Original Paper

Contrary to T. Hirai, Ramsey's Critiques of Keynes in 1922 and 1926 Were Completely Wrong

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Abstract

This paper covers Harai's analysis, contained in his section, titled "Keynes as a philosopher", of Keynes's logical theory of probability. Harai spends too much of his time repeating Ramsey's claims about having uncovered serious errors in the structure of Keynes's relational propositional theory.

.For 100 years, Keynes's logical theory has been interpreted and misevaluated through the eyes of an ignorant 18 year old teenager. The result has been the proliferation and spread of what can be called the "Ramsey myth".

The "Ramsey myth" is that an 18 year old teenager appeared at Cambridge University in 1921.This 18 year old teenager was a genius who wrote a three page review in the Jan., 1922 issue of Cambridge Magazine, which supposedly destroyed, devastated and demolished the logical foundations of Keynes's A Treatise on Probability, which was his relational, propositional logic founded on Boole's relational, propositional logic. Russell countered this in his review, but was ignored (See Brady, 2016a). In 1931, it is further supposed that Keynes then capitulated to Ramsey and repudiated his own logical theory of probability, accepting some version of Ramsey's subjectivist theory. This myth is what Hirai's paper is based on. It was false in 1921 and it is false today in 2022.

1. Introduction

The paper will be organized in the following manner. Section Two will present a very brief overview of Keynes's logical theory of probability. Section Three will show how Edgeworth's two vastly superior reviews could have saved Hirai from making the many errors that he made in his paper, where he simply repeats all of Ramsey's claims. Section Four will show that all of Hirai's analysis concerning Keynes's A Treatise on Probability (TP, 192) is wrong. Section Five will conclude the paper.

T. Harai's recent paper in the Review of Keynesian Studies, 2021,contains a section called "sketch 2 Keynes as a philosopher" (pp. 192-198). All seven pages are completely wrong as they are based on what Ramsey claimed, without any textual support, Keynes's logical theory of probability entailed (See Brady 1990, Brady, 2004a, Brady, 2004b, Brady, 2012, Brady, 2021a, Brady, 2021b, Brady, 2015, Brady and Arthmar, 2012, Arthmar & Brady, 2010, 2017, 2018). All of Ramsey's definitions of Keynes's relational, propositional logic, made in 1922 and 1926, appear nowhere in anything written by Keynes in his Collected Writings of John Maynard Keynes or in his lifetime. Ramsey basically made things up as he went along in the years 1922-1926. Ramsey grossly distorted Keynes's analysis until you end up with the very distorted assessments made by C. Misak (2020), nearly 100 years later, about Keynes purportedly making the following claims :

"Ramsey peppered the Treatise with problems. He objected to the attempt to provide a logical foundation for the Principle of Indifference. He objected to the idea of an unmeasurable, non-numerical probability and would later, in the 1926 "Truth and Probability", offer an account of how all probabilities are measurable. And he objected to the very idea of Keynes's objectively fixed probability relations—the idea that all statements stand in logical relations to each other. As Ramsey put it, there is no such probability as the probability that "my carpet is blue" given that 'Napoleon was a great general'." (Misak, 2020, p. 114).

First of all, Keynes never attempted "… to provide a logical foundation for the Principle of Indifference." (Misak, 2020). Second, Keynes's supposed "idea of an unmeasurable, non-numerical probability." (Misak, 2020) is actually Keynes's interval valued, imprecise probabilities that involve inexact measurement and approximation, as covered in great mathematical detail by Keynes in chapters 15 and 17 in Keynes's Part II of the A Treatise on Probability. Neither Ramsey or Braithwaite or Misak or Hirai ever read Part II. Third, Keynes never stated Misak's claim concerning "the idea that all statements stand in logical relations to each other. As Ramsey put it, there is no such probability as the probability that 'my carpet is blue' given that 'Napoleon was a great general'" (Misak, 2020) at any time during his life.

