University of BRISTOL
Bristol Composites Institute

Manufacturing of complex 3D surfaces inspired by biological growth mechanics

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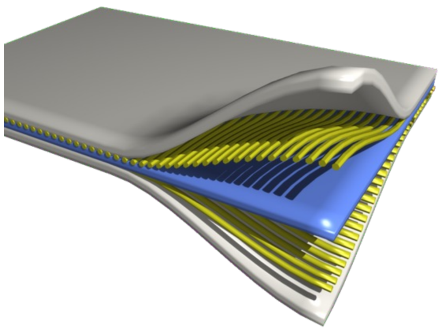
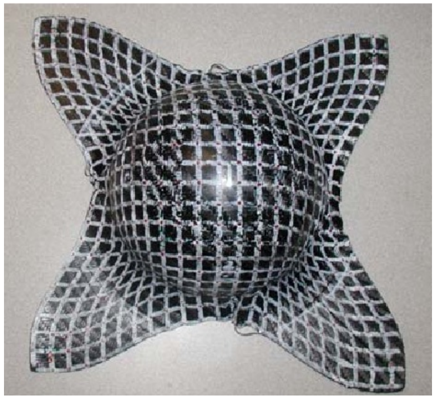
Key Question



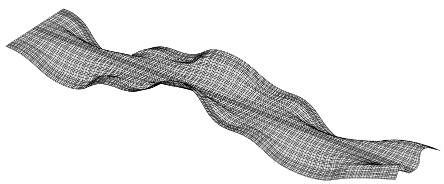


- Draping composites on doubly curved surfaces is inherently difficult
- Euclidean onto non-Euclidean surface
- Gauss: induces stretching

- How does nature produce much more complicated shapes?
- Curved material deposition? Or transition from Euclidean to non-Euclidean?

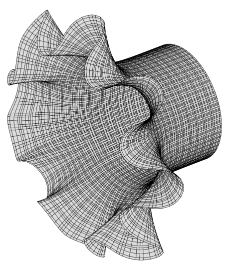
Outline

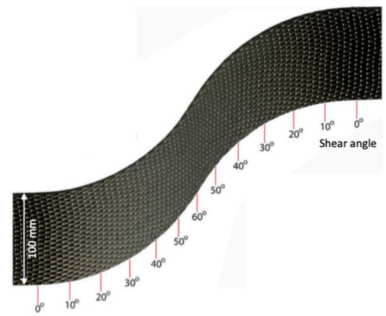
From Biomechanics → To Bioinspired Manufacturing

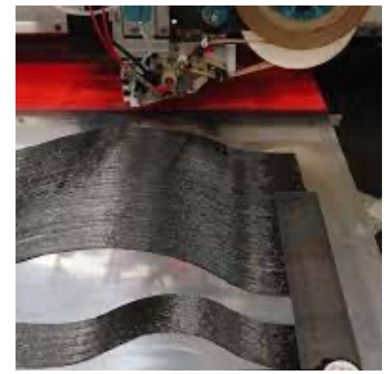


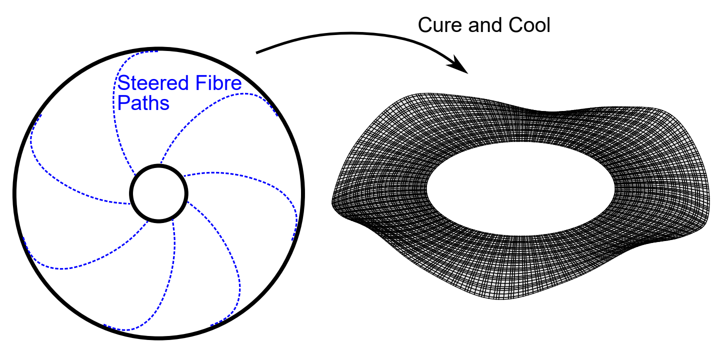
Rumex Crispus Leaf

Narcissus Petal









Crinkling of a Stretched Edge

- Analogous to tearing a plastic sheet
- Due to plastic deformation the sheet's metric changes

