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Radial hydraulic machinery four-quadrant performance curves dependent on specific speed and applied in transient calculations

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Abstract. The paper presents research analysis of possible analytical relationships that could exist in pointwise data for four-quadrant performance curves of one radial pump and seven pump-turbines for which the authors collected the basic data and performed the calculation. All the performance curves are for different specific speeds. Additionally, the examples of transformation calculation from the four-quadrant curves into Suter form curves shown in [1] - Thorley (1996) and [2] - Stepanoff (1959) have been presented. A numerical model has been developed for the calculation of transient processes in hydropower plants with installed pumps and pump-turbines. As an input data, the analytical relationships found in the previous research analysis have been used, as well as the pointwise curves for various specific speeds. The multi comparison of the results has been made and comprehensive conclusions given.

1. Introduction

A review of the literature shows that a small number of papers and books analyzed the influence of the hydraulic machine specific speed (nq) on the results obtained during the calculation of the transient processes. This paper aims to do that for installations with radial pumps and (radial) pump-turbines.

In [3] and [4], the characteristics Wh and Wm are shown for the three models of a pump (radial, semi-axial and axial). However, a given example of the calculation of the transient process in the installation with two parallel pumps has been done only for one type of pump, so the analysis how the specific speed (nq) influences the results obtained has not been shown. In [5], the same characteristics for Wh and Wm are shown for the same three pump models (radial, semi-axial and axial), but there is a discussion (analytical background) on the influence of the use of different complete characteristics Wh and Wm for predicting the behavior of the hydraulic machine during the transient processes.

In [6], the influence of specific speed (nq) on the values obtained during the calculation of the transient processes in pump installations has been analyzed for the first time. Three sets of characteristics Wh and Wm of pumps (radial, semi-axial and axial) were compared, and it is shown that the influence of the specific speed (nq) exists. In [7], the relationship between the specific speed (nq) of the model pumps (radial, semi-axial and axial), and Wh and Wm characteristics, were analyzed, as well as their influence on the values obtained during the measurement and calculation of transient processes on the installation with pumps in the system South Nevada. Significant conclusions were



reached, that the Wh and Wm characteristics of radial pumps do not vary proportionally to the specific speed (nq), while for semi-axial and axial pumps the Wh and Wm characteristics vary proportionally to the specific speed (nq). The numeric code was performed to analyze the influence of a specific speed (nq) on the characteristics of Wh and Wm for eleven pump models. The results are in accordance with the results of the measurement on the site. It was also found that for radial pumps, the impact of a specific speed (nq) and characteristics Wh and Wm is such that tends to increase the pressure in the system during the calculation of transient processes. They also came to the conclusion that the pumps with the same specific speed (nq) have a markedly different characteristics Wh and Wm .

In this paper, the Suter curves Wh and Wm are presented for the optimum opening of guide vanes for seven models of radial pump-turbines with the following specific speeds: $nq=24.8$ –USA; $nq=27$ –Serbia; $nq=28.6$ –USA; $nq=41.6$ –Austria; $nq=43.83$ –Russia; $nq=50$ –China; $nq=56$ –China (a brief explanation of designation: $nq=24$ -USA means that non-dimensional specific speed number of the pump turbine is 24 and that it is installed in PSPP-pump storage power plant in USA, this explanation applies for all other tags), as well as the set of Sutter curves Wh and Wm obtained from a set of four-quadrant characteristics for one radial pumps with the following specific speed: $nq=25$ [2]. Also shown are diagrams with the Suter curves Wh and Wm for these models of radial pump-turbines and radial pumps, which are obtained from the Universal Equations Wh i Wm , for which the authors of this paper developed numerical models in the Matlab program. The paper presents the complete process of obtaining the numerical model that has been developed for obtaining the Universal Equations Wh i Wm .

The calculation of the transient process is done for a defined pump installation consisting of: lower reservoir, two parallel pumps, valve, downstream pressure pipes, downstream reservoir, directly taken from the book [3] - Chaudhry. In the numerical code for the calculation of the transient processes, the values for the Wh and Wm characteristics obtained from model tests and from the Universal Equations Wh i Wm , were used, for the above-mentioned seven models of radial pump-turbines and one model of radial pumps. The paper analyses the influences of the specific speed (nq) on the value of the results obtained during the calculation of the transient processes.

2. The method of obtaining the Universal Equations Wh and Wm

The procedure of polynomial regression - the least squares method [8] was used in this paper to develop numerical models in the Matlab program (the process of obtaining a developed numerical model in the Matlab program consists of five steps, which are explained in detail in Chapter 2.1 of this paper), to obtain Universal Equations Wh and Wm (the wording of Universal Equation is used to denote an equation that includes the total surface of all points of model curves of various values of specific speed (nq) of radial machines listed in this paper, ie. equation applicable for all values of nq).