Hirai, much like Misak, has absolutely no idea about what he is

2. Method: Keynes's Probability (P) Relation and the Weight (V) Relation. A Decision (D) Is a Function of Both, so That D = f (P,V)

Keynes's actual decision function included the time constraint, T. The Time constraint concerned how much time the decision maker had available in order to make a decision. If the decision must be made within a fixed, short run period of time, as in a rated chess tournament involving over the board chess games, then this constraint would become binding. However, if the chess game was a correspondence (postal) game, then the time constraint would not be binding Only Savage also considered such a binding time constraint in his subjectivist approach because he recognized the severe problem of vagueness that impacted a decision maker's assessment of his personal probability. In other words,

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Savage recognized Keynes's concern with the importance of weight, but felt that it could not be formally integrated into his theory, although he hinted at seriously taking a long look at interval valued probability. These concerns made his subjective approach to decision making vastly superior to Ramsey's very inferior subjective approach. On the other hand, if the decision can be postponed until the long run, as it can be in a rated tournament correspondence (Postal) chess game, then the time constraint, as noted above, is not binding and Keynes's concern with weight becomes much less relevant. Savage recognized the binding nature of this constraint when he restricted the application of his theory to the short run and micro level only.

Therefore, suppressing the T variable yields Keynes's two logical relations, P and V, where $P(a/h)= \alpha, 0 \leq \alpha \leq 1$, where α is a rational degree of belief and $V(a/h) = w$, $0 \leq w \leq 1$, where w measures the degree of the completeness of the relevant information (See Keynes, 1921, TP, p. 313, pp. 314-315; p. 315) supporting Keynes's argument form. This logical relation, V, was severely misinterpreted in Runde (1990, pp. 278-283) as being a mathematical variable which was being used to measure weight. Runde relies completely on chapter 6 alone for his evaluation. This had been done previously by IJ Good and I Levi. Keynes made it very clear on p. 315 of the TP, which Runde overlooked, that weight, V, is measured by w. It should be clear from $V(a/h)=w$, $0 \leq w \leq 1$, that V can't measure anything, as it is the logical relation of weight between the a and h propositions that connects them(See Brady,2017c,d),. All heterodox, Post Keynesian, neo -Keynesian and Institutionalist economists, working in this area in the 20th and 21st centuries, continue to cite Runde's 1990 article (just under 300 citations according to Google Scholar).

The result is that it is impossible for them to make use of Keynes's conventional coefficient, c, of weight and risk, as it is asserted that it must be V that is used to measure weight and not Keynes's w. It is also asserted that Keynes erroneously presented three different definitions of weight in the TP when Keynes actually only presented one, which was that $V(a/h)=w, 0 \leq w \leq 1$.

Runde's errors were first introduced into the academic literature by the philosopher I J Good in the early 1950's and then emphasized by I. Levi in the late 1960's.Both Good and Levi overlooked, as did Runde, Keynes's clear definition on p.315 of the TP that only w measures V, weight (See Brady, 2018).

Runde's error, of confusing and conflating V with w, was repeated one year later in Runde (1991, pp. 128-129), where Runde confuses and conflates P with α,which is a mathematical variable that can take on a numerical value from the interval (0, 1):

"The probability relation may be written as a/h=P. This relation is objective, on Keynes's account, and expresses the degree of belief P is *rational* to hold in a in the circumstances h.If a is implied by h, P=1,if the relation between a and h is contradictory, P=0.In all other cases, where a is only partially entailed by h, P will lie somewhere somewhere between these two extremes…" (Runde, 1991, pp. 128-129).

It is mathematically impossible for a logical relation to equal 0, 1 or a number between 0 and 1.It is α that is equal to 0, 1 or a number between 0 and 1 once the probability relation has been normalized on

the unit interval [0,1].The probability relation has to be written as a/h, and not as a/h=P.

Runde errs again on the same page when he writes the following: "This can be seen if we again write a/h=P (1) where a is the 'primary proposition 'and relation (1) the secondary proposition." (Runde, 1991, p. 129).

One can suppress the notation, denoting "relation", as Keynes did on p.4, where he writes (a/h) =α or on p.40 of the TP, where he writes (a/h)=P. However, then P is defined as the probability P, not the relation P.

Runde has confused and conflated P with α. See above. This is identical to his conflation of V with w.

Vercelli spotted Runde's errors in 2011. Vercelli spots the fundamental error that it is obvious is contained in the technical analysis of Runde He fixes the flaw, but, again, does not see the contradiction when he talks about Keynes's c coefficient. Consider the following :

"Therefore, most interpreters believe that a satisfactory measure of the weight of argument may be given simply by

$V(x/h) = K$ (1)

Runde (1990) suggests the following measure: $V(x/h)=K/I$ (2)

We may overcome these shortcomings by introducing the following measure, which is derived from the third definition [author's note-Vercelli accepts Runde's illusion that Keynes had three separate definitions of weight in his TP]:

$V(x/h)= K/(K+I)$ " (3)"

(Vercelli, 2011, pp. 153-154).

