Application of multivariate regression on magnetic data to determine further drilling site for iron exploration

Research Article

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Abstract: In this study, a new approach of the multivariate regression model has been applied to make a precise mathematical model to determine further drilling for the detailed iron exploration in the Koohbaba area, Northwest of Iran. Furthermore, to figure out the additional drilling locations, the ore length to the total core ratio for the drilled boreholes has been used based on the geophysical exploration dataset. Hence, different regression analyses including linear, cubic, and quadratic models have been applied. In this study, the ore length to the total core ratio of the chosen drilled boreholes has been considered as a dependent variable; besides, the outputs of the magnetic data using the UP10 (10m upward-continuation), RTP (reduction to the pole), and A.S. (analytic signal) techniques have been designated as independent variables. Based on probability value (p-value), coefficients of determination (R^2 and R^2_{adj}), and efficiency formula (EF), the fourth regression model has revealed the best results. The accuracy of the model has been confirmed by the defined ratio of boreholes and demonstrated by four additional drilled boreholes in the study area. Therefore, the results of the regression analysis are reasonable and can be used to determine the additional drilling for the detailed exploration.

Keywords: multivariate regression, mathematical model, drilling, iron exploration, magnetic data

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1 Introduction

The modern mineral exploration is the definitive aim for the geophysical examination. Hence, several geophysical maps should generate to examine the underground mineral perception [1]. To determine the best zone, drilling of some boreholes is fundamental. Although the most reliable examination for the deposit potential is the drilling [2], it is the most expensive procedure of the mineral exploration [3]. Therefore, using proper methods is essential to decrease the drilling risks and improve the accuracy of the drilling sites [4,5]. Statistical methods play an important role to enhance the success rate and overcome the cost of mineral exploration [6–8].

In the past few years, the quantitative study of geoscientific data has increased rapidly [9]. There are several probabilistic, statistical, and mining models proposed for mineral exploration [4,5], such as logistic regression [2,10], ridge regression [11], multitemporal nonlinear regression [12], weight of evidence [13,14], artificial neural networks [15], Bayesian network classifiers [16], and multiple regression analyses [17,18]. All of these methods and techniques have shown promising results and have applied successfully to mineral resource appraisal [1].

Multivariate regression analysis is used successfully to model subsurface mineralization based on the geochemical dataset [19,20], iron mineral mapping [5], and rock properties predictions [21] and to improve prediction model to estimate the sediment yield [22]. In this study, a novel application of the multivariate regression method is proposed to determine additional drilling based on the geophysical exploration dataset. To this end, six different types of multivariate regression have been employed for the iron exploration by using several techniques including RTP (reduction to the pole), A.S. (analytic signal), and UP10 (Upward Continuation) to designate magnetic susceptibility concentrations. The outcomes of these methods have been compared with the log reports from eight different boreholes in the Koohbaba (Qoja-Kandi) area, and the results have demonstrated the proper accuracy of the technique. The log reports of the boreholes have been used to perform further drilling for iron exploration. To achieve this goal, multivariate regression analysis of the ground magnetic data layers has been performed.

1.1 Geological study

This study has been conducted at the Koohbaba area within the Urumieh-Dokhtar Magmatic Arc (UDMA), located in the Northwest of Iran. The study location has a surface area of 1.44 km^2, positioned between 46°59′40″ and 47°1′50″ east longitude and 36°51′30″ and 36°53′40″ north latitude, East Azerbaijan (Figure 1). Magmatic activity in UDMA was originated in the Eocene and continued to Pliocene with its climax in the Middle Eocene [23]. Moreover, concerning some recent research works [24,25], UDMA was dominated by the Eocene magmatic rocks, and this fact was confirmed by the geochemical analysis [26].

The UDMA has been dominated by plutonic rocks together with felsic volcanic rocks [23] and forms intrusive–exclusive complex with over 4 km thickness [28,29] and [30]. There is a wide range of composition in the study area, such as schist and shale (Kahar formation), dolomite and limestone (Elika formation), shale, sandstone and limestone (Shemshak formation), limestone, marl, sand stone, conglomerate, and andesite. Omrani et al. [26] have explained that UDMA volcanic rocks form a wide range in composition, which include andesite, minor basalts, and dacites. Magnetite associated with andesite units caused iron mineralization in the study area. The UDMA hosts large metal deposits such as iron and copper [32,33], as shown in Figure 2. Moreover, Mansouri et al. [31] have introduced some iron ore deposits close to the research area.

