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PERIODIC AND AUTOREGRESSIVE MODELS OF GROUNDWATER LEVEL DYNAMICS

Iva Guranov[1], Srđan Kostić[2], Nebojša Vasović[3]

[1] Faculty of Mechanical Engineering
University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35
e-mail: iva.guranov@gmail.com
[2] Institute for the Development of Water Resources "Jaroslav Černi",
Jaroslava Černog 80, 11226 Belgrade
e-mail: srdjan.kostic@jcerni.co.rs
[3] Faculty of Mining and Geology
University of Belgrade, Đušina 7, 11000 Belgrade
e-mail: nebojsa.vasovic@rgf.bg.ac.rs

Abstract:

In this paper, authors examine the groundwater level oscillation at groundwater station Bogatić near Šabac (Serbia), for the period 2002-2014. Analysis is done by transforming the observation data into Fourier series within fundamental period of 5 years, and by deriving a model based on the autocorrelation properties of the recorded data. One should note that this is the first time that such analyzes are preformed for the groundwater level oscillation observed in Serbia. The derived model based on groundwater level periodicity is represented by a combination of sine and cosine waves. Analysis shows rather satisfying statistical accuracy of the model, with R^2=0.707 and mean absolute error of 0.34m, which fails to capture the highest values of the groundwater level for the analyzed period. Second model is expressed in a form of a simple correlation between the two successive observations, with a time delay τ=1 and 4. Both models are derived for the period 2003-2009, and their prediction accuracy is checked for the period 2010-2014. Both of these models provide more accurate predictions in comparison to the model based on the groundwater level periodicity.

Key words: Fourier series, periodicity, autocorrelation, groundwater level, time delay

1. Introduction

Groundwater level (GWL) oscillation represents a significant input parameter for the analysis of stability of earth and man-made slopes. Slope instabilities, which include different forms of landslides, commonly occur due to sudden increase or decrease of GWL. In this sense, derivation of model for estimation of the trend and extreme (peak / low) values of GWL is of great significance for engineering practice. In order to create a convenient model for GWL forecasting, authors utilize two main properties of GWL in the paper: periodicity (oscillation) and autoregression.

Analysis of the periodic dynamics of groundwater level, as well as the formulation of sufficiently good model forecasts of GWL, is necessary from the standpoint of engineering practice today. In this way, one can reduce the number of required measurements in situ, but also draw conclusions necessary for engineers of different fields. Therefore, the research in the field of modeling the changes of groundwater levels on the basis of recorded data is very relevant today. Fourier analysis, or the representation of the time series in the form of a Fourier series, is a significant method for modeling periodic time series. Fourier series shows the time series in the form of elementary functions and their combinations. In this paper, data are presented as a combination of sine and cosine, and this model shows the fluctuations of time series at different frequencies [1]. Although widely accepted, this model shows its disadvantages, too. In case of involving all terms of the model in data representation, according to the methodology of Fourier series representation, for the time series with a large number of terms some difficulties can occur in representation of terms of the Fourier series. Because of that, most often, as it will be the case in this paper, Fourier model is shown in shorter form, with the appropriate number of terms, which adequately describes the behavior of time series.

Forecasting of groundwater level using known data for a chosen period of time is considered in [2] and [3]. In order to display the model that describes the rainfall change, based on their time series, Fourier series are used in [1]. For modeling time series of groundwater level in the work [4] authors proposed the integrated time series model, where the periodic component of the model is expressed using the Fourier series. The paper [5] used three different models based on Fourier analysis to predict future flows of the Kershan river, Iran. It was concluded that all three models are reliable and they are strongly applicable in the modeling of periodic time series. The prediction of groundwater levels with Fourier series integrated with the method of least squares is applied in the work [6] and a satisfying level of accuracy of the predictions with reasonable mean square error is achieved.

At the last Congress of Mechanics, that was held in Arandjelovac in 2015, authors applied methods of nonlinear time series analysis in order to examine the groundwater level oscillation recorded at two piezometric stations: Borča and Čuvarnica, near Belgrade, for the period 2007-2013. Authors concluded that recorded oscillations belong to the class of stochastic linear processes and that further analysis should include longer period of time and recordings observed at stations in different aquifers [7]. Present paper could be regarded as a continuation of the previous work, with the final goal to establish a reliable model that accurately describes the irregularity of the groundwater level oscillations. In this way, a main mechanism that governs the dynamics of groundwater level will be recognized and well defined.

