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MATHEMATICAL MODELING OF ROCK CRUSHING AND MULTIPHASE FLOW OF DRILLING FLUID IN WELL DRILLING

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The aim of the work is a mathematical modeling of the rock crushing during drilling and removal of the drilling cuttings (sludge) to the surface by drilling fluid. The process of rock destruction is described using the mathematical theory of fragmentation. The distribution of sludge particles in size and mass depends on such factors as the properties of the drilled rock, the rate of penetration, the type of bit, and the output power. After the formation of sludge, the process of its removal to the surface is modeled. The drilling fluid together with the rock particles is considered as a heterogeneous multiphase medium in which the carrier phase – the drilling fluid – is a non-Newtonian fluid. The flow of such a medium is described using a mixture model in the framework of the multi-fluid approach. This results in a system of nonlinear partial differential equations, for which a new closure relation is derived. To solve the system, the SIMPLE algorithm is used. As a result, the flow properties are studied with the inclusion of particles of various sizes. In particular, for particles of small size due to the action of plastic stresses in a non-Newtonian drilling fluid, an equilibrium mode arises in which the particles move with the drilling fluid without slipping. This is the fastest mode of delivery of sludge to the surface. The specific dimensions of such particles depend on the parameters of the drilling process. In particular, the appropriate size range can be adjusted by changing the parameters of the drilling fluid.

Key words: mathematical modeling; rock crushing; drilling fluid; multiphase flow; non-Newtonian fluid; drilling of the wells

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Introduction. One of the key technological processes in the drilling of wells is the circulation of drilling fluid. At the same time, we must solve the main practical task, which is the removal of particles of the drilled rock to the surface. When moving down the wellbore, the fluid loses its initial pressure, which in the end may turn out to be insufficient for the subsequent removal of the drilling cuttings. This can lead to jamming of the drill string and serious emergencies. The purpose of this work is a mathematical modeling of the formation of drilling cuttings and its removal to the surface with drilling fluid. This model will allow to calculate the various modes of drilling and predict the occurrence of complications.

To simulate the rock crushing, the general theory of fragmentation is used [3, 5]. This mathematical theory describes the grinding of the initially existing set of objects, regardless of their physical nature. The examples of its specific application are various explosive processes, spraying of fuel droplets, cell division, etc. Mathematical modeling of this process during drilling has not been previously considered. An additional difficulty here is to associate abstract mathematical concepts with specific tools used in the drilling process. The first part of this paper is devoted to this issue.

The mixture of drilling fluid and particles resulting from crushing is a two-phase medium in which the carrier phase – the drilling fluid – is usually a non-Newtonian fluid. The flow of such a medium is described by a set of nonlinear partial differential equations, which are usually solved by using simplifying assumptions, discarding part of the terms. Therefore, the issue of drilling cuttings removal from the well is solved only at the assessment level (for example, [10]). In this case the correctness of the obtained results is difficult to estimate. In this paper, the complete set of equations is solved numerically without simplifying assumptions, which makes it possible to more reliably consider the issue of drilling cuttings removal.

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Rock crushing simulation. Conventionally, we divide all possible soils into three types: 1) solid soil is the soil that is crushed during the drilling process; 2) sandy soil – its particles are not crushed; 3) mixed soil consists of a mixture of soils of the first and second type, which are considered separately.

Of particular interest for further research and at the same time the most mathematical complexity is the modeling of the redistribution of the masses of particles in the process of solid ground crushing. To describe the distribution of particles by mass (size), the natural approach would be the one based on the concept of a branching process [9]. In this approach, it is assumed that the particles are crushed independently from each other through some random intervals, and the evolution of the fragmentation process is determined only by the current state and does not depend on its history. Thus, the resulting branching process will have a Markov property.

Mathematical model of crushing. Suppose that over a period $[0, T]$ the rock with a mass M_0 is crushed, after it is carried out with a solution of drilling fluid into the annular space of the well. Let at the moment of time t the masses of particles x be distributed in accordance with the law specified by the probability density function $m(x, t)$. The fragmentation of particles occurs with an intensity $\lambda(y)$, independent of time, but dependent on the particle mass y. In other words, on average, for each particle of mass y, fragmentation occurs after a time of $1/\lambda(y)$. In this case, a particle of mass y splits into several particles, the masses of which with respect to the mass of the particle being crushed are distributed in accordance with the law defined by the conditional probability density $f(x|y)$.

