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OPTIMIZATION OF SIZING, LOCATION AND ORIENTATION OF PIEZOELECTRIC ACTUATOR-SENSOR PAIRS ON COMPOSITE PLATE

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Abstract: *Piezoelectric actuators and sensors have a wide range of application in the active vibration control of flexible structures. Control performances of the smart structures depend on the size and position of piezoelectric actuators and sensors on a smart structure. This paper deals with the optimization of sizing, location and orientation of piezoelectric actuators-sensors pairs on thin-walled composite plate. Optimization criteria are based on eigenvalues of the controllability Grammian matrix. The optimization problem is formulated by integration of finite element method based on the third-order shear deformation theory and the particle swarm optimization method. Numerical examples are provided for symmetric cross ply cantilever quadratic composite laminates. Linear quadratic regulator has been implemented for active vibration control of the composite plates with the optimized piezoelectric actuator-sensor pairs in order to show the efficiency of presented optimization method.*

Keywords: *Active vibration control, Composite plate, Piezoelectric actuators, Optimization.*

1. INTRODUCTION

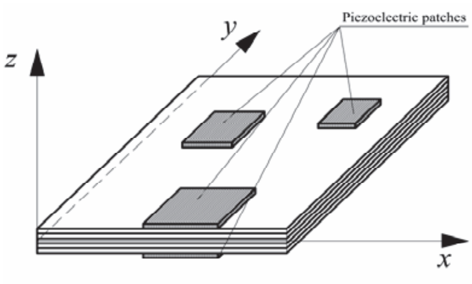
Optimization of sizing and location of the actuators and sensors for the active vibration control of flexible structures has been shown as one of the most important issues in design of active structures since these parameters have a major influence on the performance of the control system. Review of various optimization criteria for piezoelectric sensors and actuators location and sizing is presented in [1].

There are many papers which deal with the optimal placement of piezoelectric actuators / sensors on a composite beam and composite plate, but to the best of our knowledge, none of these papers does not deal with the simultaneous optimization of sizing, location and orinetation of fiber-reinforced acuator-sensor pairs on a composite plate.

This paper deals with the optimization of sizing, location and orientation of piezoelectric actuators-sensors pairs on a thin-walled composite plate. Optimization criteria are based on eigenvalues of the controllability Grammian matrix. The optimization problem is formulated by the integration of the finite element method based on the third-order shear deformation theory and the particle swarm optimization method. Numerical examples are provided for symmetric cross ply cantilever quadratic composite laminates. Linear quadratic regulator has been implemented for active vibration control of the composite plates with the optimized piezoelectric actuator-sensor pairs in order to show efficiency of the presented optimization method.

2. COUPLED EQUATIONS OF MOTIONS

The plate under consideration is composed of a finite number of layers of uniform thickness covered by piezoelectric patches at the top and the bottom (Picture 1). The *x*-*y* plane coincides with a mid-plane of the plate and *z* axis is defined as normal to the mid-plane according to the right-hand rule. Both elastic and piezoelectric layers are supposed to be thin, such that a plane stress state can be assumed. Elastic layers are obtained by setting their piezoelectric coefficients to zero. The equivalent single layer theory is used, so the same displacement field is considered for all layers of the plate. The formulation results in a coupled finite element model with mechanical (displacement) and electrical (potentials of piezoelectric patches) degrees of freedom.



Picture 1: Laminated composite plate with piezoelectric actuators and sensors

The displacement fields for the laminated plate based on the third-order shear deformation theory (TSDT) proposed by Reddy [2, 3] is given

$$\begin{aligned} u(x,y,z,t) &= u_0(x,t) + z\psi_x(x,t) - \frac{4}{3h_{\mathrm{pl}}^2} z^3 \left(\psi_x + \frac{\partial w_0}{\partial x}\right) \\ v(x,y,z,t) &= v_0(x,t) + z\psi_y(x,t) - \frac{4}{3h_{\mathrm{pl}}^2} z^3 \left(\psi_y + \frac{\partial w_0}{\partial y}\right), \quad (1) \\ w(x,z,t) &= w_0(x,t) \end{aligned}$$

where u, v and w are displacement components in the x, y and z directions respectively, u_0, v_0, w_0 are mid-plane ($z=0$) displacement, ψ_x and ψ_y are cross-sections rotations at the mid-planeand h_{pl} is total thickness of the beam.

