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# COMPARISON OF VARIOUS OPTIMIZATION CRITERIA FOR ACTUATOR PLACEMENT FOR ACTIVE VIBRATION CONTROL OF SMART COMPOSITE BEAM

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**Abstract**:

Position of piezoelectric actuators and sensors on a smart structure directly affects the control performances of a smart structure. In order to improve efficiency of active vibration control of a smart structure, optimization of piezoelectric actuators and sensors placement has been performed. There are various optimization criteria for optimal placement of piezoelectric actuator. The 'state-of-the-art' of optimization criteria is presented in [1]. The aim of this paper is to compare control effectiveness of smart composite cantilever beam, where optimal configurations of actuator-sensor pairs were found by using four optimization criteria (LQR based optimization, grammian matrices, performance index and fuzzy optimization strategy). The problem is formulated as multi-input-multi-output (MIMO) model. The beam is discretized by using the finite element method (FEM). The particle swarm optimization (PSO) method is used to find optimal configurations for each configuration.

**Key words**: active vibration control, smart composite beam, piezoelectric actuator, optimization criteria

## 1. Introduction

The optimal placement of piezoelectric actuators and sensors on a smart structure for active vibration control has been shown as the one of the most important issue in design of active structures since these parameters have a major influence on the performance of the control system. There are many papers which deal with optimal placement of piezoelectric actuators and sensors. A comprehensive review is presented in [1, 2].

There are many criteria for optimal actuator placement on a smart structure. The most used optimization criteria can be divided in two approaches. The first approach consists of combination of optimal location sensors and actuators and controller parameters. In [3, 4, 5] quadratic cost function based on linear quadratic regulator (LQR) was used to taking into account the measurement error and control energy. The energy dissipation method as criterion for the optimization has been presented on [6, 7, 8]. The second approach deals with optimal location and size of sensors and actuators independently of controller definition. Optimization using objective function based on grammian matrix is presented in [9, 10, 11]. In [12, 13, 14, 15] has been presented modal controllability index based on singular value analysis of control vector. Fuzzy optimization approach is presented in [16].

The aim of this paper is to compare control effectiveness of smart composite cantilever beam, where optimal configurations of actuator-sensor pairs were found by using optimization criteria mentioned in the previous paragraph. The beam is discretized by using the finite element method (FEM). The particle swarm optimization (PSO) method is used to find optimal configurations for each configuration.

## 2. Equations of active vibration control

A laminated composite beam with integrated piezoelectric sensors and actuators is considered (Fig. 1).

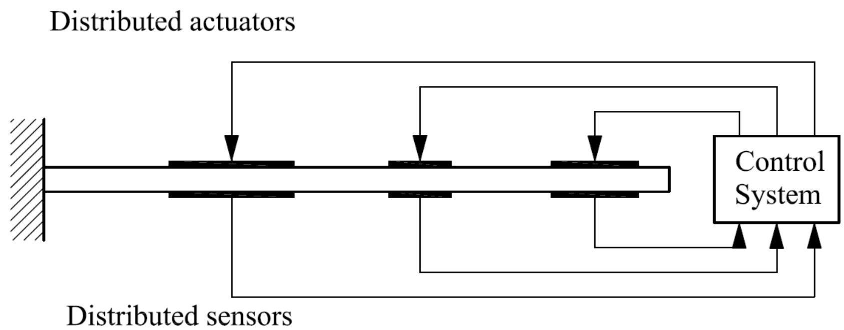


Fig. 1. General layout of the system.

The beam is discretized using the finite element method based on the third-order shear deformation theory [17, 18]. Equation of motion [16] can be expressed in state-space form as:

$$\{\dot{X}\} = [A]\{X\} + [B]\{\phi\}_{\mathrm{AA}} + \left[\hat{B}\right]\{F_{\mathrm{m}}\} \quad (1)$$

where $\{X\}$ represents the state vector, $[A]$ represents the system matrix, $[B]$ is the control matrix, $\left[\hat{B}\right]$ is the disturbance matrix and $\{\phi\}_{\mathrm{AA}}$ is the vector of external applied voltage on actuators.

