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SIMULATION OF OPERATION OF PNEUMATIC COMPENSATOR WITH QUASI-ZERO STIFFNESS IN THE ELECTRIC CENTRIFUGAL SUBMERSIBLE PUMP UNIT

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The ECSPU pneumatic compensators with quasi-zero stiffness are proposed. The pneumatic compensator with quasi-zero stiffness is suggested to be made in the form of pneumatic spring assemblies having a power characteristic with a positive stiffness working area and a set of successively connected Belleville springs and a power characteristic with a working area of negative stiffness. Structurally, a set of Belleville springs is located inside the air spring and supports pneumatic compensator piston. As a result of adding the negative stiffness of the disc spring washers set and the positive stiffness of the pneumatic spring, the resulting system (the proposed pneumatic compensator) acquires a quasi-zero or specified low stiffness.

The efficiency of the suggested pneumatic compensator was determined by the possibility of moving its piston from the effects of various pressure drops. It was assumed that the greater the distance the piston can move under a given action, the more effective the pneumatic compensator is. The effect of various forces acting on the piston in the case of pressure drops on the discharge line of the electric centrifugal submersible pump units (ECSPU) is simulated: a rapidly decreasing load; a sudden increase in the force acting on the piston and vibration impact. In all the considered examples, the displacement of the piston was several meters, which corresponds to the length of the working area of the power characteristic of the considered pneumatic compensator with quasi-zero stiffness. It is shown that existing pneumatic compensators, which are like gas caps, are in principle unable to provide the same displacement of the piston under the same effects on it. For their effective operation, the size of the gas cap should be several tens of meters, which is impossible in the conditions of the well. In the calculations, it is shown that it is possible to manufacture the necessary disk spring washers from various materials: steel; fiberglass FGM; beryllium bronze. Of particular interest are disk spring washers made of beryllium bronze, which are capable of withstanding up to 20 billion load cycles.

Key words: pneumatic compensator, pneumatic spring, electric centrifugal submersible pump units, quasi-zero stiffness, negative stiffness, disk spring washers

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To compensate for pressure fluctuations in the discharge line of the electric centrifugal submersible pump units (ECSPU), a pneumatic compensator has been developed that has a power characteristic (the dependence of the displacement restoring force) with the working area of quasi-zero stiffness. In this area, the restoring force is almost constant and does not depend on the displacement of the pneumatic compensator piston [1].

Systems with quasi-zero stiffness have been known since the 1970s and were proposed for the first time by the Soviet scientist P.M. Alabuzhev mainly for protection against vibrations [4]. In the system of P.M.Alabuzhev, as in most existing systems with quasi-zero stiffness [8], the working area is obtained by adding positive and negative stiffness. To achieve negative stiffness (the angle of tangent tilt to the power characteristic is less than zero [9]), in the system of P.M. Alabuzhev a Mises girder is used. To obtain a positive stiffness (the angle of tangent tilt to the power characteristic is greater than zero), an ordinary spring is used. The parameters of the system are chosen so that when the power characteristics of a Mises girder with negative stiffness are combined with the spring with positive stiffness, the total power characteristic with a working area of a quasi-constant force with quasi-zero stiffness is obtained-the angle of tangent tilt to the power characteristic in this area is approximately equal to zero.

In existing quasi-zero stiffness systems ensuring protection against vibration, the force characteristic of the working area is provided by loading these systems with the weight of the object to be protected against vibration [7].

In our case, the force acting on the piston of the pneumatic compensator at the pump intake is directed upward and is determined by the formula

$$F_{\Sigma}^{AB} = pS\,,$$

where p – pressure at the pump intake, MPa; S – pneumatic compensator piston square area, m^2 (Fig.1).

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Because of its small size the weight of the piston was not taken into account. To obtain negative stiffness in the proposed pneumatic compensator with quasi-zero stiffness, a set of series-connected Belleville springs is used, which is located inside the pneumatic compensator (pneumatic spring) (Fig.1).