The main idea was to have the Universal Equations in a form of,

$$W_{h,m} = f(nq, \theta) \quad (1)$$

where the influence of nq would be explicitly shown. An angle (θ) is defined by the equation $\theta = \arctg v/\alpha$ where is: $v = Q/Q_R$ - dimensionless discharge variable, $\alpha = N/N_R$ - dimensionless speed of rotation, $\beta = M/M_R$ - dimensionless moment and $h = H/H_R$ - dimensionless head. The Suter curves $Wh(\theta) = h/(\alpha^2 + v^2)$ and $Wm(\theta) = \beta/(\alpha^2 + v^2)$ are expressed as a function of the angle θ , as shown in figure 1 [4]. The obvious main influence of θ would be implicit and given in coefficients in function f . However, after a lot of numerical experiments, it turned out that the better fitting of the points by the curve is when the influences of nq and θ are given vice-versa: θ - explicit and nq - implicit in coefficients. So, the final aimed function (universal equation) to be resolved was,

$$W_{h,m} = a_0 + \sum a_i(nq) \cdot \theta^i \quad (2)$$

As it was difficult to fit the points as much precisely as wished, the higher order of polynomials had to be used unwillingly for the better fitting, so the overall function f got more complex than intended (polynomials with exponents 3, 3, 7).

2.1. Finding analytical connection with regression procedure for Wh and Wm curves for seven models of pump-turbines and one pump model

The following steps were used to obtain the Universal Equations for Wh and Wm in the Matlab program:

In the first step, in two separate diagrams with coordinates $Wh - \theta$ and $Wm - \theta$ (angle theta) through seven Suter curves Wh and Wm for seven models of pump-turbines: $nq=24.8$ –USA; $nq=27$ –Serbia; $nq=28.6$ –USA; $nq=41.6$ –Austria; $nq=43.83$ –Russia; $nq=50$ –China; $nq=56$ –China, (which was calculated by the author of this paper on the basis of collected data of four-quadrant curves) and one pump model $nq=25$ [2], the mean mathematical curve (the third-degree polynomial) has been determined.

In the second step, in the Matlab program the values of ΔWh and ΔWm (distances of each of the eight curves Wh and Wm to the mean curve) were calculated, and then the values obtained for ΔWh and ΔWm and exported from the Matlab program in Excel table.

In the third step, the vertical cross-sections of eight nq curves in the range $\theta=(0^0-270^0)$, with step 5^0 (in total 55 cross sections) were made, and in each of these 55 sections on each of the eight nq curves the readings of values for ΔWh and ΔWm have been done. After that, separately in 55 diagrams for each of the above cross sections with the coordinates $\Delta Wh - nq$ and $\Delta Wm - nq$, such a data obtained has been subjected to determination of the mean mathematical curve (third degree polynomial). Figures 2-5 show only a part of the 55 diagrams, two diagrams with $\Delta Wh - nq$ and two diagrams with $\Delta Wm - nq$, through which the polynomials of the third order were passed.

In the fourth step, the values for four coefficients ($K1, K2, K3, K4$), which exist in a polynomial of the third degree (a total of 55 polynomials and 220 coefficient values), are determined and used for determination of the mean curve of the third degree polynomial (total of 55 polynomials). These values are exported from the developed numerical model in the Matlab program to Excel tables.

In the fifth step, and for Wm and Wh specifically to the four separate diagrams with coordinate coefficient - θ (angle theta), the values for four coefficients are sorted ($K1, K2, K3, K4$ - total of 220 coefficient values) from 55 polynomial of the third order, and through all these points (for each of these four coefficients separately), new mathematical curves (polynomial of seventh degree) were passed. At figure 6, figure 7, figure 8, figure 9, are displayed diagrams with the values of the coefficients ($K1, K4$) - θ (angle theta), for Wh and Wm .

After all of the above, from the developed numerical model in the Matlab program, the Universal Equations for Wh and Wm are obtained, consisting of a collection of a mathematical equations (a third-degree polynomial) - the mean curve and mathematical equations (third-order polynomial) - delta, where the delta consists of mathematical equations (polynomials of the third degree) are passed through points with values for $\Delta Wh - nq$ and $\Delta Wm - nq$ for each of the seven nq models of pump-turbines and one model pump, and polynomials of the seventh degree of each of four coefficients ($K1, K2, K3, K4$) which exists in the above-mentioned third-degree polynomials for $\Delta Wh - nq$ and $\Delta Wm - nq$.

Also in the developed numerical model in the Matlab program, the part was programmed that allows (in the obtained Universal Equations Wh and Wm) insertion of new values for nq and obtaining of the new curves for Wh and Wm which are compared with the Wh and Wm curves of seven models of pump-turbines and one pump model. On the comparative diagrams of these curves, the values for the regression accuracy r^2 are presented for each new nq . Figures 10 and 11 show two diagrams (one for Wh and one for Wm) showing the point-wise curve model and mean curve plus delta for one nq with the highest values for accuracy r^2 . Also, figure 12 shows together diagrams with the curves Wh and Wm obtained on model tests and the Wh and Wm curves obtained from Universal Equations Wh and Wm , for seven model nq pump-turbines and one model pump.

