Adaptive Artificial Time Delay Control for Quadrotors under State-dependent Unknown Dynamics

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***Abstract*— Quadrotors are becoming more and more essential for applications such as payload delivery, inspection and search-and-rescue. Such operations pose considerable control challenges, especially when various (a priori unbounded) state-dependent unknown dynamics arises from payload variations, aerodynamic effects and from reaction forces while operating close to the ground or in a confined space. However, existing adaptive control strategies for quadrotors cannot handle unknown state-dependent uncertainties. We address such unsolved control challenge in this work via a novel adaptive method for artificial time delay control, where *unknown dynamics is robustly compensated by using input and state measurements collected at immediate past time instant (i.e., artificially delayed)*. Closed-loop stability is established via Lyapunov theory. The effectiveness of this controller is validated using experimental results.**

I. INTRODUCTION

The last couple of decades have seen intense research interest in quadrotors due to their applications in surveillance, disaster management, smart aerial transportation, etc [1], [2]. Nevertheless, control of quadrotors is a challenging task under uncertainties stemming from system parameters (e.g. payload variations) and external forces (e.g. aerodynamic effects, reaction forces during operation close to the ground).

To tackle uncertainties, many notable robust control [3]–[5] and adaptive control methods [2], [6]–[10] have been reported. However, while robust control designs [3]–[5] rely on a priori knowledge of the uncertainity bounds, the adaptive control methods [2], [6]–[10] require a priori parametrization (i.e. structural knowledge) of uncertainty. Unfortunately, most of the aforementioned uncertainties are state dependent (i.e. not a priori bounded) and their parametrization is often unknown [11], [12].

Artificial time delay control (a.k.a. *time delay estimation* (TDE)-based control) was conceptualised to reduce a priori knowledge on system structure and uncertainties [13], [14] by approximating the unknown dynamics via state and input measurements of immediate past time instant (i.e. *with an artificial delay*). *Therefore, TDE-based control should not be confused with the field of control of time-delay systems*.

The simplicity in implementation and the significantly low computation burden helped TDE-based methods to find remarkable acceptance in the control literature of robotics in the past decade [15]–[21] including in quadrotors [22]–[24], showing improved performances compared to conventional methods of robust and adaptive control. Yet, a formal stability analysis of TDE-controlled quadrotors with state-dependent uncertainities is missing. Therefore, a relevant question arises whether *the existing TDE methods can tackle the unknown state-dependent uncertainties in quadrotors*, as, left unattended, such uncertainties can cause instability [11]. Unfortunately, the TDE methods for quadrotors [22]–[24] (and relevant references therein) rely on the assumption of a priori bounded approximation error (a.k.a. *TDE* error), which is quite common in TDE literature [15]–[20] and hence, these methods are conservative for quadrotors to negotiate state-dependent uncertainties (please refer to Remark 4 later). Further, being an underactuated system, the adaptive TDE works [15]–[19], [21] are not directly applicable to a quadrotor.

In light of the above discussions, artificial delay based adaptive control for quadrotors, which is also robust against *unknown state-dependent uncertainty* is still missing. In this direction, this work has the following major contributions:

- The proposed adaptive TDE method for a quadrotor system, to the best of the authors' knowledge, is a first of its kind, because the existing TDE methods for quadrotors [22]–[24] are non-adaptive solutions.
- Unlike the existing adaptive TDE solutions [15]–[19], the proposed adaptive TDE method can provide *robustness* (hence termed as adaptive-robust TDE, ARTDE) against state-dependent unmodelled dynamics.
- The closed-loop system stability is established analytically. Further, experimental results suggest significant improvement in tracking accuracy for the proposed method compared to the state of the art.

The rest of the paper is organised as follows: Sect. II outlines the quadrotor dynamics and the control problem; Sect. III presents the proposed control design and the closed-loop stability analysis; Sect. IV presents comparative experimental results and Sect. V concludes the work.

The following notations are used in this paper: $\lambda_{\min}(\cdot)$ and $||(\cdot)||$ denote minimum eigenvalue and 2-norm of $(\cdot)$, respectively; $(\cdot)_L$ denotes that $(\cdot)(t)$ is time-delayed as $(\cdot)(t-L)$; an identity matrix is denoted by I.