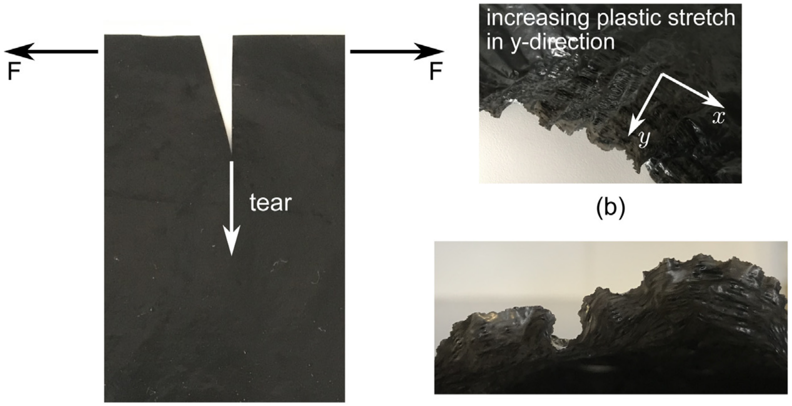
$$\mathrm{d}l^2 = f(y)^2 \mathrm{d}x^2 + \mathrm{d}y^2$$

- Associated Gaussian curvature

$$K(y) = -1/f \cdot (\mathrm{d}^2 f/\mathrm{d}y^2) \longrightarrow$$ Generally negative