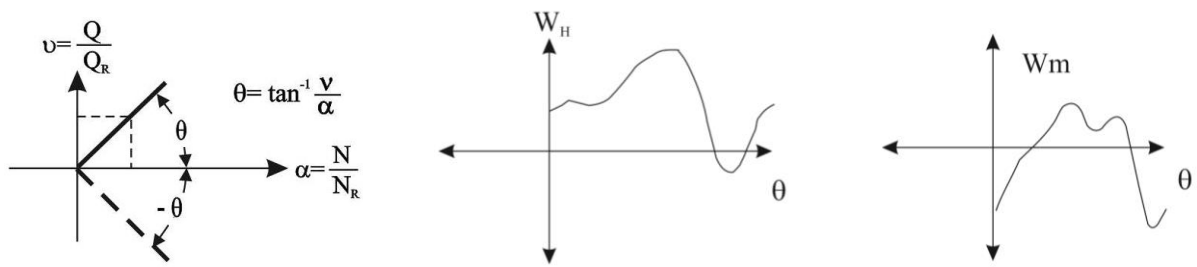


Figure 1. Showing the angle θ , and Suter curves W_h and W_m expressed in function of the angle θ .

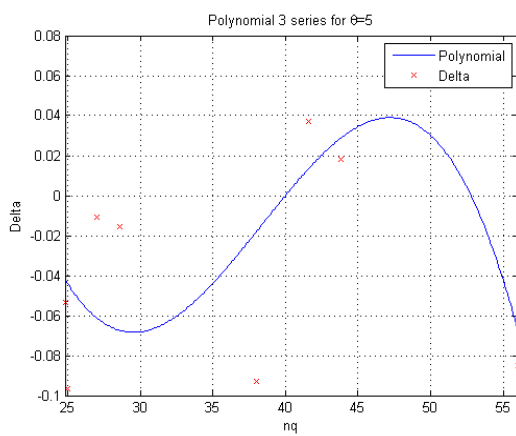


Figure 2. Values for ΔW_h at vertical cross-sections of curves (seven nq pump-turbines and one nq pump) at the angle $\theta=5^\circ$.

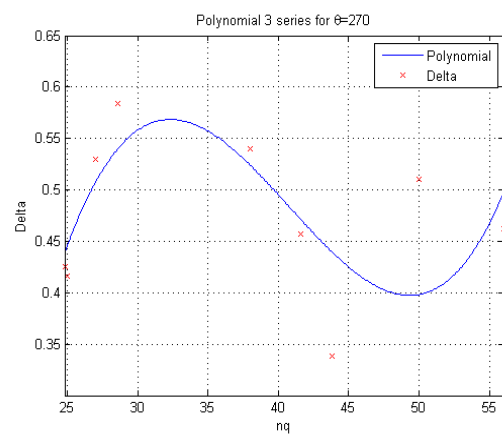


Figure 3. Values for ΔW_h at vertical cross-sections of curves (seven nq pump-turbines and one nq pump) at the angle $\theta=270^\circ$.

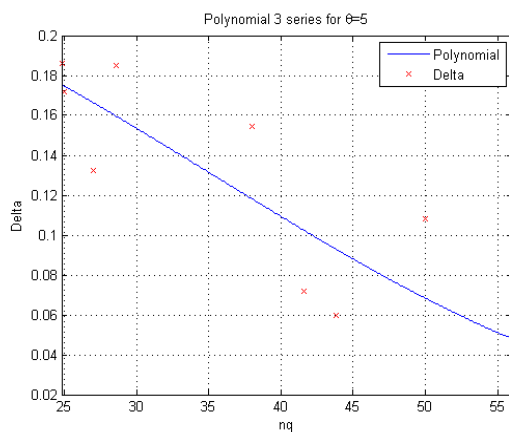


Figure 4. Values for ΔW_m at vertical cross-sections of curves (seven nq pump-turbines and one nq pump) at the angle $\theta=5^\circ$.

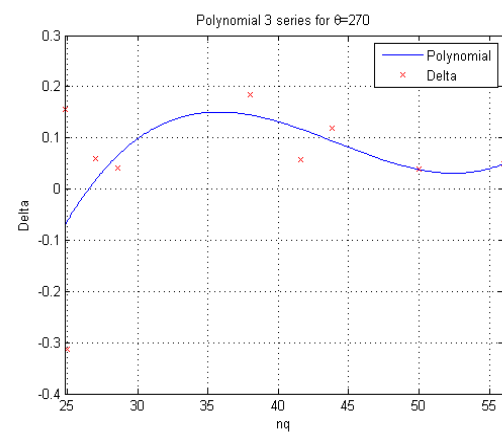


Figure 5. Values for ΔW_m at vertical cross-sections of curves (seven nq pump-turbines and one nq pump) at the angle $\theta=270^\circ$.

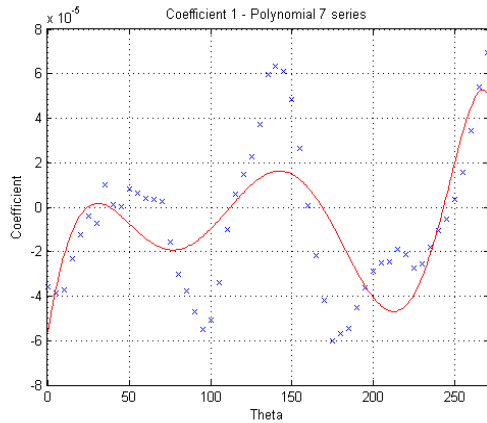


Figure 6. Polynomial of seventh degree is passed through 55 points with values coefficient $K1 - \theta$ (theta), for Wh .