Regarding the formulation of the model based on autoregression properties, such an approach is commonly applied in the time series analysis [8]. In particular, real observed time series, independent of their origin (rainfall, river flow, river level, groundwater level) usually exhibit strong degree of autocorrelation, which could be conveniently used for derivation of adequate forecasting model [9,10].

This paper consists of the following sections. Observation results are shown in Section 2, while section 3 is devoted to the results of the performed research. In particular, in Section 3.1 authors show the results of decomposition of the observed groundwater level changes into Fourier series, while model based on autocorrelation properties is given in Section 3.2. Main conclusions with the directions for further research are given in the final Section.

2. Analyzed data

In this paper, authors analyze oscillation of groundwater level recorded at the piezometer B-3, Bogatić, near Šabac in the Western Serbia (Fig. 1 and Table 1). This piezometer, as a part of the station of the underground monitoring network of the Republic Hydrometeorological Service of Serbia, started to work on 1st August 2002 and data are available until the 31st December 2014 [11]. Monitoring piezometer Bogatić B-3 belongs to the group of piezometers which provide daily readings of groundwater level. Within the specified period of observation, there are missing data for the months of March 2006, December 2008 and July 2009, as well as for 10th October 2012 and 9th April 2013. In order to avoid interruption of continuous time series, missing data were replaced with the average values for the following and previous month (in cases where data for the entire month are missing), and with the values obtained as the average value for the day before and after the one that is missing (when the missing datum is for only one day).

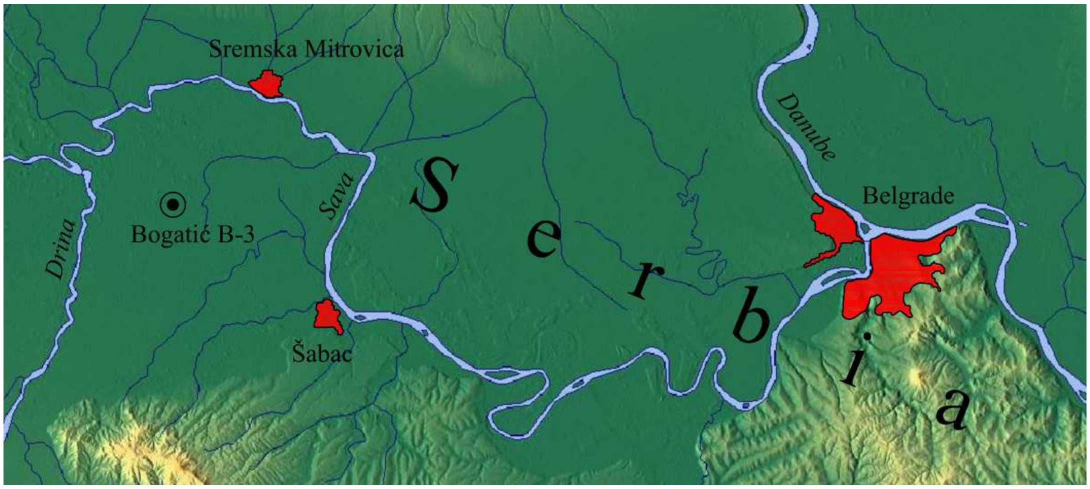

Fig. 1. Geographical position of the piezometer station Bogatić, whose recorded data are examined in present paper.

Name and type	Bogatić – underground water station
Station ID	B-3
Starting date of recording	01/08/2002
Latitude (° ‘ ’’)	44 50 18
Longitude (° ‘ ’’)	19 29 06
The height above the ground (m)	0.4
Construction length (m)	7
Level “0” (m)	81

Table 1. Information about piezometer at Bogatić.

3. Results

3.1. Periodic model based on groundwater regular oscillations

First approach for modeling of groundwater level dynamics utilizes its periodical character. Since GWL is strongly affected by the climatic factors, low values of GWL are expected in winter and summer, while peak values are recorded in spring and autumn. In order to use this property for derivation of prediction model, authors employ the method of decomposing the observed groundwater level changes into Fourier series.

Fourier trigonometric series is one that can be expressed in the following form:

$$\mu(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos(\omega n t) + \sum_{n=1}^{+\infty} b_n \sin(\omega n t) \tag{1}$$

where coefficients a_n and b_n are real and t is any real number [12]. It is assumed that the series converges for all ωt values under consideration. The second and the third term on the right side of equation (1) when n=1 represent the sine and cosine terms of the base period T, and the following terms for n=2 represent the sine and cosine waves with a period of T/2, etc. According to the Fourier theorem, practically any function which is periodical can be represented by a Fourier series as indicated in equation (1). Thereby, any non-periodic function can be viewed as a periodic function with an infinite period.