## 3. Optimization criteria

### *3.1 LQR based optimal placement*

In the LQR optimal control, the feedback gains are chosen to minimize a cost function

$$J = \frac{1}{2}\int_0^{\infty}\left(\{X\}^T[Q]\{X\} + \{\phi\}_{AA}^T[R]\{\phi\}_{AA}\right)dt\,, \quad (2)$$

where $[Q]$ and $[R]$ are symmetric semi-positive definite and positive definite matrices selected to provide suitable performance. Assuming full-state feedback, the control law is given on following way

$$\{\phi\}_{AA} = -[G]\{X\} \quad (3)$$

where feedback gain $[G]$ is

$$[G]=[R]^{-1}[B]^{\mathrm{T}}[P] \tag{4}$$

$[P]$ is determined by solving the following algebraic Riccati equation

$$[A]^{\mathrm{T}}[P]+[P][A]-[P][B][R]^{-1}[B]^{\mathrm{T}}[P]+[Q]=0 \tag{5}$$

Optimal value of the cost function (2) can be expressed as [19]

$$J_{opt}(x)=\{Y_S(0)\}^{\mathrm{T}}[P(x)]\{Y_S(0)\}, \tag{6}$$

where $x$ presents the location of the actuator. It can be seen from equation (6) that functional depends on both actuator locations and the initial condition $\{Y_S(0)\}$. The dependence on the initial codition can be removed by minimizing the trace of the solution of the Riccati equation (5) [13]

$$J_{opt}(x)=\operatorname{trace}[P(x)]. \tag{7}$$

Thus, the LQR based optimization criteria can be expressed on following

$$\text{minimize} \quad J(x)=\operatorname{trace}[P(x)], \tag{8}$$

or

$$\text{maximize} \quad \overline{J}(x)=\frac{1}{1+J(x)}=\frac{1}{1+\operatorname{trace}[P(x)]} \ . \tag{9}$$

*3.2 Controllability index*

In [12], a controllability index for actuator is proposed, which is obtained by maximizing global control force. The modal control force applied to the system can be written as

$$\{f_{\mathrm{C}}\}=[B]\{\phi\}_{\mathrm{AA}}. \tag{10}$$

It follows from (10) that

$$\{f_{\mathrm{C}}\}^{\mathrm{T}}\{f_{\mathrm{C}}\}=\{\phi\}_{\mathrm{AA}}^{\mathrm{T}}[B]^{\mathrm{T}}[B]\{\phi\}_{\mathrm{AA}}. \tag{11}$$

Using singular value analysis, $[B]$ can be written as $[B]=[M][S][N]^{\mathrm{T}}$, where $[M]^{\mathrm{T}}[M]=[\mathrm{I}]$, $[N]^{\mathrm{T}}[N]=[\mathrm{I}]$ and

$$[S]=\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{N_{\mathrm{A}}} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{12}$$

where $N_{\mathrm{A}}$ presents number of actuators. Equation (11) can be written as

$$\{f_{\mathrm{C}}\}^{\mathrm{T}}\{f_{\mathrm{C}}\}=\{\phi\}_{\mathrm{AA}}^{\mathrm{T}}[N][S]^{\mathrm{T}}[S][N]^{\mathrm{T}}\{\phi\}_{\mathrm{AA}}, \tag{13}$$

or

$$\|f_{\mathrm{C}}\|^2 = \|\phi\|_{\mathrm{AA}}^2 \|S\|^2 . \tag{14}$$

Thus, maximizing this norm independently of the applied voltage $\{\phi\}_{\mathrm{AA}}$ induces maximizing $\|S\|^2$. The magnitude of $\sigma_i$ is a function of location and size of piezoelectric actuator. In [13] is proposed controllability index which is defined by

$$\Omega_{\mathrm{C}} = \prod_{i=1}^{N_{\mathrm{A}}} \sigma_i . \tag{15}$$

The higher controllability index indices the smaller electrical potential will be required for control.

*3.3 Grammian matrices*

The controllability of a system can be expressed quantitatively by using controllability Grammian matrix defined as [20]

$$[W_{\mathrm{C}}(t)] = \int_0^t e^{[A]\tau} [B][B]^{\mathrm{T}} e^{[A]^{\mathrm{T}}\tau} \mathrm{d}\tau . \tag{16}$$

In modal coordinates controllability Grammian is diagonally dominant [20]

$$[W_{\mathrm{C}}] = \begin{bmatrix} W_{\mathrm{C}11} & 0 & \cdots & 0 \\ 0 & W_{\mathrm{C}22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_{\mathrm{C}nn} \end{bmatrix} \tag{17}$$

and each diagonal term of controllability Grammian matrix can be expressed in a closed form eliminating time dependence of the solution

$$W_{\mathrm{C}ii} = \frac{1}{4\zeta_i \omega_i} \left(\overline{B}\right)_i \left(\overline{B}\right)_i^{\mathrm{T}} , \tag{18}$$

where $\left(\overline{B}\right)_i$ is *i*-th row of matrix $\left[\overline{B}\right]$. The value of $W_{\mathrm{C}ii}$ gives information about the energy transmitted from the actuators to the structure for the *i*-th mode. To maximize each diagonal term of controllability Grammian matrix, maximization of following function can be done [20, 21]

$$J_C = \operatorname{trace}\left([W_C]\right)\left(\det(W_C)\right)^{1/(2n)} \tag{19}$$

where $n$ presents the number of controlled modes.