For the mass distribution density of particles $m(x, t)$ the integro-differential equation (fragmentation equation) holds [5]:

$$\frac{\partial m(x,t)}{\partial t} = -\lambda(x)m(x,t) + \int_x^{M_0} f(x|y)\lambda(y)m(y,t)dy\,; \quad (1)$$

$$m(x,0) = m_0(x), \quad x \in [0, M_0], \;\; t \in [0, T],$$

where $m_0(x)$ − particles mass distribution density at the initial time.

Usually (for example, [4]) the power dependence of $\lambda(x)$ on the mass x is used. In this study, the dependency was modified as follows:

$$\lambda(x) = \begin{cases} \lambda_0 x^{\beta}, & x > x_{\min} \\ 0, & x \le x_{\min} \end{cases}, \quad \beta \ge 0, \quad (2)$$

to ensure that only particles of mass greater $x_{\min}$ are crushed.

Often (see, for example, [8]) the power type of the conditional distribution density function is used.

$$f(x|y) = \gamma \frac{x^{\gamma-1}}{y^{\gamma}}, \quad \gamma > 0.$$

It is assumed that with an increase in particle size by a certain amount, the sizes of particles formed during crushing will change in the same amount. Thus, $m(x, t)$ will depend on a set of parameters $q = \{x_{\min}, \lambda_0, \beta, \gamma\}$.

Note that, having the function $m(x, t)$, it is easy to restore the average radius of particles with radii in a given interval $[r_1, r_2]$:

$$\bar{r}(r_1, r_2, t) = \int_{r_1}^{r_2} r^3 m(x(r),t)dr \Big/ \int_{r_1}^{r_2} r^2 m(x(r),t)dr\,. \quad (3)$$

Formula (3) allows us to obtain a piecewise constant approximation of the particle size distribution.

Estimation of model parameters. In the practical application of the described model, it is necessary to set the numerical values of parameters $q = \{x_{\min}, \lambda_0, \beta, \gamma\}$ for specific soils and bits.

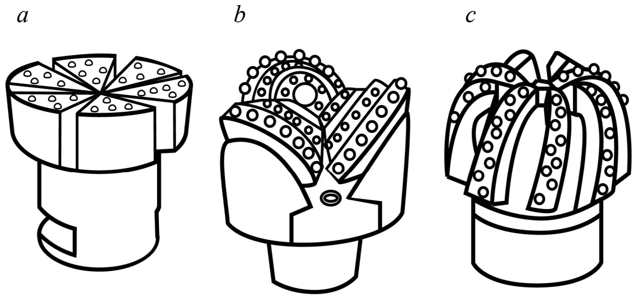

Fig.1. Bit types: *a* – blade; *b* – roller cone;
c – diamond (PDC –Polycrystalline Diamond Bits)

Suppose that the particles are spherical, then x_{min} can be calculated as the mass of a particle of maximum radius r washed out of the bit by the fluid. The parameter r can be defined as double distance between the cutting edges of the bit and depends on its type.

The parameter β, which is in formula (2) and determines the intensity of crushing, depends on the type of bit and allows to set the dependence of the crushing intensity on the mass of the soil particle. When the design of the bit is such that its entire working surface is in contact with the soil (Fig.1, *a*), we assume $\beta = 0$, which means the same crushing intensity for particles of all masses. With a more complex bit shape, when pieces of different sizes are simultaneously captured (Fig.1, *b*, *c*), the crushing intensity depends on the particle mass.