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II. Quadrotor System Dynamics

The well established Euler-Lagrangian system dynamics of a quadrotor model is given by [6]

$$m\ddot{p}(t)+G+d_p(p(t),\dot{p}(t),t)=\tau_p(t), \tag{1a}$$

$$J(q(t))\ddot{q}(t)+C_q(q(t),\dot{q}(t))\dot{q}(t)+d_q(q,\dot{q},t)=\tau_q(t), \tag{1b}$$

$$\tau_p(t)=R_B^W(q)U(t), \tag{1c}$$

where (1a) and (1b) are the position dynamics and the attitude dynamics, respectively; (1c) converts the input vector $\tau_p \in \mathbb{R}^3$ in Earth-fixed frame to $U \triangleq \begin{bmatrix}0 & 0 & u_1\end{bmatrix}^T \in \mathbb{R}^3$ in body-fixed frame via the rotational matrix R_B^W given by

$$R_B^W=\begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & s_\phi c_\theta & c_\theta c_\phi \end{bmatrix}, \tag{2}$$

where $c_{(\cdot)}, s_{(\cdot)}$ are abbreviations for $\cos(\cdot), \sin(\cdot)$ respectively. Various other symbols in (1) are described as follows: the mass and inertia matrix are represented by $m \in \mathbb{R}^+$ and $J(q) \in \mathbb{R}^{3\times 3}$ respectively; the center-of-mass of the quadrotor is denoted by the position vector $p \triangleq \begin{bmatrix}x & y & z\end{bmatrix}^T \in \mathbb{R}^3$; the orientation/attitude (roll, pitch, yaw angles respectively) is denoted via $q \triangleq \begin{bmatrix}\phi & \theta & \psi\end{bmatrix}^T \in \mathbb{R}^3$; $G \triangleq \begin{bmatrix}0 & 0 & mg\end{bmatrix}^T \in \mathbb{R}^3$ denotes the gravitational force vector with gravitational constant g; $C_q(q,\dot{q}) \in \mathbb{R}^{3\times 3}$ is the Coriolis matrix; the unmodelled disturbances, which can be both state and time dependent, are denoted by d_p and d_q; $\tau_q \triangleq \begin{bmatrix}u_2 & u_3 & u_4\end{bmatrix}^T \in \mathbb{R}^3$ are the control inputs for roll, pitch and yaw;

Property 1. Standard Euler-Lagrange mechanics implies the inertia matrix $J(q)$ is uniformly positive definite $\forall q$ [6].

In the following, we highlight the available system parametric knowledge for control design:

Assumption 1: The exact knowledge of m, J is not available, and only some upper bounds are known (cf. Remark 6 later); meanwhile, the system dynamics term $C_q(q,\dot{q})$, unmodelled (possibly state-dependent) dynamics d_p, d_q and their bounds are unknown for control design.

Remark 1 (Validity of Assumption 1): In practice, maximum allowable payload is always known; hence, a priori upper bound knowledge of m and J is plausible for control design, while handling other unknown state-dependent dynamics terms is a control challenge solved in this work.

Assumption 2 ([6], [25], [26]): The desired position $p_d \triangleq \begin{bmatrix}x_d & y_d & z_d\end{bmatrix}^T$ and yaw trajectories ψ_d are designed such that they are smooth and bounded.

Remark 2 (Desired roll and pitch): As standard in literature [25], the desired roll (ϕ_d) and pitch (θ_d) angle trajectories are computed using τ_p and ψ_d.

III. Proposed Controller Design and Analysis

A. Control Problem and Objective

Under Property 1 and Assumption 1, the aim is to design an adaptive robust TDE (ARTDE) controller for quadrotors to track a desired trajectory (cf. Assumption 2).

Since the aim is to design controller for the complete six degrees-of-freedom quadrotor dynamics, the standard position and attitude co-design method is followed (cf. [6], [25]) instead of the reduced-order model one (cf. [7], [8]). Such approach (cf. Fig. 1 later) relies on simultaneous design of an outer loop controller for (1a) and of an inner loop controller for (1b) as described in the following subsections.

B. Outer Loop Controller

Let the position tracking error be defined as $e_p(t) = p^d(t) - p(t)$. The variable dependency will be removed subsequently for brevity whenever it is obvious. The position dynamics (1a) is re-arranged by introducing a constant $\bar{m}$ as

$$\bar{m}\ddot{p}+N_p(p,\dot{p},\ddot{p})=\tau_p, \tag{3}$$

$$\text{with} \quad N_p(p,\dot{p},\ddot{p})=(m-\bar{m})\ddot{p}+G+d_p$$

and the selection of $\bar{m}$ is discussed later (cf. Remark 6). Note that, N_p contains unknown state-dependent (via $(p,\dot{p})$) dynamics d_p (cf. Assumption 1) and hence, it is not a priori bounded.

The control input τ_p is proposed as

$$\tau_p=\bar{m}u_p+\hat{N}_p(p,\dot{p},\ddot{p}), \tag{4a}$$

$$u_p=u_{p0}+\Delta u_p,\ u_{p0}=\ddot{p}^d+K_{1p}\dot{e}_p+K_{2p}e_p, \tag{4b}$$

where $K_{1p}, K_{2p} \in R^{3\times 3}$ are two user-defined positive definite matrices; Δu_p is the adaptive control term responsible to tackle uncertainties in position dynamics and it will be designed later; $\hat{N}_p$ is the estimation of N_p derived from the past state and input data as

$$\hat{N}_p(p,\dot{p},\ddot{p}) \cong N_p(p_L,\dot{p}_L,\ddot{p}_L)=(\tau_p)_L-\bar{m}\ddot{p}_L, \tag{5}$$

where $L>0$ is a small time delay which and its choice is discussed later. The notation $(\cdot)_L$ is defined in Sect. I.