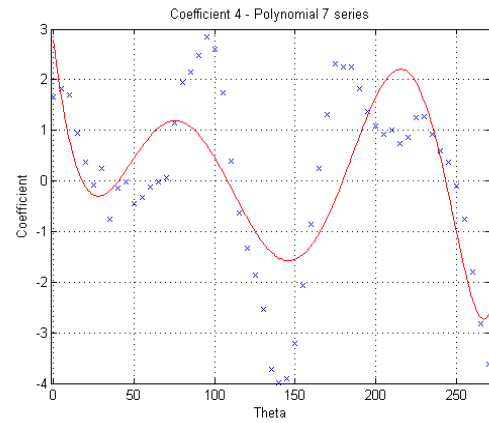


Figure 7. Polynomial of seventh degree is passed through 55 points with values coefficient $K4 - \theta$ (theta), for Wh .

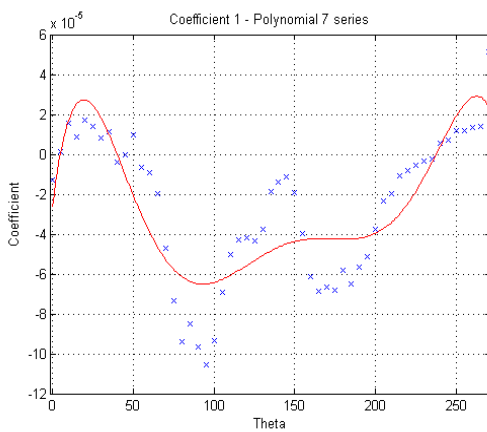


Figure 8. Polynomial of seventh degree is passed through 55 points with values coefficient $K1 - \theta$ (theta), for Wm .

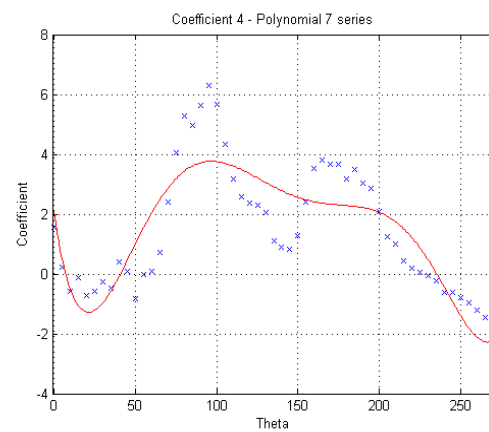


Figure 9. Polynomial of seventh degree is passed through 55 points with values coefficient $K4 - \theta$ (theta), for Wm .

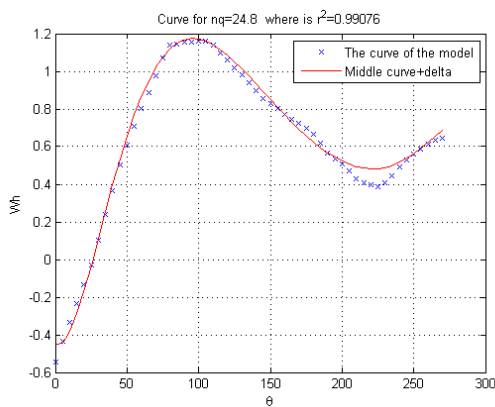


Figure 10. Wh - Model curve and mean curve plus delta for $nq=24.8$ with $r^2=0.99$.

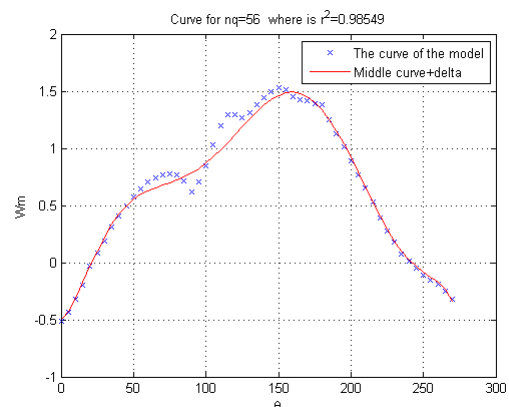


Figure 11. Wm - Model curve and mean curve plus delta for $nq=56$ with $r^2=0.98$.

