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ALGORITHM FOR THE PROJECTION OF FOREST GROWTH AND PRODUCTION

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Resumo

Algoritmo para projeção de crescimento e produção de florestas. A modelagem do crescimento e produção florestal é uma ferramenta essencial para o manejo florestal, pois permite realizar simulações e projetar variáveis biométricas da floresta no futuro, auxiliando assim no planejamento de estoque, bem como análises econômicas. Neste trabalho é proposto um modelo de crescimento e produção por distribuição de diâmetros com a aplicação da função Weibull, baseado na recuperação de parâmetros por meio de funções simplificadas entre os atributos da floresta e os parâmetros da função Weibull. O algoritmo foi desenvolvido em linguagem VBA do Excel. A validação foi realizada com dados de Inventário Florestal Contínuo (IFC) em um povoamento de *Khaya grandifoliola* e em renques de *Eucalyptus* spp. em sistema ILPF, que foram organizados ordinalmente em sete combinações de datas, das mais afastadas às mais próximas da data de projeção. Os resultados foram avaliados pelo Erro Padrão Percentual (EP%) aplicado aos volumes projetados e observados, e pelo teste de Kolmogorov-Smirnov aplicado as distribuições de diâmetro para verificação de aderência. Foi possível identificar uma relação exata para o parâmetro *c* da função Weibull em função dos percentis e do parâmetro *b*, aprimorando o método de recuperação de parâmetros. Outro aprimoramento metodológico foi o uso do diâmetro máximo e altura máxima por idade para ajuste da função hipsométrica. O algoritmo apresentou resultados para volume total com erros de até 20% em 85% dos testes.

Palavras-chave: Prognose, distribuições probabilísticas, modelagem.

Abstract

The modeling of forest growth and production is an essential tool for forestry management because it allows us to perform simulations and project forest biometric variables in the future, thus assisting in stock planning and economic analyses. In this work, a growth and production model by diameter distribution was proposed with the application of the Weibull function based on the recovery of parameters through simplified functions between the forest attributes and the parameters of the Weibull function. The algorithm was developed in Excel's VBA language. Validation was performed with data from the Continuous Forest Inventory (CFI) in a stand of *Khaya grandifoliola* and in rows of *Eucalyptus* spp. in the ILPF system, which were ordinarily organized into seven date combinations, from the most distant from to the closest to the projection date. The results were evaluated by the percentage standard error (SE%) applied to the projected and observed volumes and by the Kolmogorov–Smirnov test applied to the diameter distributions to verify adherence. It was possible to identify an exact relationship for parameter *c* of the Weibull function as a function of the percentiles and for parameter *b*, improving the parameter recovery method. Another methodological improvement was the use of maximum diameter and maximum height for age to adjust the hypsometric function. The algorithm presented results for total volume with errors up to 20% in 85% of the tests.

Keywords: Prognosis, probability distributions, modeling.

INTRODUCTION

A forest growth and production model (FG&P) aims to efficiently predict future tree yields, which will depend on genetic, climatic, pedological, silvicultural tract and phytosanitary factors. As all modeling is a simplification of reality, to meet this objective, it is necessary to evaluate the cost–benefit relationship, especially of data acquisition, when considering how many factors should be used, together with the efficiency required to predict the future dimensions of the trees.

One of the solutions usually employed uses a continuous forest inventory (CFI), which aims to capture the interference of these factors intrinsically, modeling the growth trend observed in a given period, to predict production at a future age following this trend. This is the basis of most empirical models, with some biological characteristics (POGODA *et al.*, 2019; RIBEIRO *et al.*, 2014; LEITE *et al.*, 2013; RETSLAFF *et al.*, 2012; MIGUEL *et al.*, 2010; CAO, 2004) and the use of artificial intelligence, when there is no need to observe biophysical relationships between the variables (DIAMANTOPOULOU *et al.*, 2015).

Among the empirical models, Retslaff *et al.* (2012) presented and tested a set of functions that integrate a FG&P model for estimating and projecting forest attributes (basal area, minimum, maximum, mean and quadratic diameters, and number of remaining trees). The attributes most correlated with the parameters of a probability distribution will be projected to the future age as input variables, estimating the probability distribution, which informs the number of trees per diameter class at a future age. CAO (2004) adopted a generic function in one of the tested methods, considering the parameters *a*, *b* and *c* of the Weibull function and the diameter at the percentile or the mean diameter as the dependent variables and the spacing, density, dominant height and age as the independent variables. Leite *et al.* (2013) tested two methods, parameter prediction and projection. In the prediction method, functions are fitted involving parameters of the probability distribution and inventory attributes with all ages, while in the projection method, functions are fitted with a combination of later and earlier ages associated with the parameters.

These are the main characteristics of a FG&P model with diameter distribution used to obtain forest inventory attributes that are correlated with the parameters of a probabilistic distribution and to propose functions that obtain greater efficiency in the estimation of the parameters at a future age.

Thus, the functionality of a model lies in its functions and not in the equations, which will be useful only for the conditions under which they were generated. For growth and production prognosis, the distribution model by diameter classes using the diameter at breast height (DBH) of the trees is the most common metric (OGANA *et al.*, 2020; MIRANDA *et al.*, 2018; JESUS *et al.*, 2018; JESUS *et al.*, 2017; AZEVEDO *et al.*, 2016; POUDEL; CAO, 2013; LEITE *et al.*, 2013; RETSLAFF *et al.*, 2012; MIGUEL *et al.*, 2010) used for the information on the forest structure with only dendrometric measurement variables (CAP and Ht).