It is known that the power characteristic of a disk spring washer with a certain ratio of dimensions (Fig.2) can have a section of a power characteristic with negative stiffness [1].

The number of successively located disk spring washers provides the length of the working area of the power characteristic with quasi-zero stiffness. The total power characteristic of a pneumatic compensator is determined by the formula

$$F_{\Sigma}=\frac{p_0(\frac{\pi D^2}{4})H}{(H-x)}+8\pi Es(\frac{x}{N})\times$$

$$\times\frac{\left\{(f-(\frac{x}{N}))(f-(\frac{x}{N})/2)\left[\frac{(D+d)}{2(D-d)}-\frac{1}{\ln(D/d)}\right]+s^2\ln(\frac{D}{d})/12\right\}}{(D-d)^2}, \quad (1)$$

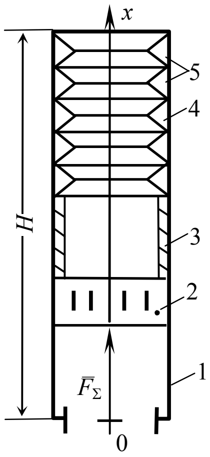

Fig.1. Scheme of pneumatic compensator with quasi-zero stiffness

1 – body; 2 – piston; 3 – bush;
4 – disk spring washers;
5 – sealing

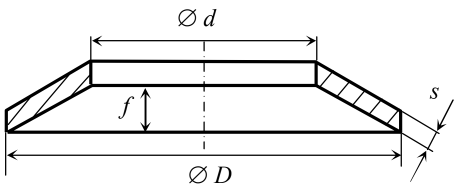

Fig.2. Disk spring washer

where p_0 – initial pressure in the pneumatic compensator; H – height of the pneumatic compensator cylinder (see Fig. 1); N – number of disk spring washers; E – module of elasticity the first type of disk spring washers made of a certain material; s – thickness of the cone of disk spring washer (Fig.2); x – compression of disk spring washer; f – height of the disk spring washer cone; D – external diameter of disk spring washer; d – internal diameter of disk spring washer.

The first summand determines the restoring force of the air spring, the second defines the restoring force of the set of series-connected Belleville springs.

Taking up $f = 0.005$ m; $D = 0.056$ m; $E = 2.1\cdot10^{11}$ N/m^2, it is necessary to choose such values of p_0, s, N, d, that, on the one hand, the dependency $F_{\Sigma}(x)$ has an area with quasi-zero stiffness (quasi-constant force F_{Σ}^{AB}), and, from the other hand, it matches the beginning of the working area with quasi-zero stiffness (point A, Fig.3) and is provided by the predetermined pressure p at the discharge line of the ECSPU $(F_{\Sigma}^{AB}\approx pS)$.

Figure 3 shows dependencies F_{Σ}, F, P, calculated using formula (1), where F – the first summand of the equation, and P – the second one. The parameters of the pneumatic compensator are chosen in such a way that this force moves the piston to the working area AB of the power characteristic with quasi-zero stiffness. The parameters p_0, s, N, d are chosen so that when adding the areas of power characteristic with negative $A'B'$ and positive stiffness $A''B''$ a section with quasi-zero stiffness AB is obtained.

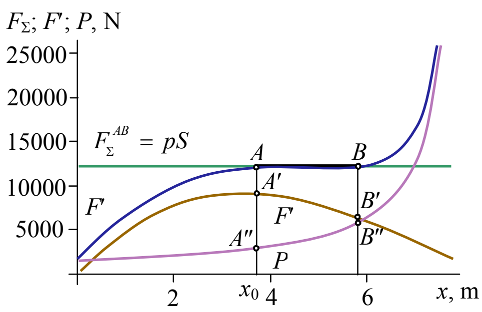

Fig.3. Power characteristic of pneumatic compensator with quasi-zero stiffness (F_{Σ}^{AB} at AB section):