In order to obtain the Fourier coefficients of the trigonometric series authors use the orthogonality conditions, from which it follows:

$$a_0 = \frac{1}{T}\int_0^T x(t)dt$$

(2)

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(\omega nt)dt, \text{ for } n > 0,$$

(3)

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(\omega nt)dt, \text{ for } n > 0.$$

(4)

The listed terms (2-4) are cases when one considers continuous signals. As time series are examined in the paper, it is necessary to obtain expressions for the coefficients of discrete signals [12,13]:

$$a_0 = \frac{1}{T}\sum_t x_t = \bar{x},$$

(5)

$$a_n = \frac{2}{T}\sum_t x_t \cos(\omega_n t),$$

(6)

$$b_n = \frac{2}{T}\sum_t x_t \sin(\omega_n t),$$

(7)

where $x(t)=x_t$, and $\omega n = \omega_n$, and the valid term is $\omega=2\pi/T$.

The aforementioned equations represent periodic component of the model. However, apart from the periodic components, each model is made of random components, while the trend under consideration is already the part of the periodic component. In accordance with all previously given, final equation can be written in the following form:

$$x(t) = \mu(t) + e(t)$$

(8)

or

$$x(t) = a_0 + \sum_{n=1}^{k} a_n \cos(n\omega t) + \sum_{n=1}^{k} b_n \sin(n\omega t) + e(t)$$

(9)

where $e(t)$ represents the residual, and k can take values:

$$k = \begin{cases} T/2, \text{ if } T \text{ is even,} \\ (T-1)/2, \text{ if } T \text{ is odd.} \end{cases}$$

From expressions (8) and (9) it follows that:

$$e(t) = x(t) - a_0 - \sum_{n=1}^{+\infty} a_n \cos(n\omega t) - \sum_{n=1}^{+\infty} b_n \sin(n\omega t).$$

(10)

The resultant residuals can be analyzed using the mean square error, which can be calculated using the formula:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(e(t))^2$$

(11)

and

$$RMSE = \sqrt{MSE}$$

(12)

The coefficient of determination R^2 is determined by the expression

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}},$$

(13)

whereby the sum of squared residuals is SS_{res}:

$$SS_{res} = \sum_t (e(t))^2,$$

(14)

while total sum of squares is SS_{tot}, or:

$$SS_{tot} = \sum_t (x(t) - \bar{x})^2.$$

(15)

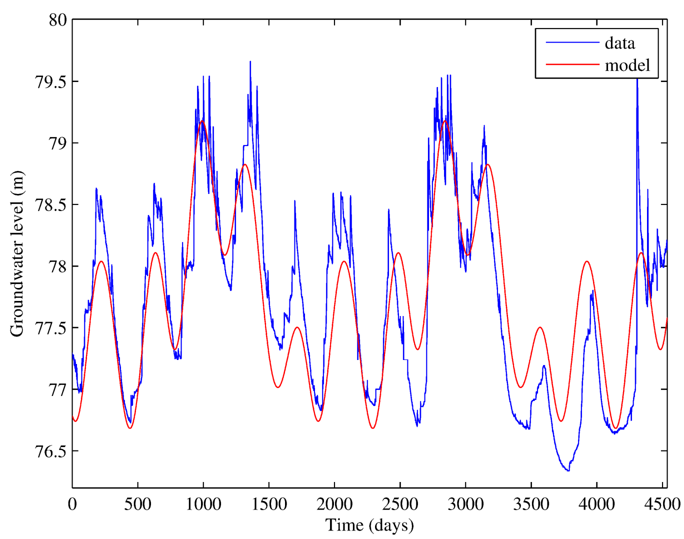

Fig. 2. Comparison of the observed and predicted data using model (17) in temporal coordinates – station Bogatić.