- Saddle-like configurations. If *f(y)* pronounced: saddles on saddles

Flower Petal



Torn bin bag: crinkled edge

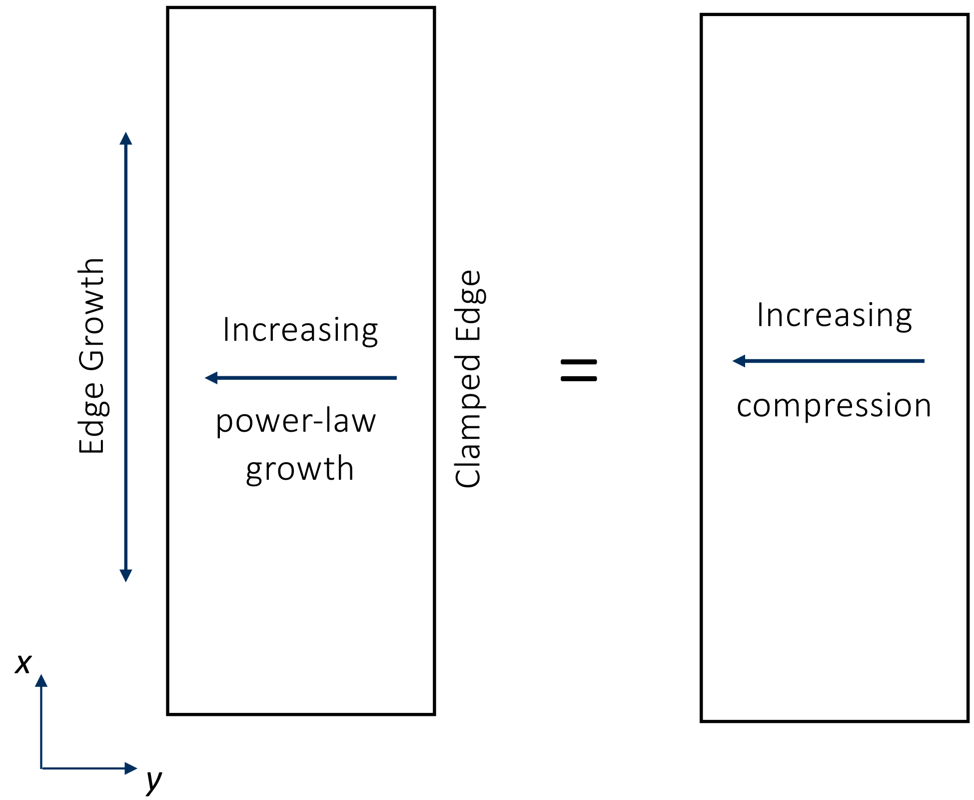
Power-law Growth Towards Edge

- Flat sheet:
  - clamped at one edge
  - lateral growth towards free edge

$$g_x(y, \lambda_g) = 1 + \lambda_g(1 + y/l)^{-a}$$

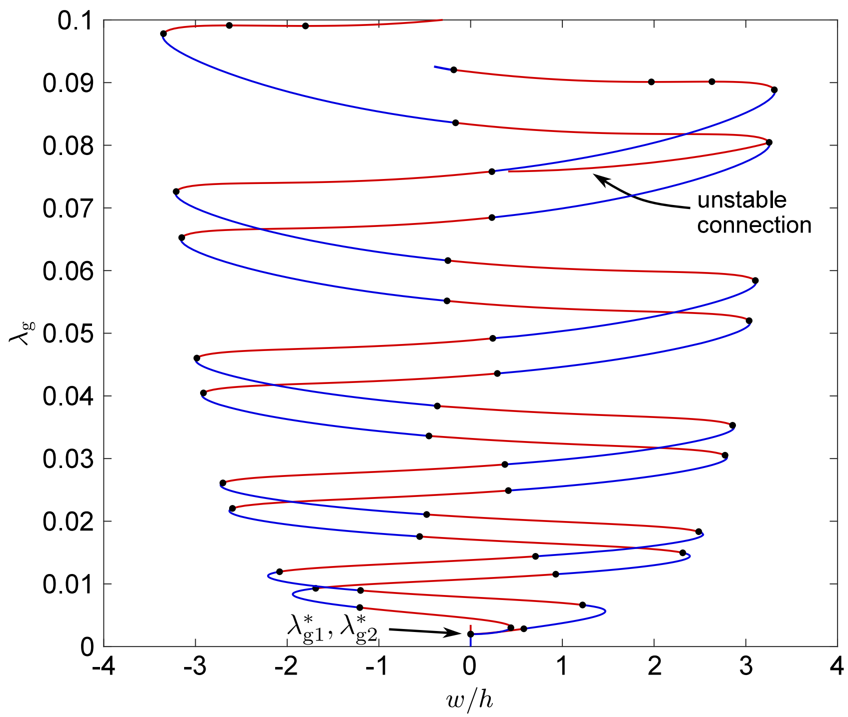
- Model using multiplicative decomposition of deformation gradient:

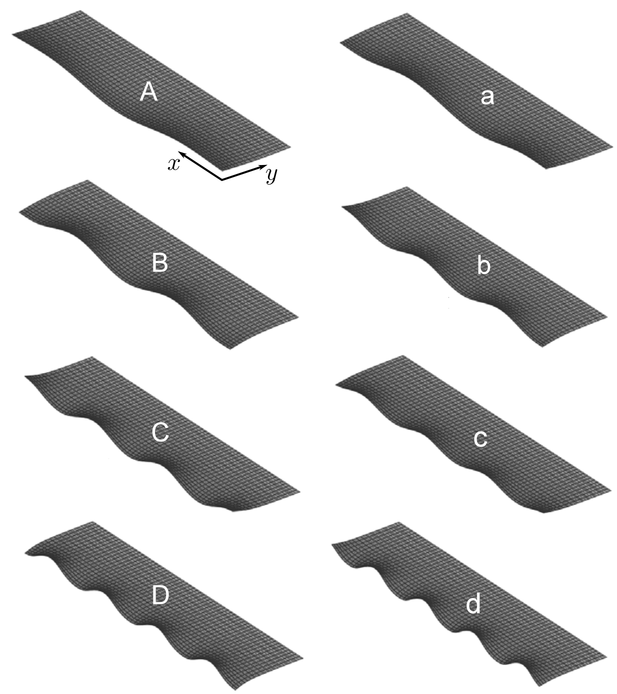
$$\boldsymbol{F} = \boldsymbol{F}_e \boldsymbol{F}_g \qquad \boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}$$