Remark 3 (Artificial time delay): The estimation of uncertainty as in (5) is based on intentionally (or artificially) introducing a time delay (a.k.a TDE) in the form of past data; some literature (cf. [15]–[20] and referenced therein) terms this mechanism as artificial time delay based design.

Substituting (4a) into (3), one obtains

$$\ddot{e}_p=-K_{1p}\dot{e}_p-K_{2p}e_p+\sigma_p-\Delta u_p, \tag{6}$$

where $\sigma_p=\frac{1}{\bar{m}}(N_p-\hat{N}_p)$ is the *estimation error originating from (5)* and it is termed as the *TDE error*.

In the following, we first derive the upper bound structure of $||\sigma_p||$, based on which the adaptive control term $\Delta\mathbf{u}_p$ is designed subsequently.

1) Upper bound structure of $||\sigma_p||$: From (3) and (6), once can derive the following:

$$\hat{N}_p=(N_p)_L=[m(p_L)-\bar{m}]\ddot{p}_L+G_L+(d_p)_L, \tag{7}$$

$$\sigma_p=\ddot{p}-u_p. \tag{8}$$

Using (7), the control input τ_p in (4a) can be rewritten as

$$\tau_p=\bar{m}u_p+[m(p_L)-\bar{m}]\ddot{p}_L+G_L+(d_p)_L. \tag{9}$$

Multiplying both sides of (8) with m and using (3) and (9) we have

$$\begin{aligned} m\sigma_p &= \tau_p-N_p-mu_p \\ &= \bar{m}u_p+[m(p_L)-\bar{m}]\ddot{p}_L+G_L+(d_p)_L-N_p-mu_p. \end{aligned} \tag{10}$$

Using (6) we have

$$\ddot{p}_L = \ddot{p}_L^d - \ddot{e}_L = \ddot{p}_L^d + K_p \xi_L - \sigma_L + (\Delta u_p)_L, \tag{11}$$

where $K_p \triangleq [K_{1p} \ \ K_{2p}]$, $\xi_L = [e_L{}^T \ \ \dot{e_L}{}^T]^T$. Substituting (11) into (10), and after re-arrangement yields

$$\begin{aligned}\sigma_p = & \underbrace{m^{-1}\bar{m}(\Delta u_p - (\Delta u_p)_L)}_{\chi_1} + \underbrace{m^{-1}(m_L(\Delta u_p)_L - m(\Delta u_p)_L)}_{\chi_2} \\ & + \underbrace{m^{-1}\{(\bar{m}-m)\ddot{p}^d - (m-m_L)\ddot{p}_L^d + G_L + (d_p)_L - G - d_p\}}_{\chi_3} \\ & + \underbrace{m^{-1}(m_L - \bar{m})K_p\xi_L}_{\chi_4} - \underbrace{m^{-1}(m_L - \bar{m})\sigma_L}_{\chi_5} \\ & + \underbrace{(\bar{m}/m - 1)K_p\xi_p}_{\chi_6}. \end{aligned} \tag{12}$$

Using the relation $(\cdot)_L = (\cdot)(t) - \int_{-L}^{0} \frac{\mathrm{d}}{\mathrm{d}\theta}(\cdot)(t+\theta)\mathrm{d}\theta$ and the fact that integration of any continuous function over a finite interval (here $-L$ to 0) is always finite [27], the following conditions hold for unknown constants δ_i, $i = 1, \cdots, 5$:

$$||\chi_1|| = ||\frac{1}{m}\bar{m}\int_{-L}^{0}\frac{d}{d\theta}\Delta u_p(t+\theta)d\theta|| \leq \delta_1 \tag{13}$$

$$||\chi_2|| = ||\frac{1}{m}\int_{-L}^{0}\frac{d}{d\theta}m(t+\theta)\Delta u_p(t+\theta)d\theta|| \leq \delta_2 \tag{14}$$

$$\begin{aligned}||\chi_3|| = ||\frac{1}{m}\{&(\bar{m}-m)\ddot{p}^d - (m-m_L)\ddot{p}_L^d \\ & G_L + (d_p)_L - G - d_p\}|| \leq \delta_3\end{aligned} \tag{15}$$

$$\begin{aligned}||\chi_4|| = ||\frac{1}{m}\int_{-L}^{0}&\frac{d}{d\theta}(m(t+\theta)-\bar{m})K_p\xi_p(t+\theta)d\theta \\ & + (\bar{m}/m - 1)K_p\xi_p|| \leq ||E_pK_p||||\xi_p|| + \delta_4\end{aligned} \tag{16}$$

$$\begin{aligned}||\chi_5|| = &||E_p\sigma_p + \frac{1}{m}\int_{-L}^{0}\frac{d}{d\theta}\{(m(t+\theta)-\bar{m})\sigma_p(t+\theta)\}d\theta|| \\ \leq & ||E_p||||\sigma_p|| + \delta_5\end{aligned} \tag{17}$$