After finite element discretization [3], the following equation of motion can be obtained

$$[M]\{\ddot{u}\} + [C_{\mathrm{d}}]\{\dot{u}\} + [K^*]\{u\} = \{F_{\mathrm{m}}\} - [K_{\mathrm{me}}]_{\mathrm{A}}\{\varphi\}_{\mathrm{AA}} \quad (2)$$

where $\{u\}$ presents the vector of generalized mechanical displacements, $[M]$ presents the mass matrix, $[K_{\mathrm{me}}]_{\mathrm{A}}$ is the piezoelectric stiffness matrix of actuator, $[C_{\mathrm{d}}]$ is the damping matrix, $\{\phi\}_{\mathrm{AA}}$ is the vector of external applied voltage on actuators, $\{F_{\mathrm{m}}\}$ is the vector of external forces and $[K^*]$ is the coupled stiffness matrix given as

$$\begin{aligned} [K^*] &= [K_{\mathrm{m}}] + [K_{\mathrm{me}}]_{\mathrm{A}}[K_{\mathrm{e}}]_{\mathrm{A}}^{-1}[K_{\mathrm{me}}]_{\mathrm{A}}^{\mathrm{T}} + \\ &+ [K_{\mathrm{me}}]_{\mathrm{S}}[K_{\mathrm{e}}]_{\mathrm{S}}^{-1}[K_{\mathrm{me}}]_{\mathrm{S}}^{\mathrm{T}} \end{aligned}$$

where $[K_{\mathrm{m}}]$ presents the elastic stiffness matrix, $[K_{\mathrm{me}}]_{\mathrm{S}}$ is the piezoelectric stiffness matrix of sensor and $[K_{\mathrm{e}}]_{\mathrm{A}}$ and $[K_{\mathrm{e}}]_{\mathrm{S}}$ are the dielectric stiffness matrices of actuator and sensor, respectively.

For practical implementation the obtained model needs to be truncated, where only the first few modes are taken into account. Thus, the displacement vector can be approximated by the modal superposition of the first r modes as

$$\{u\} \approx [\Psi]\{\eta\} \quad (4)$$

where $[\Psi]$ presents the modal matrix, and $\{\eta\}$ the vector of modal coordinates. Using Equation (4), Equation (3) can be transformed in the reduced modal space as

$$\begin{aligned} \{\ddot{\eta}\} + [\Lambda]\{\dot{\eta}\} + [\omega^2]\{\eta\} &= [\Psi]^{\mathrm{T}}\{F_{\mathrm{m}}\} - \\ -[\Psi]^{\mathrm{T}}[K_{\mathrm{me}}]_{\mathrm{A}}\{\varphi\}_{\mathrm{AA}} \end{aligned} \quad (5)$$

where $[\omega^2]$ presents the diagonal matrix of the squares of the natural frequencies, and

$$[\Lambda] = \operatorname*{diag}_{i=1,r}(2\zeta_i\omega_i) \quad (6)$$

presents the modal damping matrix in which ζ_i is natural modal damping ratio of the i -th mode.

Equation (5) can be expressed in a state-space form as

$$\{\dot{X}\} = [A]\{X\} + [B]\{\varphi\}_{\mathrm{AA}} + \{d\} \quad (7)$$

where

$$\{X\} = \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix}, \quad [A] = \begin{bmatrix} [0] & [\mathrm{I}] \\ -[\omega^2] & -[\Lambda] \end{bmatrix},$$
$$[B] = \begin{bmatrix} [0] \\ -[\Psi]^{\mathrm{T}}[K_{\mathrm{me}}]_{\mathrm{A}} \end{bmatrix}, \quad \{d\} = \begin{bmatrix} [0] \\ [\Psi]^{\mathrm{T}}\{F_{\mathrm{m}}\} \end{bmatrix} \quad (8)$$

present the state vector, the system matrix, the control matrix disturbance respectively, where $[\mathrm{I}]$ and $[0]$ are the appropriately dimensioned identity and zero matrix.