As the expression (1) on the right side has an infinite sum of sine and cosine terms, authors, for the sake of simplicity, take into account only the terms n=0,1,2,…,8. Based on this, the following model could be proposed:

$$\mu(t) = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_2 \cos(2\omega t) + b_2 \sin(2\omega t) + a_3 \cos(3\omega t) + b_3 \sin(3\omega t) + a_4 \cos(4\omega t) + b_4 \sin(4\omega t) + a_5 \cos(5\omega t) + b_5 \sin(5\omega t) + a_6 \cos(6\omega t) + b_6 \sin(6\omega t) + a_7 \cos(7\omega t) + b_7 \sin(7\omega t) + a_8 \cos(8\omega t) + b_8 \sin(8\omega t)$$

(16)

and by applying expressions (5-7) the final model has the form:

$$\mu(t) = 77.75 - 0.5885\cos(0.003394t) - 0.3435\sin(0.003394t) + 0.06343\cos(0.006788t) + 0.3416\sin(0.006788t) - 0.02066\cos(0.010182t) + 0.1181\sin(0.010182t) - 0.04588\cos(0.013576t) + 0.07139\sin(0.013576t) - 0.3421\cos(0.01697t) - 0.424\sin(0.01697t) + 0.03247\cos(0.020364t) + 0.05569\sin(0.020364t) - 0.0387\cos(0.023758t) - 0.02341\sin(0.023758t) - 0.01947\cos(0.027152t) - 0.01801\sin(0.027152t)$$

(17)

whereby the fundamental frequency is ω=0.003394 days^{-1}. From fundamental frequency one can conclude that fundamental period is T=1851.26 days, e.g. 5 years (Fig. 2).

One can see from Fig.1 that derived model (17) follows the trend of the groundwater level dynamics reasonably well. If one compares the estimation of model (17) and the real observed data, a satisfying statistical correlation is obtained, with the moderately high value of determination coefficient, R^2=0.707 and mean absolute error of 0.344m (Fig. 3). Nevertheless, derived model (17) does not capture the peak values of groundwater level, as it could be clearly seen in Fig. 2.

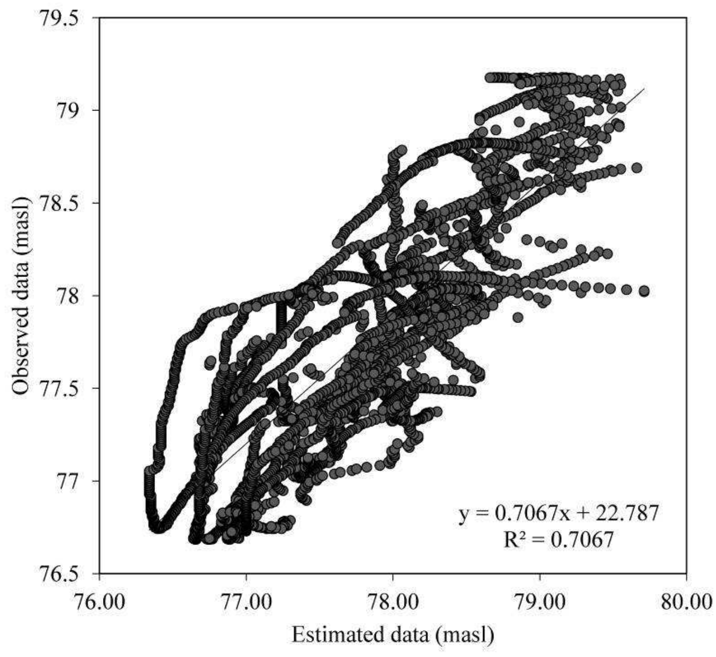

Fig. 3. Comparison of observed data and estimated data using the derived model (17).

3.2. Autoregressive models based on autocorrelation of observed time series

Second approach for modeling the groundwater level dynamics relies on the strong autocorrelation properties of the GWL. This property could also be used for derivation of a convenient prediction model. In particular, for the observed data, there is a strong autocorrelation between the successive observations, for τ=1 (Fig. 4).

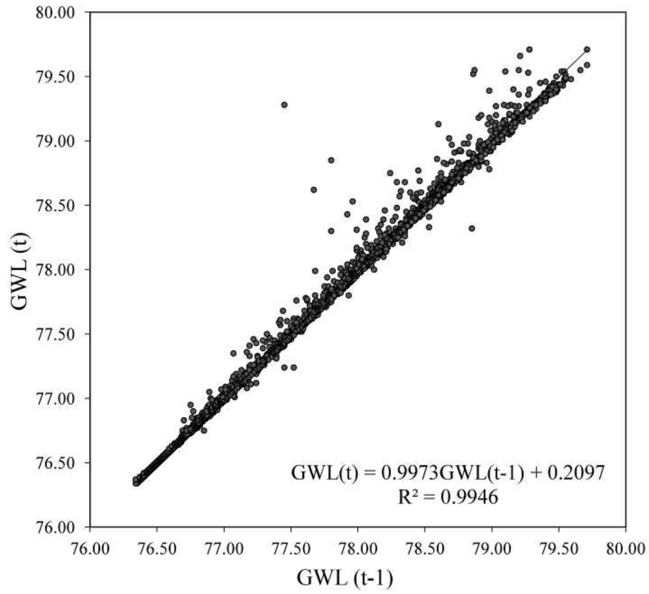

Fig. 4. Strong autocorrelation between the two successive observations of GWL at the Bogatić station.