F' – power characteristic of a set of disk springs;
x_0 – A point coordinate;
P – power characteristic of pneumatic spring;
F_{Σ} – resulting power characteristic

The power characteristic (Fig.3) of the pneumatic compensator is calculated for discharge line of the ECSPU with pressure of $p = 50$ bar, piston area $S \approx 0.0025$ m^2 and piston height of $H = 8$ m $(F_{\Sigma}^{AB} = pS \approx 12315\,\text{H})$. It turned out that power characteristics for the predetermined pressure at the dis-

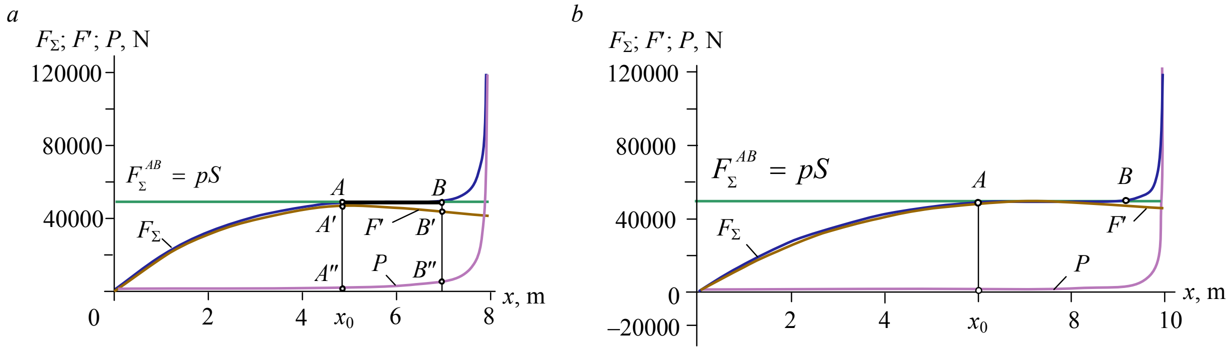

Fig.4. Two power characteristics of pneumatic compensator with quasi-zero stiffness: a – $p_0 = 3 \cdot 10^5$ N/m^2, $s = 0.028$ m, $N = 1594$, $d = 0.026$ m, $H = 8$ m; b – $p_0 = 10^5$ N/m^2, $s = 0.027$ m, $N = 2260$, $d = 0.032$ m, $H = 10$ m

For legend signs see Fig.3

charge line of the ECSPU can be obtained with one and the same quasi-constant force F_Σ^{AB} with different parameters of p_0, s, N, d, H (f, D, E values are constant).

Figure 4 shows two power characteristics with quasi-zero stiffness with one and the same quasi-constant force $F_\Sigma^{AB} \approx 49260$ N for pump discharge pipe pressure of $p = 200$ bar, but with different values of p_0, s, N, d, H (with constant f, D, E) (calculated using formula (1)). When the discharge line pressure of the electric centrifugal submersible pump acts on the pneumatic compensator piston the coordinate origin moves to point A (Fig.3, 4).

Then the equation defining the restoring force will have the form

$$\hat{F}_\Sigma = F_\Sigma(x - x_0) - pS\,, \tag{2}$$

where F_Σ is calculated using formula (1).

To determine coordinate x_0 the force $F_\Sigma^{AB} = pS$ is equated to the force F_Σ:

$$pS = \frac{p_0\left(\frac{\pi D^2}{4}\right)H}{(H - x_0)} + 8\pi Es\left(\frac{x_0}{N}\right)\frac{\left\{\left(f - \left(\frac{x_0}{N}\right)\right)\left(f - \left(\frac{x_0}{N}\right)/2\right)\left[\frac{(D+d)}{2\cdot(D-d)} - \frac{1}{\ln(D/d)}\right] + s^2\ln\left(\frac{D}{d}\right)/12\right\}}{(D-d)^2}\,. \tag{3}$$

The power characteristic described by equation (2) is shown in Fig.5. Here is shown the section of the power characteristic with quasi-zero stiffness AB (see Fig.4, b) in other coordinate system, taking into account the effect of pressure at the discharge pipe of the on the ECSPU. The effect of dry friction forces F_{fr} form a hysteresis loop. When the piston of the pneumatic compensator moves, its coordinates will change in the direction of the arrows.