3. OPTIMIZATION CRITERIA FOR PIEZOELECTRIC ACTUATOR SIZING AND LOCATION

Controllability is a function of system dynamics and the location, size and number of actuators. The controllability can be expressed quantitatively by using controllability Grammian matrix [4] defined as

$$[W_{\mathrm{C}}(t)] = \int_0^t e^{[A]\tau}[B][B]^{\mathrm{T}} e^{[A]^{\mathrm{T}}\tau} \mathrm{d}\tau \quad (9)$$

In modal coordinates controllability Grammian is diagonally dominant [4]

$$[W_{\mathrm{C}}]=\begin{bmatrix} W_{\mathrm{C}11} & 0 & \cdots & 0 \\ 0 & W_{\mathrm{C}22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_{\mathrm{C}nn} \end{bmatrix} \tag{10}$$

and each diagonal term of controllability Grammian matrix can be expressed in a closed form eliminating time dependence of the solution

$$W_{\mathrm{C}ii}=\frac{1}{4\zeta_i\omega_i}\left(\overline{B}\right)_i\left(\overline{B}\right)_i^{\mathrm{T}} \tag{11}$$

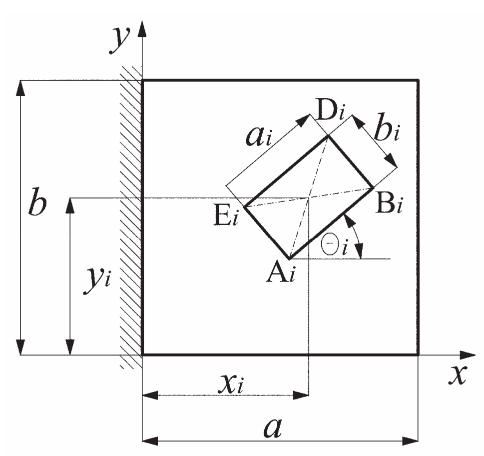
where $\left(\overline{B}\right)_i$ is i-th row of matrix $\left[\overline{B}\right]$. The value of $W_{\mathrm{C}ii}$ gives information about the energy transmitted from the actuators to the structure for the i-th mode. In other words, larger i-th eigenvalue of controllability Grammian matrix leads to smaller control efforts for suppression of the i-th mode. Consequently, if any eigenvalue of controllability Grammian is very low, the corresponding mode is very difficult to control and would require a huge energy for suppression. In [4] the performance index is presented

$$J_{\mathrm{e}}=\mathrm{trace}\left([W_{\mathrm{C}}]\right)\left(\det\left([W_{\mathrm{C}}]\right)\right)^{1/(2N\mathrm{c})} \tag{12}$$

where N_{C} presents the number of controlled modes. According to the equation (12), the objective function is:

$$OBJ=\mathrm{maximize}(J_{\mathrm{e}}). \tag{13}$$

In this paper, the actuator and sensor are conventionally collocated [3], thus, only optimization of sizing, location and orientation of the actuators will be performed and corresponding sensor has the same size, location and orientation but it is placed on the opposite side of the plate. Picture 2 presents composite plate with i-th actuator-sensor pair.



Picture 2: Composite plate with i-th actuator-sensor pair

Parameters which determine size, location and orientation of the i-th actuator-sensor pair are following:

- x_i, y_i : position of the center of the i-th actuator-sensor pair with respect to coordinate system of the plate
- a_i, b_i : length and width of the i-th actuator-sensor pair
- Θ_i : orientation of the actuator-sensor pair

Constrains of this optimization problem are:

- constraints in dimensions:

$$a_{i\,\min}\le a_i\le a_{\max},\quad b_{i\,\min}\le b_i\le b_{\max},\qquad i=1,\ldots,N_{\mathrm{P}}$$

where $a_{i\,\min}$ and $a_{i\,\max}$ present minimum and maximum length of i-th actuator-sensor pair, $b_{i\,\min}$ and $b_{i\,\max}$ is its minimum and maximum width while N_{P} presents the number of atuator-sensor pairs.

