THE DETERMINATION OF THE MEAN VALUE OF THE NON-HOMOGENOUS BACKGROUND RADIATION AND THE MEASUREMENT UNCERTAINTY USING WELCH-SATTERTHWAITE EQUATION

by

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In this paper, the procedure for determination of the mean value of non-homogenous background radiation and the expression of measurement uncertainty is considered. The background radiation is described using the Student's distribution, and the measurement uncertainty using the Welch-Satterthwaite equation. The proposed algorithm was experimentally verified under well-controlled laboratory conditions and satisfactory results were obtained.

Key words: GM counter, non-homogenous background radiation, measurement uncertainty

INTRODUCTION

One of the basic characteristics of the measurement uncertainty concept is that some distribution functions are assigned to each influential factor. Concerning this, there is a clear difference between the concept of measurement uncertainty and the classical error theory. This is particularly true in the determination of the combined measurement uncertainty [1-7]. Namely, in the case of combined measurement uncertainty, the standard uncertainty u_c (which has the meaning of the standard deviation) is defined, as well as the expanded measurement uncertainty $U_c = ku_c$, where k is the expansion factor. The standard combined uncertainty is then calculated using the existing relations. However, a much larger problem is the determination of the combined measurement uncertainty distribution, and hence the determination of the expansion factor k.

In the experimental nuclear physics, there is a problem of combined measurement uncertainty distribution function determination for the experiments in which the background radiation is taken into account. Namely, in the standard procedures, the background radiation is calculated on the basis of a small statistical sample, where for the measured radiation (the number of counted impulses by the GM counter or the like) the normal distribution is assumed to be valid. It can then be assumed that the number of degrees of freedom of result is infinitely large. Such a procedure is not statistically correct, since the normal distribution can not be applied to a small statistical sample, but it is necessary to apply the Student's t-distribution [1-7].

The Student's distribution is used in case where the measured quantity belongs to the normal distribution, but the number of experiments n in the statistical sample is relatively small. In this case, the Student's distribution has $n_s = n - 1$ degrees of freedom. In nuclear physics, when measuring radioactivity, the measured quantity is determined on the basis of several influential quantities that are subject to normal distribution, but with different degrees of freedom. In this case, the distribution of the combined measurement uncertainty can be represented in a good approximation by the Student's distribution whose number of degrees of freedom is obtained by applying the Welch-Satterthwaite equation.

In laboratories for the radiation examination, the background radiation is not homogeneously distributed. That, in the end, is not the case with secondary cosmic radiation. Because of that, the measurement of the background radiation using the measuring device in just one position does not provide the satisfactory information. The aim of this paper is to propose the

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method that makes it possible to determine the background radiation and express the measurement uncertainty in the room with a non-homogenous distribution of radiation sources, using just two or three measuring devices in a relative short time period [2-7].

STUDENT'S DISTRIBUTION

In the case when there is a little number of measurements of a certain value, the Gaussian distribution does not prove to be adequate, so the Student's distribution is being used. The typical example of such case is the recording of the background radiation using the Geiger-Muller (GM) counter [6]. The Student's distribution is obtained if two random variables are observed, one of which (marked with number 2) belongs to the Gaussian distribution, and the other one has a $\chi^2_{n_s}$ distribution with n_s degrees of freedom. Using z and $(\chi^2_{n_s} / n_s)^{1/2}$, a new random variable t is formed

$$t = \frac{z}{\sqrt{\dfrac{\chi^2_{n_s}}{n_s}}} \tag{1}$$

Following the general Gaussian and $\chi^2_{n_s}$ distribution, the distribution of the new random variable t is obtained in the form

$$p(t) = \frac{1}{\sqrt{\pi n_s}} \frac{\Gamma\left(\dfrac{n_s+1}{2}\right)}{\Gamma(n_s)} \left(1+\frac{t^2}{n_s}\right)^{-\frac{n_s+1}{2}} \tag{2}$$

The distribution described with eq. (2) represents the Student's distribution.

Like the Gaussian, the Student's distribution also has a symmetrical shape around the y-axis. An important characteristic of Student's distribution is that when the number of degrees of freedom n_s is rising, then this distribution is transformed into the general Gaussian distribution. By reducing the number of degrees of freedom n_s, the standard deviation of Student's distribution is rising.

THE DETERMINATION OF THE EFFECTIVE NUMBER OF DEGREES OF FREEDOM OF THE COMBINED MEASUREMENT UNCERTAINTY

It is already stated that the Student's distribution is applied when the measurement value belongs to the Gaussian distribution, but the number of random variables n is relatively small. In that case, the Student's distribution has $n_s = n - 1$ degrees of freedom. There is a case in practice when the measured value y is determined by the couple of influential values x_i, each subject to the Gaussian distribution, but the numbers of degrees of freedom are different. In that case, the distribution of combined uncertainty can be represented with good approximation [7] by the Student's distribution, having in mind the question of determining the effective number of degrees of freedom. That number is obtained using the Welch-Satterthwaite equation.