There are growth and production models called individual trees, which include variables that simulate factors that affect tree growth, such as tree size, distance between trees, age, site index, canopy coverage rate and basal area, and these are used to generate competition indices. Although they were developed for application in native forests, they have been applied in planted forests (CASTRO *et al.*, 2014). These models require a greater number of variables from forest inventories than empirical models require.

Other models, such as process-based models (PBMs), which simulate ecophysiological processes such as radiation use efficiency, carbon balance models, and partitioning and simple stand nutritional parameters, are deterministic models. An example is 3-PG (physiological processes predicting growth), which calculates photosynthesis, transpiration, biomass partitioning to tree parts and litter production from rainfall, temperature, solar radiation, soil water, local quality, and other biophysics variables (GUPTA and SHARMA, 2019; OLIVEIRA *et al.*, 2018). These models are more robust because they implement functions that simulate plant growth; however, they have a high demand for data entry compared to empirical models.

Among the software that implement empirical models for the prognosis of planted forests is SigmaE, which contains a module for the growth and production of trees based on the diameter distribution, which includes percentage inflows, with a simulator of thinning and conversion of stems into logs (DynaTree) (COSTA *et al.*, 2020), and the Sis family (OLIVEIRA, 2011) for several forest species, which simulates forest growth and production with or without thinning, with economic evaluation of classes of wood products. A specialty of its methodology is a set of internal information, which does not require the continuous forest inventory (CFI) data or consolidated data on the stand, such as basal area, density, site index, and age.

However, this work implements a model by diametric distribution, in which it is necessary to adopt a probability function, such as SB Johnson, Beta, Gamma, Log-Normal, or Weibull. However, in methodologies for estimating prognosis (POGODA *et al.*, 2019; RETSLAFF *et al.*, 2012; MIGUEL *et al.*, 2010; CAO, 2004), the Weibull function is regularly present, and it has even applied in native forests (ORELLANA *et al.*, 2017; LIMA *et al.*, 2015) due to the good results of adherence to the observed data (RIBEIRO *et al.*, 2014; LEITE *et al.*, 2013).

To estimate the parameters (location, scale and shape) of the Weibull function, techniques of percentiles, maximum likelihood, method of moments, or linear regression are used (HUDAK; TIRYAKIOGLU *et al.*, 2009; CAO, 2004; BERGER and LAWRENCE, 1974). To project the diameter distribution at a future age, prediction techniques are adopted when the parameters of the probability density function are obtained by regression with the attributes of the forest (WANG *et al.*, 2011) or by recovery of the parameters using the method of moments or percentiles (POUDEL; CAO, 2013).

The methodologies for projecting the parameters of the Weibull function use 1st- and 2nd-order statistics of variables extracted from remeasured samples of the CFI, such as the quadratic diameter, mean diameter, minimum and maximum diameters, dominant height and age. These variables obtained from the samples usually correlate with the location, scale and shape parameters of the Weibull function (AZEVEDO *et al.*, 2016; LEITE *et al.*, 2013; MIGUEL *et al.*, 2010).

The hypothesis is that the forest growth and production model developed using linear functions, the Weibull function and the parameter recovery method obtains results with errors less than or equal to 20% accuracy (i.e., the error level threshold usually required by forest inspection agencies, such as the State Institute of Forests of Minas Gerais). For validation, data from continuous forest inventories in different production systems of approximately 9 years of age were used (a stand of *Khaya grandifoliola* C. DC. 1907 and rows of *Eucalyptus* spp.

in an ILPF system), and these data were grouped into minimal sets of three measurements to generate estimates of tree diameter and height growth, which were then compared to the observed data.

MATERIALS AND METHODS

Study area

The plantations of African mahogany (*Khaya grandifoliola*) and eucalyptus rows (*E. urophilla clone x E. grandis*) in the ILPF systems used for validation were located in Sete Lagoas, in the central region of Minas Gerais, in the Cerrado biome, and the fertilization methods followed the recommendations for potential production. The mahogany seedlings were drip irrigated in the first two years because the climate of the region is seasonal, Cwa type, with a dry season in winter (May to October). The average annual rainfall is 1,335 mm, approximately 70% of which occurs between October and February (PEEL *et al.*, 2007). Eucalyptus plantations in this region do not require irrigation, but in the case of planting in 2009, it was necessary to irrigate in the first year because planting occurred late, in early February.

Mahogany was planted on 12/01/2009 at 5 × 5 m spacing (25 m²). For this study, a plot with 50 trees was selected from the CFI. The diameter at the 1.3 m (DBH in cm) and total height (h in m) in the respective months are reported in Table 1.

Eucalyptus was planted at 15 × 2 m spacing in the ILPF system on two dates: the first on 02/05/2009 and the second on 10/24/2011. In each row, one tree was selected for every 10 trees, and 40 trees per system were sampled. The diameter at 1.3 m (DBH in cm) and total height (ht in m) were measured. The measurement periods of both systems are reported in Table 1.