$$||\chi_6|| = ||(\bar{m}/m - 1)K_p\xi_p|| \leq ||E_pK_p||||\xi_p||, \tag{18}$$

and the following holds

$$|E_p| = |1 - \bar{m}/m| < 1. \tag{19}$$

Using (12) and (13)-(18), one derives

$$\|\sigma_p\| \leq \beta_{0p} + \beta_{1p}\|\xi_p\|, \tag{20}$$

$$\text{where } \beta_{0p} = \frac{\sum_{i=1}^{5}\delta_i}{1-|E_p|}, \ \beta_{1p} = \frac{2\|E_pK_p\|}{1-|E_p|}. \tag{21}$$

2) Designing Δu_p: The term Δu_p is designed as

$$\Delta u_p = \alpha_p c_p (s_p/||s_p||), \tag{22}$$

where $s_p = B^T U_p \xi_p$, $\xi_p = \begin{bmatrix} e_p{}^T & \dot{e_p}{}^T \end{bmatrix}^T$ and U_p is the solution of the Lyapunov equation $A_p{}^T U_p + U_p A_p = -Q_p$ for some $Q_p > 0$, where $A_p = \begin{bmatrix} 0 & I \\ -K_{2p} & -K_{1p} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$; $\alpha_p \in \mathbb{R}^+$ is a user-defined scalar; c_p is the switching gain which provides robustness against the TDE error and it is constructed from the upper bound structure of $||\sigma_p||$ as

$$c_p = \hat{\beta}_{0p} + \hat{\beta}_{1p}||\xi_p||, \tag{23}$$

where $\hat{\beta}_{0p}, \hat{\beta}_{1p}$ are the estimates of β_{0p}, β_{1p}, respectively and they are evaluated as follows:

$$\dot{\hat{\beta}}_{ip} = \begin{cases} \|\xi_p\|^i\|s_p\|, & \text{if any } \hat{\beta}_{ip} \leq \underline{\beta}_{ip} \text{ or } s_p^T\dot{s}_p > 0 \\ -\|\xi_p\|^i\|s_p\|, & \text{if } s_p^T\dot{s}_p \leq 0 \text{ and all } \hat{\beta}_{ip} > \underline{\beta}_{ip} \end{cases}, \tag{24}$$

with $\hat{\beta}_{ip}(0) \geq \underline{\beta}_{ip} > 0$, $i = 0, 1$ are user-defined scalars. Combining (4), (31) and (35), we have

$$\begin{aligned}\tau_p = & \underbrace{(\tau_p)_L - \bar{m}\ddot{p}_L}_{\text{TDE part}} + \underbrace{\bar{m}(\ddot{p}^d + K_{1p}\dot{e}_p + K_{2p}e_p)}_{\text{Desired dynamics injection part}} \\ & + \underbrace{\bar{m}c_p(s_p/||s_p||)}_{\text{Adaptive-robust control part}}.\end{aligned} \tag{25}$$

Eventually, U is applied to the system using τ_p as in (1c).

C. Inner Loop Controller

The orientation/attitude error is defined as [25]

$$e_q = ((R_d)^T R_B^W - (R_B^W)^T R_d)^v \tag{26}$$

$$\dot{e}_q = \dot{q} - R_d^T R_B^W \dot{q}_d \tag{27}$$

where R_d is the rotation matrix as in (2) evaluated at $(\phi_d, \theta_d, \psi_d)$ and $(.)^v$ is the *vee* map converting elements of $SO(3)$ to $\in \mathbb{R}^3$ [25] (ϕ_d, θ_d are generated as per Remark 2).

Introducing a constant matrix $\bar{J}$ (cf. Remark 6 for its choice), the attitude dynamics (1b) is re-arranged as

$$\bar{J}\ddot{q} + N_q(q, \dot{q}, \ddot{q}) = \tau_q, \tag{28}$$

$$\text{with} \quad N_q(q, \dot{q}, \ddot{q}) = [J - \bar{J}]\ddot{q} + C_q\dot{q} + d_q. \tag{29}$$

The control input τ_q is designed as

$$\tau_q = \bar{J}u_q + \hat{N}_q(q, \dot{q}, \ddot{q}), \tag{30a}$$

$$u_q = u_{q0} + \Delta u_q, \ u_{q0} = \ddot{q}^d + K_{1q}\dot{e}_q + K_{2q}e_q, \tag{30b}$$

where $K_{1q}, K_{2q} \in R^{3\times 3}$ are two user-defined positive definite matrices; the adaptive control term Δu_q, used for tackling uncertainties in attitude dynamics, would be designed later; $\hat{N}_q$ is the estimation of N_q computed via TDE as

$$\hat{N}_q(q, \dot{q}, \ddot{q}) \cong N_q(q_L, \dot{q}_L, \ddot{q}_L) = (\tau_q)_L - \bar{J}\ddot{q}_L. \tag{31}$$

Substituting (30a) into (28), we obtain the error dynamics as

$$\ddot{e}_q = -K_{1q}\dot{e}_q - K_{2q}e_q + \sigma_q - \Delta u_q, \tag{32}$$

where $\sigma_q = \bar{J}^{-1}(N_q - \hat{N}_q)$ represents the attitude *TDE error*.