The parameter γ depends on the characteristics of the soil and describes the mass distribution of breakaway faction. If particles are spherical, the surface area of a particle of mass y can be calculated using its density ρ by the formula $s(y) = 4\pi(3y/(4\rho\pi))^{2/3}$. The surface area of all particles formed during a single crushing of a particle of mass y is determined by the expression

$$s_{\Sigma}(y) = \int_0^y x^{-1} y s(x) f(x|y) dx .$$

The work A of the bit for soil crushing can be calculated as the power received by the bit from the downhole motor or drill string in time T. At the same time, according to the law of crushing of Rittinger [6],

$$A = k\Delta s, \tag{4}$$

where A – work for crushing, J; Δs – particle surface area change, m^2; k – coefficient of proportionality, N/m. Then the parameter γ can be calculated by substituting $\Delta s = s_{\Sigma}(y) - s(y)$ in equation (4) and averaging of this equation over a period $[0; T]$.

The parameter λ_0 is associated with the rate of penetration, which is assumed to be constant. The estimate of the parameter λ_0 can be found by solving equation (4). However, this requires multiple solutions to equation (1) for different values of the parameter λ_0, which is a laborious process from a computational point of view. On the other hand, it is easy to see that the solution of equation (1) for an arbitrary λ_0 is obtained from the corresponding solution for the case of $\lambda_0 = 1$ by simply scaling of time.

Thus, to simulate the crushing of soil with bits, the following parameters should be set: soil density, proportion of solid rock (first type of soil) of the total crushed mass, penetration rate, borehole diameter, bit type (roller cone, blade, etc.) and the distance between the blades of the bit, the proportionality coefficient of the law of crushing of Rittinger for the considered soil and the power at the bit.

To illustrate the algorithm execution, a specific example was considered. We chose mixed soil with a density of 2650 kg/m^3 (quartz aleurolite), a penetration rate of 0.01 m/s, a borehole diameter of 0.4445 m, a proportionality coefficient of the Rittinger crushing law 0.28. The share of sandy soil was 25 % and was supposed to be distributed according to the normal law in the range of 0.1-0.5 mm. We determined the hardness and abrasion categories for the considered type of rock (according to L.A. Schreiner and GOST 12288-66). When drilling such rocks, roller cones and PDC bits are used, so for specific calculations one PDC drill bit with six blades was chosen (with $\gamma = 1.5$, $\beta = 2$) and the second – a roller cone bit with three nozzles ($\gamma = 1.1$, $\beta = 2$). Figure 2 shows the percentage distribution of particle radii at specified intervals for two selected bits.

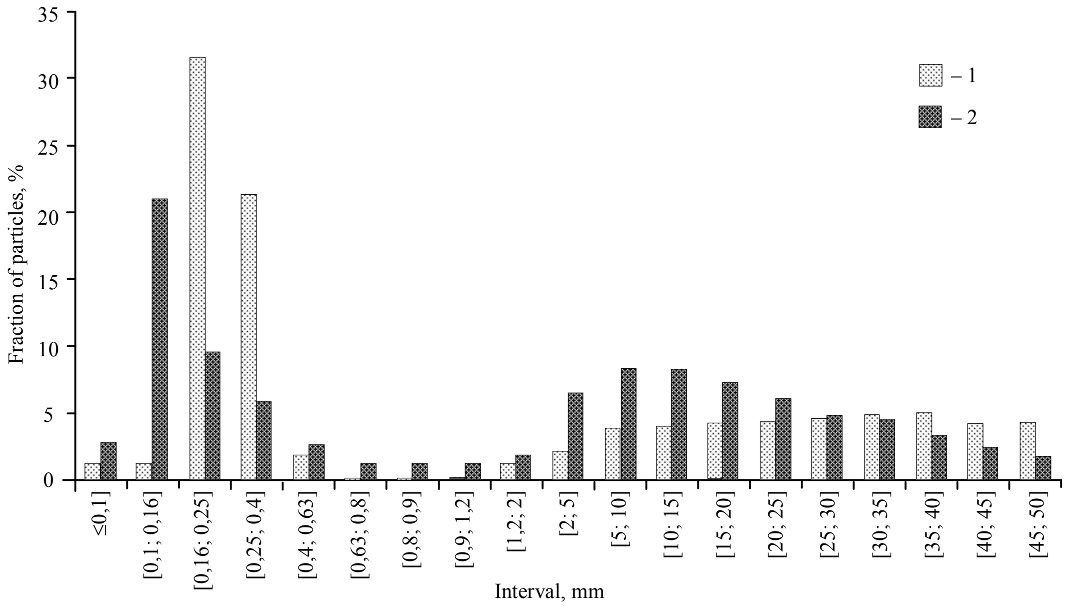

Fig.2. Histogram of the distribution of particle radii at specified intervals for two types of bits
1 – PDC; 2 – roller cone

Analysis of such histograms allows to select the most appropriate drilling parameters from a particular point of view and the corresponding tools.