1.2 Multivariate regression

The regression analysis shows the relation between one or more responses (dependent variables) and one or more predictors (independent variables) and also predicts the values of the responses for a given set of predictors. Usually, these variables are quantitative, i.e., interval or ratio. The following mathematical formula can express the simple relationship between those variables:

$$Y = F(X_i), \tag{1}$$

where Y represents dependent variables and X expresses the independent variables. It would be a linear regression if Y and X_i have a linear relation; otherwise, it would be a nonlinear regression. The main aim of the regression analysis is to represent the dependent variables as an independent variable function [34]. In this section, the mathematics approach of this technique is presented, and an in-depth presentation of this subject is available in the literature [35].

Multivariate regression is a beneficial statistical method to evaluate the linear relationships between several independent and dependent variables [5]. Therefore, it is the multiple regression expansion with an equal number of equations as the number of response variables. One advantage of using multivariate analysis is that the type 1 error can be determined, and it does not cover the number of variables [34]. Consequently, this method is applied in this study. The linear regression model can be expressed as follows [36,37]:

$$Y_i = \beta(X_i) + \varepsilon_i \quad i = 1, 2, \ldots, n \tag{2}$$

where β is a p-dimensional column vector of unknown regression coefficients, Y_i is the ith component of the n-dimensional column vector Y, X_i is the ith row of the $n \times p$ design matrix X, and ε is the random error. The random error value indicates the amount of dispersion in the estimation of the Y value [19]. With n independent observation on Y and the associated values of Z_i, the complete model can be expressed as follows [38,39]:

$$\begin{aligned} Y_1 &= \beta_0 + \beta_1 X_{11} + \beta_2 X_{12} + \ldots + \beta_k X_{1k} + \varepsilon_1 \\ Y_2 &= \beta_0 + \beta_1 X_{21} + \beta_2 X_{22} + \ldots + \beta_k X_{2k} + \varepsilon_2 \\ &\cdots \\ Y_n &= \beta_0 + \beta_1 X_{n1} + \beta_2 X_{n2} + \ldots + \beta_k X_{nk} + \varepsilon_n , \end{aligned} \tag{3}$$

In the matrix form, it becomes:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} & \ldots & X_{1k} \\ 1 & X_{21} & X_{22} & \ldots & X_{2k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & X_{n1} & X_{n2} & \ldots & X_{nk} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}. \tag{4}$$

The error assumptions are as the following:

1. $E(\varepsilon) = 0$
2. $\mathrm{Cov}(\varepsilon) = \sigma^2 I$

where I is the $n \times n$ identity matrix. The ordinary least-square method has been used to estimate the β matrix [40,41], which is the regression coefficient matrix (Formula 5).

$$[\beta] = ([X]^T [X])^{-1} [X]^T [Y] \tag{5}$$

In multivariate regression, the relationship between the dependent variable Y and the independent variables X_i is measured by the coefficients of determination [35].