Therefore, Vercelli was able to see the nature and scope of the mathematical problems that existed with the Runde, Cr articles. He attempts to remedy this in his footnote 1:

By introducing the usual criterion of normalization of probability measures K+I=1 (a criterion that Keynes himself has utilized in the TP, for example, on page 348)….(Vercelli, 2011, p. 168).

This fixes one problem, but simply creates another problem that takes the place of the failure of Runde to normalize on the unit interval [0, 1].

Vercelli's requirement, that K +I =1, must actually be written as

$0 \leq V \leq 1$,

which now states that the logical relation, V, is restricted to applications between 0 and 1,which makes no sense at all because you can't restrict a logical relation in this fashion.

Keynes's position on belief thus depends on both P and V, not P alone, as was argued by Ramsey. Ramsey further confused the issue by claiming that there was something called a non numerical probability relation. This so called non numerical probability relation does not exist anywhere in Keynes's A Treatise on Probability.

Keynes's P and V relations are independent, since $\alpha =p/(p+q)$ and $w=K/(K+I)$,where p is the probability of success, q is the probability of failure, K is the absolute amount of Knowledge and I the absolute amount of Ignorance.

However, V constrains the type of probability that α is, either numerical or non numerical. If w=1, so that all of the α values are numerical and additivity holds, then it is numerical. It does not hold in the general case[1], where w<1,where the α values are interval valued probabilities so that non additivity holds. A decision maker can accept, and have a different opinion about, any value within the boundary set up by the upper and lower probabilities defining the interval.   A w<1 creates very complicated, complex, severe and intricate problems concerning/about one's beliefs because of the non- linearities introduced by w in Keynes's decision theory, which is a function of both α(probability) and w(weight) or the relations P and V. This can best be seen by using the Mathematica program or MathLab to generate three dimensional contours of Keynes's conventional coefficient of risk and weight, c, involving c, p and w. Everything is substantially simplified if w=1.

Ramsey does not have any idea about what V represents. Hence, he has no inkling about the role that confidence plays in belief. There is no simple, direct, linear connection between probability and belief for Keynes as there is for Ramsey because Ramsey has conflated confidence with probability. The one exception[1] is if w=1, so that V drops out and one is left with α.Only in this very special case would Ramsey be correct. However, Keynes never disputed this outcome, correctly seeing that it would be rare in the real world of actual decision making, as opposed to Ramsey's imaginary academic world where everyone uses only the purely mathematical theory of the calculus of probability to base their decisions on.

[Place Footnote at bottom of this page]

The exception to the general case occurs, as discussed by Keynes on p. 160 (Keynes, TP, 1921, p. 160), if the principle of Indifference can be applied, where the existing evidence is "…perfectly symmetrical in its bearing on the various alternatives". This would be the case in Keynes's discussion of urn models on pp. 75-76 of chapter 6 of the TP. The uncertain urn has numerical probabilities of ½, which are identical to the risky urn. The important point made by Keynes, which was that the risky urn has more and stronger evidence than the uncertain urn, has nothing to do with risk allowing quantitative estimates and uncertainty requiring non quantitative estimates of probability.

Keynes specified an exact argument form that all of the propositions needed to satisfy in his, propositional logic. Keynes's fundamental requirement is that the propositions must be related, connected or similar: "Let our premises consist of any set of propositions h, and our conclusion consist of any set of propositions a, then, if a knowledge of h justifies a rational belief in a of degree α, (then-author's insert) we say that there is a probability-relation of degree α between a and h." (Keynes, 1921, p. 4).

Only if a knowledge of h justifies a can there be a "…probability-relation of degree α between a and h." Otherwise, there is NO SUCH LOGICAL RELATION between a and h. Ramsey's fanciful belief that there can be only one h proposition and one a proposition is nonsense, given Keynes's very clear statement throughout his TP that his relational propositional logic works with sets of h and a propositions.