Consider the movement of the pneumatic compensator piston when the bottom hole pressure is changed. Typical effects on the piston from pressure drops at the discharge line of the ECSPU can be divided into three types: rapidly decreasing load (Fig.6, a); a sudden increase in the force acting on the piston (Fig.6, b); vibration impact (Fig.6, c).

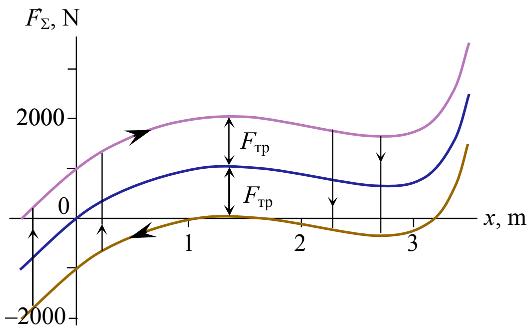

Fig.5. Working area of power characteristic of PC with a hysteresis loop

Each proposed impact can be described analytically. The function that describes a rapidly decreasing load is as following, $F_1 = (A_1 th[\omega(t - t_0)] + A_1)$ $(A_1 - A_1 th[\omega_1(t - t_0)])$.

The function describing the sudden change of the force acting on the piston is $F_2 = (A_2 th[\omega_2 t])$. The function describing vibration impact is, $F_3 = (A_3\cos[\omega_3 t])$. The amplitude values A_1, A_2, A_3 were chosen so that ($A_1 = 50$ N$^{1/2}$; $A_2 = 2500$ N; $A_3 = 2500$ N) maximum force acting on the pneumatic compensator piston were equal to:

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$$F_{1_{\max}} = F_{2_{\max}} = F_{3_{\max}} = 2500 \text{ N}.$$

The coefficients ω_1, ω_2 determine values Δ1, Δ2, shown at Figure 6. The value Δ1 defines the extent of the rapidly decreasing load; the value Δ2 – change rate of the force acting on the pneumatic compensator piston (Fig.6, *b*). The coefficient ω_3 determines the period of vibration forces *T*, acting on pneumatic compensator piston (Fig.6, *c*).

Because of the difficulty in determining the delivery head of the ECSPU during its joint operation with a pneumatic compensator with quasi-zero stiffness, the efficiency of the proposed device was determined by the possibility of its movement when pressure drops act on it. It was assumed that the more the piston of the pneumatic compensator moves when subjected to the predetermined pressure drops on the ECSPU discharge line, the more efficient is the operation of the pneumatic compensator.

Let us consider the motion of the piston of the pneumatic compensator with quasi-zero stiffness when a rapidly changing force is applied to it (Fig.6, *a*). The differential equation describing this motion has the following form

$$m\ddot{x} = (A_1 th[\omega_1(t-t_0)] + A_1)\times$$

$$\times(A_1 - A_1 th[\omega_1(t-t_0)]) - F_{\Sigma}(x) - F_{\text{fr}}\text{sign}[\dot{x}], \quad (4)$$

where $m = 0.2$ kg – piston weight (arbitrary value); $t_0 = 2$ s – time shift of a rapidly changing force; $F_{\Sigma}(x)$ – resulting restoring force in the area of quasi-zero stiffness [see formula (1), Fig.5].