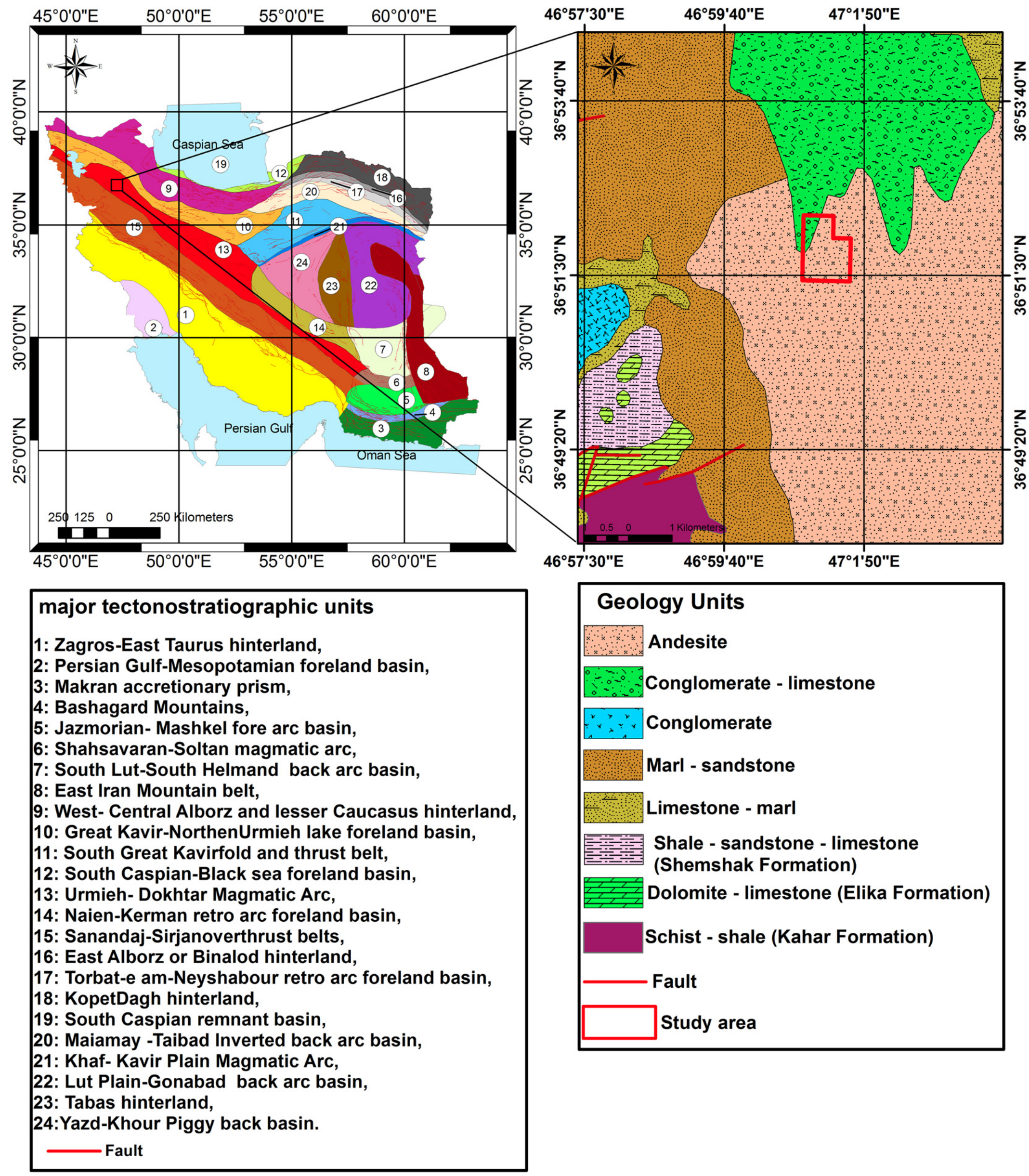

Figure 1: The Koohbaba position in the physiographic-tectonic zoning map [27].

This is the most frequent statistical approach to estimate the fitness of the model [42]. The mathematical expressions are as follows:

$$R_2 = \frac{\mathrm{SSR}}{\mathrm{SST}} = 1 - \frac{\mathrm{SSE}}{\mathrm{SST}}. \tag{6}$$

where SSR shows the regression sum of squares, SSE represents the residual (error) sum of squares, and SST is the total sum of squares total. The model can fit the data more reliable as the value of R^2 increases [43]. The R^2 value may increase by adding a new independent variable to the function, although its presence can be required or not. Therefore, it looks complicated whether an increase in R^2 value is significant. In that case, the adjusted determination of coefficients is expressed as follows [19]:

$$R^2_{\mathrm{adj}} = 1 - \frac{(1 - R^2)(n - 1)}{(n - p)} \tag{7}$$

where n is the number of samples (data) and k represents the number of coefficients. The adjusted coefficient of determination R^2_{adj} has been submitted to decrease the bias in R^2 [42]. By adding a different independent variable to the regression model, if the mean square residual (MSR) reduces, the R^2_{adj} will increase.