- constraints in position:

$$0\le x_{\mathrm{A}i},x_{\mathrm{B}i},x_{\mathrm{D}i},x_{\mathrm{E}i}\le a,\quad 0\le y_{\mathrm{A}i},y_{\mathrm{B}i},y_{\mathrm{D}i},y_{\mathrm{E}i}\le b,$$
$$i=1,\ldots,N_{\mathrm{P}},$$

- constraints in covered surface of the plate by the actuators-sensors pairs:

$$\frac{\sum_{i=1}^{N_P}a_ib_i}{ab}\le\varepsilon$$

where ε presents tolerance of the coverage of the surface

- constraints that do not allow overlapping of the actuator-sensor pairs.

Taking into account constraints, the optimization problem is finding parameters x_i, y_i, a_i, b_i, Θ_i ($i=1,\ldots,N_{\mathrm{P}}$), such objective function

$$\overline{J}_{\mathrm{e}}=\begin{cases} J_{\mathrm{e}}, & \text{If constraints are not violated} \\ 0, & \text{If constraints are violated} \end{cases} \tag{14}$$

is maximized.

Presented optimization method will be solved by using the Particle swarm optimization (PSO) method [5]. A particle changes its velocity and position in the following way

$$\begin{aligned} v_{id}^{k+1}&=\chi v_{id}^{k}+c_1 rand_1\left(lbest_{id}-p_{id}^{k}\right)+\\ &+c_2 rand_2\left(gbest_d-p_{id}^{k}\right)\\ p_{id}^{k+1}&=p_{id}^{k}+v_{id}^{k+1}\\ i&=1,\ldots,n_{\mathrm{POP}}\quad d=1,\ldots,m \end{aligned}, \tag{15}$$

where χ is the inertia weight, c_1 is the cognition factor, c_1 is the social learning factor, $rand_1$ and $rand_2$ are random numbers between 0 and 1, the superscript k denotes the iterative generation, n_{POP} is the population size and lastly, $lbest$ and $gbest$ are the best local and global position of the particle. The cognition factor and social learning factors are usually set as $c_1=c_2=1.5$.

In this paper, each actuator is determined with x_i, y_i, a_i, b_i, Θ_i. According to that, coordinates of the i-th particle in the k-th iteration is

$$
\left[p_i^k \right] = \begin{bmatrix} x_{i1}^k & y_{i1}^k & a_{i1}^k & b_{i1}^k & \Theta_{i1}^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{iNp}^k & y_{iNp}^k & a_{iNp}^k & b_{iNp}^k & \Theta_{iNp}^k \end{bmatrix}. \qquad (16)
$$

4. NUMERICAL EXAMPLE

In this example, the quadratic cantilever symmetric laminated plate is considered. The dimensions of the plate are 0.5mx0.5m. The plate consists of eight graphite-epoxy layers. The thickness of each layer is 0.25mm and orientations are $\left(90^0/0^0/90^0/0^0\right)_S$. Piezoelectric patches are made of PZT5A fiber composite. Material properties of the graphite-epoxy layer and PZT are given in Table 1.

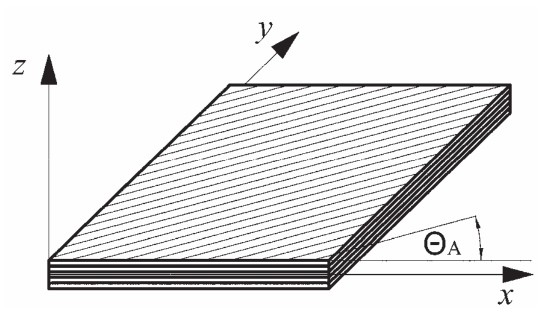
Table 1: Material properties of graphite-epoxy and PZT

Material properties	Graphite-Epoxy	PZT5A Fiber composite
$E_1(\text{GPa})$	174	30.2
$E_2(\text{GPa})$	10.3	14.9
$G_{13}(\text{GPa})$	7.17	5.13
$G_{23}(\text{GPa})$	6.21	5.13
ν_{12}	0.25	0.45
$\rho\left(\text{kg/m}^3\right)$	1389.23	4600
$e_{31}\left(\text{C/m}^2\right)$	/	9.41
$e_{32}\left(\text{C/m}^2\right)$	/	0.166
$k_{33}(\text{F/m})$	/	$6.1\text{x}10^{-9}$

In the first case, the top and bottom surface of the plate is covered fully with piezoelectric layers (Picture 3) (actuator layers is on the top and sensor layer is at the bottom of the plate). The influence of orientation of the actuator layer on controllability of each mode will be analyzed. The number of the controlled modes is 6, and they are presented in Table 2. For this analysis, the plate is discretized into 50x50 finite elements.