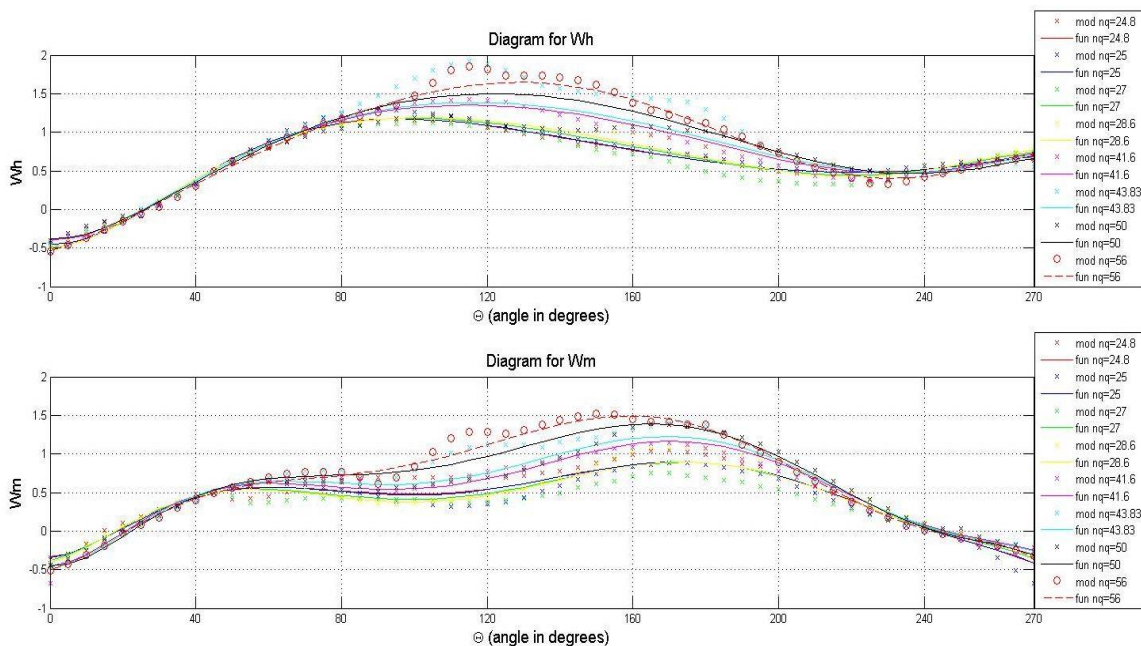


Figure 12. Comparison Wh and Wm curves obtained on model tests and from the Universal Equations Wh and Wm , for seven nq model pump-turbines and one nq model pump.

Based on the results obtained from the developed numerical models for Universal Equations Wh and Wm , the following conclusion can be made: that the results are solid, which will improve in the coming period, and encouraging that there is a universal analytical dependence which could describe all the Wh and Wm curves with different nq , and that these dependencies do not depend on the reference curve and that confirmed a clear existence of dependence on nq .

2.2. Universal Equations Wh and Wm (Polynomials 3, 3, 7)

The Universal Equations - Wh (3) and Wm (4), as well as the values of the coefficients obtained from the numerical models developed in the Matlab program are fully presented as follows:

Universal Equations - Wh has the look:

$$\begin{aligned}
 Wh &= S1 * \Theta^3 + S2 * \Theta^2 + S3 * \Theta + S4 + \Delta \\
 Wh &= S1 * \Theta^3 + S2 * \Theta^2 + S3 * \Theta + S4 + [D11 * \Theta^7 + D12 * \Theta^6 + D13 * \Theta^5 + \\
 &D14 * \Theta^4 + D15 * \Theta^3 + D16 * \Theta^2 + D17 * \Theta + D18] * nq^3 + [D21 * \Theta^7 + \\
 &D22 * \Theta^6 + D23 * \Theta^5 + D24 * \Theta^4 + D25 * \Theta^3 + D26 * \Theta^2 + D27 * \Theta + \\
 &D28] * nq^2 + [D31 * \Theta^7 + D32 * \Theta^6 + D33 * \Theta^5 + D34 * \Theta^4 + D35 * \Theta^3 + \\
 &D36 * \Theta^2 + D37 * \Theta + D38] * nq + [D41 * \Theta^7 + D42 * \Theta^6 + D43 * \Theta^5 + \\
 &D44 * \Theta^4 + D45 * \Theta^3 + D46 * \Theta^2 + D47 * \Theta + D48]
 \end{aligned} \quad (3)$$

Coefficients of Θ at Mean curve: $S1= 0.03439$; $S2= -0.4854$; $S3= 1.681$; $S4= -0.5254$.

Coefficients of Θ at nq^3 : $D11= -1.011105964928527e-06$; $D12= 1.363502390576926e-05$;
 $D13= -6.265399572245447e-05$; $D14= 9.376290433443766e-05$; $D15= 8.228704736142970e-05$;
 $D16= -3.367886394897410e-04$; $D17= 2.541572423593976e-04$; $D18= -5.579500435221324e-05$.

Coefficients of Θ at nq^2 : $D21= 1.234621094204153e-04$; $D22= -1.679156279207369e-03$;
 $D23= 7.854627550157659e-03$; $D24= -1.254673718956816e-02$; $D25= -7.314242717756788e-03$;
 $D26= 3.792655081648275e-02$; $D27= -2.934789296713252e-02$; $D28= 6.418470191702393e-03$.

Coefficients of Theta at nq : D31= -4.847151220618230e-03; D32= 6.615509045646753e-02;
D33= -3.113356290997428e-01; D34= 5.076047891242924e-01; D35= 2.456486447646135e-01;
D36= -1.419666418017583e+00; D37= 1.095964979882342e+00; D38= -2.344992263329560e-01.

Coefficients of Theta (free term - without nq): D41= 5.695860505133595e-02;
D42= -7.616430236007363e-01; D43= 3.403083506635225e+00; D44= -4.408679669381974e+00;
D45= -7.192069950118181e+00; D46= 2.182833742436747e+01; D47= -1.493842324756119e+01;
D48= 2.793951552816559e+00.