1) Upper bound structure of $||\sigma_q||$: The upper bound structure for $||\sigma_q||$ can be derived in a similar way to that of $||\sigma_p||$ in Sect. III-B.1 as (omitted due to lack of space)

$$\|\sigma_q\| \leq \beta_{0q} + \beta_{1q}\|\xi_q\|, \tag{33}$$

$$\text{where } \beta_{0q} = \frac{\sum_{i=1}^{5}\bar{\delta}_i}{1-\|E_q\|}, \ \beta_{1q} = \frac{2\|E_qK_q\|}{1-\|E_q\|}, K_q \triangleq [K_{1q} \ \ K_{2q}]$$

where $\bar{\delta}_i$ are unknown constants and the following holds

$$||E_q|| = ||J^{-1}\bar{J} - I|| < 1. \tag{34}$$

2) Designing Δu_q: The term Δu_q is designed as

$$\Delta u_q = \alpha_q c_q (s_q/||s_q||), \tag{35}$$

where $s_q = B^T U_q \xi_q$, $\xi_q = \begin{bmatrix} e_q{}^T & \dot{e_q}{}^T \end{bmatrix}^T$ and U_q is the solution of the Lyapunov equation $A_q{}^T U_q + U_q A_q = -Q_q$ for some $Q_q > 0$, where $A_q = \begin{bmatrix} 0 & I \\ -K_{2q} & -K_{1q} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$; $\alpha_q \in \mathbb{R}^+$ is a user-defined scalar; c_q is the adaptive gain designed as

$$c_q = \hat{\beta}_{0q} + \hat{\beta}_{1q} ||\xi_q||, \tag{36}$$

where $\hat{\beta}_{0q}, \hat{\beta}_{1q}$ are the estimates of $\beta_{0q}, \beta_{1q} \in \mathbb{R}^+$, respectively. The gains are evaluated as follows:

$$\dot{\hat{\beta}}_{iq} = \begin{cases} ||\xi_q||^i ||s_q||, & \text{if any } \hat{\beta}_{iq} \leq \underline{\beta}_{iq} \text{ or } s_q^T \dot{s}_q > 0 \\ -||\xi_q||^i ||s_q||, & \text{if } s_q^T \dot{s}_q \leq 0 \text{ and all } \hat{\beta}_{iq} > \underline{\beta}_{iq} \end{cases}, \tag{37}$$

with $\hat{\beta}_{iq}(0) \geq \underline{\beta}_{iq} > 0$, $i = 0, 1$ are user-defined scalars. Finally, the inner loop control becomes

$$\tau_q = \underbrace{(\tau_q)_L - \bar{J}\ddot{q}_L}_{\text{TDE part}} + \underbrace{\bar{J}(\ddot{q}^d + K_D \dot{e}_q + K_P e_q)}_{\text{Desired dynamics injection part}} + \underbrace{\bar{J} c_q (s_q/||s_q||)}_{\text{Adaptive-robust control part}}. \tag{38}$$

Figure 1 depicts the overall proposed control framework.

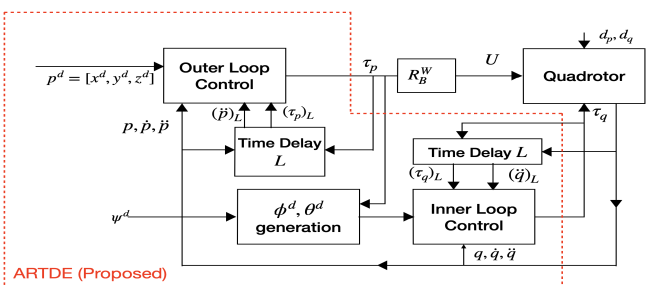

Fig. 1. Block diagram of the proposed ARTDE control framework via the outer- and inner loop co-design approach.

Remark 4 (State-dependent TDE error bound): Note from (20) and (33) that the TDE errors σ_p, σ_q depend on states via ξ_p, ξ_p respectively and hence cannot be bounded a priori: for this reason, the standard TDE-based methods [15]–[18], [20] are not applicable for quadrotors as they rely on a priori bounded TDE error (i.e., they assume $\beta_{1p} = 0$). ARTDE mainly differs from the state-of-the-art in the way its switching gain is adapted to tackle state-dependent uncertainty and the consequent stability analysis.

Remark 5 (Choice of L): It can be noted from (13)-(17), that high value of time delay L will lead to high values of δ_i, i.e., larger TDE error: therefore, one needs to select the smallest possible L, which is usually selected as the sampling time of the low level micro-controller [15]–[20], [22]–[24].