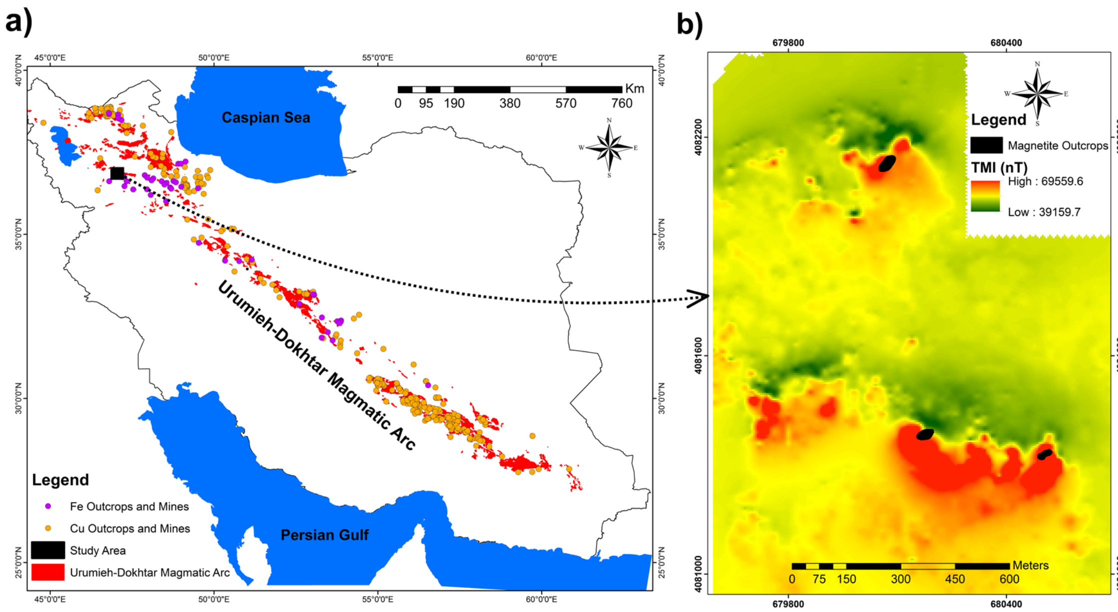

Figure 2: (a) Cu and Fe outcrops of the Urumieh-Dokhtar Magmatic Arc (UDMA) and (b) magnetite outcrops in the study area.

2 Materials and methods

2.1 Data analysis

Ground magnetic data were collected by following the same method used by Mansouri et al. [31] in the Kooh-baba area. The required geophysical data have provided by magnetometer GSM-19T in the research region (±0.2 nT absolute accuracy). As shown in Figure 3, the total magnetic intensity (TMI) represents the magnetic anomalies in the E–W direction in the north and center of the site. In general, there are three dipolar magnetic anomalies (one magnetic anomaly in the north and two magnetic anomalies in the west and east of the center). These three dipolar magnetic anomalies are 130 related to magnetite dikes in andesite units [5]. Accordingly, the multivariate regression method has been used to determine further drilling sites for iron exploration.

2.1.1 Preparing dependent variables

The output of the drilled boreholes has been considered as the dependent variable because this variable reveals the accuracy for defining the drilling points. The value for the ratio $\left(\frac{\text{Ore Length}}{\text{Total Core}}\right)$ is between 0 and 1, and for the best boreholes, it is closer to one. The log reports have been collected, and the magnetite thickness of each has been measured respect to the total core (Table 1). The accepted boundary for the ore length is 20% of the total Fe. Besides, Table 2 presents the statistical factors of the ratio using the regression analysis.

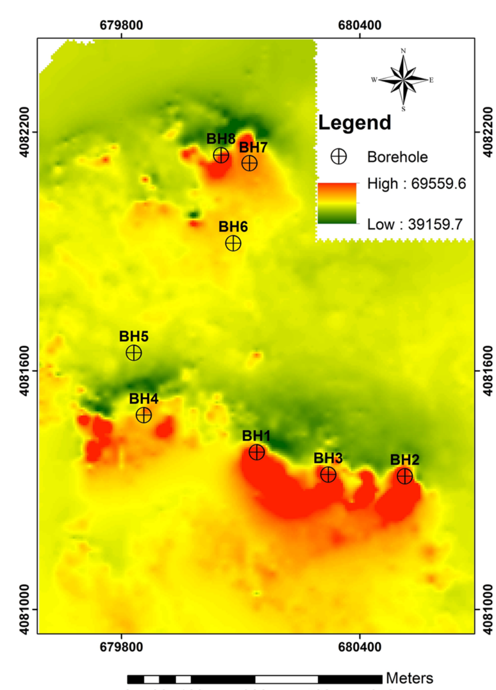

Figure 3: TMI map of the study area with the location of eight drilled boreholes.