Universal Equations - W_m has the look:

$W_m = S1 * \Theta^3 + S2 * \Theta^2 + S3 * \Theta + S4 + \Delta$

$W_m = S1 * \Theta^3 + S2 * \Theta^2 + S3 * \Theta + S4 + [D11 * \Theta^7 + D12 * \Theta^6 + D13 * \Theta^5 + D14 * \Theta^4 + D15 * \Theta^3 + D16 * \Theta^2 + D17 * \Theta + D18] * nq^3 + [D21 * \Theta^7 + D22 * \Theta^6 + D23 * \Theta^5 + D24 * \Theta^4 + D25 * \Theta^3 + D26 * \Theta^2 + D27 * \Theta + D28] * nq^2 + [D31 * \Theta^7 + D32 * \Theta^6 + D33 * \Theta^5 + D34 * \Theta^4 + D35 * \Theta^3 + D36 * \Theta^2 + D37 * \Theta + D38] * nq + [D41 * \Theta^7 + D42 * \Theta^6 + D43 * \Theta^5 + D44 * \Theta^4 + D45 * \Theta^3 + D46 * \Theta^2 + D47 * \Theta + D48]$ (4)

Coefficients of Theta at Mean curve: S1= 0.03849; S2= -0.5294; S3= 1.692; S4= -0.6158.

Coefficients of Theta at nq^3 : D11= -5.091972710880069e-08; D12= -1.957355540069362e-06;
D13= 3.676288603313958e-05; D14= -2.260908584785088e-04; D15= 6.384864869414088e-04;
D16= -8.286104209122147e-04; D17= 3.734383705735456e-04; D18= -2.611432411599402e-05.

Coefficients of Theta at nq^2 : D21= 1.578339559917061e-05; D22= 5.173578230403051e-05;
D23= -3.076565044673074e-03; D24= 2.248064671098574e-02; D25= -6.894637720160464e-02;
D26= 9.453934853035186e-02; D27= -4.429494906758778e-02; D28 = 3.311490749639990e-03.

Coefficients of Theta at nq : D31= -1.154528602923698e-03; D32= 7.813451215025517e-03;
D33= 5.043610133229166e-02; D34= -6.378629391879377e-01; D35= 2.284440502342860e+00;
D36= -3.389996194018964e+00; D37= 1.672121493835456e+00; D38= -1.385783736122681e-01.

Coefficients of Theta (free term - without nq): D41= 8.415005200618253e-03;
D42= 1.564118036533019e-03; D43= -1.258123259337855e+00; D44= 9.834228431155022e+00;
D45= -3.104375854338123e+01; D46= 4.359159633165154e+01; D47= 2.142980694381100e+01;
D48= 2.076207470562215e+00.

3. Analysis of the impact of a specific speed (nq) on the results obtained by the calculation of the transient processes at the pumping installation

In this paper for the calculation of the transient processes, the pump installation for example from the book [3] – Chaudhry was used (see figure 13). Pump installation consists of: lower reservoir, two parallel pumps (which, during the transient state, simultaneously run out of power), valve, outlet pipe (consists of a pipe 1 and pipe 2), downstream reservoir. Data for pipe 1: L_1 -length=450 m, D_1 -diameter=0.75 m, a_1 -wave speed 900 m/s, f_1 -friction factor=0.01, and Q_1 -discharge=0.5 m³/s. Data for pipe 2: L_2 -length=550 m, D_2 -diameter=0.75 m, a_2 -wave speed 1100 m/s, f_2 -friction factor 0.012, Q_2 -discharge=0.5 m³/s. Data for the pump ($Q_R=0.25$ m³/s, $H_R=60$ m, $N_R=1100$ rpm, $WR^2=16.85$ kg-m², pump efficiency at rated conditions $\eta=0.84$). The calculation of the transient processes was done on the numerical code developed in the Matlab program, numerical code is developed using the boundary conditions for parallel pumps. Initially both pumps are operating at rated conditions, and the transient-state conditions are caused by simultaneous failure of power of both pumps. Numerical code was developed using the boundary conditions derived in Section 4.5 for Parallel Pumps and the flowchart

shown in figure 4.4. from [3]. The method of characteristics and the boundary conditions for the reservoir and series junctions were used to analyze the transient conditions in the discharge line. The water-hammer wave velocity for various sections of the discharge line was determined using the equations presented in Section 2.6 from [3]. During the calculation of the transient processes at the specified pump installation, the values for Wh and Wm of the characteristics obtained in model tests and from Universal Equations Wh and Wm for seven models of pump-turbines and one model pumps with different specific speeds ($nq=24.8, nq=25, nq=27, nq=28.6, nq=41.6, nq=43.83, nq=50, nq=56$) were used. Results obtained (changes in pressure, discharge and speed) at the pump during the calculation of the transient processes are shown on the diagrams in figure 14, figure 15, figure 16.

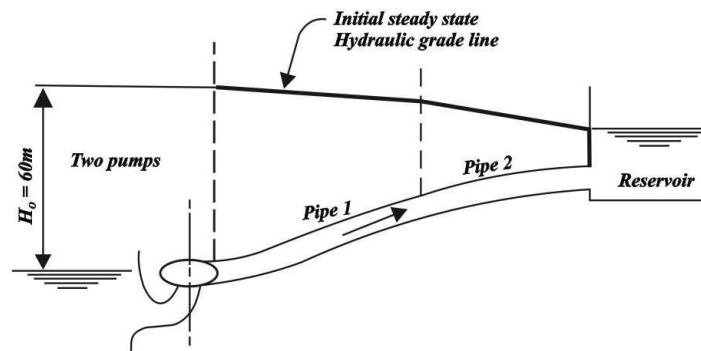


Figure 13. Pump installation with piping system.

