The cost function without reward for compliance

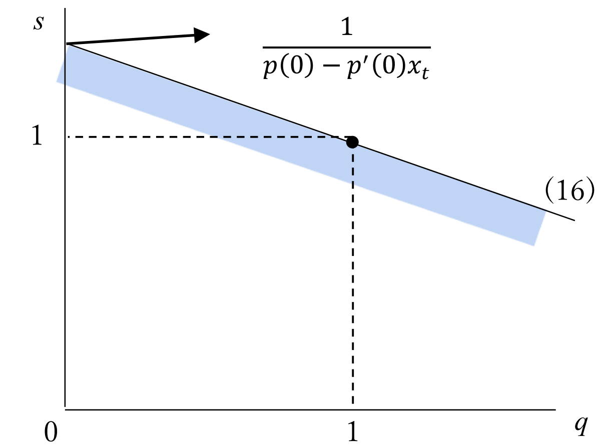

Fig. 3 The zero-effort constraint with $p(0)-p'(0)x_t<1$

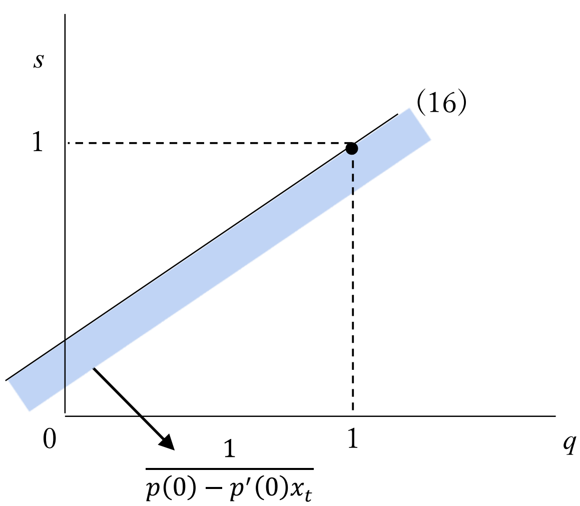

Fig. 4 The zero-effort constraint with $p(0)-p'(0)x_t>1$

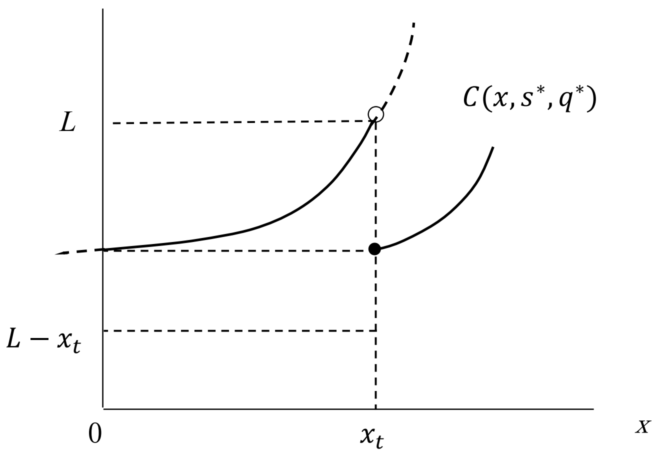

Fig. 5　The cost function under the optimal policy

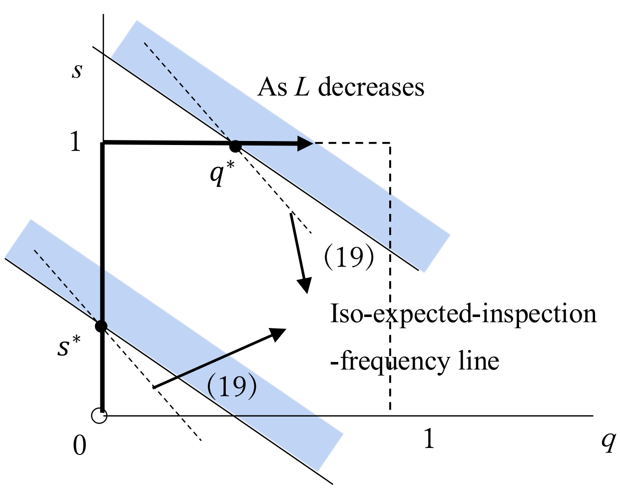