Table 1: Log report of boreholes with RTP classification based on fractal method [31]

Borehole id	Total core (m)	Magnetite range (m) (grades greater than 20% Fe total)		Magnetite thickness (m) in total core (grades greater than 20% Fe total)	Ore length/ total core
		From	To		
BH1	136.5	19.3	25.2	52.4	0.38
		60.7	85.2		
		109.4	131.4		
BH2	171.2	4	12.2	47.2	0.27
		50.2	53.5		
		130.6	166.3		
BH3	151.2	80	102	32	0.21
		112	122		
BH4	106	44	48	12.5	0.11
		81	89.5		
BH5	58.9	—	—	0	0
BH6	136.5	69	72	3	0.02
BH7	172	44	47	14	0.08
		61.5	63.5		
		156	164		
BH8	157	70	90	29	0.18
		133	142		

2.1.2 Preparing independent variables

In the regression analysis, selecting the independent variable is essentially important as these variables must be relevant to the models. Therefore, to make the model, three geophysical raster maps such as upward continuation (UP10), reduce to pole (RTP), and analytic signals (A.S.) have been generated by using Oasis Montaj V.8.4. The upward continuation method is a proper method for deep and semi-deep iron and porphyry-Cu deposit exploration [44]. This method distinguishes the magnetic field far away from the source, and it can decrease the effect of shallow magnetic frequency to create a better map. Figure 4a shows the UP10 map in the Koohhbaba area.

The RTP approach converts magnetic anomalies to a symmetrical pattern, and it can make the magnetic anomalies shape with higher accuracy to the spatial site. Therefore, the magnetic anomalies can interpret much easier [44,45]. In this study, the TMI map has been converted to RTP by using a magnetic declination (4.93) and inclination (55.43). Figure 4b shows the RTP raster map of the study location. The A.S. technique is a wildly known filter to enhance the magnetic field and for locating the magnetic anomalies edges. The basic concept of this method has been discussed in the literature [29,46,47]. The A.S. raster map is shown in Figure 4c.

After generating maps, the values of UP10, RTP, and A.S. raster layers have been extracted in the exact location of eight boreholes (dependent variable); consequently, the values of three unique independent variables (UIVs) have been obtained. The statistical parameters of UIVs (UP10, RTP, and A.S.) have been used in the regression modeling (Table 2).

2.2 Regression analysis

In the regression analysis such as linear regression, to have the best model, it is essential to create many models [19]. In this study, six types of multivariate regression analyses, including linear, quadratic, and cubic models, have been employed (Table 3) to find out the best drilling points for iron exploration.

The models became more complex, from Y_1 to Y_6 as the coefficients increased too. To select the best model, there are some criteria considered for the regression analysis. The values for R^2, R^2_{adj}, and p-value (ANOVA test) for different regression models is presented in Table 4. Besides, the unknown regression coefficients values are presented in Tables 5 and 6. Other independent variables, which are not presented in tables, are excluded, and they do not affect the statistical models.

Table 2: The statistical parameters of dependent (ore length/total core) and unique independent (UP10, RTP, and A.S.) variables used in the regression modeling

Variables				*N*	Mean	St. Dev	Skewness	Kurtosis
Dependent	Sample	Y	ore length/total core	8	0.156	0.13	0.522	−0.436
Unique independent	Raster maps (pixel)	X_1	UP10	8	52261.8	3464.8	0.065	−1.273
		X_2	RTP	8	56116.5	7494.7	0.170	−1.793
		X_3	A.S.	8	279.7	276.04	0.356	−2.035