Keynes gives an excellent example of his argument form on pp.5-6 of chapter II of the TP:

"These general ideas are not likely to provoke much criticism. In the ordinary course of thought and argument, we are constantly assuming that knowledge of one statement, while not proving the truth of a second, yields nevertheless some ground for believing it.

We assert that we ought on the evidence to prefer such and such a belief. We claim rational grounds for assertions which are not conclusively demonstrated. We allow, in fact, that statements may be unproved, without, for that reason, being unfounded. And it does not seem on reflection that the information we convey by these expressions is wholly subjective. When we argue that Darwin gives valid grounds for our accepting his theory of natural selection, we do not simply mean that we are psychologically inclined to agree with him; it is certain that we also intend to convey our belief that we are acting rationally in regarding his theory as probable. We believe that there is some real objective relation between Darwin's evidence and his conclusions, which is independent of the mere fact of our belief, and which is just as real and objective, though of a different degree, as that which would exist if the argument were as demonstrative as a syllogism**.** We are claiming, in fact, to cognise correctly *a logical connection between one set of propositions* which we call our evidence and which we suppose ourselves to know, and *another set which we call our conclusions,* and to which we attach more or less weight (author's insert-the word "weight" refers to chapters 6 and 26 of the TP analysis of Keynes's evidential weight of the argument) according to the grounds supplied by the first. It is this type of objective relation between sets of propositions—the type which we claim to be correctly perceiving when we make such assertions as these—to which the reader's attention must be directed." (Keynes, TP, 1921, pp. 5-6; italics added; this is expanded upon on by Keynes on pp. 160-161).

Ramsey's 1922 example, "…for example between 'My carpet is blue' and 'Napoleon was a great general'…", that was given on page 3 of his *Cambridge Magazine* review in 1922, is false because knowledge of the color of a carpet can't ever supply a connection (or evidence) to justify the conclusion that Napoleon was a great general, where one proposition is regarded as providing the information or evidence upon which to accept the other proposition provisionally. The same conclusion holds for Ramsey's "this is red; that is round" and his "this is red; that is blue "examples. There is no connection between the propositions.

I believe that Ramsey had to have been on some type of medication, narcotic or drug treatment or been suffering from some unknown mental disorder at the time he was writing his review as his entire paragraph (Ramsey, 1922, p. 3) represents a complete contradiction of what Keynes presented very clearly on pp. 4-6 and pp. 54-56 of the TP. The same holds for his very similar characterization of Keynes's theory made in 1926.

Finally, Keynes would not conclude that "…the probability is not numerical." (Ramsey, 1922, p. 3). Keynes would conclude that Ramsey's example violated his argument form and was irrelevant. Therefore, there is no conditional probability because neither of the propositions provides any relevant evidence for the other proposition. It is extraordinary that so many philosophers and economists have

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described Ramsey's 1922 article as brilliant. In fact, it makes absolutely no sense whatever.

How could have Hirai have avoided the intellectual mess that he created in his article? Hirai needed to have read chapters I, XI and XII of Boole's 1854 The Laws of Thought while simultaneously reading chapters I and II of Keynes's TP. Doing this would have provided him with hard evidence that Ramsey's many, many attacks on Keynes's logical theory of probability were also attacks on Boole's logical theory of probability. It is easy to then conclude that Ramsey's claims needed to be evaluated from a critical standpoint as no serious scholar had impugned Boole's theory

3. Discussion I: A Brief View of Edgeworth's Superior Understanding of Keynes's *A Treatise on Probability*

Consider Edgeworth's very brief summary of the TP: "Suffice it that many probabilities which are incapable of numerical measurement can be placed "between two numerical measures" (p. 32). We thus obtain the idea of a "finite probability" as one which exceeds some numerical probability (p. 257). Such measurements play a leading part in induction. To establish a generalization it is necessary that "with the experience we have" actually had there are finite probabilities, however small, derived "from some other source, in favour of the generalization" (p. 238). Round this nucleus of finite probability, through the operation of repetition and likeness, science grows. "An argument from" induction must always involve some element of analogy, and, 'on the other hand, few arguments from analogy can afford to "ignore altogether the strengthening influence of pure induction "[repetition]" (p. 255)." (Edgeworth, JRS, 1922b, pp. 107-108; also see Edgeworth, 1922a, Brady, 2016b, c, 2017a).