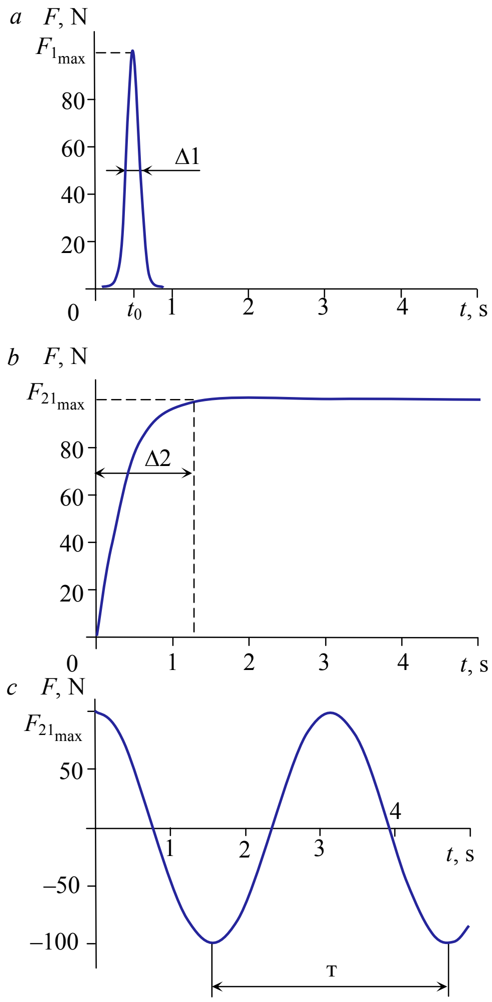

Fig.6. Typical forces acting on ECSPU pneumatic compensator piston:
a – a rapid decreasing load; *b* – a sudden i ncreasing force acting on the piston;
c – vibration impact

The following parameter values are accepted: $H = 10$ m; $f = 0.005$ m; $D = 0.056$ m; $d = 0.032$ m; $E = 2.1\cdot10^{11}$ N/m^2; $p_0 = 10^5$ N/m^2; $N = 2260$; $s = 0.0027$ m; $F_{\text{fr}} = 200$ N – constant force of dry friction acting on the pneumatic compensator piston (arbitrary value).

Figure 7 shows the dependence of the pneumatic compensator piston coordinate from time under the action of a rapidly decreasing load (see Fig.6, *a*) calculated numerical solution of the differential

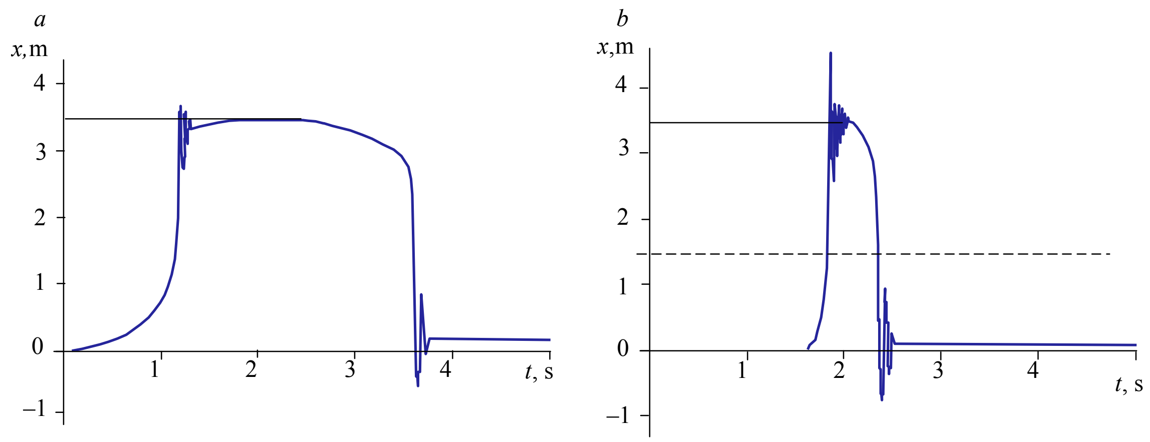

Fig.7. Dependences of the pneumatic compressor piston coordinate with quasi-zero stiffness from time under the action of a rapidly decreasing load: *a* – $\omega_1 = 1$ s^{-1}; *b* – $\omega_1 = 5$ s^{-1}

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equation (4). As can be seen from the figure, for different values of ω_1 the maximum displacement of the piston is approximately 3.5 m. With other values of ω_1 the maximum displacements of the piston will stay the same (not mentioned here).