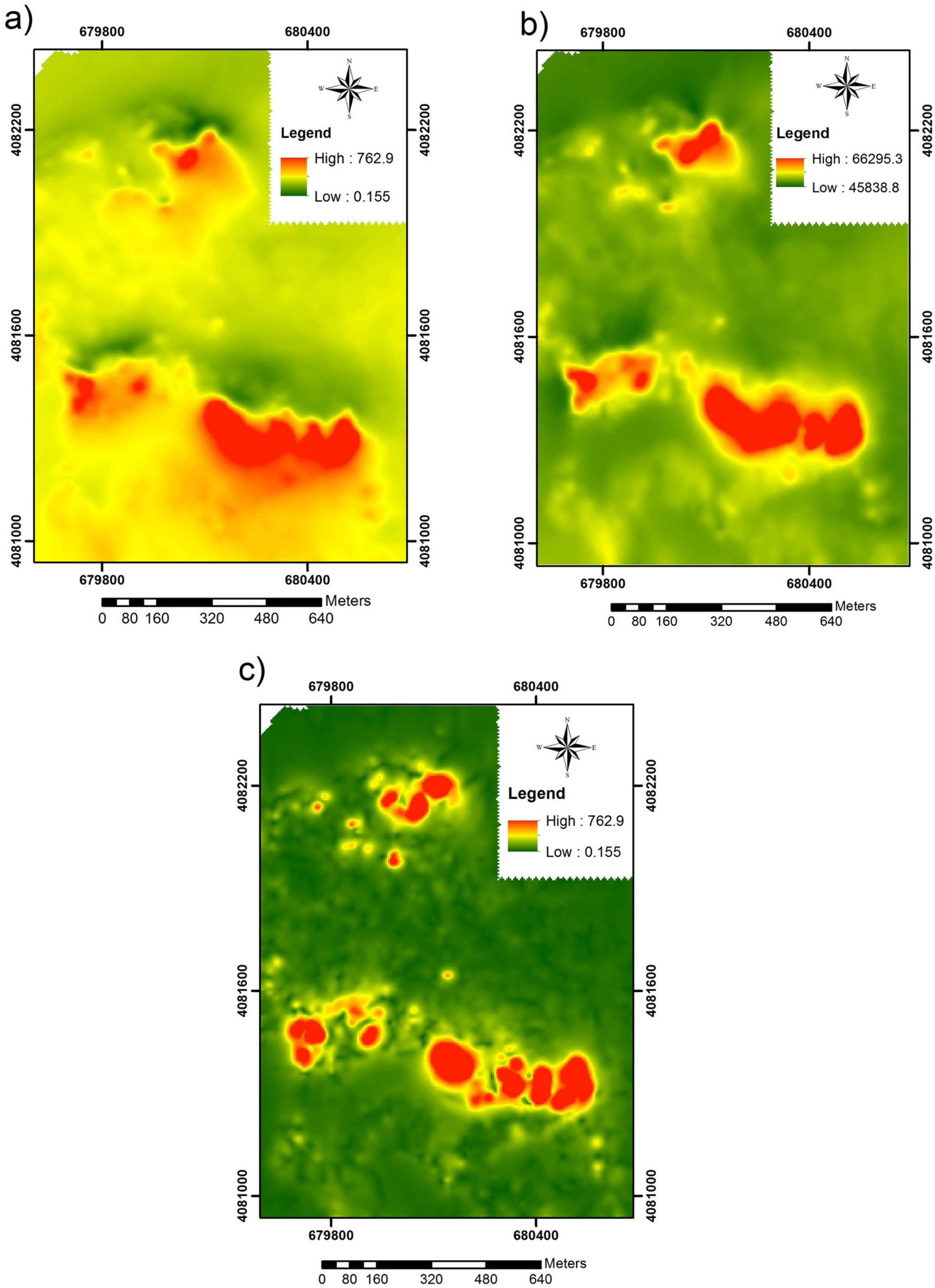

Figure 4: Raster maps of the Koohbaba area: (a) upward continued to 10 m (UP10), (b) reduction to the pole (RTP), and (c) analytic signal (A.S.).

3 Results and discussions

To select the best model, some criteria have to been considered. First, the computed variance and random error mean values confirmed the acceptable value for all regression models. Furthermore, based on Table 3, the p-value (ANOVA test) of the models is acceptable ($\ll 0.05$). Therefore, all these values demonstrated the accuracy of the regression models.

As Table 4 represents, the lowest value for R^2 has been obtained by the first model (Y_1), and the highest one has been achieved by the last three models (Y_4, Y_5, and Y_6).