Remark 6 (On the choice of $\bar{m}$ and $\bar{J}$): From the upper bounds on m and J (cf. Assumption 1), one can always design $\bar{m}$ and $\bar{J}$ which satisfy (19) and (34) respectively, which is standard in TDE literature [15]–[20]: hence, we do not introduce any additional constraint.

D. Stability Analysis

Theorem 1: Under Assumptions 1-2 and Property 1, the closed-loop trajectories of (3) and (28) using the control laws (25), (38), the adaptive laws (24), (37) and design conditions (19) and (34), are Uniformly Ultimately Bounded (UUB).

Proof: See Appendix. ■

Remark 7 (Continuity in control law): In practice, the terms $(s_p/||s_p||)$ and $(s_q/||s_q||)$ in the control laws are generally replaced by continuous saturation functions without affecting the overall UUB stability result, albeit minor modifications in the analysis (cf. [28]).

IV. Experimental Results and Analysis

The proposed ARTDE is tested on a quadrotor setup (Q-450 frame, Turnigy SK3-2826 motors, approx. 1.4 kg excluding accessories and payload) which uses Raspberry Pi-4 as a processing unit and one electromagnetic gripper (0.03 kg approx. 0.03 kg). Optitrack motion capture system (at 60fps) and IMU data were used to obtain the necessary state and state-derivatives feedback. To properly verify the importance of the proposed controller, ARTDE is compared with a conventional adaptive TDE (ATDE) [19], and also with a non-TDE adaptive method, adaptive sliding mode control (ASMC) [8].

1) Experimental Scenario and Parameter Selection: The objective is the quadrotor should follow an infinity-shaped 2-loop path in 3D plane (cf. Figs. 2-3); the height from the ground is purposefully kept relatively small, as it is well-known that near-ground operations are more challenging to control since unknown ground-reaction forces are created by displaced wind from propellers. In addition, a fan is used to create external wind disturbances. The experimental scenario consists of the following sequences (cf. Fig. 2):

- The quadrotror starts from the center of the path (where the loops intersect) with a payload (0.35kg), it completes one loop with the payload and drops it at its starting position ($t = 35$s).
- Then, the quadrotor completes another loop without the payload and comes back to the origin.

The gripper is operated via a remote signal, which is separate from the proposed control design. For experiment, the control parameters of the proposed ARTDE are selected as: $\bar{m} = 1$ kg, $\bar{J} = 0.015I$ (kgm^2), $K_{2p} = K_{2q} = 10$, $K_{1p} = K_{1q} = 5$, $L = 0.015$, $Q_p = Q_q = I$, $\varepsilon_p = \varepsilon_q = 5 \times 10^{-5}$, $\underline{\beta}_{ip} = \underline{\beta}_{iq} = 0.01$, $\hat{\beta}_{ip}(0) = \hat{\beta}_{iq}(0) = 0.01$, $i = 0, 1$. For parity in comparison, same values of $\bar{m}, \bar{J}$, K_{ip}, K_{iq} and $L = 1/60$ are selected for ATDE [19], sliding variables for ASMC [8] are selected to be s_p and s_q. Other control parameters for ATDE and ASMC are as per [19] and [8], respectively.

2) Experimental Results and Analysis: The performances of the controllers are compared via Figs. 5-4 and via Table I in terms of root-mean-squared (RMS) error and peak error (absolute value). The error plots reveal that both ATDE and ASMC show turbulent behaviour while following the trajectory, specifically in the y direction. The quadrotor also swayed after dropping the payload, while the proposed

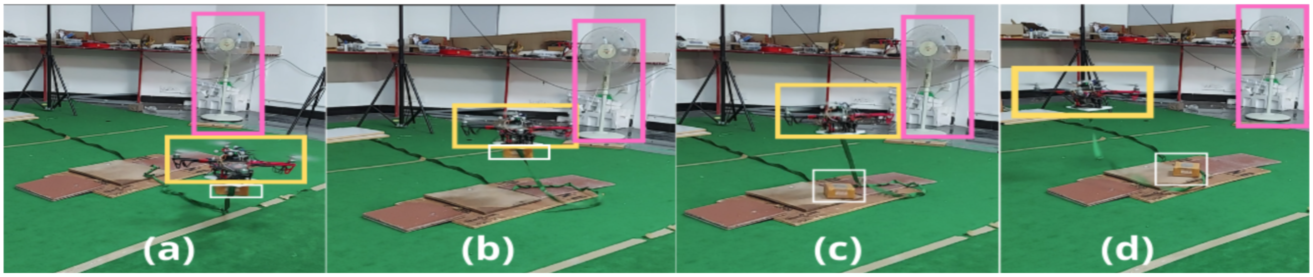

Fig. 2. Snapshots from the experiment with proposed ARTDE: (a) flying one loop of the trajectory with payload and (b) dropping it at the starting point; (c) initiating the other loop without payload and (d) completing the trajectory.