This very powerful summary by Edgeworth is still the best short summary by far that has ever been made of Keynes's *A Treatise on Probability* (TP, 1921), even though it is now one hundred years old. Edgeworth's summary is the perfect antidote to the extremely poor coverage made of Keynes's approach by R. B. Braithwaite in his editorial foreword to the TP that was placed at the front of Volume 8 of the 1973 Collected Writings of John Maynard Keynes's TP by Donald Moggridge, as well as the many false claims made by the 18 year old "boy genius", Frank Ramsey.

Reading Edgeworth would also inoculate readers of the TP from the extremely poor assessments made of Keynes's work by economists, who simply do not know what an interval valued probability is, like R. Dimand, R. Skidelsky, A. Carabelli, J. Runde, B. Davis, T. Lawson, G Meeks, R. O' Donnell and many others. Their unanimous acceptance of Ramsey's false claim that Keynes's theory was primarily an ordinal theory that would be useless in science is probably their greatest error.

Hirai could have saved himself from his errors if he had carefully read Edgeworth's two reviews; however, there is no evidence from his references that he had ever read Edgeworth's reviews. Hirai thus made the fundamental mistake of putting all of his intellectual eggs into one basket, a very inferior basket based on Ramsey comprising the first page (p. 115, 1921) of Chapter 10 of Part II and several pages taken from Keynes's Introductory Chapter 18 (pp. 117-121, 1921) of Part III of the TP. He relied completely on his belief that the Ramsey's reviews were the only one's worth reading because only

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Ramsey had supposedly spotted alleged flaws in Keynes's logical structure about propositions and the objective, logical, probability relation connecting them. In fact, Ramsey never correctly identified even one, single flaw in Keynes's TP in either of his reviews. Ramsey's understanding about Keynes's propositional logic, which follows directly from Boole's approach, is nil.

The two crucial pieces of information that Hirai could have gleaned from a reading of Edgeworth were (a) Keynes's theory of probability is an interval valued theory of imprecise probability and (b) the very important role played by finite probabilities in Keynes's mathematical theory of induction and analogy explained by Keynes in chapters 20 and 22 on pp.233-238 and pp. 253-27, respectively,where a finite probability was defined by Keynes as being greater than both a numerical probability and/or a non numerical probability, where a non numerical probability is an interval valued probability.

4. Discussion II: Hirai's Mistaken Reliance on Ramsey for His Understanding of the Foundations of Keynes's Logical Theory of Probability

Consider the following claims made by Ramsey that is the foundation for Hirai's 2021 work on Keynes's logical theory of probability in the *Review of Keynesian Studies:* see

"Mr. Keynes starts from the supposition that we make probable inferences for which *we claim objective validity*; we proceed from full belief in one proposition to partial belief in another, and we claim that this procedure is *objectively right*, so that if another man *in similar circumstances* entertained a different degree of belief, *he would be wrong* in doing so. Mr. Keynes accounts for this by supposing *that between any two propositions*, taken as premiss and conclusion, there holds *one and only one relation* of a certain sort called probability relations; and that if, *in any given case*, the relation is that of degree α, from full belief in the premiss, we should, if we were rational, proceed to a belief of degree α in the conclusion." (Ramsey, 1926; In Kyburg & Smokler, 1980 (2nd ed.), pp. 26-27; italics added; see also Bateman, 1987, p. 106).

The italics emphasis involve Ramsey in severe error; the second to last italics emphasis holds only for the special case where Keynes's Principle of Indifference (POI) is applicable or one is dealing with a least upper bound or a greatest lower bound for interval valued probability.

First, nowhere in the TP does Keynes claim objective validity, which can only occur in a deductive logic, as opposed to practical certainty (Keynes, TP, p. 160). Second, nowhere does Keynes claim that his procedure is objectively right. Keynes claims that it is rational and reasonable because it is based on all of the relevant, available evidence which appears in the h propositions. Third, the circumstances must be identical, not similar. Fourth, Keynes never claimed that a decision maker would be wrong. Keynes stated that such a decision maker would be non rational, as opposed to being irrational. Thus, for Keynes, the only rational answer for both of the Ellsberg two urn ball (red and black ball) problems is that the probability of a red or black is ½, based on a correct application of Keynes's POI, as based on Keynes's summary on pp. 54-56 of the A Treatise on Probability (TP; 1921), and not on Keynes's restatement of the erroneous Laplace-Bernoulli version on p. 42. Fifth, Keynes's relational,