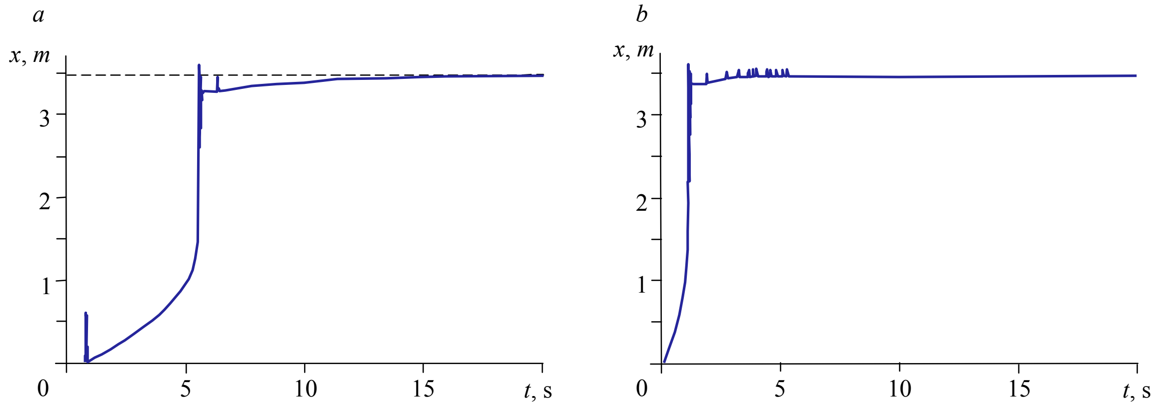

Fig.8. Dependences of the pneumatic compressor piston coordinate with quasi-zero stiffness from time under the action of a sudden increasing force: *a* – $\omega_2 = 0.1\ s^{-1}$; *b* – $\omega_2 = 0.5\ s^{-1}$

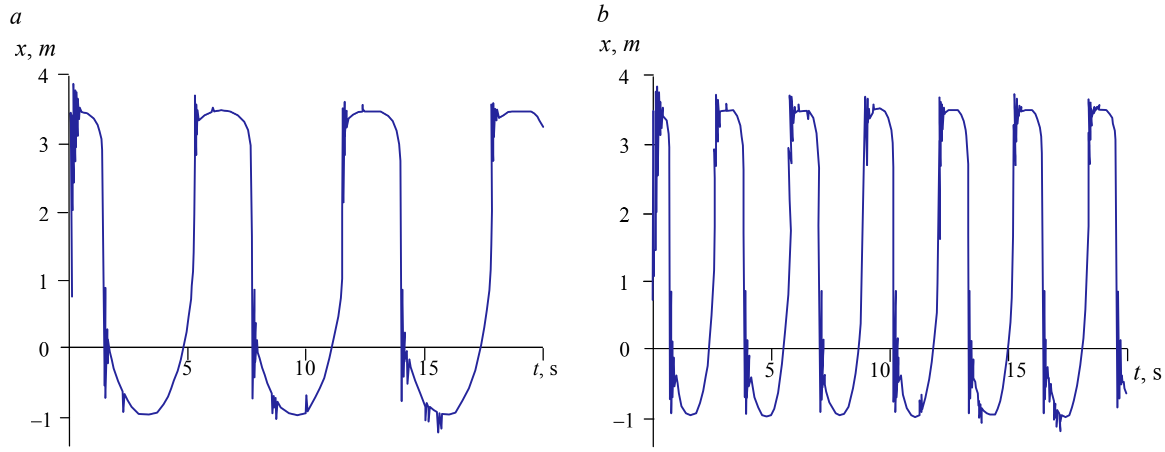

Fig.9. Dependences of the pneumatic compressor piston coordinate with quasi-zero stiffness from time under the action of vibration impact with different frequencies: *a* – $\omega_3 = 1\ s^{-1}$; *b* – $\omega_3 = 2\ s^{-1}$