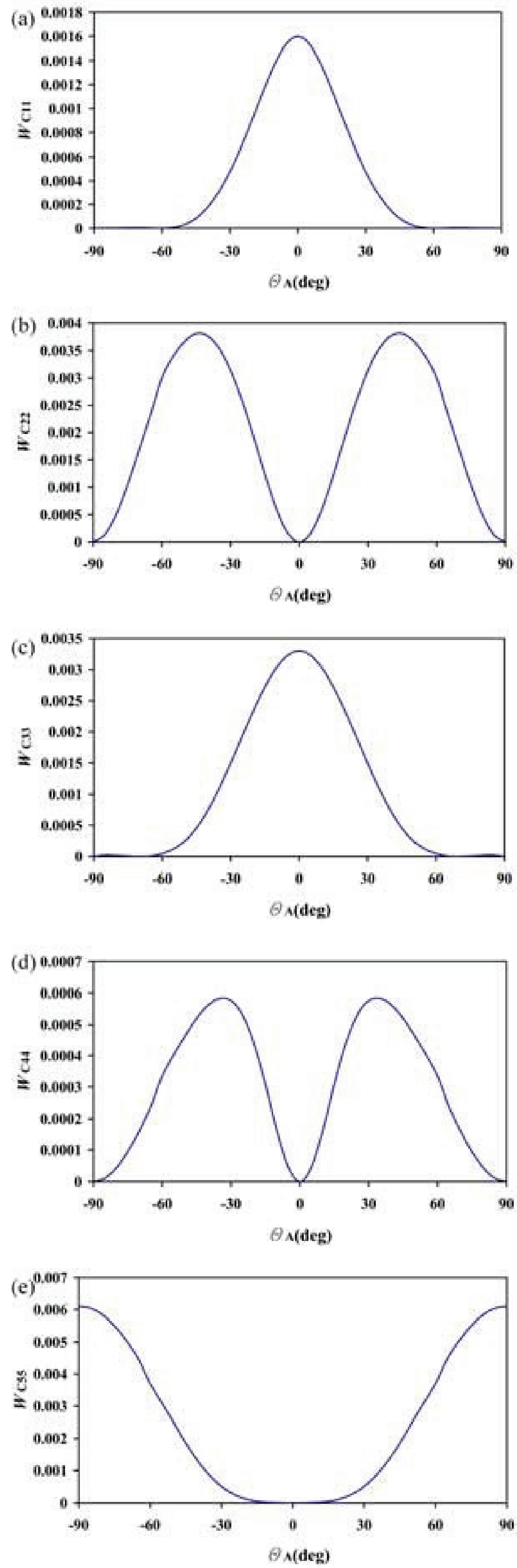
Table 2: Table name

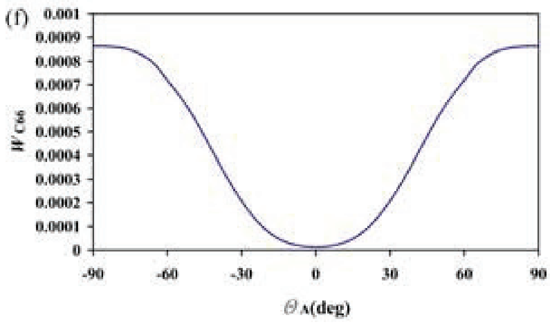
Mode	Frequency (Hz)
1	8.628
2	14.54
3	54.069
4	62.908
5	81.181
6	114.663