Table 1. Dates and months of forest inventory measurement (CFI) on the forest systems.
Tabela 1. Datas e meses de mensuração do IFC para os sistemas florestais.

Mahogany		Euc2009		Euc2011	
Date	Months	Date	Months	Date	Months
01/05/2012	29.0	04/06/2012	39.9	06/12/2013	25.8
01/05/2013	41.0	24/04/2013	50.6	13/11/2014	37.2
01/05/2014	53.0	27/05/2014	63.7	26/11/2015	49.8
17/05/2015	65.5	09/07/2015	77.1	03/10/2016	60.2
12/06/2016	78.4	03/10/2016	92.0	21/11/2017	74.0
04/06/2017	89.7	14/08/2017	102.3	27/11/2018	86.4
05/10/2018	106.2	29/05/2018	111.8	12/11/2019	98

Volume models

To model the shape of the bole, 13 mahogany trees were cubed in 2018 using a digital dendrometer RD 1000 Criterion, with an accuracy of 6 mm (NICOLETTI *et al.*, 2015). To model the shape of the bole in the two eucalyptus plantations, rigorous cubing was performed in 2014, using 29 trees thinned at planting in 2009 and 24 trees thinned at planting in 2011.

Algorithm and functions

The algorithm was developed in Visual Basic Application (VBA) with the purpose of projecting the DBH and the total height of trees in a future age. Table 2 presents a summary of the use cases. The first step was to obtain the thinning and volume equations per tree from the tree shape data (obtained by cubing one or a set of trees). This was performed by the first button (Shape Factor, Shape Eq., Cubage.txt) by reading the Taper.txt file to adjust the shape equation according to the Kozak model (1969) (Eq. 1). The form factor per tree was generated, and the cubage file “Cubagem.txt” was automatically created. The second button (Volume Equation) requests the reading of the Cubagem.txt file to adjust the volume equation of the Schumacher and Hall model (1933) (Eq. 2). Finally, the coefficients b_0, b_1 and b_2 of Eq. 1 and Eq. 2 were calculated.

$$(\mathrm{d_i}/\mathrm{dap})^2 = \mathrm{b_0} + \mathrm{b_1} * + \frac{h_i}{ht} + b_2 * \left(\frac{h_i}{ht}\right)^2 \quad (1)$$

$$\mathrm{v} = \exp\left(\mathrm{b_0} + \mathrm{b_1} * \ln(\mathrm{dap}) + b_2 * \ln(ht)\right) \ (2)$$

where b_0, b_1, and b_2 are estimated parameters; d_i and DBH (cm) are the diameter along the trunk and the diameter at breast height, respectively; eh_i (m) is the height along the trunk; and ht (m) is the total height.

The third button “Projection” executes the other procedures in an integrated manner. With the data from the CFI, the attributes of the sample were calculated, including the dominant height (hd) by the Assmann method, the basal area of the sample (G), the quadratic diameter (Dg), minimum (D_{min}) and maximum (D_{max}), the annual minimum height (H_{min}) and maximum height (H_{max}), the periodic increment (IPA) and the current increment (ICA). The number of trees per hectare (N/ha), the basal area per hectare (Gm^2/ha) and the volume per hectare (Vm^3/ha) were reported.

For the Weibull function, the diameters (x_1 and x_2) at the 24th and 93rd percentiles, adopted by Wendling *et al.* (2011), and the location, scale and shape parameters were estimated using the percentile method (RETSLAFF *et al.*, 2012; WENDLING *et al.*, 2011) from observed distributions, with a diameter class width of 1 cm.

The linearizable functions were then fitted: for dominant height, hd (m), the inverse of age (1/I) was used (Eq. 3); to project the future basal area (G), Eq. 5 was used; and the dominant height at a future age (hdf), which was calculated by the guide curve method using S (hd at index age (Ii)) and future age (If) was calculated using Eq. 4. The equations are as follows:

$$ln(hd) = b_0 + b_1 * \left(\frac{1}{I}\right) \qquad (3)$$

$$ln(\text{hdf}) = \ln(S) + b_1 * \left(\frac{1}{If} - \frac{1}{Ii}\right) \qquad (4)$$

$$ln(G) = b_0 + b_1 * ln(\text{I} * \text{hd}) \qquad (5)$$

where b_0 and b_1 are the parameters to be estimated, I is the age, Ii is the index age, If is the future age, S is the dominant height at the index age, hd is the dominant height, hdf is the dominant height at the future age, and G is the basal area of the sample.