Table 3: Multivariate regression formula used for detailed iron exploration

Types of regression	Number of coefficients	Formula
First degree	4	$Y_1 = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + \varepsilon$
First degree	7	$Y_2 = Y_1 + a_4x_1x_2 + a_5x_1x_3 + a_6x_2x_3$
Second degree	10	$Y_3 = Y_2 + a_7x_1^2 + a_8x_2^2 + a_9x_3^2$
Second degree	13	$Y_4 = Y_3 + a_{10}x_1^2x_2^2 + a_{11}x_1^2x_3^2 + a_{12}x_2^2x_3^2$
Third degree	16	$Y_5 = Y_4 + a_{13}x_1^3 + a_{14}x_2^3 + a_{15}x_3^3$
Third degree	19	$Y_6 = Y_5 + a_{16}x_1^3x_2^3 + a_{17}x_1^3x_3^3 + a_{18}x_2^3x_3^3$

Table 4: Values for multivariate regression models

Models	R^2	R^2_{adj}	p-value (ANOVA)
Y_1	0.910	0.887	0
Y_2	0.93	0.912	0.003
Y_3	0.937	0.917	0.001
Y_4	0.986	0.966	0.001
Y_5	0.986	0.955	0.001
Y_6	0.986	0.948	0.005

The fourth model is a second-degree function, and it has lower complexity than other models (Table 3). Even though R^2 is a proper parameter for examining the model with the same number of independent variables, it cannot be adequate for comparing the models with different numbers of independent variables as increasing the number of independent variables will increase the R^2 values. Hence, R^2_{adj} has been computed, and model number 4 has indicated a fitted model with the highest value (0.966). Therefore, based on the results obtained by coefficients of determination, model Y_4 is the most appropriate model in this study, and it can be applied to determine further drilling exploration sites in the study area.

To approve this fact, initially, the efficiency formula (Formula 8) has been computed as follows [48]:

$$\mathrm{EF} = \frac{\sum_{i=1}^{n} (P_i - O_i)^2}{\sum_{i=1}^{n} (\bar{O}_i - O_i)^2}, \tag{8}$$

where n is the number of observations, O is the observed values, $\bar{O}$ mean of absolute value, and P_i is the estimated value. The mean value of the observation becomes more reliable than the estimated values if the model efficiency (EF) approaches zero; therefore, to avoid the model limitations, EF values should be closer to 1 to have a workable model [49].

According to the EF values (Table 4), Y_4 is considered as the best model, followed by Y_5 and Y_6. This result confirmed the output result from the regression analysis by considering the coefficients of determination and p-values (ANOVA test). To determine the further drilling sites in the study area (Koohbaba), the raster map is obtained from Y_4. To generate the intended figures, ArcGIS V.10.1 has been employed (with using the raster calculator toolbox). Figure 5a represents the final raster map of the study by considering the fourth regression model.

Moreover, four new boreholes (borehole No. 9–12) have been drilled in the study area, belonging to class 5. The result is presented in Table 7 and Figure 5. The accepted boundary for the magnetic thickness is 20% of the total Fe. Concerning this information, the final results are very promising and appropriate.

Table 5: The calculated coefficients of regression models (1–3)

Model 1		Model 2		Model 3	
Variables	Coefficients (a_i)	Variables	Coefficients (a_i)	Variables	Coefficients (a_i)
CST	−0.866	CST	−0.59	CST	−6.333
x_1	5.504	x_1	−3.351	x_2	−0.014
x_2	4.016	x_2	1.628	x_3	−0.022
x_3	4.016	x_3	−3.354	x_2^2	−1.469
—	—	x_1x_3	2.051	x_3^2	2.292
—	—	—	—	x_1^2	−7.266

Table 6: The calculated coefficients of regression models (4–6)

Model 4		Model 5		Model 6	
Variables	Coefficients (a_i)	Variables	Coefficients (a_i)	Variables	Coefficients (a_i)
CST	−4.481	CST	−2.210	CST	−1.413
x_1	−1.540	x_2	6.206	x_1	−2.839
x_2	0.018	x_3	0.016	x_2	6.188
x_3	−0.024	$x_1^2x_3^2$	−8.798	x_3	−2.539
$x_1^2x_2^2$	−2.012	x_1^3	−5.654	$x_1^2x_3^2$	−7.294
$x_1^2x_3^2$	6,215	x_2^3	−9.229	$x_1^3x_2^3$	−1.299
—	—	x_3^3	4.354	$x_1^3x_3^3$	−1.974

