Journal of Physics Special Topics
An undergraduate physics journal

A3_10 Correcting the Aim

T. Leversha, L. Flood, K. Holland, M. Manley
Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 14, 2022

Abstract
This paper takes a deeper look at the situation proposed in "Dont aim at him!" by J.F Barker (et al) and concludes that their assumption of air resistance being negligible for a fired bullet makes their model inapplicable to real-world examples like that of Craig Harrison.

Introduction

In the paper "Dont aim at him!"[1] published for the University of Leicester Physics Special Topics Journal, a simple kinematic model is constructed for investigating a record-holding shot taken by Craig Harrison. There is a large inaccuracy in the paper arising from the assumption that air resistance was negligible and so we set out to use Python to explore how important drag is for this system and to the investigation objectives of that original paper.

Theory

The groundwork of the model was built from the kinematic equation. We split the bullet's motion into "chunks" of time duration Δt where the velocity is assumed to be constant. The distance travelled in this time was denoted by ΔS. As such the kinematic equation became:

$$\Delta S = u\Delta t + \frac{1}{2}a\Delta t^2$$

Where u and a are the velocity and acceleration respectively at the beginning of the time step Δt. At the end of each time step, the deceleration was calculated from the drag acting on the bullet at velocity u. It was assumed that once the bullet leaves the muzzle it has no acceleration caused by the firing of the rifle and so the acceleration was calculated using:

$$F_D = \frac{1}{2}\rho C_D A u^2$$

Where the coefficient of drag C_D for a streamlined body was used (0.04)[2] due to a bullet being designed as such, the cross-section area A was 0.233×10^{-3} m^2 as calculated from the dimensions of a .338 Lapua Magnum[3], in which the mass of the bullet m was also sourced from, and the fluid density ρ was taken to be the value for air at surface pressures (1.225 kgm^{-3}[4]). In the horizontal axis, the drag is the only force acting on the bullet and so the deceleration on the bullet is:

$$a = \frac{F_D}{m} = \frac{1}{2m}\rho C_D A u^2$$

This model requires an initial horizontal velocity to be inputted which was calculated using trigonometry and the initial muzzle speed of 936 ms^{-1}[5]; this velocity for multiple angles (above the horizontal plane) was calculated and fed into the model. This results in Figure 1 where we have shown the horizontal trajectories for each angle and marked the known measurements from the Craig Harrison shot[6] as dashed lines. From this, we want to find an angle at which the bullet travels 2475 m in 6.015 s to match Craig Harrison's.

Results & Conclusion

From Figure 1 it can be seen that the "real" curve would lie between 45° and 47.5° as the point at which 2475 m intersects 6.015 s is between these two curves. This shows that this

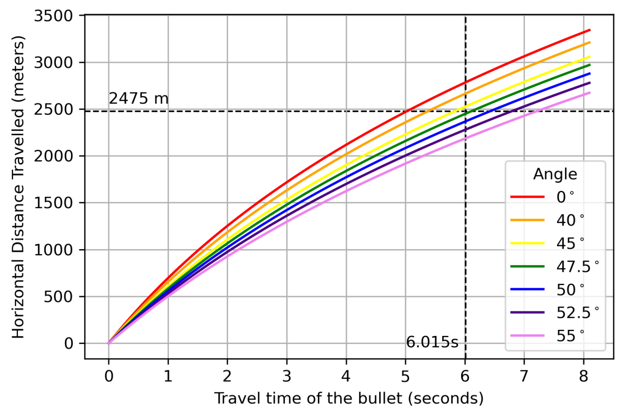

Figure 1: A graph showing the horizontal distance travelled by the bullet from different initial firing angles.

simplistic model is able to replicate results as seen in the real world.

If we were to compare the model used in the J.F Barker's (et al) paper with our drag model (as shown in Figure 2) we can begin to see why their model obtains only a small vertical displacement. Firstly, their model would reach the target horizontal distance in less than half the time as Craig Harrison's actual shot which would give the bullet less time to deflect due to gravity. Secondly, as their model is angled to the horizon, there would be no contribution to the vertical displacement by the vertical component of the initial velocity (as there is none). However, with a drag inclusive model the rifle must be angled above the horizon and thus there will be a significant vertical displacement caused by that vertical component of the velocity.

In conclusion when analysing kinematic systems (especially ones at great speeds like a bullet)

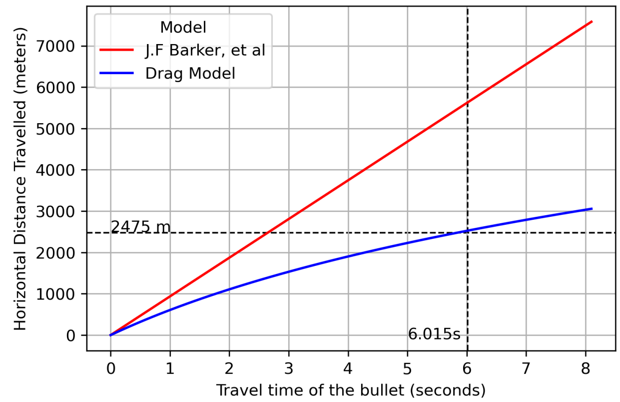

Figure 2: A graph showing the comparison between the J.F Barker (et al) model and our 45° elevation drag model.

drag is an essential component that must be considered otherwise the results obtained will differ greatly from real-world examples of those systems.

References

[1] `https://journals.le.ac.uk/ojs1/index.php/pst/article/view/1924/1825` [Accessed 30 November 2022]

[2] `https://www.engineeringtoolbox.com/drag-coefficient-d_627.html` [Accessed 30 November 2022]

[3] `https://en.wikipedia.org/wiki/.338_Lapua_Magnum` [Accessed 30 November 2022]

[4] `https://www.engineersedge.com/calculators/air-density.htm` [Accessed 30 November 2022]

[5] `http://www.military-today.com/firearms/l115a3.htm` [Accessed 30 November 2022]

[6] `https://en.wikipedia.org/wiki/Craig_Harrison_(British_Army_soldier)` [Accessed 30 November 2022]