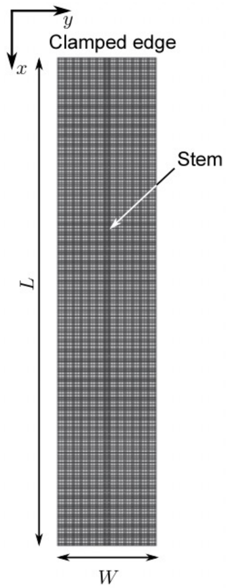
Pattern Formation

Intertwined symmetric and anti-symmetric patterns of increasing wave number

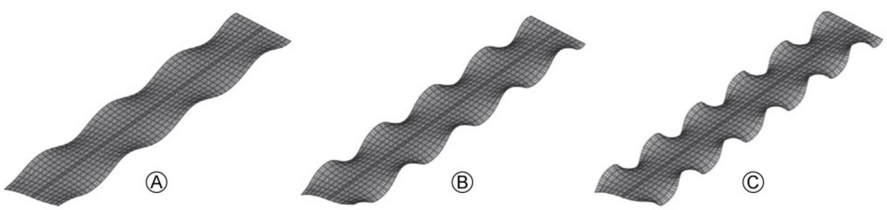


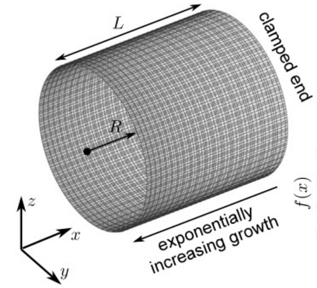


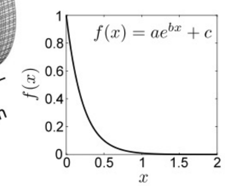
Edge Growth → Various Shapes



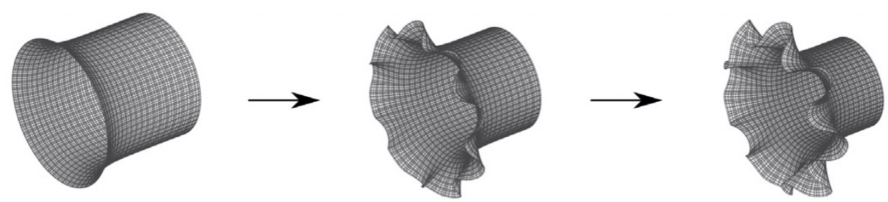
Hart’s Tongue Fern





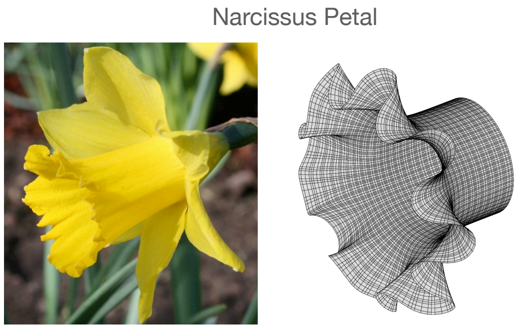


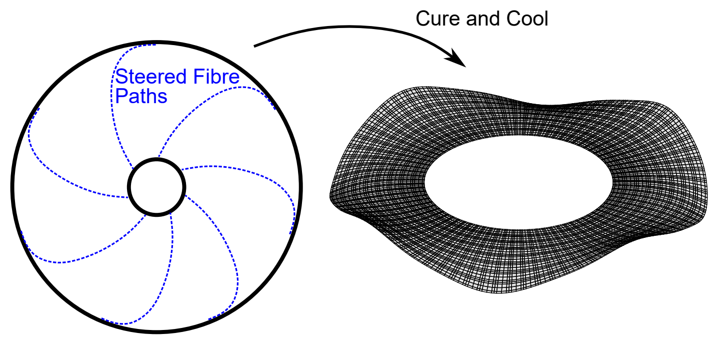
Common Daffodil



Bioinspiration

From Understanding Biomechanics → To Bioinspired Manufacturing





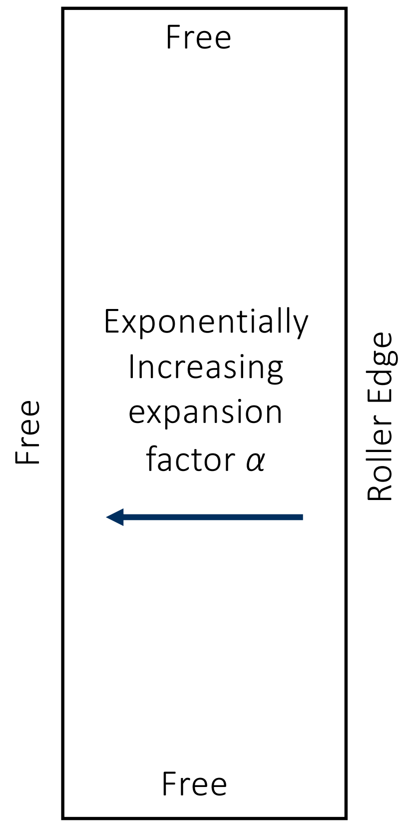
- Pattern formation not “just” patterned deposition of material
- Varying planar growth laws lead to complex doubly curved shapes