Knowing this, one could establish a reliable model for future predictions of GWL. In present case, we derived a model based on the autocorrelation for the period 2003-2009, and confirmed its accuracy for the period 2010-2014.

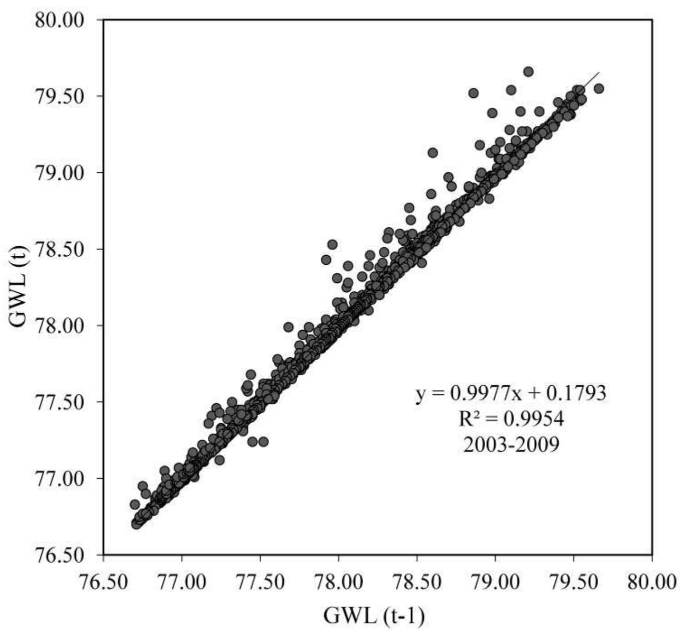

Fig. 5. Strong autocorrelation between the two successive observations of GWL at the Bogatić station for the period 2003-2009, time delay τ=1.

As one can see from Fig. 5, derived model could be expressed in the following form:

$$GWL(t) = 0.9977 \cdot GWL(t-1) + 0.1793$$

(18)

This expression could be used for prediction of future values from the series 2010-2014 (Fig. 6).

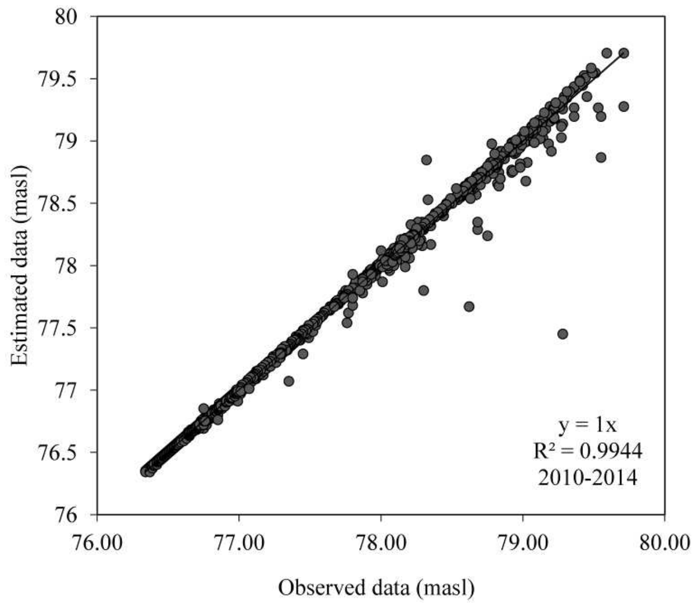

Fig. 6. Estimated vs observed data using the derived model (18) for the period 2010-2014.