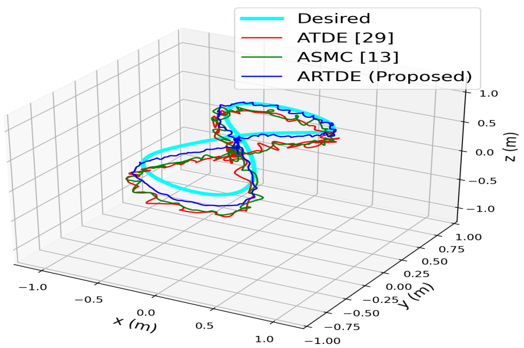

Fig. 3. Tracking performance of the Infinity-shaped loop.

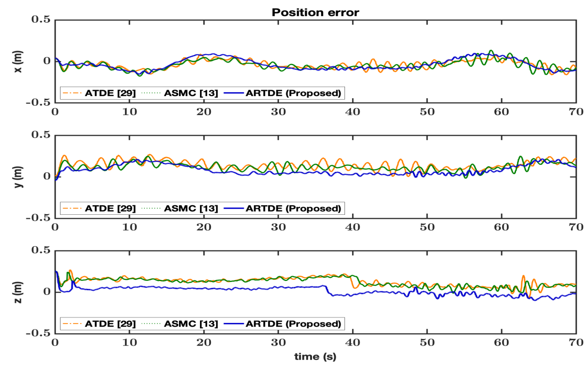

Fig. 5. Position tracking error comparison.

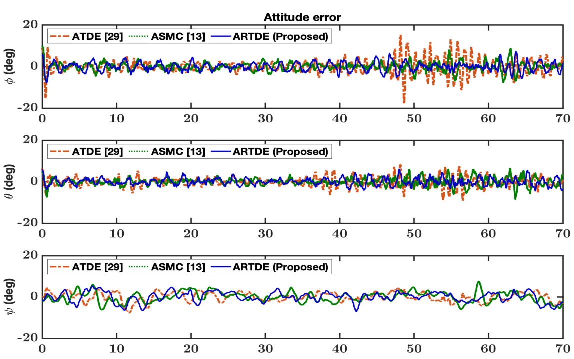

Fig. 4. Attitude tracking error comparison.

TABLE I
PERFORMANCE COMPARISON

	RMS error (m)			RMS error (degree)		
Controller	x	y	z	ϕ	θ	ψ
ATDE [19]	0.07	0.15	0.13	3.37	2.28	2.22
ASMC [8]	0.07	0.13	0.13	2.04	1.81	2.46
ARTDE (proposed)	0.06	0.10	0.04	2.12	1.72	2.50
	Peak error (m)			Peak error (degree)		
Controller	x	y	z	ϕ	θ	ψ
ATDE [19]	0.09	0.27	0.27	15.32	8.29	6.01
ASMC [8]	0.13	0.25	0.25	9.53	6.46	7.50
ARTDE (proposed)	0.09	0.22	0.25	7.46	6.25	5.76

ARTDE could maintain its position after dropping of payload. This can be verified from Table I, where ARTDE provides more than 30% improved accuracy compared to ATDE and ASMC in y and z directions. These results clearly show the benefit of considering state-dependent error-based adaptive control law as opposed to the conventional a priori bounded adaptive control structures.

V. CONCLUSION

In this paper, an artificial delay based adaptive controller for quadrotors was proposed to tackle state-dependent uncertainties. Closed-loop system stability was analytically verified. The experimental results under uncertain scenario showed significant performance improvements for the proposed controller against state-of-the-art methods.

APPENDIX

Proof of Theorem 1: The stability analysis of ARTDE is carried out using the following Lyapunov function candidate:

$$V = (1/2)(\xi_p^T U_p \xi_p + \xi_q^T U_q \xi_q) + \frac{1}{2}\sum_{i=0}^{1}(\hat{\beta}_{ip}-\beta_{ip})^2 + (\hat{\beta}_{iq}-\beta_{iq})^2. \quad (39)$$

The error dynamics (6) and (32) can be written as

$$\dot{\xi}_p = A_p\xi_p + B(\sigma_p - \Delta u_p), \dot{\xi}_q = A_q\xi_p + B(\sigma_q - \Delta u_q) \quad (40)$$

Positive definiteness of K_{ip} and K_{iq} $i = 1,2$ guarantee that A_p and A_q are Hurwitz. The first condition in the adaptive laws (24), (37) reveal that gains $\hat{\beta}_{ip}$ and $\hat{\beta}_{iq}$ increase if they attempt to go below $\underline{\beta}_{ip}$ and $\underline{\beta}_{iq}$ respectively; this yields $\hat{\beta}_{ip}(t) \geq \underline{\beta}_{ip}$, $\hat{\beta}_{iq}(t) \geq \underline{\beta}_{iq}$ $\forall t \geq 0$ $\forall i = 0,1$. Further, these adaptive laws enumerate the following four possible cases.