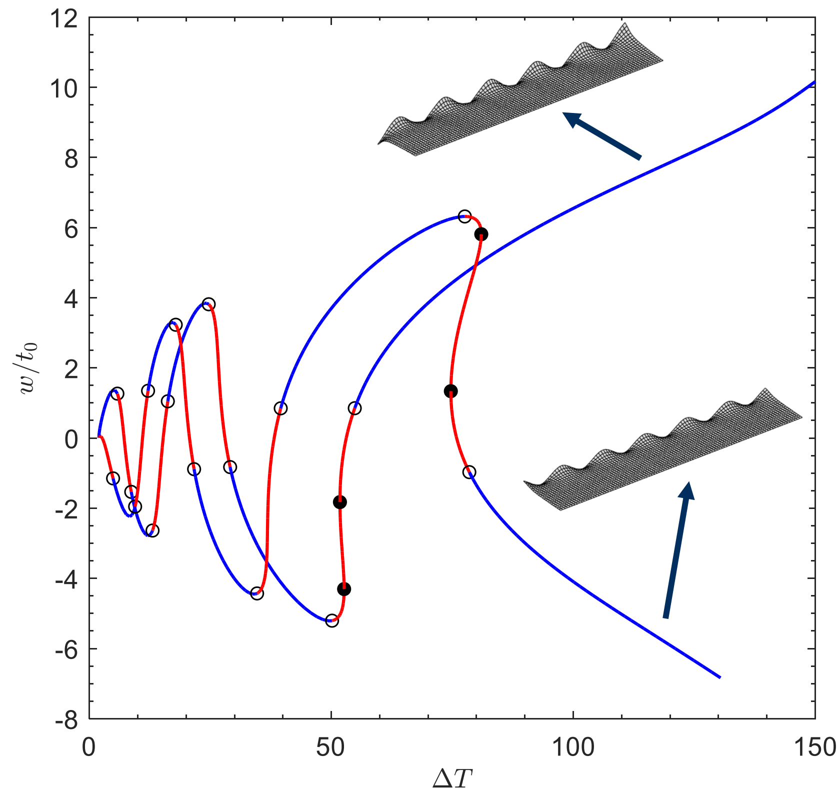
- Straightforward mechanical analogy between growth & thermal expansion
- *Use planar variations of expansion coefficient to create complex shapes?*

Feasibility study

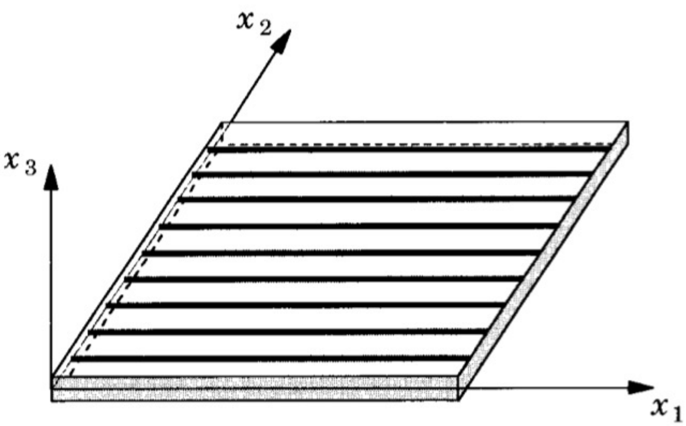
- Isotropic plate with varying expansion factor
- Exponential distribution of CTE from roller edge to free edge

$$\alpha(\bar{y}) = \alpha_0 \exp(-50\bar{y})$$





Orthotropic expansion factors of composites



- In the fibre direction, larger Young's modulus but essentially zero CTE
- When we cool down composites from curing, composites tend to
  - expand slightly in the fibre direction
  - contract across the fibre direction

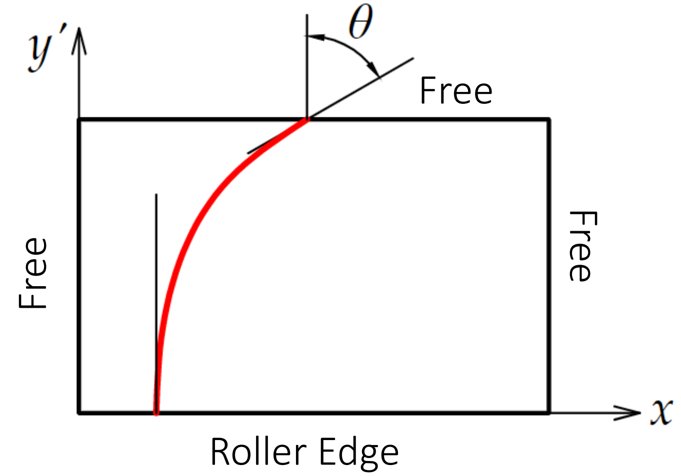
Typical CFRP lamina properties

E_{11} [GPa]	E_{22} [GPa]	υ_{12} [-]	G_{12} [GPa]	α_{11} $[K^{-1}]$	α_{22} $[K^{-1}]$
161	11.38	0.32	5.17	-1.810×10^{-8}	31.0×10^{-6}