Fig. 6　Optimal path of inspection frequency with $p(0) - p'(0)x_t < 1$

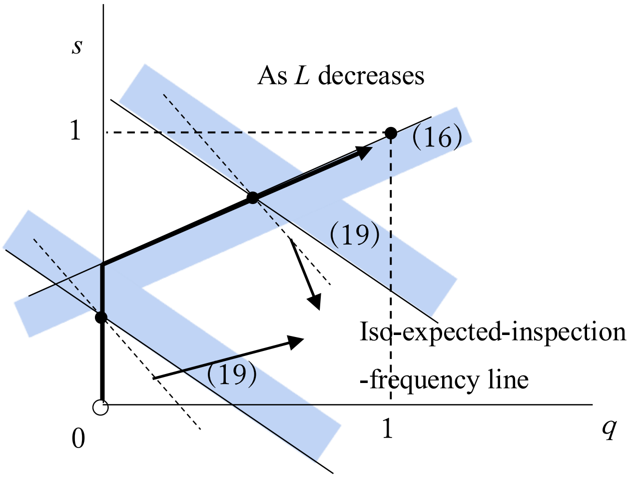

Fig. 7　Optimal path of inspection frequency with $p(0) - p'(0)x_t > 1$

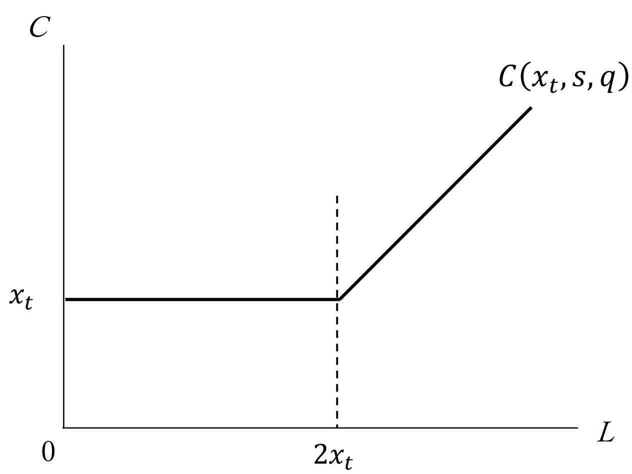

Fig. 8　The cost on the equilibrium

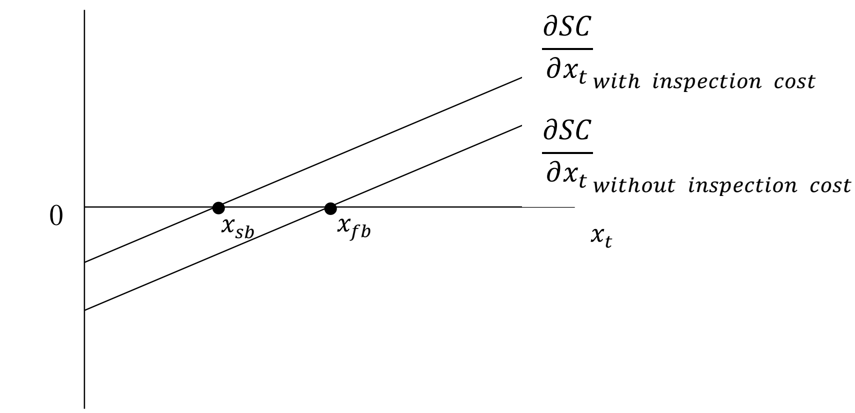

Fig. 9 The second-best effort

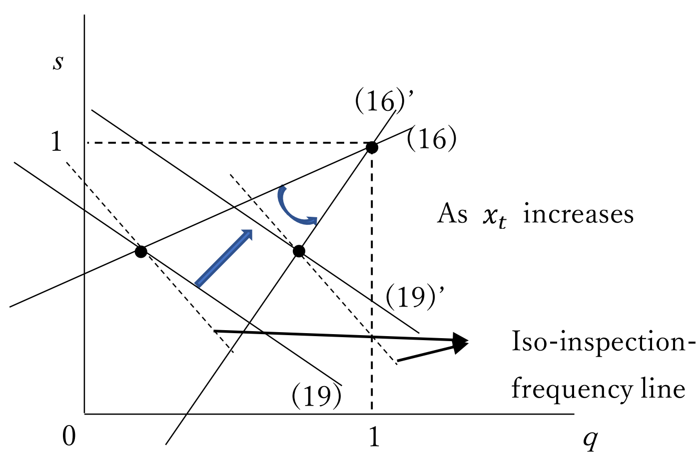

Fig. 10 The effect of increasing x_t on the feasible area