Case (i): Both $\dot{\hat{\beta}}_{ip}(t) > 0, \dot{\hat{\beta}}_{iq}(t) > 0$
Using the Lyapunov equations $A_p^T U_p + U_p A_p = -Q_p$, $A_q^T U_q + U_q A_q = -Q_q$, and the adaptive laws (24), (37) yield

$$\begin{aligned}\dot{V} \leq& -(1/2)(\xi_p^T Q_p \xi_p + \xi_q^T Q_q \xi_q) - c_p||s_p|| - c_q||s_q|| \\ &+ (\beta_{0p} + \beta_{1p}||\xi_p||)||s_p|| + (\beta_{0q} + \beta_{1q}||\xi_q||)||s_q|| \\ &+ \sum_{i=0}^{1}\{(\hat{\beta}_{ip} - \beta_{ip})||\xi_p||^i||s_p|| + (\hat{\beta}_{iq} - \beta_{iq})||\xi_q||^i||s_q||\} \\ &\leq -(1/2)(\lambda_{\min}(Q_p)||\xi_p||^2 + \lambda_{\min}(Q_q)||\xi_q||^2) \leq 0. \quad (41)\end{aligned}$$

Case (ii): Both $\dot{\hat{\beta}}_{ip}(t) < 0, \dot{\hat{\beta}}_{iq}(t) < 0$
For this case, following the steps as in Case (i), one has

$$\begin{aligned}\dot{V} \leq& \leq -(1/2)(\lambda_{\min}(Q_p)||\xi_p||^2 + \lambda_{\min}(Q_q)||\xi_q||^2) \\ &+ 2\{(\beta_{0p} + \beta_{1p}||\xi_p||)||s_p|| + (\beta_{0q} + \beta_{1q}||\xi_q||)||s_q||\}. \quad (42)\end{aligned}$$

The second laws of (24) and (37) yield $s_p^T \dot{s}_p \leq 0, s_q^T \dot{s}_q \leq 0$ which imply $||s_p||, ||s_q||, ||\xi_p||, ||\xi_q|| \in \mathscr{L}_\infty$ (cf. the relation $s_p = B^T U_p \xi_p$, $s_q = B^T U_q \xi_q$). Thus, $\exists \varsigma_p, \varsigma_q \in \mathbb{R}^+$ such that

$$2(\beta_{0p} + \beta_{1p}||\xi_p||)||s_p|| \leq \varsigma_p,\ 2(\beta_{0q} + \beta_{1q}||\xi_q||)||s_q|| \leq \varsigma_q.$$

The fact that $\hat{\beta}_{ip}, \hat{\beta}_{iq} \in \mathscr{L}_\infty$ in Case (i) and decrease in Case (ii) implies $\exists \varpi_p, \varpi_q \in \mathbb{R}^+$ such that $\sum_{i=0}^{1}(\hat{\beta}_{ip} - \beta_{ip})^2 \leq \varpi_p$, $\sum_{i=0}^{1}(\hat{\beta}_{iq} - \beta_{iq})^2 \leq \varpi_q$. Therefore, from (39) we have

$$V \leq \lambda_{\max}(U_p)||\xi_p||^2 + \lambda_{\max}(U_q)||\xi_q||^2 + \varpi_p + \varpi_q. \quad (43)$$

Using the relation (43), (42) can be written as

$$\dot{V} \leq -\upsilon V + \varsigma_p + \varsigma_q + \upsilon(\varpi_p + \varpi_q), \quad (44)$$

where $\upsilon \triangleq \frac{\min\{\lambda_{\min}(Q_p), \lambda_{\min}(Q_q)\}}{\max\{\lambda_{\max}(U_p), \lambda_{\max}(U_q)\}}$.

Case (iii): $\dot{\hat{\beta}}_{ip}(t) > 0, \dot{\hat{\beta}}_{iq}(t) < 0$
Following the derivations for Cases (i) and (ii) we have

$$\begin{aligned}\dot{V} \leq& -(1/2)(\lambda_{\min}(Q_p)||\xi_p||^2 + \lambda_{\min}(Q_q)||\xi_q||^2) \\ &+ 2(\beta_{0q} + \beta_{1q}||\xi_q||)||s_q|| \leq -\upsilon V + \varsigma_q + \upsilon\varpi_q. \quad (45)\end{aligned}$$

Case (iv): $\dot{\hat{\beta}}_{ip}(t) < 0, \dot{\hat{\beta}}_{iq}(t) > 0$
Following the previous cases yields

$$\begin{aligned}\dot{V} \leq& -(1/2)(\lambda_{\min}(Q_p)||\xi_p||^2 + \lambda_{\min}(Q_q)||\xi_q||^2) \\ &+ 2(\beta_{0p} + \beta_{1p}||\xi_p||)||s_p|| \leq -\upsilon V + \varsigma_p + \upsilon\varpi_p. \quad (46)\end{aligned}$$

The stability results from Cases (i)-(iv) reveal that the closed-loop system is UUB.

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