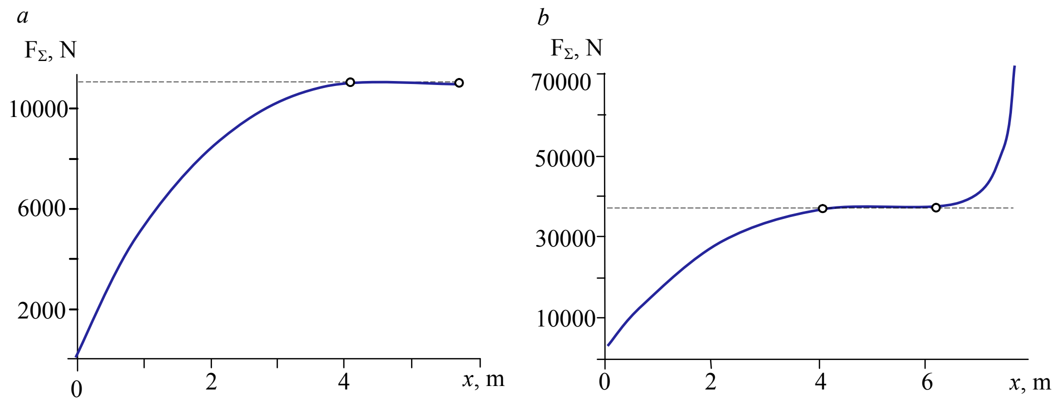

Fig.10. Power characteristics of a pneumatic compensator with quasi-zero stiffness, with the of the different disk springs material: *a* – fiberglass FGM (fiberglass material); *b* – beryllium bronze

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The differential equation describing the motion of the piston with a sudden increase in the force acting on it (see Fig.6, *b*) has the form

$$m\ddot{x} = A_2 th[\omega_2 t] - F_\Sigma(x) - F_{\text{fr}}\,\text{sign}[\dot{x}]. \tag{5}$$

Figure 8 shows the dependencies of the pneumatic compensator piston coordinate from time, obtained by numerical solution of the differential equation (5). As can be seen from the figure, for different values of ω_2 the maximum displacement of the piston is also 3.5 m. With other coefficients of ω_2 the maximum displacements of the piston will stay the same (not mentioned here).

Differential equation (6) describes the motion of the pneumatic compensator piston with vibration change in the force acting on it (see Fig. 6, *c*)

$$m\ddot{x} = A_3 \cos[\omega_3 t] - F_\Sigma(x) - F_{\text{fr}}\,\text{sign}[\dot{x}]. \tag{6}$$

Figure 9 shows the dependence of the pneumatic compensator piston coordinate from time under vibrational action (see Fig.6, *c*) at different frequencies, obtained by numerical solution of the differential equation (6). The maximum displacement of the piston here is also approximately 3.5 m. Resonance frequencies, very small for quasi-zero stiffness, were not considered here. At other non-resonance angular frequencies ω_3 the maximum displacement of the piston is also 3.5 m (not mentioned here).

In the considered examples, the maximum displacement of the piston is approximately equal to the length of the working area (≈ 4 m) of the power characteristic of the proposed pneumatic compensator with quasi-zero stiffness (Fig.6, *b*). Consequently, under the considered impacts (Fig.6), the piston will move almost the entire length of the working area and compensate for the pressure drop on the discharge line of the ECSPU. Since the length of the working area with quasi-zero stiffness can be changed by properly selecting the parameters of the pneumatic compensator [see formula (1)], for any real values of the amplitudes A1, A2, A3 [see formulas (4)-(6)], one can obtain the necessary length of working area.