propositional logic does not hold' between any two propositions', but only between sets of propositions that are related or connected by a degree of similarity (or likeness or resemblance). Ramsey's claim is simply a regurgitation of the false claim he first made four years earlier in 1922 in his Jan., *Cambridge Magazine* review on p. 3 that "…between any two non contradictory propositions there holds a logical relation…", which Ramsey then illustrated with his completely irrelevant 'my carpet is blue; Napoleon was a great general 'example. Neither of these propositions are related or connected. All of Ramsey's 1922 and 1926 examples involve unconnected or unrelated propositions. Sixth, there holds one and only one relation of logical probability for exact, precise, numerically connected probabilities based on the POI or for the calculation of a least upper bound (LUB) and /or Greatest Lower Bound (GLB) for Keynes's interval valued probabilities. Seventh, Keynes's approach does not hold 'in any given case'; it holds in some cases. Finally, Ramsey is very confused in his belief that Keynes's approach only holds for two propositions, i.e. for one h proposition and one a proposition. In fact, Keynes works with sets of h and a propositions throughout the TP. The best example of this would be his Darwin example on pp. 5-6 and its extension on p. 160-161 (TP, 1921).

Hirai takes an approach to reading the TP which can only be described as reading "little bits and pieces" of Keynes's TP. Hirai's coverage of Part II of Keynes's TP is a mirage. The two pages he cites from, pp.125-126 (Hirai, pp. 192-194) from the 1973 CWJMK version (p. 115, TP, 1921) have nothing to do with Keynes's main conclusions in Part II, which were that (a) additivity is a special case,(b)interval valued probability is the general case, and (c) the normal distribution can't be simply assumed, but must be shown to hold by the examination of preliminary data (Chapter 17, TP). The researcher must consider the use of the geometric and harmonic means, the median, and the mode and not just assume the arithmetic mean.

Hirai's discussions of Part III on induction and analogy on pp.241,243 and 245 (Hirai, pp. 195-196) from the 1973 CWJMK edition of the TP (pp. 217, 219 and 221, TP, 1921) ignore Keynes's crucial discussions of finite probabilities on pp.233-238 and 254-257,as well as Keynes's qualification that inductive(practical) validity is NOT the same as deductive(universal)validity (pp. 244-245, TP, 1921).

I have already pointed out that a reading of Edgeworth's two reviews in 1922 would have allowed Hirai to actually have come to grips with what Keynes was presenting in Parts II and III of the TP.

However, it is on pp.196-198 of Hirai, where Hirai's assessments of Ramsey's empty criticisms of Keynes's *A Treatise on Probability* are simply accepted as being true, that Hirai's failure to absorb Parts II and III of the TP become obvious.

Hirai's belief, that he can take one small paragraph out of context from p. 245 (p. 221, TP, 1921) of Part III's introductory chapter 18 and present it as the major conclusion of Part III of the TP, is comparable to Ramsey's total confusions about Keynes's Boolean, relational, propositional logic.

We will consider Hirai's confusions paragraph by paragraph.

Consider the following:

"Here "probability" is defined as the degree of rational belief between propositions, and, moreover,

"inductive arguments" are defined, being drawn by this probability. Furthermore, it is argued that induction should be a problem of formal logic (the existence of probability relation) rather than a factual problem." (Hirai, 2021, p. 196).

If the a and h propositions are related and connected, then there exists an objective, logical, relation of probability (similarity between them such that there is a degree of rational belief α. However, it is quite impossible that induction "….should be a problem of formal logic (the existence of probability relation) rather than a factual problem." since the best outcome for an inductive argument is "practical certainty "only, which is NOT the deductive result of universal certainty (the conclusion is always true. See pp.160-161 and 244-245, TP, 1921).

Consider now Hirai's next confusion: "To sum up, Keynes argues that (i) induction is to be seen as the most important element in the argument on probability, (ii) although induction is composed of "analogy" and "pure induction", analogy is more important, (iii) prior probability can be obtained through analogy." (Hirai, 2021, p. 196). Of course, this leaves out of the discussion Keynes's "finite probabilities"π and η as emphasized by Edgeworth. Without these finite probabilities, pure induction can't be used to support a generalization.