Tow-Steered Design

- Smoothly vary fibre trajectory to:
  - ➢ Induce spatially varying residual stresses during cooling
  - ➢ Create doubly curved shapes from flat preform through buckling

- General guidelines
  - ➢ Fibre direction aligned with the **x'** axis where wrinkling occurs (no contraction)
  - ➢ Fibre variation in the **y'** direction to produce contraction internally

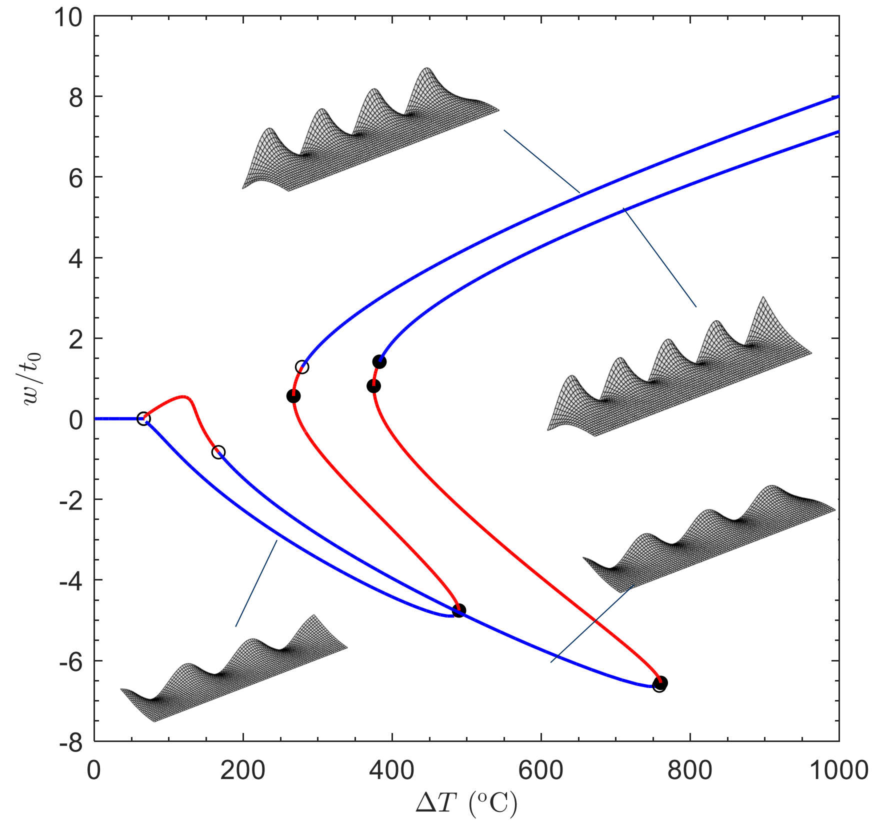


- Rectangular plate with 8-layer (balanced, symmetric layup)
- Fibre angle variation:

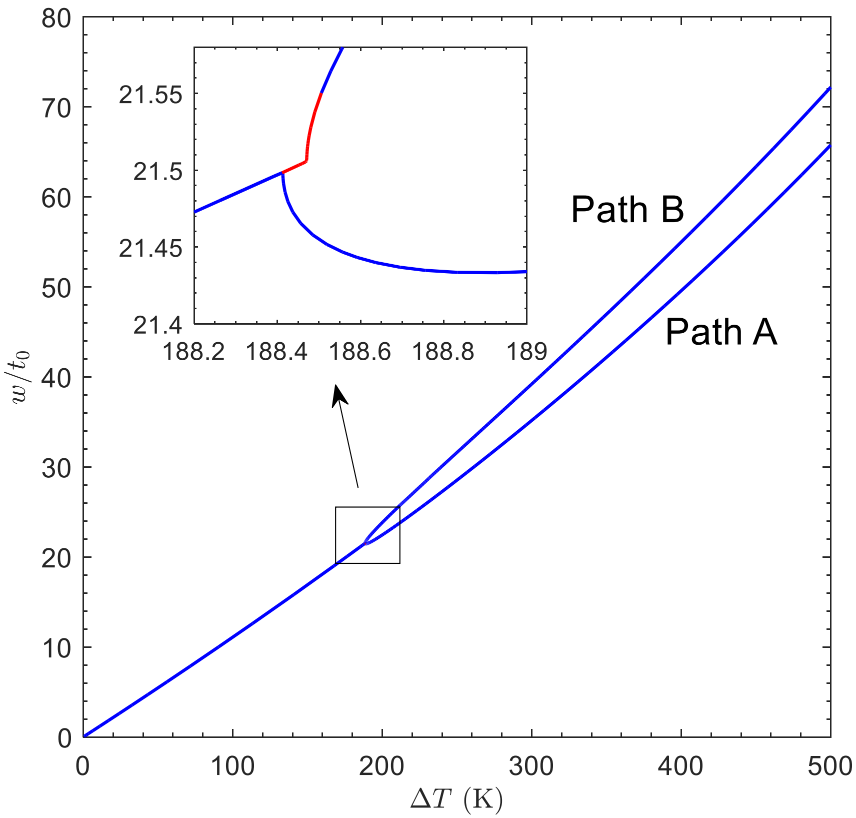
$$\theta(\bar{y}) = 30\exp(-50\bar{y})$$

Results

- Stable symmetric and anti-symmetric wrinkling modes.
- Critical wrinkling pattern is a symmetric mode with 3 waves.
- Further mode progression is not observed due to the relatively large thickness at the free edge.

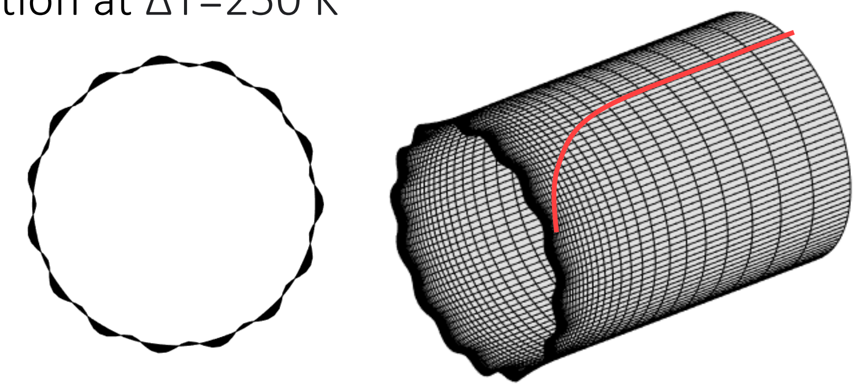


Cylindrical shell inspired from daffodil's corona

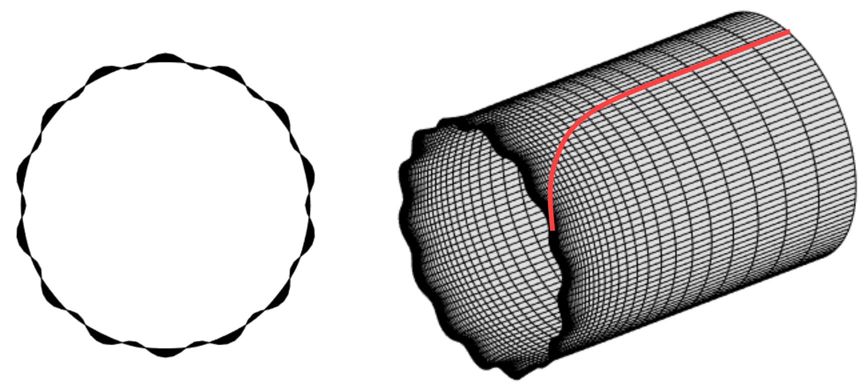


- Deformation at ΔT=250 K

Path A:
(15 waves)



Path B:
(14 waves)



Conclusion and Future Work

- Complex patterns can form in growing materials through:
  - spatially varying growth laws
  - excess length leads to compressive stresses and buckling
  - shape is doubly curved (usually saddles)
- Analogy between growth and thermal expansion:
  - use planar variations in expansion coefficients to induce residual stresses → tow-steered composites
  - can we create other shapes (not just saddles)?
  - scope for inverse design?

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Questions?

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