*3.4 Fuzzy optimization method*

Fuzzy optimization method for optimal placement and sizing of piezoelectric actuator/sensor pairs is presented in [16]. Using fuzzy set theory [22], membership function of the *i*-th objective function can be written as follows

$$\mu_{W\,i}(\boldsymbol{p}) = \begin{cases} \dfrac{W_{\mathrm{C}ii}(\boldsymbol{p})}{W_{\mathrm{C}ii\,\max}}, & W_{\mathrm{C}ii}(\boldsymbol{p}) < W_{\mathrm{C}ii\,\max} \\ 1, & W_{\mathrm{C}ii}(\boldsymbol{p}) \geq W_{\mathrm{C}ii\,\max} \end{cases} \tag{20}$$

where $W_{\mathrm{C}ii\,\max}$ denote the maximum ***i***-th eigenvalue of $W_{\mathrm{C}}$, and $\boldsymbol{p}$ presents the design variables set. The optimum solution $\boldsymbol{p}^*$ can be selected by maximizing the smallest membership function

$$\mu_{\mathrm{D}}\left(\boldsymbol{p}^*\right) = \max\ \mu_{\mathrm{D}}(\boldsymbol{p}) \tag{21}$$

where

$$\mu_{\mathrm{D}} = \min_{i=1,\ldots,N_{\mathrm{C}}} \mu_{W\,i} \tag{22}$$

presents the membership function of the optimal decision function.

## 4. Optimization results

In the numerical example, cantilever laminated beam is considered. Dimensions of the beam are 500mm x 25mm. The beam is made of eight Graphite-Epoxy (Carbon-Fibre Reinforced) layers. The thickness of each layer is 0.25mm and orientations are $\left(90^0/0^0/90^0/0^0/0^0/90^0/0^0\ /\ 90^0\right)$. Piezoelectric actuators and sensors are made of PZT, their number is five and they are collocated. Their thicknesses are 0.2mm and length is 20mm. Material properties of Graphite-Epoxy and PZT are given in Table 1.

| Material properties | Graphite-Epoxy | PZT |
|---|---|---|
| $E_1$(GPa) | 174 | 63 |
| $E_2$(GPa) | 10.3 | 63 |
| $G_{13}$(GPa) | 7.17 | 24.6 |
| $G_{23}$(GPa) | 6.21 | 24.6 |
| $v_{12}$ | 0.25 | 0.28 |
| $\rho$(kg/m$^3$) | 1389.23 | 7600 |
| $e_{31}$(C/m$^2$) | / | 10.62 |
| $k_{33}$(F/m) | / | $0.1555\text{x}10^{-7}$ |

Table 1. Material properties of Graphite-Epoxy and PZT

Optimization results (objective function and locations of actuators) are presented in Table 2. For LQR based optimal placement, weighting matrices $Q = 10^5[\mathrm{I}]_{12x12}$, $R = [\mathrm{I}]_{5x5}$ are used.

| | LQR | Controllability index | Grammian matrices | Fuzzy optimization method |
|---|---|---|---|---|
| Objective function | $1.6079 \cdot 10^{-8}$ | $2.5453 \cdot 10^{-9}$ | $1.1027 \cdot 10^{-6}$ | 0.6484 |
| Location $x_i$ (mm) | 0 | 0 | 0 | 0 |
| | 30 | 170 | 70 | 30 |
| | 60 | 240 | 330 | 70 |
| | 330 | 320 | 400 | 140 |
| | 410 | 400 | 430 | 400 |

Table 2. Objective function values and optimal locations of actuator / sensor pairs for LQR based optimal placement.

Positions of actuator/sensor pairs are presented in Figure 2, for LQR based optimal placement, Figure 3 for controllability index, Figure 4 for Grammian matrices and Figure 5 for fuzzy optimization method.