It is when Hirai explicitly considers Ramsey's extraordinarily weak critique of Keynes's theory that Hirai goes astray: "And in the process, he criticized Keynes's Treatise on Probability (Ramsey (1926) [Ramsey [1931] pp. 156-98]). His criticism runs as follows. (1) There does not exist anything like a probabilistic relation between propositions. (Rejection of the definition of "probability"). (2) Consistency is not maintained in the arguments regarding the main principles. (Ambiguity of the objectivity and subjectivity of "probability" found in the TP). (3) The attempt to absorb the world of induction into the world of deduction is nonsense. Ramsey argues (3) as follows. The logical relation which justifies the inference is that the sense or import of the conclusion is contained in that of the premisses. But in the case of an inductive argument this does not happen in the least; it is impossible to represent it as resembling a deductive argument and merely weaker in degree; it is absurd to say that the sense of the conclusion is partially contained in that of the premisses. (Ramsey [1931] p. 186) Keynes responded to it with acceptance to a large extent as follows (Keynes, 1931). He states that the development of formal logic by Russell and [early] Wittgenstein (1922) gradually emptied out the content, eventually eliminating not only any experience, but the most rational thinking principles. Ramsey tried to counter this with a kind of pragmatism and turned to human logic. "Thus, he was led to consider "human logic" as distinguished from "formal logic". Formal logic is concerned with nothing but the rules of consistent thought. But in addition to this we have certain "useful mental habits" for handling the material with which we are supplied by our perceptions and by our memory and perhaps in other ways, and so arriving at or towards truth; and the analysis of such habits is also a sort of logic. The application of these ideas to the logic of probability is very fruitful. "(JMK.10, p. 338) Keynes went on to admit, "So far I yield to Ramsey—I think he is right" (JMK. 10, p. 339). This is a brief commentary, but the matter is weighty and deep. Keynes agrees with Ramsey's theory of probability.

We may reasonably conclude that he is critical of his own theory of probability, which belongs to a stream of formal logic." (Hirai, 2021, pp. 196-198).

Hirai's (1) fails completely, since Keynes's relational, propositional logic used in the TP comes directly from G. Boole's relational, propositional logic as contained in chapters XVI-XXI of the Laws of Thought (see TP, 1921, p. 5, ft. 2).

Hirai's (2) fails completely, as he never establishes any inconsistency in any part of the TP. Hirai's (3) makes no sense, because Keynes never tried to absorb induction into deduction. Where this is supposed to have occurred in the TP is never cited by Hirai. I await Hirai's citing of the exact page and paragraph where Keynes made such an attempt in the TP. Hirai's reliance on Ramsey's assessment of the value of induction is completely erroneous. Let us consider Ramsey's assessment: "The logical relation which justifies the inference is that the sense or import of the conclusion is contained in that of the premises. But in the case of an inductive argument *this does not happen in the least; it is impossible to represent it* as resembling a deductive argument and merely weaker in degree; *it is absurd to say* that the sense of the conclusion is partially contained in that of the premises. (Ramsey, 1931, p. 186; p. 44, Kyburg & Smokler, 1980, (2nd ed); italics added).

Of course, Ramsey's assessment of induction relative to deduction is identical to J.S. Mill's assessment as pointed out by Keynes on pp.267-268: "An induction has no validity….unless it is absolutely certain" (Keynes, 1921, p. 267).

Keynes had already countered Ramsey's attack on induction with his discussion of "practical certainty" on p.161 of the TP, where such "practical certainty" (Darwin's theory or Einstein's theories or the work of Linus Pauling)is based on approximation. See also Keynes's discussions of Darwin's theory on pp.5-6 of the TP (1921). Ramsey's expressions, "this does not happen in the least", "it is impossible to represent it", and "it is absurd to say", as far as physics, biology, chemistry and engineering go, would be rejected by practically all physical, life and biological scientists.