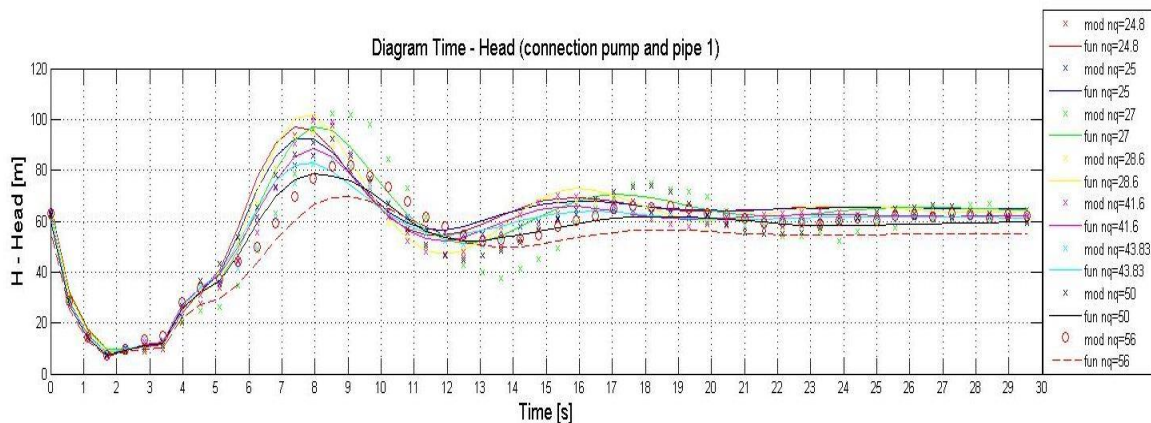


Figure 14. Pressure changes, at the connection of the pump and pipe 1.

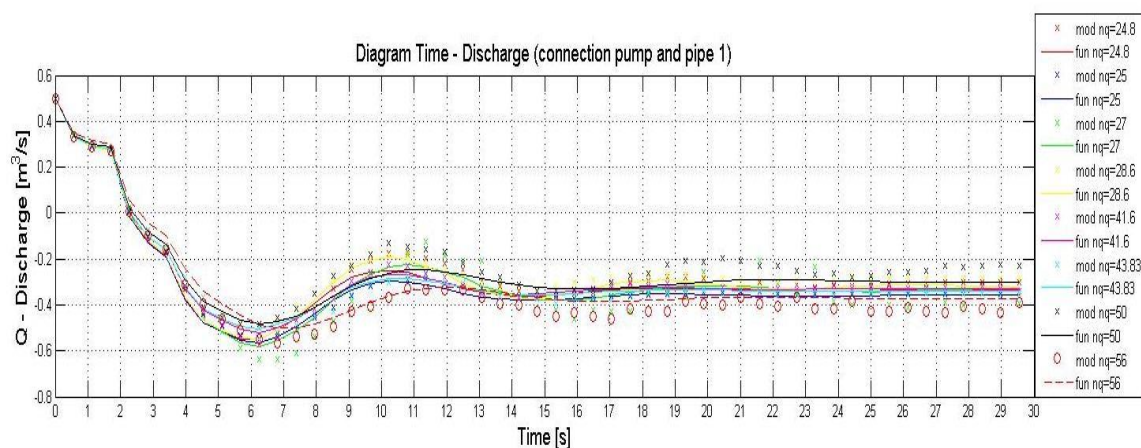


Figure 15. Discharge changes, at the connection of the pump and pipe 1.

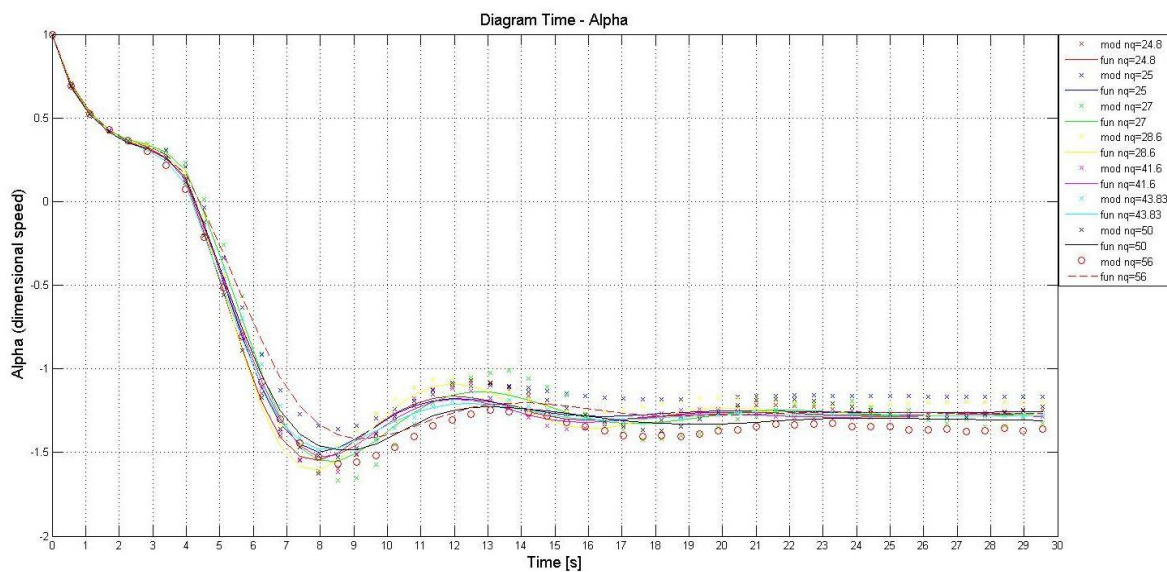


Figure 16. Speed changes, at the pump during the transient.