Multiphase flow of drilling fluid, considering its non-Newtonian properties. After passing through the pipes of the drill string, the fluid enters the annular space together with particles of the drilled rock. Further, under the action of the existing pressure, this mixture should rise to the collar and eventually be removed from the well for subsequent cleaning and regeneration. Note that at the same time, formation fluids and gas bubbles from the rock surrounding the well may appear in the well space. If necessary, the model can be modified to include these effects.

The paper describes a mixture that consists of a continuous (drilling fluid) and a dispersed phase (drilling cuttings). The particles of the dispersed phase can have the same size, determined by the formula (3), or be of different sizes. In the latter case, a piecewise constant approximation is introduced using the formula (3) and each set of particles of the same size is considered as a separate phase.

One of the possible ways to simulate a multiphase medium is to present it as a combination of several interpenetrating continuums (multi-fluid model).

Basic equations. In the most general model, the Navier-Stokes equations, averaged in one way or another (by volume, time, or ensemble), are used to represent the flow of individual components. When using volume averaging [11], the system consists of the equations written for each phase of the flow,

$$\frac{\partial}{\partial t}(\alpha_k\rho_k)+\nabla(\alpha_k\rho_k\mathbf{u}_k)=0\,; \tag{5}$$

$$\frac{\partial}{\partial t}(\alpha_k\rho_k\mathbf{u}_k)+\nabla(\alpha_k\rho_k\mathbf{u}_k\mathbf{u}_k)=-\alpha_k\nabla p_k+\nabla(\alpha_k\boldsymbol{\tau}_k)+\alpha_k\rho_k\mathbf{g}+\mathbf{M}_k\,. \tag{6}$$

Here index k is associated with a given phase; ρ_k – phase density; $\mathbf{u}_k$ – velocity vector; p_k – pressure; $\boldsymbol{\tau}_k$ – viscous stress tensor; $\mathbf{g}$ – gravity factor; $\mathbf{M}_k$ – force acting on this phase from other phases. In both equations there is an important parameter α_k, the interpretation of which depends on the applied averaging type. When using volume averaging, this value characterizes the phase fraction in the individual volume of the multiphase medium ($0 < \alpha_k \le 1$).

To calculate the flows in the well, the system of equations (5), (6) can be significantly simplified, given the fact that the longitudinal size of the well is much larger than the transverse one. This allows the use of a narrow channel approximation [2].

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A further simplification was made by averaging by cross section [6], which made it possible to switch to a one-dimensional description. To reduce the number of solved differential equations, a mixture model was applied [12]. The main idea is the concept of a mixture – a model environment, the movement of which describes the behavior of a multiphase system as a whole. Its equations are obtained by adding the equations of continuity and momentum balance for each phase. As a result, we get the system of equations (index *m* refers to the parameters of the mixture):

• phase continuity equation

$$\frac{\partial}{\partial t}(\alpha_k\rho_k A)+\frac{\partial}{\partial z}(\alpha_k\rho_k u_k A)=0,\ k=1...n, \tag{7}$$

where A – well cross-sectional area;

• mixture continuity equation

$$\frac{\partial}{\partial t}(\rho_m A)+\frac{\partial}{\partial z}(\rho_m u_m A)=0; \tag{8}$$

• momentum balance equation for a mixture

$$\rho_m\left(\frac{\partial u_m}{\partial t}+u_m\frac{\partial u_m}{\partial z}\right)=-\frac{\partial p_m}{\partial z}+F_w+\rho_m g \tag{9}$$

where the z coordinate is measured from the collar and is directed down the well depth.