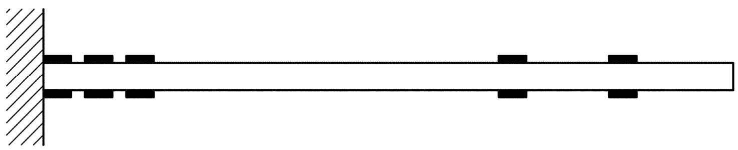

Figure 2. Position of actuator/sensor pairs found by using LQR based optimal placement

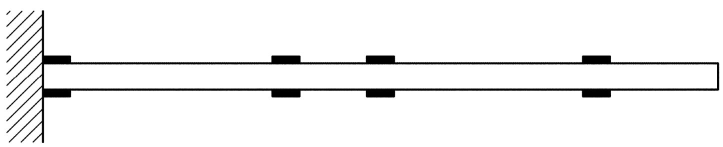

Figure 3. Position of actuator/sensor pairs found by using controllability index.

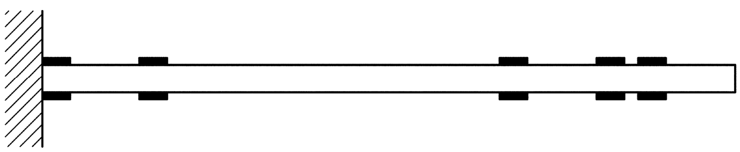

Figure 4. Position of actuator/sensor pairs found by using Grammian matrices.

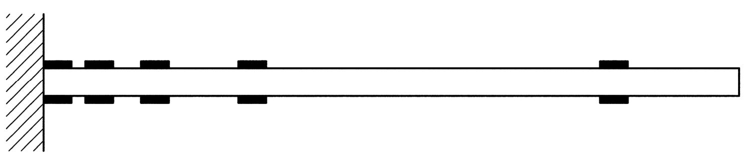

Figure 5. Position of actuator/sensor pairs found by using fuzzy optimization method.

## 4 Active vibration control using particle swarm optimized LQR control

In this section, control performances of smart composite beam with configurations of actuator – sensor pairs obtained by optimizations, is performed. LQR control algorithm is employed.

Ang, Wang and Quek [23] have proposed that $[Q]$ and $[R]$ could be determined by considering the weighted energy of the system as

$$[Q] = \begin{bmatrix} \alpha_2[\omega^2] & [0] \\ [0] & \alpha_1[\mathrm{I}] \end{bmatrix} \tag{23}$$

and

$$[R] = \alpha_3[K_\mathrm{e}]_\mathrm{A} \tag{24}$$

where $[K_\mathrm{e}]_\mathrm{A}$ presents dielectric stiffness matrix of the actuator and $\alpha_1$, $\alpha_2$, $\alpha_3$ present energy-weighting parameters. Parameters $\alpha_1$, $\alpha_2$, $\alpha_3$ are obtained by using the particle swarm optimization method in order to achieve maximum damping ratio for each controlled mode [16]. Optimization is performed for the case when the beam is subjected to an impulse load of 10 N at the tip a for duration of 0.1ms. Maximum allowable actuator voltage is 200 V. Optimized parameters $\alpha_1$, $\alpha_2$, $\alpha_3$, obtained damping ratio for each controlled mode and maximum actuator voltages for each optimization criterion are presented in Table 3.

| | LQR weighted | Controllability index | Grammian matrices | Fuzzy optimization method |
|---|---|---|---|---|
| $\zeta_{\mathrm{d}1}$ (%) | 6.78 | 4.84 | 5.17 | 7.19 |
| $\zeta_{\mathrm{d}2}$ (%) | 6.39 | 6.06 | 4.99 | 5.87 |
| $\zeta_{\mathrm{d}3}$ (%) | 6.31 | 6 | 5.72 | 5.99 |
| $\zeta_{\mathrm{d}4}$ (%) | 5.67 | 5.86 | 5.96 | 5.88 |
| $\zeta_{\mathrm{d}5}$ (%) | 6.06 | 6.23 | 6.67 | 5.95 |
| $\zeta_{\mathrm{d}6}$ (%) | 6.57 | 6.77 | 6.95 | 6.1 |
| $\alpha_1$ | 209.248 | 8407.201 | 761.96 | 218.89 |
| $\alpha_2$ | 2.797 | 4.527 | 79.787 | 4.077 |
| $\alpha_3$ | 119 | 4758.191 | 489.316 | 128.243 |
| $\phi_{\mathrm{AA1\,max}}$ (V) | 141.01 | 154.302 | 149.664 | 147.278 |
| $\phi_{\mathrm{AA2\,max}}$ (V) | 94.152 | 130.671 | 99.929 | 95.173 |
| $\phi_{\mathrm{AA3\,max}}$ (V) | 91.043 | 137.184 | 145.895 | 97.949 |
| $\phi_{\mathrm{AA4\,max}}$ (V) | 139.304 | 148.777 | 200 | 113.855 |
| $\phi_{\mathrm{AA5\,max}}$ (V) | 200 | 200 | 157.315 | 200 |