4 Conclusion

The conclusions are presented as follows:

1. Regression analysis is a proper and direct statistical method to identify the potential favorable drilling exploration sites with high accuracy in Koohbaba, Northwest of Iran.
2. The application of the multivariate regression analysis has been confirmed in this area. In this research study, multivariate regression has been developed to create a mathematical model (with reasonable accuracy) for iron mineral exploration by using geophysical data as a new approach.
3. Six different types of multivariate regression models such as two linear, two quadratic, and two cubic equations have been employed to identify the additional drilling area. According to the results of the coefficients of determination (R^2 and R^2_{adj}), p-value, and EF, the fourth regression model (quadratic equation) has been the best response, and it has been confirmed by the ratio (ore length/total core) values of the former drilled boreholes and the further drilled boreholes.
4. The accuracy of the model has been approved by drilling four new boreholes. This additional field investigation has shown promising results.

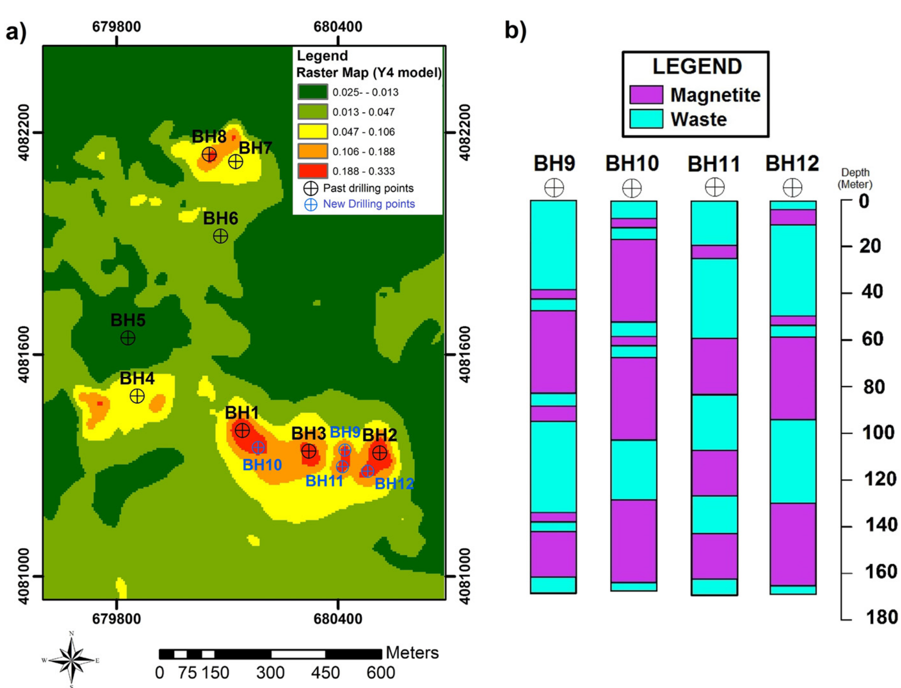

Figure 5: The final raster map of the study area based on the Y_4 regression model with drilled boreholes.

Table 7: Log report of new drilled boreholes

Borehole ID	Total core (m)	Magnetite range (m)		Magnetite thickness (m)	Ore length/ total core
		From	To		
BH9	170.8	39.3	41.8	69.9	0.41
		43.5	81.2		
		82.6	92.3		
		136.5	138.4		
		142.4	160.5		
BH10	169.2	9.4	12.1	107.7	0.63
		18.2	52.5		
		59.3	62.1		
		68.3	102.4		
		129.2	163		
BH11	171.3	19.8	23.2	65.5	0.38
		59.8	82.3		
		108.2	128.4		
		143.2	162.6		
BH12	170.7	3.3	11.1	77.1	0.45
		51.1	53.4		
		59.2	93.4		
		130.4	163.2		

5. The results confirm that the regression analysis model using the geophysical data is an effective approach as it can reduce the time and cost of exploration.
6. Conclusively, 17092.11 m^2 of the study area has been considered as a suitable candidate zone for the detailed studying and determining additional drilling for iron exploration in the area of interest.

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