The functions to obtain the minimum (D_{min}) and maximum (D_{max}) diameters were used for the designed diameter classes (Eqs. 6 and 7). The quadratic diameter (D_g) of the sample was informative (Eq. 8), and the functions of the diameters at the 24th (x_1) and 93rd (x_2) percentiles, Eqs. 9 and 10, were used to recover the diameter distribution parameters at a future age. All these attributes were regressed by the variable Naperian logarithmic age, *ln* (I), by the characteristic of linearity of the parameters as a function of time.

$$\left(D_{min}, D_{max}, D_g, x_1, x_2\right) = b_0 + b_1 * ln(\text{I}) \qquad (6\text{-}10)$$

The origin or location parameter (a) was adjusted by the diameter at the 24th percentile (x_1), and the scale parameter (b) was adjusted by the diameter at the 93rd percentile subtracted from a (x_2-a), as expressed in Eqs. 11 and 12. The parameter of form *c* has a power function: $y = b_0 * k^{-b_1}$, where $k = \frac{(x_2\text{-}x_1)}{b}$, which can be linearized according to Eq. 13. This relationship behaves as deterministically when applying the estimate of the parameters of the Weibull function by age (Figure 1). The recovery of parameters *a*, *b* and *c* of the Weibull function depends only on the diameters in the percentiles (x_1 and x_2) at a future age.

$$a = b_0 + b_1 * x_1 \qquad (11)$$

$$b = b_0 + b_1 * (x_2 - a) \qquad (12)$$

$$\ln(c) = b_0 + b_1 * \ln\left[\frac{(x_2 - x_1)}{b}\right] \qquad (13)$$

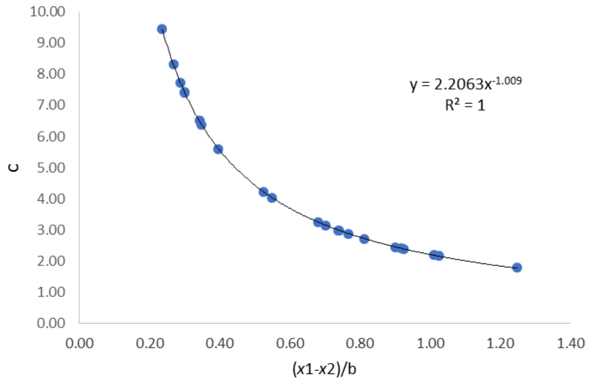

Figure 1. Scatterplot of the c parameter and k ratio from the adjustment of the Weibull function for the three forest systems with all measured data (data not shown).
Figura 1. Dispersão entre o parâmetro c e a razão *k* resultantes dos ajustes da função Weibull para os três sistemas florestais com todas as datas mensuradas (dados não mostrados).

To obtain the cumulative distribution, F_x, of future trees by diameter class using the Weibull function formula (Eq. 14), the upper limit of the diameter class (*cl*) and the projected parameters of the Weibull function were used:

$$F_x = 1 - exp\left\{-\left[\left(\frac{cl-a}{b}\right)^c\right]\right\} \qquad (14)$$

The hypsometric function (Eq. 15) was fitted with the data of maximum height, H_{max} (m), and maximum diameter, D_{max} (cm), of all ages from the CFI, eliminating the interference-suppressed trees and the need for an equation by age. This function was used to calculate the total height by diameter class of the designed trees:

$$ln(H_{max}) = \mathrm{b}_0 + \mathrm{b}_1 * ln(D_{max}) \qquad (15)$$

With the number of trees per class with a diameter of 1 cm and the total height corresponding to the class, the file of future trees was generated in a ‘name.txt’ file, which was the final objective of the algorithm.

Table 2. Use cases
Tabela 2. Casos de uso

Button	**Spreadsheet**	**Auxiliary worksheet**	**File**	**Purpose**
Fator Forma Eq.Forma Cubagem.txt	EqFormaVol	Report_Fshape	Taper.txt	Generate shape function Calculate volume and form factor Generate Cubagem.txt file
Equação Volume	EqFormaVol		Cubing.txt	Generate volume function
PROJEÇÃO	Projection	Weibull_D Weibull_D_aux	Projection.txt	Eq. basal area (G) Eq. minimum, maximum and quadratic diameters (D_{min}, D_{max}, D_g) Eq. for recovery of Weibull parameters (x_1, x_2, a, b, c) Eq. dominant height (Hd) Projection of Hd and G Projection of D_{min}, D_{max}, D_g Projection of x_1 , x_2 , a, b, c Estimated future diameter distribution Estimated height by Eq. Hypsometric Generate future ‘Tree.txt’ file

Validation

To aid in the validation of the total volume calculated by the projected diameter distribution, the total volume was also calculated using the linearized Gompertz equation (Eq. 16), where v_{max} (ha) is the maximum volume reached or the stagnation point of the curve. As v_{max} will vary with the genetic material and the site, the user needs to define a proportional factor in the projection age to reach v_{max} from the projected volume. In this study, v_{max} = 1.3 * volume observed at the last age measured in the CFI.

$$ln\left[ln\left(\frac{v_{max}}{v}\right)\right] = ln(b_0) + b_1 * I \qquad (16)$$

where b_0 and b_1 are the parameters to be estimated, I is the age, v_{max} is the volume at the age of stagnation of growth, and v is the volume at age I.

To make the projections, the CFI data were divided into seven combinations (six for three dates and one for all dates), not including the projection date.

The CFI date combinations for mahogany and eucalyptus planted in 2009 were 2012, 2013, and 2014; 2013, 2014, and 2015; 2014, 2015, and 2016; 2015, 2016, and 2017; 2012, 2014, and 2016; 2013, 2015, and 2017; and 2012, 2013, 2014, 2015, 2016, and 2017.

The combinations used for eucalyptus planted in 2011 were 2013, 2014, and 2015; 2014, 2015, and 2016; 2015, 2016, and 2017; 2016, 2017, and 2018; 2013, 2015, and 2017; 2014, 2016, and 2018; and 2013, 2014, 2015, 2016, 2017, and 2018.