Hirai severely misquotes Keynes's discussion of Ramsey in *The New Statesman and Nation* in 1931 (Vol. 10, 1973, CWJMK, pp. 337-338; 1931, p. 407):

"Ramsey argues, as against the theory that I had put forward, that probability is concerned not with objective relations between propositions but (in some sense) with degrees of belief, and he succeeds in showing that the calculus of probabilities simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a consistent system. Thus the calculus of probabilities belongs to formal logic. But the basis for our degrees of belief—or the a priori probabilities, as they used to be called—is part of our human outfit, perhaps given us by natural selection, analogous to our perceptions and our memories rather than to formal logic. So far I yield to Ramsey—I think he is right. But…." (Keynes, Vol. 10, 1973, CWJMK, pp. 337-338; 1931, New Statesman and Nation, p. 407).

The only part of this paragraph where Keynes "yields" to Ramsey is the following part:

"…and he succeeds in showing that the calculus of probabilities simply amounts to a set of rules for ensuring that the system of degrees of belief which we hold shall be a consistent system. Thus the

calculus of probabilities belongs to formal logic."

However, the vast majority of Keynes's probabilities are interval valued, which Keynes described as being non numerical because they required two numerals, not one, the laws of the purely mathematical theory of probability do not apply. Since Keynes had always agreed that instances where numerical probabilities held could be analyzed using the calculus of probability, Keynes's "yielding "to Ramsey has no import for the theory presented in Keynes's A Treatise on Probability. Finally, Hirai's summary makes no sense: "Keynes went on to admit, "So far I yield to Ramsey——I think he is right" (JMK. 10, p. 339). This is a brief commentary, but the matter is weighty and deep. Keynes agrees with Ramsey's theory of probability. We may reasonably conclude that he is critical of his own theory of probability, which belongs to a stream of formal logic (Hirai, 2021, p. 198).

Keynes does not agree with Ramsey's theory of probability unless the evidential weight of the evidence, w,=1 or the lower limit = the upper limit of an interval valued probability. Keynes was not critical of his own theory as demonstrated in the Keynes-Townshend (see references on Townshend, 1937-1938) correspondence in 1937-38.Finally, Keynes's theory does not belong to a stream of formal logic. It is Ramsey's theory that belongs to formal logic (the purely mathematical theory of

Hirai's article directly conflicts with the first article published in the 2021 issue of the RKS by Sakai. Sakai argued that Keynes's theory was an interval probability approach. Harai, as the editor of this journal, had to have read Sakai's paper as he would have been responsible for sending the paper out for review. As Professor James S Earley told me in December, 1980,before agreeing to sit on my dissertation committee, that "if, and it is a big if, Edgeworth is right about Keynes's probabilities being between two numbers, then Ramsey is wrong."

Nowhere in Harai's article is Sakai's result concerning interval probability Nor does Hirai used, listed or cited. Nor does Hirai realize that Keynes's logic is the logic of George Boole's The Laws of Thought.

5. Conclusions

Hirai bases his analysis of Keynes's work in Part II on one irrelevant page (p. 115, TP, 1921) taken from Part II of the TP, three introductory pages (pp. 217, 219, 221) taken from Part III and one page taken from the first page of chapter 33 of Part V of the TP. The vast majority of his assessment of Keynes's TP is based only on Ramsey's 1926 review that I have demonstrated to be intellectually worthless and without any merit at all (See Brady, 2021a; 2021b; 2004a; 2004b).

I will now again provide the Edgeworth quotation from his 1922 review in the *Journal of the Royal Statistical Society* so that the reader can see how inferior Hirai's assessment of Keynes is compared to a 100 year old reviews by Edgeworth that have never been used by any heterodox or orthodox economist in the 20th or 21st centuries. Both of Edgeworth's 100 year old reviews combined completely refute all heterodox claims made about Keynes's logical theory of probability. The work of Bateman (1987, 1989, 1990, 2016, 2021a, b), Davis (1994), Dimand (2021), Faulkner, Fed-uzi, McCann and Runde (CJE,

2021), Meeks (2003), Lawson (2012), O'Donnell (2021a, b), Skidelsky (1992) and Weatherson (2002) is completely refuted because all of these articles rely on Ramsey's flawed claims. It should be noted that both Edgeworth reviews appear in the references of the Faulkner, Feduzi, McCann and Runde paper, but none of Edgeworth's analysis is ever used or quoted anywhere in their paper.

It is interesting to see that Bateman, Skidelsky, Meeks, Davis, O'Donnell, Runde and Weatherson all rely, either completely or partially, on Ramsey's claims about Keynes's relational, propositional logic being wrong, while ignoring everything actually written about it by Keynes in the TP.

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