One should note that, even though model (18) provides accurate results, it could serve only for a single day prediction. This is the reason why authors developed additional model, with a time delay τ=4. As in the previous case, model is developed for the period 2003-2009 (R^2=0.9739), and it was verified for the subsequent period 2010-2014 (Fig. 7 and 8):

$$GWL(t) = 0.9869 \cdot GWL(t-4) + 1.0229 \quad (19)$$

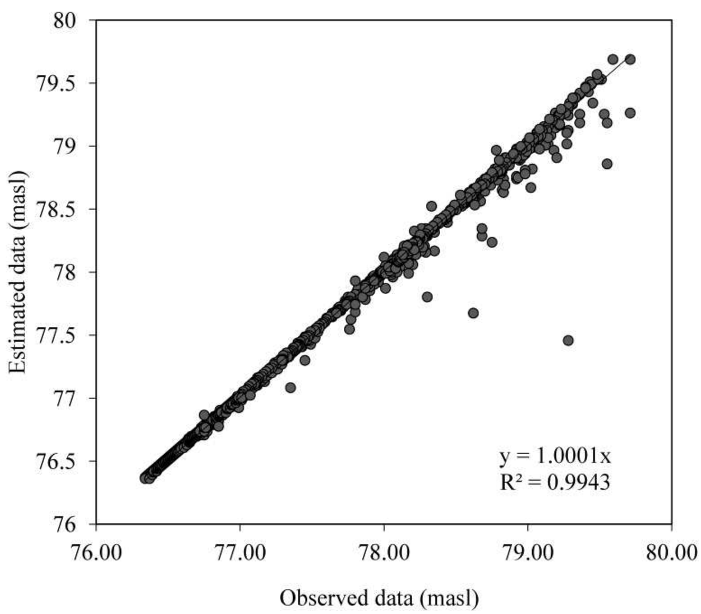

Fig. 7. Estimated vs observed data using the derived model (19) for the period 2010-2014.

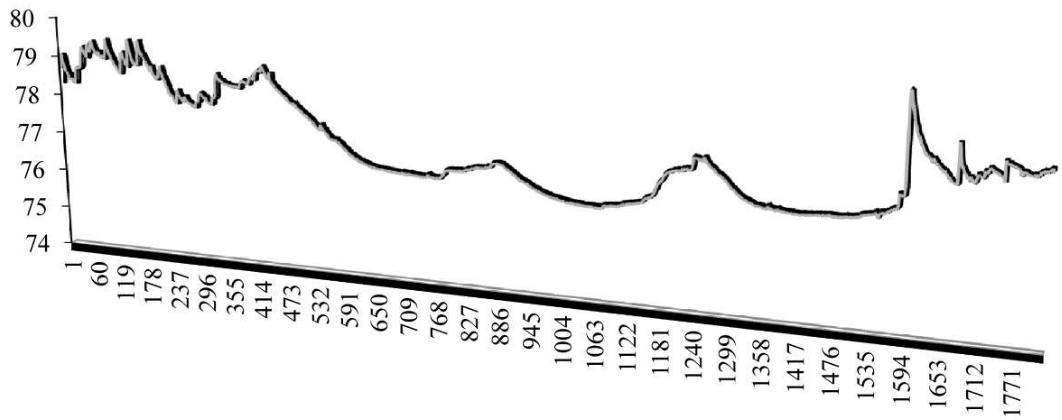

Fig. 8. Predicted (dark gray) vs. observed data (light gray) using the derived model (19) for the period 2010-2014. Almost exact matching is obtained.

4. Conclusion

In this paper authors derived two models for estimation of groundwater level dynamics. First model utilizes the property of periodicity, by decomposing the observed array into the Fourier time series. Such model provides solid estimation of a general trend (R^2=0.707 and mean absolute error ≈0.35m). This model is presented as a combination of sine and cosine waves, within the fundamental period of T=1851.26 days, e.g. 5 years. The main disadvantage of this model lies in its incapability of capturing the peak values of groundwater level. In this sense, it would be very useful if future analyzes could result in a more accurate model regarding the highest values, or in a separate model for the peak values of groundwater level.

On the other hand, second model, which is based on the strong autocorrelation of the observed data, provides very accurate predictions of groundwater level. In this case, authors developed a model which correlates two successive observations, in a general form: $x(t)=f(x(t-\tau))$, where τ takes values 1 and 4. Such models could be very beneficial, especially from the engineering viewpoint, since they enable accurate estimation of groundwater level for several days ahead. Concerning this, it would certainly be interesting to examine the maximum values of time delay τ for which a reliable model for groundwater level forecasting could be developed.

Besides the present approaches, in the future research, one could try to derive models which would correlate groundwater level with the effect of rainfall and the level of surface waters. In that case, GWL forecasting would be independent of the actual previous measurements of the GWL, and it could serve as a convenient prediction model.

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