The combinations have an ordinal nature, starting with more distant dates and ending with all dates, and two combinations (5 and 6) have dates with distances of approximately 2 years.

To identify the date combinations with the best approximation between the projected data and the CFI data, the percentage standard error (SE%) between volumes per hectare (Eq. 17) was obtained, and the Kolmogorov–Smirnov test was applied with 5% significance for adherence between the projected and observed diameter distributions:

$$EP = \frac{(projetado - observado)}{observado} *100 \qquad (17)$$

RESULTS

The coefficients of determination (R^2) for the equations of shape (Eq. 1) and volume per tree (Eq. 2) were 0.85 and 0.95 for mahogany; 0.93 and 0.94 for eucalyptus planted in 2009; and 0.97 and 0.98 for eucalyptus planted in 2011, respectively.

Table 3 provides the coefficients of determination (R^2) of the equations for basal area, minimum, maximum and quadratic diameters, diameters at the 24th and 93rd percentiles (Eqs. 5-10), parameters *a, b, and c* (Eqs. 11-13), dominant height (Eq. 4), total height per tree (Eq. 15), and the Gompertz function (Eq. 16). Table 4 shows the projected values of the same parameters for each combination of CFI dates.

The function that presented an exact fit (R^2=1) in all tests was the shape parameter (c), showing that $c = b_0 * \left(\frac{x_2 - x_1}{b}\right)^{-b_1}$, where c is expressed as a power ratio of the amplitude between the diameters at the 24th and 93rd percentiles divided by the scale parameter b.

The high R^2 values of the linear fit of the maximum and square diameters and of the percentiles with the logarithm of age (except for combinations 3 and 4 of the 2009 eucalyptus) indicated that the linear relationships of these variables with age were highly correlated, and the estimation of these parameters was not particularly complex in this case.

The variable that showed the lowest correlation in the adjustment for mahogany was the minimum diameter. For the 2009 eucalyptus, combinations 3 and 4 presented very low R^2 values. Nevertheless, in the diameter distribution of eucalyptus in 2009, parameter a showed a low quality in the fit of combinations 3 and 5-7. For the 2011 eucalyptus, no inconsistencies or R^2 values lower than 0.78 were observed.

Table 3. Coefficient of determination (R^2) of the fitted equations with data from the combinations.
Tabela 3. Coeficiente de determinação (R^2) das equações ajustadas com dados das combinações.

Dates of Comb.	**G**	**D_{min}**	**D_{max}**	**D_g**	**x_1**	**x_2**	**a**	**b**	**c**	**S**	**hipsm**	**VGptz**
						Mahogany						
2012-13-14	0.98	0.83	1.00	1.00	1.00	1.00	0.99	1.00	1.00	0.96	0.93	1.00
13-14-15	0.98	1.00	1.00	1.00	0.98	1.00	0.99	0.99	1.00	0.95	0.94	0.99
14-15-16	1.00	0.79	0.97	1.00	0.97	0.99	0.95	0.98	1.00	1.00	1.00	1.00
15-16-17	1.00	0.69	0.91	1.00	0.97	1.00	0.99	0.99	1.00	0.99	1.00	1.00
12-14-16	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.99
13-15-17	0.99	0.97	1.00	1.00	1.00	1.00	0.98	1.00	1.00	0.98	0.96	1.00
12-13-14-15-16-17	0.99	0.95	0.99	1.00	0.99	1.00	0.99	1.00	1.00	0.98	0.96	0.99
						Euc. 2009						
2012-13-14	1.00	0.79	0.99	0.99	0.99	0.99	0.88	0.97	1.00	0.96	1.00	0.96
13-14-15	0.98	0.76	0.95	0.93	0.95	0.93	0.93	0.99	1.00	0.85	1.00	0.93
14-15-16	0.53	1.00	0.12	0.72	0.75	0.58	0.00	0.97	1.00	1.00	0.26	0.72
15-16-17	0.01	0.99	0.31	0.45	0.58	0.03	0.87	1.00	1.00	0.78	0.99	0.66
12-14-16	0.99	0.99	0.93	1.00	1.00	1.00	0.16	1.00	1.00	0.92	0.87	1.00
13-15-17	0.88	0.93	0.69	0.94	0.96	0.88	0.30	1.00	1.00	1.00	0.94	0.93
12-13-14-15-16-17	0.92	0.95	0.81	0.96	0.97	0.93	0.10	0.99	1.00	0.95	0.86	0.95
						Euc. 2011						
2013-14-15	0.97	0.98	0.98	0.99	0.98	1.00	1.00	1.00	1.00	0.98	1.00	0.99
14-15-16	0.83	0.98	0.97	0.88	0.92	0.87	1.00	0.98	1.00	1.00	0.98	0.91
15-16-17	0.86	0.98	0.97	0.88	0.99	0.86	1.00	0.79	1.00	0.99	0.78	0.98
16-17-18	0.95	0.96	0.98	0.94	0.88	0.86	0.98	0.99	1.00	0.99	0.97	0.96
13-15-17	0.97	0.99	1.00	0.95	0.95	0.94	0.99	1.00	1.00	1.00	1.00	0.95
14-16-18	0.99	0.98	0.99	1.00	1.00	0.99	1.00	0.97	1.00	1.00	0.95	1.00
13-14-15-16-17-18	0.97	0.98	0.98	0.97	0.97	0.96	0.99	0.99	1.00	0.98	0.98	0.97

Legend: Eq. for G, basal area (m); Dmin, minimum diameter; Dmax, maximum diameter; Dg, mean diameter; x_1, diameter at the 24th percentile; x_2, diameter at the 93rd percentile; a, location parameter; b, scale parameter; c, shape parameter of the Weibull function; S, dominant height at the index age; hypsom, full height; VGptz, volume per hectare calculated by the Gompertz function.