The mixture density ρ_m is calculated using the formula $\rho_m=\sum_{k=1}^{n}\alpha_k\rho_k$, and u_m – velocity of the center of mass of the individual volume of the mixture $u_m=\sum_{k=1}^{n}\alpha_k\rho_k u_k/\rho_m$, where n – total number of phases. It is also assumed that all phases of the mixture share the same pressure, which coincides with the pressure of the mixture p_m. The forces of friction on the borehole wall F_w can be calculated using known hydraulic formulas for the flow of single-phase fluid with the parameters of the mixture.

Algebraic slipping model for viscoplastic fluid. In the resulting system of equations (7)-(9), the number of unknowns is greater than the number of equations. To close the system in [12], the so-called algebraic slip model was constructed. In this paper, it is supplemented for the case of non-Newtonian carrier phase.

The strength of the interfacial interaction is determined by the formula [12]

$$M_k=\alpha_k(\rho_m-\rho_k)\left(g-\frac{\partial u_m}{\partial t}-u_m\frac{\partial u_m}{\partial z}\right). \tag{10}$$

We introduce the drag coefficient β_k and write $M_k=-\beta_k u_{vk}$, where u_{vk} – slip velocity of phase k in relation to the carrier phase, $u_{vk}=u_k-u_v$, u_k – velocity of the dispersed phase, u_v – velocity of the continuous (carrier) phase. For a Newtonian fluid, the resistance coefficient $\beta_k=\alpha_k\rho_k f(u_{vk})/t_k$, where $t_k=\rho_k d_k^2/(18\mu_c)$ – particle relaxation time; d_k – equivalent diameter of particles; $f(u_{vk})$ – drag function, which was calculated by the Schiller – Nauman formula:

$$f=\begin{cases}1+0.15\,\mathrm{Re}^{0,687}, & \mathrm{Re}\le 1000,\\ 0.0183\,\mathrm{Re}, & \mathrm{Re}>1000,\end{cases} \tag{11}$$

where $\mathrm{Re}=\rho_c d_k|u_{vk}|/\mu_c$.

Often, the drilling fluid is a clayey substance that behaves not as Newtonian, but as a viscoplastic (Bingham) fluid. For this type of fluid the rheological law is as follows: $\tau=\tau_0+\eta_c\frac{\partial u}{\partial y}$, where τ_0 – ultimate shear stress; η_c – coefficient of structural viscosity [7]. In this case, structural viscosity is used to determine the Reynolds number.

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Each particle of the dispersed phase in a Bingham fluid, in addition to the force of viscous resistance, is additionally affected by the force F_B associated with plastic stresses on the surface of the particle. Following [3], $F_B = -\tau_0 A_{k,d}$, where $A_{k,d}$ – surface area of a particle of the dispersed phase.

Consider the individual volume V, assuming that it contains n_k particles of disperse phase k. Let us set $n_k = V_k/V_{k,d}$, where V_k – volume occupied by phase k in volume V and $V_{k,d}$ – volume of one particle of phase k. The total force associated with plastic stresses acting on the dispersed phase in volume V, is equal to $F_B n_k = -\tau_0 A_{k,d} V_k / V_{k,d}$. Dividing it by the volume, we get $M_{k,B} = -\alpha_k \tau_0 A_{k,d} / V_{k,d}$. For spherical particles with equivalent diameter d_k we have $M_{k,B} = -\alpha_k 6\tau_0 / d_k$.

The total strength of the interfacial interaction $M_k = M_{k,N} + M_{k,B}$, where $M_{k,N}$ – strength of viscous resistance due to the presence of the structural viscosity of the Bingham fluid; $M_{k,B}$ – force arising due to plastic stress. As a result, we have

$$M_k = -\beta_k u_{vk} - \alpha_k \frac{6\tau_0}{d_k}.$$

In view of (10), we obtain the equality

$$-\beta_k u_{vk} - \alpha_k \frac{6\tau_0}{d_k} = \alpha_k (\rho_m - \rho_k)\left(g - \frac{\partial u_m}{\partial t} - u_m \frac{\partial u_m}{\partial z} \right).$$

Hence considering the equation $\rho_m - \rho_k = -\alpha_c(\rho_k - \rho_c)$ we have the slip velocity valid for a two-phase medium,

$$u_{vk} = \frac{\alpha_k}{\beta_k}\left(\alpha_c(\rho_k - \rho_c)\left(g - \frac{\partial u_m}{\partial t} - u_m \frac{\partial u_m}{\partial z} \right) - \frac{6\tau_0}{d_k} \right). \quad (12)$$

Note that u_{vk} is not calculated directly from the equation (12), since β_k depends on u_{vk} through drag function (11). Consequently, (12) is an equation for u_{vk}, which needs to be solved by some approximation method.