In figure 14, the diagram shows the pressure change at the connection of the pump and pipe 1. In 8 seconds of the transient processes, the greatest increase in pressure occurs, and it varies in the range of 65 to 100 m.w.c. The values for Wh and Wm are characteristics obtained from the model tests and from the Universal Equations Wh and Wm . From the above, it can be concluded that specific speed ($nq=24.8$, $nq=25$, $nq=27$, $nq=28.6$, $nq=41.6$, $nq=43.83$, $nq=50$, $nq=56$) have the influences on the change of pressure increase at the pump installation during the transient processes.

Figure 15 shows the diagram where the discharge changes are displayed at the connection of the pump and pipe 1 (after 2.5 seconds of the transient processes, the discharge becomes reversible (flow changes direction) and in 10 seconds the amplitude of oscillation of the discharge is most pronounced, as well as the difference between reversible discharges of the mentioned models of pump-turbines and pumps (varies in the range -0.15 to -0.4 m^3/s). After 19 seconds, the amplitudes of discharge oscillation are reduced to a minimum and reversible discharges stabilize around the value of -0.45 m^3/s . From the above, it can be concluded that the specific speed number ($nq=24.8$, $nq=25$, $nq=27$, $nq=28.6$, $nq=41.6$, $nq=43.83$, $nq=50$, $nq=56$) has a certain influence on the change of discharge increase at the pump installation during the transient processes. For all the above mentioned models of pump-turbines and pumps, in the program for water hammer, the values for Wh and Wm characteristics obtained from model tests and from Universal Equations Wh and Wm , were used to yield the above results.

Figure 16 shows a diagram with a change in the speed of rotation during the transient process for the listed models of pump-turbines and pumps. These results were obtained from the program for water hammer based on values for Wh and Wm characteristics (obtained from model tests and from the Universal Equations Wh and Wm). After 4.5 seconds of the transient processes there is a reversible direction of rotation, and in 8.5 seconds of the transient process, the most pronounced difference is among the reversible rotational speeds in the range -1.3 to -1.7 for dimensional speed. After 19 seconds, the amplitude of oscillation of reversible speed of rotation, has been reduced to a minimum and stabilized around value -1.25 for non-dimensional speed, with certain deviations of reversible rotational speed among the listed models of pump-turbines and pumps in the range -1.1 to -1.4 non-dimensional speed. From the above, it can be concluded that specific speed ($nq=24.8$, $nq=25$, $nq=27$, $nq=28.6$, $nq=41.6$, $nq=43.83$, $nq=50$, $nq=56$) have the influences on the change of reversible speed of rotation at the pump installation during the transient processes.

4. Conclusion

The paper presents for the first time the Universal Equations for Wh and Wm , from which the values for the Wh and Wm characteristics may be obtained for different values of specific speed (nq). In order to obtain the given equations in the Matlab program, the authors have developed numerical models, which are used as input data the values for the characteristics Wh and Wm with model testing for seven models of pump-turbine and one pump model. In the numerical model in the Matlab program the calculations of the transient process at the pump installation were made, where the values for Wh and Wm were taken separately, first with data of model tests (points) and then from the Universal Equations for Wh and Wm . An analysis of the obtained results from the calculation of the transient process was performed, and found that there is an influence of specific speed (nq) on the results of the calculation of the transient process.

5. Nomenclature

a_1	[m/s]	wave speed in pipe 1	Q	[m ³ /s]	discharge
a_2	[m/s]	wave speed in pipe 2	Q_R	[m ³ /s]	rated discharge
D_1	[m]	diameter pipe 1	Q_1	[m ³ /s]	discharge in pipe 1
D_2	[m]	diameter pipe 2	Q_2	[m ³ /s]	discharge in pipe 2
f_1	[-]	friction factor in pipe 1	θ	[⁰]	angle
f_2	[-]	friction factor in pipe 2	Wh	[-]	characteristic head
H	[m]	head	Wm	[-]	characteristic moment
H_R	[m]	rated head	ΔWh	[-]	error of characteristic head
L_1	[m]	length of pipe 1	ΔWm	[-]	error of characteristic moment
L_2	[m]	length of pipe 2	WR^2	[kg•m ²]	moment of inertia
N	[rpm]	speed of rotation	η	[-]	pump efficiency
N_R	[rpm]	rated speed of rotation	M	[Nm]	shaft moment
nq	[-]	specific speed	M_R	[Nm]	rated shaft moment
v	[-]	dimensionless discharge variable			
α	[-]	dimensionless speed of rotation			
h	[-]	dimensionless head			
β	[-]	dimensionless moment			

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