Table 4. Values projected by the equations in Table 3 for each combination of dates and the observed CFI values for the parameters.
Tabela 4. Valores projetados pelas equações da Tabela 3 para cada combinação de datas e valores observados do IFC para os parâmetros.

Dates of Comb.	**G**	**D_{min}**	**D_{max}**	**D_g**	**x_1**	**x_2**	**a**	**b**	**c**	**S**	**Vm3ha**	**VGptz**
						Mahogany						
2012-13-14	10.4	6.4	22.1	16.5	14.8	20.7	0	17.9	6.8	19.6	68.9	93.9
13-14-15	10.0	10.4	21.4	17.9	16.8	20.7	0	18.9	10.8	22.7	84.2	92.0
14-15-16	11.0	7.8	24.2	18.3	16.6	21.6	0	19.2	8.7	16.0	79.6	80.2
15-16-17	11.1	7.7	23.2	18.5	16.4	22.2	0	19.5	7.6	15.9	80.1	79.6
12-14-16	10.8	7.2	23.5	17.6	15.7	21.3	0	18.7	7.4	18.8	76.2	86.4
13-15-17	10.7	9.0	22.3	18.4	16.8	21.6	0	19.4	9.0	17.7	81.0	82.8
12-13-14-15-16-17	10.8	8.0	22.8	18.0	16.3	21.4	0	19.0	8.3	17.7	74.3	85.5
IFC	**10.3**	**8.3**	**23.6**	**18.3**	**16.2**	**22.0**	**0**	**19.3**	**7.4**	**15.3**	**76.1**	

Dates of Comb.	**G**	**D_{min}**	**D_{max}**	**D_g**	**x_1**	**x_2**	**a**	**b**	**c**	**S**	**Vm3ha**	**VGptz**
Euc. 2009												
2012-13-14	16.7	20.8	31.8	24.6	23.7	28.1	21.0	4.9	2.5	27.7	214.5	236.0
13-14-15	19.4	20.1	33.8	27.0	24.9	31.1	20.2	7.6	2.7	30.1	256.4	282.7
14-15-16	15.5	21.9	28.8	25.6	23.7	28.6	17.6	8.6	4.0	34.9	200.9	231.6
15-16-17	14.8	22.4	27.2	24.5	23.1	26.9	10.0	15.0	8.9	32.3	271.5	213.1
12-14-16	15.0	21.4	29.2	24.9	23.4	27.9	17.2	8.6	4.3	31.6	206.3	227.1
13-15-17	17.5	21.7	30.2	25.8	24.1	28.9	12.7	14.1	6.5	31.3	232.3	239.1
12-13-14-15-16-17	16.5	21.7	30.0	25.5	23.9	28.6	14.8	11.7	5.5	31.4	227.3	237.6
IFC	**18.5**	**22.3**	**31.7**	**26.6**	**24.9**	**29.8**	**20.1**	**7.2**	**3.2**	**31.6**	**252.2**	
Euc. 2011												
2013-14-15	15.7	17.1	27.6	23.4	21.8	27.0	16.6	7.7	3.2	25.5	156.3	181.3
14-15-16	12.2	15.2	27.2	21.3	20.6	24.3	16.2	6.1	3.6	26.1	129.8	143.6
15-16-17	10.0	15.1	26.6	19.5	18.2	22.0	15.1	5.0	2.9	25.8	98.6	114.6
16-17-18	11.4	15.8	28.2	20.8	19.5	24.3	15.8	6.0	2.76	26.8	130.4	133.1
13-15-17	11.8	15.6	27.0	20.9	19.5	23.7	15.6	5.9	3.1	25.5	114.1	129.4
14-16-18	11.5	15.8	28.2	21.0	19.9	24.6	16.0	6.1	2.8	26.7	125.1	136.6
13-14-15-16-17-18	11.6	15.8	27.6	20.9	19.7	24.2	15.8	6.0	2.9	25.8	119.5	134.2
IFC	**12.7**	**16.7**	**30.55**	**22.1**	**20.7**	**25.2**	**16.7**	**6.1**	**3.0**	**27.9**	**153.8**	

Legend: Vm³ha, total volume calculated with the projected diameter distribution and the hypsometric equation.

The standard error of volume per hectare projected by the diameter distribution and by the Gompertz function in relation to that of the CFI is presented in Table 5. For mahogany, the diameter distribution generated better volume per hectare results than did the Gompertz function, with a maximum error of 11% (13-14-15). In the 2009 eucalyptus, the maximum error for volume was 20% (15-14-16). For the 2011 eucalyptus, the best results were estimated by the Gompertz function, but both underestimated the volume per hectare with higher errors, reaching 36% (14-15-16).