The algorithm for selecting the parameters of the proposed pneumatic compensator for obtaining the required power characteristic under the condition that the quasi-zero stiffness matches the working area at a given pressure on the discharge line of the ECSPU is extremely complicated, so it is practically impossible to mathematize it. The selection of the parameters of the pneumatic compensator for each particular pressure on the discharge line of the ECSPU was carried out in such a way that the power characteristic with a section of quasi-zero stiffness could be obtained visually. The calculations showed that this is sufficient.

In the cases discussed above, the disk spring washers were made of steel ($E = 2.1\cdot 1011$ N/m^2). Fig.10, *a* shows the power characteristic of the pneumatic compensator, obtained with the use of disk spring washers made of fiberglass FGM (modulus of elasticity of the first type is $E = 35$ GPa, the material was chosen arbitrarily) [7]. Fig.10, *b* shows the power characteristic of the pneumatic compensator with quasi-zero stiffness, in which the disk spring washers are made of beryllium bronze (modulus of elasticity of the first type $E = 135.1$ GPa) [3].

This material has a high endurance limit. It is known that springs of beryllium bronze can withstand up to 20 billion cycles of load. The shortcomings of this material include its high cost and toxicity. The ECSPU pressures to which these power characteristics were calculated, they are 45 and 150 bar.

At present, the parameters of the pneumatic compensators, which are like gas caps, are determined from the following relationship [2]:

$$\frac{P_{\max} - P_{\min}}{P_{av}} = \frac{V_{\max} - V_{\min}}{V_{av}}, \tag{7}$$

where $P_{\max}$ – maximal pressure of discharge pipe of the ECSPU; $P_{\min}$ – minimal pressure of discharge pipe of the ECSPU; $V_{\max}$ – maximal volume of the gas cap; $V_{\min}$ – minimal volume of the gas cap; P_{av} – average pressure of discharge pipe of the ECSPU; V_{av} – average volume of the gas cap,

$$P_{av} = \frac{P_{\max} + P_{\min}}{2};\ V_{av} = \frac{V_{\max} + V_{\min}}{2}.$$

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Let us take up $P_{max} – P_{min}$ = 2 bar; P_{av} = 30 bar. For a classical pneumatic compensator located in the well, which is a gas cap (pneumatic spring), the volume will be determined by the product of the pneumatic compensator piston square area S times by the height of the pneumatic compensator cylinder H. Then, on the right-hand side of the relation (7), the area S can be canceled out:

$$\frac{2}{30} = \frac{H_{max} - H_{min}}{H_{av}}. \tag{8}$$

If the value of $H_{max} – H_{min}$ = 4 m and is comparable to the length of the working section of the above-mentioned power characteristic with quasi-zero stiffness (see Fig.6, *b*), then H_{av} = 60 m. With other values $P_{max} – P_{min}$ and P_{av} the value of H_{av} remains not very large. Therefore, it is practically impossible to obtain an effective pneumatic compensator in the traditional way.

At present, there are activities to create a laboratory bench for assessing the adequacy of the obtained theoretical results. It is supposed to create real units and test them in production. This direction in the creation of pneumatic compensators having new operating principle seems promising.

Conclusions

1. The ECSPU pneumatic compensator has been proposed for the first time, it has a power characteristic with a working area of quasi-zero stiffness, which is a combination of a pneumatic spring and a set of series-connected Belleville springs, which is located inside the pneumatic spring and supports the piston of a pneumatic compensator.
2. The efficiency of the proposed pneumatic compensator was determined by the displacement of the pneumatic compensator piston when various consequences from the pressure drop on the discharge line of the ECSPU were acting on it: the more displacement was, the more effective the pneumatic compensator was.
3. The displacement of the pneumatic compensator piston, when it was subjected to forces from the pressure drops of the discharge line of the ECSPU was several meters, which corresponds to the length of the working area of the power characteristic of the considered pneumatic compensator with quasi-zero stiffness.
4. The possibility of creating Belleville springs made of various materials: steel; fiberglass FGM; beryllium bronze was proved.

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