The initial and boundary conditions should be added to the obtained equations. In particular, we propose to add the initial distribution of the volumetric content of each phase along the entire annular space of the well.

To calculate the flow in the drill string and in the annular space, we set the volumetric flow rates and, consequently, the velocities of all phases at the entrance to the drill pipe or at the bottom. The volume of the drilling return at the bottom is determined from the ratio between rate values $Q_{cut}/(Q_{cut} + Q_c)$, where Q_{cut} and Q_c – flow-rates of drilling return and drilling fluid respectively.

Verification of the model. The solution of the problem of drilling cuttings removal. To solve the resulting system of equations, a numerical method was constructed. The continuity equation for a separate phase (7) was solved according to the scheme using upwind differences. For the joint solution of the equations of continuity and impulse balance for the mixture (8), (9), the iterative SIMPLE algorithm was used [13]. To determine the relative phase velocity, we solved the nonlinear equation (12). The calculation was made to obtain a steady-state solution for the volume fraction of each phase.

The execution of the constructed algorithm was verified in the particular case of a Newtonian fluid by the example of solving the test problem of deposition of drilling return particles after the termination of the circulation of drilling fluid. It was found that in the special case when the density of drilling cuttings phase and drilling fluid are close, the system of equations (7)-(9), (12) allows an analytical solution for the volumetric content of particles. In a computational experiment, it was shown that a numerical solution converges to an analytical one.

Let us consider some results of calculations that reveal the properties of a drilling fluid as a viscoplastic one. The defining parameters were chosen from the examples given in [10]. We considered drilling return particles with density $\rho_k = 2560$ kg/m^3, drilling fluid with density $\rho_c = 1300$ kg/m^3 and structural viscosity $\eta_c = 0.01$ Pa·s (it is assumed that $\rho_k > \rho_c$). The flow rate of the fluid at the bottom is

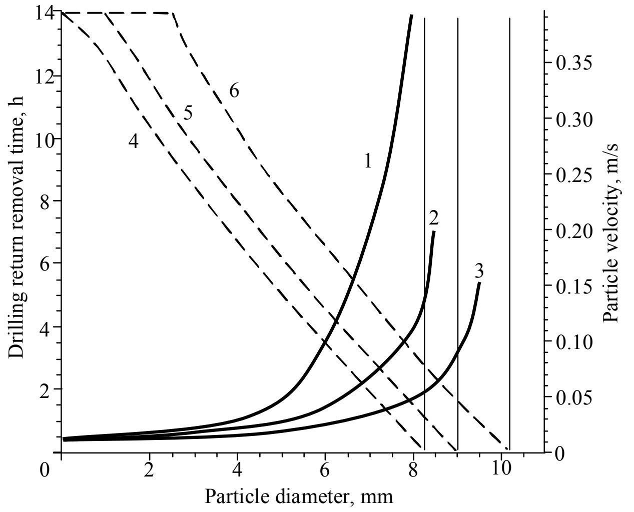

Fig.3. Dependence of drilling return removal time and particle velocity from diameter for different values τ_0

Solid lines mark removal time, dotted – particle velocity; 1; 4 – τ_0 = 0; 2; 5 – τ_0 = 2 Pa; 3; 6 – τ_0 = 5 Pa

0.06 m³/s, and the rate of removed drilling cuttings is 0.0016 m³/s and depends on the penetration rate 0.01 m/s. The volume fraction of drilling cuttings at the bottom hole was 2.6 %. The well depth is 500 m and its diameter is 0.445 m.

Figure 3 shows the dependence of the drilling cuttings removal time and the particle ascent velocity on the diameter for different values of the ultimate shear stress τ_0. In this case, $\tau_0 = 0$ corresponds to the Newtonian fluid, and $\tau_0 = 2$ Pa and $\tau_0 = 5$ Pa to a viscoplastic one. Note that for a given range of particle diameters (from 0 to 10 mm), the Reynolds number did not exceed 400, which corresponds to the case of laminar flow.