Table 3. Damping ratios for controlled modes, energy-weighting parameters and maximum actuator voltages for each optimization criterion.

The tip displacement the beam for configuration of actuator-sensor obtained by using LQR weighted fuzzy optimization method is presented in Figure 6. From Figure 6 it can be concluded that this two methods have almost equal performance for active vibration suppression.
Figure 7 shows the tip displacement of the beam for configuration of actuator-sensor obtained by using controllability index, Grammian matrices and fuzzy optimization method. It can be concluded from Figure 7 that fuzzy optimization method leads to the better vibration suppression compared to Grammian matrices and controllability index. Grammian matrices shows slightly better control performances compared to controllability index.

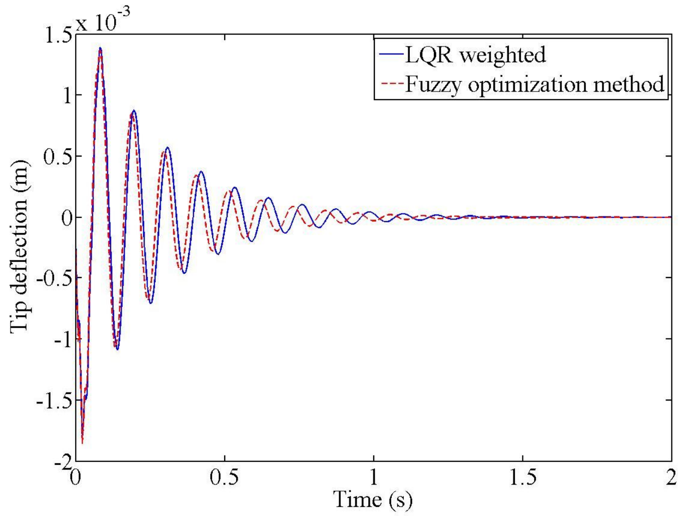


Figure 6. The tip displacement of the beam for configuration of actuator-sensor obtained by using LQR weighted fuzzy optimization method.

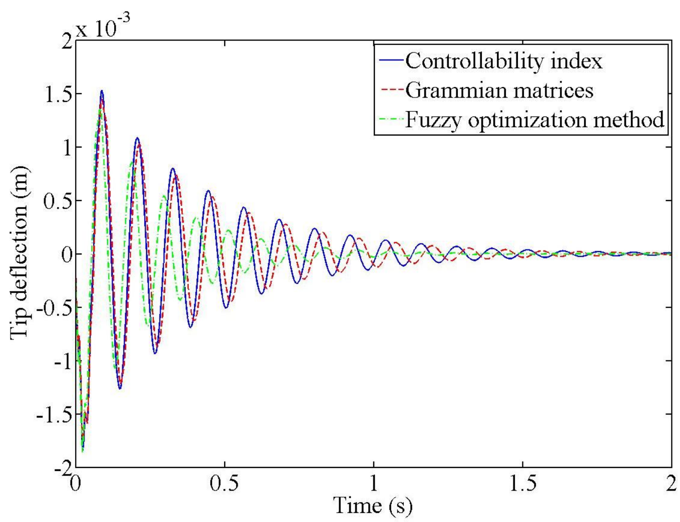


Figure 7. The tip displacement of the beam for configuration of actuator-sensor obtained by using controllability index, gramian matrices and fuzzy optimization method.

## 5. Conclusions

This paper presents comparison of control effect effectiveness of smart composite cantilever beam where optimal configurations of actuator-sensor pairs were found by using four optimization criteria (LQR based optimization, controllability index, performance index and fuzzy optimization strategy). Particle swarm optimized LQR is used as control algorithm.

Comparing control performances, it is found that the best performances provide LQR based optimization and fuzzy optimization strategy. These two methods have almost equal performance for active vibration suppression. After these two methods, Grammian matrices follows and controllability index provides minimum vibration reduction.

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