Table 5. Percentage standard error (SE%) between the projected volume per hectare by the diameter distribution and the Gompertz function and the observed volume of the CFI.

Tabela 5. Erro Padrão Percentual (EP%) entre volume por hectare projetado pela distribuição de diâmetros e pela função Gompertz, e volume observado do IFC.

	Forest System (projection date)						
	Mahogany	**(05/10/2018)**	**Euc_2009**	**(29/05/2018)**		**Euc_2011**	**(12/11/2019)**
Dates of comb.	**v_Weibull**	**v_Gptz**	**v_Weibull**	**v_Gptz**	**Dates of comb.**	**v_Weibull**	**v_Gptz**
2012-13-14	-9	23	-15	-6	2013-14-15	2	18
13-14-15	11	21	2	12	14-15-16	-16	-7
14-15-16	5	5	-20	-8	15-16-17	-36	-25
15-16-17	5	5	8	-16	16-17-18	-15	-13
12-14-16	0	13	-18	-10	13-15-17	-26	-16
13-15-17	6	9	-8	-5	14-16-18	-19	-11
12-13-14-15-16-17	-2	12	-10	-6	13-14-15-16-17-18	-22	-13

The adhesion test between the projected distributions and the observed frequency for the diameter classes of 1 cm amplitude (Figure 2) is shown in Table 6. For mahogany, the only adhesion rejection occurred in the 2012-13-14 combination. For the 2009 eucalyptus, only the 13-14-15 combination generated a projected adherent to the observed diameter distribution, and for the 2011 eucalyptus, non-adherence occurred in combinations 1 and 3.

In the 2009 eucalyptus, low coefficients of determination in the equations, especially in combinations 3 and 4 and for the parameter *a* of the Weibull function (Table 3), affected the displacement of the diameter distributions in relation to the observed data. The cause was the irregularity of the diameter distribution observed and the low frequency of data in the sample, causing greater inaccuracy in the estimation of the parameters of the Weibull function.

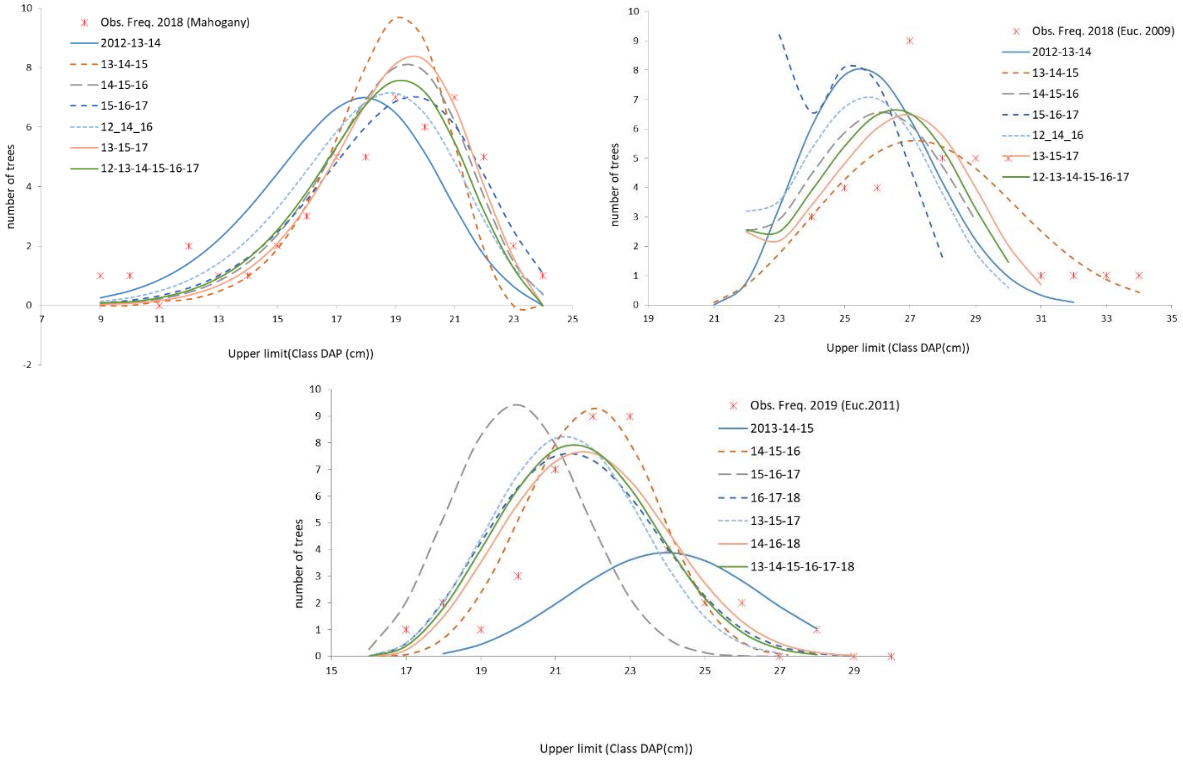

Figure 2. Diameter distributions projected by the combinations and observed data for each forest system.
Figura 2. Distribuições dos diâmetros projetadas pelas combinações, e dados observados de cada sistema florestal.