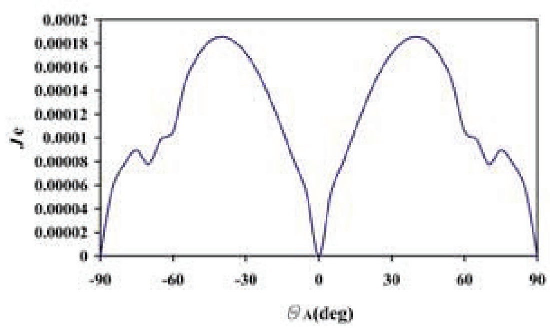
Picture 3: Composite plate with *i*-th actuator-sensor pair

Picture 4 presents the controllability of each controlled mode versus orientation angle of the actuator layer. Picture 5 presents performance index versus orientation angle of the actuator layer.



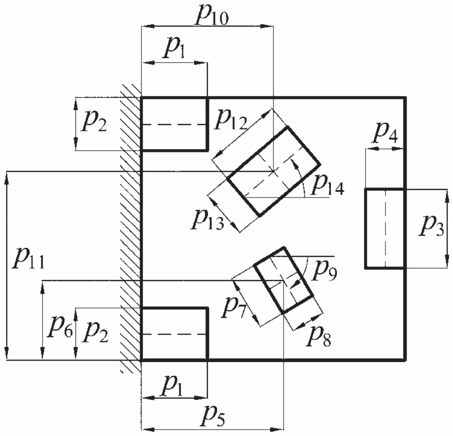


Picture 4: Controllability of controlled mode versus orientation angle of actuator layer: (a) 1st mode, (b)2nd mode, (c) 3rd mode, (d) 4th mode, (e) 5th mode, (f) 6th mode



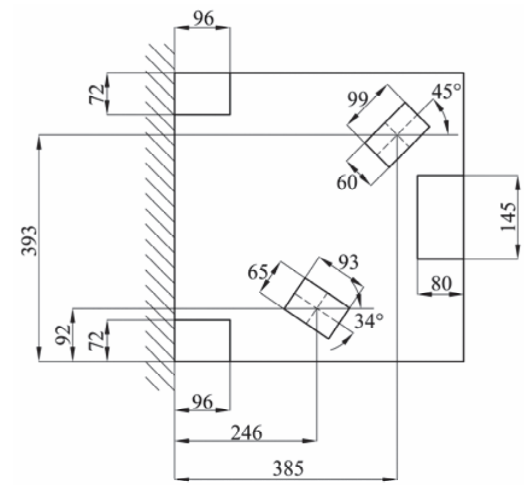
Picture 5: Performance index versus orientation angle of actuator layer

In this part, optimisation of sizing, location and orientation of the actuator-sensor pairs will be performed. The number of the actuator-sensor pairs is 5 and they are collocated. From Picture 4 it can be concluded that, for particular modes, maximum controllability is achieved when orientation of the actuator is 0°, and for other modes when orientation of the actuator is 90°. In order to reduce the number of parameters to be optimized, positions of the three actuators are fixed: 1st and 2nd actuator have equal dimensions, they are placed at the root of the plate on the corners and their orientation is 0° (Picture 6). 3rd actuator is placed in the middle of the free end of the plate and its orientation is 90° (Picture 6).

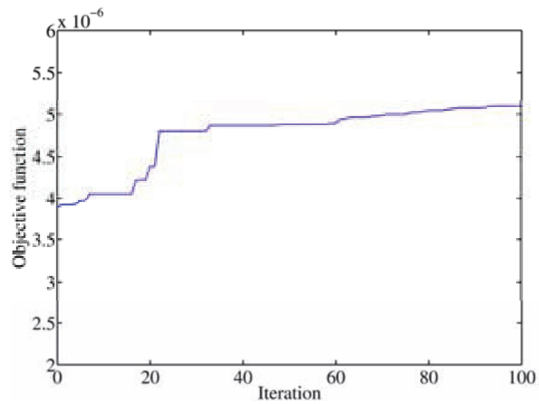


Picture 6: Parameters that are optimized

The number of initial population is 100 and the number of iteration is 100. The coverage of surface (ε) is 15%. Sizes, locations and orientations of the actuator-sensor pairs obtained by optimization iz presented in Picture 7. Picure 8 presents the convergence of the objective function. The obtained performance index is $J_e = 5.1x10^{-6}$.