Particles in a non-Newtonian medium are removed faster than in a Newtonian medium and the velocity is higher with larger the values of τ_0. At the same time, in a certain range of diameters for a non-Newtonian fluid, the particle removal time remains constant, independent of either the diameter or τ_0. This is the ultimate removal time, which is limited by the velocity of the carrier phase. The particle velocity graphs here have horizontal sections (in the range of diameters from 0 to 1 mm for τ_0 = 2 Pa and from 0 to 2.5 mm for τ_0 = 5 Pa). For these sections, the phase slip velocity is $u_{vk} = 0$, i.e. drilling fluid and drilling return particle rates are the same. This is due to the additional impact on the particles of plastic stress from the fluid. Such flow regime of an inhomogeneous medium is called an equilibrium [1]. It is the most profitable, since at a given velocity of the carrier phase it allows to remove drilling return in the shortest possible time.

If particles ascend but are left behind the carrier phase the inequality $u_{vk} > 0$ should be satisfied. For a steady flow through a pipe of constant cross section, the derivatives in (12) are zero and then from (12) it follows that the inequality $u_{vk} > 0$ holds for particles with a diameter $d_k > d_k^*$:

$$d_k^* = \frac{6\tau_0}{\alpha_c(\rho_k - \rho_c)g}. \tag{13}$$

The parameter d_k^* defines the boundary of equilibrium mode. From (13) it follows that $d_k^* \to 0$ when $\tau_0 \to 0$, i.e. for Newtonian fluid equilibrium mode in the above sense is impossible.

Obviously, too large particles at a given flow rate u_v cannot be carried to the surface. The minimum diameter of such particles is determined by the zero rate of removal. This flow rate is called levitation speed u_w [10]. Assuming $u_k = 0$, we have $u_w = -u_v$, since $u_v < 0$, then $u_w > 0$. For the steady-state flow from (12) and taking into account (13), we obtain

$$u_w = \frac{d_k \tau_0}{3\eta_c f(u_w, d_k)}\left(\frac{d_k}{d_k^*} - 1\right). \tag{14}$$

Thus, one can find the limiting diameter of the particles carried out d_k^{**} for given flow parameters $d_k^{**} > d_k^*$, moreover, since $u_w > 0$. Geometrically, the value d_k^{**} corresponds to the intersection point of the particle velocity graph with the abscissa axis. When $d_k \to d_k^{**}$ the particle removal time goes to infinity, approaching the vertical asymptotes.

When $d_k \le d_k^*$ the levitation is impossible. Indeed, since during levitation $u_k = 0$, and in this area $u_k = u_v$, then the flow rate is $u_v = 0$. We obtain an equilibrium mode in the state of static equilibrium. In this case, the particles of the drilling return appear to be «frozen» in the drilling fluid. This means that when the circulation of the fluid stops (for example, for some technical reasons), drilling return particles will not settle, contaminating the well. Note that special cases of formulas (13), (14) (for $\alpha_c = 1$, $f = 1$) were previously obtained in [10].

Conclusions

1. We implemented a mathematical model of rock crushing during drilling based on fragmentation theory and determined the relationship between the parameters of the model and the drilling process. We considered such factors as the properties of the drilled rock, the rate of penetration, the type of bit, and the supplied power. The constructed model allows us to obtain the distribution of particle sizes formed during drilling at specified intervals and calculate the average value in each interval. This is important for solving the problem of drilling cuttings removal, as well as the design of systems for cleaning and regeneration of drilling fluid.

2. Based on the averaged hydrodynamic equations, a mixture model of drilling fluid circulation as a viscoplastic fluid, moving together with particles of the cutting material, was constructed. A closing relation for the interfacial interaction force was derived considering the viscous-plastic properties of the drilling fluid. An algorithm for the numerical solution of the resulting system of equations was proposed. The problem of the removal of drilling cuttings from the well was investigated. The existence of an equilibrium flow mode was shown, which is the most beneficial to use for the removal of drilling cuttings .

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