Table 4. Kolmogorov–Smirnov test for date combinations from forestry inventory.
Tabela 4. Teste Kolmogorov-Smirnov para combinações de dadas dos inventários florestais.

Comb.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**KS tab. (α = 0.05)**
Mahogany	0.222*	0.148$^{n.s.}$	0.068$^{n.s.}$	0.057$^{n.s.}$	0.113$^{n.s.}$	0.075$^{n.s.}$	0.087$^{n.s.}$	0.190
Euc. 2009	0.365*	0.094$^{n.s.}$	0.306*	0.545*	0.399*	0.216*	0.268*	0.213
Euc. 2011	0.342*	0.123$^{n.s.}$	0.475*	0.161$^{n.s.}$	0.204$^{n.s.}$	0.103$^{n.s.}$	0.150$^{n.s.}$	0.208

* significant at 0.05, ns not significant.

DISCUSSION

The validation was performed by two tests: (1) the accuracy between the volume per ha projected by each set of measurements, from the ages farthest from the projection date to the closest, and the volume per ha observed on the projection date, measured by the percentage standard error; and (2) the test of adherence between the distributions projected, based on measurements, and that observed on the projection date. These tests allowed the following evaluations of the efficiency of the methodology implemented.

Among the studies that have addressed the volume attribute is that of Miranda *et al.* (2018), who applied the parameter prediction method, testing several models, some obtained by stepwise regression, from the correlation matrix between attributes of the continuous forest inventory with the b and c parameters of the Weibull function. Their results showed estimates of projected volume per ha with percent standard errors between -14.5% and 23.3%. Azevedo *et al.* (2016), who also applied the parameter prediction method, did not obtain satisfactory adjustments in the equations to estimate the *b* and *c* parameters of the Weibull function, which was reflected in the underestimates of the volume per ha of *Eucalyptus urophylla* at ages 48, 60 and 72 months with errors of -7.72%, -15.16 and -12.99%, respectively. The volume errors per ha obtained in this study with the projection of the Weibull distribution ranged from -36 to 11%.

Regarding the adherence between the estimated and observed diameter distributions, in the study by Ogana *et al.* (2020), the distributions estimated by the generalized Weibull function did not adhere to those observed in 16 of 35 plots, a result similar to the 9 of 21 combinations that were not adhered to by the same Kolmogorov–Smirnov test in this study (Table 6).

That is, there were some errors with low accuracy and adherence associated with the locations where the data were provided for growth and yield modeling. For these cases, it is likely that the imprecision is greater when using "ready-made" equations from other locations, even with similar genetic, pedological and climatic characteristics, evidencing the importance of continuous forest inventories to obtain reasonable levels of accuracy. In this sense, the developed algorithm worked with data from the local CFI generating equations that could be used only for the forest under study, and there was no need to maintain or disclose the coefficients of these equations.

Regarding validation, the results for mahogany were superior when compared to the results for eucalyptus in the ILPF system, and it was not possible to identify whether the cause was due to its structure or due to the spacing between trees that, in the ILPF, were under less competition. Regarding the date combinations, except for mahogany, the combinations further from the projection date did not result in greater errors, and this was an unexpected result because the closer the projection date is to the observed data date, the more precise it should be.

Another finding was the nonassociation between adherence to the projected distribution (Table 6) and the percent standard error of volume per hectare (Table 5), originating from the projected distribution between the ILPF 2009 and 2011 systems, showing that more accurate modeling of the diameter distributions may not result in more accurate volume estimates due to a combination of factors that have not yet been identified. This divergence was observed when comparing the precision of the equations (R^2) for the 2009 and 2011 eucalyptus trees (Table 3) with the volume errors per ha in Table 5, noting that the lowest projection errors were not associated with the largest coefficients of determination of the equations.

The purpose of the presented algorithm was to locally model the growth rate as a function of age, indirectly capturing the influences of site quality, climate, species, genetic material and silvicultural tracts from a minimal dataset of the CFI. Even with this control, some significant errors were observed, especially for 2011 eucalyptus, in the ILPF system, and this magnitude of errors is common in prognosis studies (OGANA *et al.*, 2020; MIRANDA *et al.*, 2018; AZEVEDO *et al.*, 2018; AZEVEDO *et al.*, 2020; MIRANDA *et al.*, 2018; AZEVEDO *et al.*, 2020; *al.*, 2016).

Among the advances in the development of the proposed methodology is the identification of the function for parameter *c* (Eq. 13), which showed an exact potential relationship with the diameters in the percentiles and the *b* parameter of the Weibull function (Table 3); the hypsometric function, constructed as the maximum height as a function of the maximum diameter, regardless of age (Eq. 15); and the application with integrated adjustment of all functions, using only taper data, DBH data and total height by age. Using a minimum of three measurements from the CFI, the results were generated for the projection of the volume per hectare with errors of up to 20% in 85%. For use of the algorithm, taper data from the study area are recommended.

CONCLUSIONS

- The algorithm presented accuracy results within the expected range, with an error up to 20% in 85% of the cases, but there was an inverse relationship between the modeling precision and the accuracy of the volume per hectare in the ILPF systems, requiring further tests in other forest production systems to evaluate its efficiency.

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