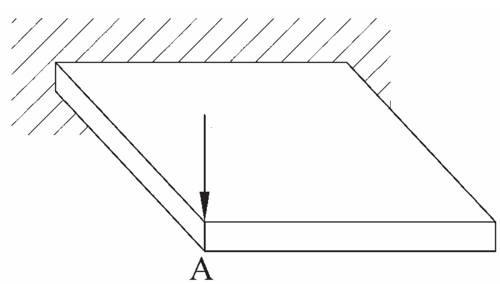


Picture 7: Sizes, locations and orientations of actuator-sensor pairs obtained by optimization



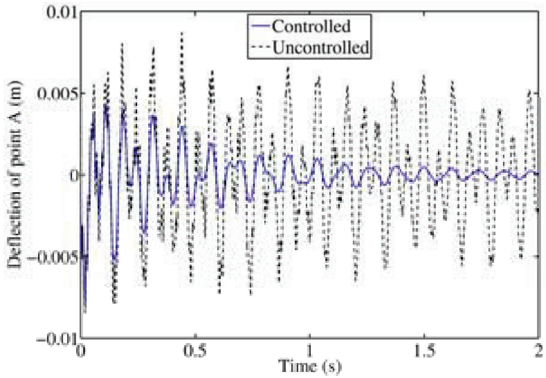
Picture 8: Convergence of objective function

After finding optimal sizes, locations and orientations of the actuator-sensor pairs, the next goal is an active vibration suppression of this plate. In this case vibrations occur due to the action of the impulse load of 300N for a period of 0.1ms at the free end of the plate (point A: Picture 9).



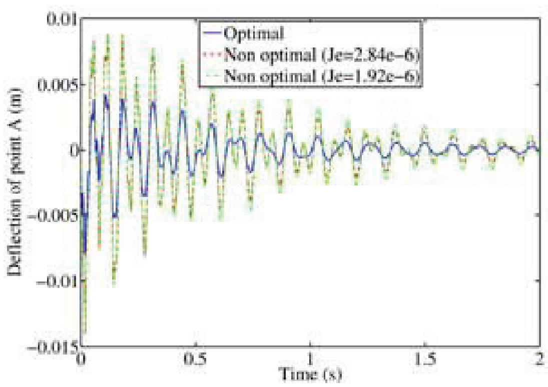
Picture 9: Action of impulse load on the plate

LQR optimal control is employed for active vibration suppression of the plate. Weighting matrices have following values: $[Q]=10^6[I]_{12x12}$, $[R]=[I]_{5x5}$. Picture 10 presents deflection of point A for the LQR optimal control and in the uncontrolled case.



Picture 10: Deflection of point A in the case of LQR optimal control and in the uncontolled case

In order to present efficiency of the proposed optimization technique, control performance of the obtained configuration of the actuator-sensor pairs is compared with two randomly generated configurations with performance index $J_e = 2.84x10^{-6}$ and $J_e = 1.92x10^{-6}$. Weighting matrices for these configurations are $[Q]=10^6[I]_{12x12}$, $[R]=2x[I]_{5x5}$. Comparison of their control performance is presented in Picture 11.



Picture 11: Deflection of point A in the case of LQR optimal control: comparison of optimal with non optimal sizing, locations and orientations of actuator-sensor pairs

From Picture 11 it can be concluded that obtained configuration with optimization has better control performances compared to randomly generated configuration.

5. CONCLUSION

This paper deals with the optimization of sizing, location and orientation of piezoelectric actuators-sensors pairs on thin-walled composite plate, where optimization criteria are based on the eigenvalues of the controllability Grammian matrix. Numerical examples are provided for a symmetric cross ply cantilever quadratic composite laminates. Also, the influence of orientation of the actuator layer on controllabilities of particular modes are examined. Comparing control performances of the obtained actuator-sensor configuration with randomly generated configurations it can be concluded that obtained configuration with optimization has better control performances than randomly generated.

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