SOLUTION OF ARBITRARY FULLY FUZZY MATRIX EQUATIONS AND PAIR FULLY FUZZY MATRIX EQUATIONS

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Abstrak

Teori sistem kawalan sering melibatkan aplikasi persamaan matriks dan pasangan persamaan matriks yang mana terdapat kemungkinan keadaan ketidakpastian boleh wujud. Dalam kes ini, persamaan matriks dan pasangan persamaan matriks klasik tidak mampu menangani masalah tersebut. Walaupun terdapat beberapa kajian lepas dalam menyelesaikan persamaan matriks dan pasangan persamaan matriks dengan keadaan ketidakpastian, kajian tersebut mempunyai beberapa batasan yang meliputi operasi aritmetik kabur, jenis pekali kabur dan juga kesingularan pekali matriks. Oleh itu, kajian ini bertujuan untuk membina kaedah baharu untuk menyelesaikan persamaan matriks dan pasangan persamaan matriks dengan semua pekali persamaan matriks adalah sebarang nombor kabur segitiga kiri-kanan (LR-TFN) samada positif, negatif atau hampir sifar. Dalam membina kaedah tersebut, beberapa pengubahsuaian pada operator penolakan aritmetik kabur dan operator pendaraban aritmetik kabur sedia ada adalah diperlukan. Dengan pengubahsuaian operator aritmetik kabur tersebut, kaedah yang dibina ini melangkaui had positif untuk membenarkan LR-TFN negatif dan hampir sifar sebagai pekali persamaan. Kaedah yang dibina juga telah menggunakan hasil darab Kronecker dan operator Vec dalam mengubah persamaan matriks kabur penuh dan pasangan persamaan matriks kabur penuh menjadi bentuk persamaan yang lebih mudah. Di samping itu, sistem linear bersekutu baharu dibina berdasarkan operator aritmetik pendaraban kabur yang diubahsuai. Kaedah yang dibina disahkan dengan mengemukakan beberapa contoh berangka. Hasilnya, kaedah yang dibina berjaya menunjukkan penyelesaian untuk sebarang persamaan matriks kabur penuh dan pasangan persamaan matriks kabur penuh, dengan kekompleksan operasi kabur yang minimum. Kaedah yang dibina boleh digunakan pada matriks singular dan matriks bukan singular untuk sebarang saiz matriks. Dengan itu, kaedah yang dibina merupakan satu sumbangan baharu dalam aplikasi teori sistem kawalan.

Kata kunci: Teori sistem kawalan, Persamaan matriks kabur penuh, Operator aritmetik kabur, Segitiga nombor kabur LR, Pasangan persamaan matriks kabur penuh.

Abstract

Control system theory often involved the application of matrix equations and pair matrix equations where there are possibilities that uncertainty conditions can exist. In this case, the classical matrix equations and pair matrix equations are not well equipped to handle these conditions. Even though there are some previous studies in solving the matrix equations and pair matrix equations with uncertainty conditions, there are some limitations that include the fuzzy arithmetic operations, the type of fuzzy coefficients and the singularity of matrix coefficients. Therefore, this study aims to construct new methods for solving matrix equations and pair matrix equations with all the coefficients of the matrix equations are arbitrary left-right triangular fuzzy numbers (LR-TFN), which either positive, negative or near-zero. In constructing these methods, some modifications on the existing fuzzy subtraction and multiplication arithmetic operators are necessary. By modifying the existing fuzzy arithmetic operators, the constructed methods exceed the positive restriction to allow the negative and near-zero LR-TFN as the coefficients of the equations. The constructed methods also utilized the Kronecker product and Vec-operator in transforming the fully fuzzy matrix equations and pair fully fuzzy matrix equations to a simpler form of equations. On top of that, new associated linear systems are developed based on the modified fuzzy multiplication arithmetic operators. The constructed methods are verified by presenting some numerical examples. As a result, the constructed methods have successfully demonstrated the solutions for the arbitrary fully fuzzy matrix equations and pair fully fuzzy matrix equations, with minimum complexity of the fuzzy operations. The constructed methods are applicable for singular and non-singular matrices regardless of the size of the matrix. With that, the constructed methods are considered as a new contribution to the application of control system theory.

Keywords: Control system theory, Fully fuzzy matrix equation, Fuzzy arithmetic operators, LR triangular fuzzy number, Pair fully fuzzy matrix equation.

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List of Abbreviations

ALS1	Associated Linear System 1
ALS2	Associated Linear System 2
BMO	Babbar Multiplication Operator
DMO	Dubois Multiplication Operator
FLS	Fuzzy Linear System
FFLS	Fully Fuzzy Linear System
FME	Fuzzy Matrix Equation
FFME	Fully Fuzzy Matrix Equation
FSE	Fuzzy Sylvester Matrix Equation
FFSE	Fully Fuzzy Sylvester Matrix Equation
KMO	Kaufmann Multiplication Operator
LR-fuzzy numbers	Left Right Fuzzy Numbers
LR-TFM	Left Right Triangular Fuzzy Matrix
LR-TFN	Left Right Triangular Fuzzy Numbers
LR-TrFN	Left Right Trapezoidal Fuzzy Numbers
MMO	Malkawi Multiplication Operator
PALS1	Pair Associated Linear System 1
PALS2	Pair Associated Linear System 2
PME	Pair Matrix Equation
PFME	Pair Fuzzy Matrix Equation
PFFME	Pair Fully Fuzzy Matrix Equation
TFN	Triangular Fuzzy Number
TrFN	Trapezoidal Fuzzy Number
WMO	Wan Multiplication Operator

List of Symbols

$\tilde{X}$	Fuzzy matrix X
$\tilde{X}_m$	Fuzzy matrix X with order $m \times m$
I_n	Identity matrix with order $n \times n$
$\in$	Element of
$\mu_{\tilde{A}}$	Membership function of fuzzy set A
$\underline{u}$	Lower bound
$\overline{u}$	Upper bound
$\otimes$	Fuzzy multiplication operator
$\oplus$	Fuzzy addition operator
$\ominus$	Fuzzy subtraction operator
$\otimes_k$	Fuzzy Kronecker product
$\ominus_d$	Fuzzy direct subtraction operator
$\ominus_{nzp}$	Fuzzy near-zero positive subtraction operator
$\ominus_{nzn}$	Fuzzy near-zero negative subtraction operator
$\cup$	Union of two set

CHAPTER ONE
INTRODUCTION

1.1 Matrix Equations and Pair Matrix Equations

A matrix is generally known as a rectangular array which consists of numbers, symbols or expression, arranged in rows and columns. Normally, a matrix is used to represent a linear system of equations, so that it can be solved analytically or numerically by using any classical linear algebra methods. In real-life applications, matrices have been used in the fields of graph theory, cryptography, computer graphic and so on (Anton & Rorres, 2010). Besides, matrices have also been used independently in the form of matrix equations. The most common matrix equation is

$$AX = B \tag{1.1}$$

which can be written in a form of a matrix equation as follows:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1p} \\ x_{21} & x_{22} & \dots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{np} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mp} \end{pmatrix} \tag{1.2}$$

where the coefficient matrix $A = (a_{ij})$, $1 \leq i \leq m$, $1 \leq j \leq n$, the right hand matrix $B = (b_{ij})$, $1 \leq i \leq m$, $1 \leq j \leq p$ and the solution matrix $X = (x_{ij})$, $1 \leq i \leq n$, $1 \leq j \leq p$. The entries for each matrix of Equation (1.2) are in the form of crisp numbers.

In addition, Equation (1.1) can be expanded to several types of matrix equations, such as

$$AXB = C \tag{1.3}$$

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