In order to get the Welch-Satterthwaite equation, it is assumed that the combined measurement uncertainty (MA) of the measuring value y is determined based on the N uncertainty components u_i, where each influential value x_i is subject to Student's distribution with number of degrees of freedom n_i. With the assumption that the uncertainty components u_i are unrelated to one another, the standard combined measurement uncertainty, MA $u_c(y)$, of the measuring value is being determined. The distribution of the combined uncertainty in the latter approximation is expressed using the Student's distribution. The effective number of degrees of freedom n_{se} is calculated using the Welch-Satterthwaite equation

$$n_{se} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \dfrac{u_i^4}{u_{si}}} \tag{3}$$

where $u_c(y) = (\sum u_i^2)^{1/2}$. The coefficient of expansion $k = t_{n_{se}}$ is obtained based on the effective number of degrees of freedom and the statistical certainty from the Student's distribution [1]. It can be shown that the effective number of degrees of freedom n_{se} is lower than the sum of individual degrees of freedom, but it is higher than the largest individual degree of freedom.

EXPERIMENT

Two experiments were performed in order to check the possibility of applying the proposed method for determining the effective number of degrees of freedom of the combined measurement uncertainty of the GM counter.

The aim of the first experiment was to find the mean value of the background radiation in the laboratory with the non-homogenous distribution of γ radiation sources. For that purpose, three identical GM counters with the operating point placed in the center of the plateau of the voltage-current characteristic A, B, and C were used. The counters were set in places where different values of background radiation values were expected. The obtained results are shown in tabs. 1-3. This experiment, besides finding the mean value, is used to determine the corresponding Student's distribution (effective number of degrees of freedom) and the measurement uncertainty of the mean value.

In the second experiment, the measurements of background radiation were performed in a room containing two radioactive sources. To this end, one source is removed from the room and then 8 measure-

Table 1. Background radiation in number of counted impulses by the GM counter A

BGR	1053	997	1012	1003	991	995	1077	1004	1033	1001

Table 2. Background radiation in number of counted impulses by the GM counter B

BGR	2103	1999	1972	2003	2091

Table 3. Background radiation in number of counted impulses by the GM counter C

BGR	4153	3997	4012	3903	4091	4125	4077	3984	4033
	4001	4032	4123	3897	3998	3983			

Table 4. Background radiation in number of counted impulses by the GM counter 1

BGR	2200	2000	1910	2260	2090	2200	1920	1940

ments (each lasted for one hour) by the GM counter were carried out. The obtained results are shown in tab. 4. After that, the previously removed source is returned to its place, and the other source is removed. Then, the background radiation is measured 500 times (each measurement lasted one hour) by the GM counter. The measured results were found to belong to the normal distribution (using the statistical κ^2 test with certainty of 95 %). The average value of this normal distribution is $F_{s2} = 1000$, and the standard deviation $\delta_2 = u_2 = 50$. Since it is a known distribution, the appropriate degree of freedom is $n_{s2} \quad \infty$. The aim of this experiment was to determine the mean value of the background radiation and the corresponding measurement uncertainty.

RESULTS AND DISCUSSION

In the first experiment, the background radiation was measured 10 times (for one hour) using the GM counter A at the place where it was located. Obtained results are listed in tab. 1.

Using the GM counter B, the background radiation was measured 5 times (for one hour) at the place where counter B was located. The measuring results are in tab. 2.

Using the GM counter C, the background radiation was measured 15 times (for one hour) at the place where counter C was located. The measurements are listed in tab. 3.

The mean values obtained by the measurement are

$$F_{\rm mA} \quad 1016.6, \; F_{\rm mB} \quad 2033.6, \text{and } F_{\rm mC} \quad 4027.3 \qquad (4)$$

The mean value of the equivalent background radiation is given by the following equation

$$F_{\rm m} \quad F_{\rm mA} \quad F_{\rm mB} \quad F_{\rm mC} \quad 7077.5 \qquad (5)$$

The standard uncertainty tip A and corresponding numbers of degrees of freedom are

$$\begin{array}{llll} u_1 \quad s_1 \quad 28.5 & u_{s_1} \quad 10 \quad 1 \quad 9 \\ u_2 \quad s_2 \quad 59.2 & u_{s_2} \quad 5 \quad 1 \quad 4 \\ u_3 \quad s_3 \quad 75.6 & u_{s_3} \quad 15 \quad 1 \quad 14 \end{array} \qquad (6)$$

The combined standard measurement uncertainty is

$$u_{\rm c} \quad \sqrt{u_1^2 \quad u_2^2 \quad u_3^2} \quad 100.2 \qquad (7)$$

The effective number of degrees of freedom $u_{\rm se}$ of the combined uncertainty is obtained using the Welch-Satterthwaite equation (rounded to the first lower integer value)

$$u_{\rm se} \quad \frac{u_c^4}{\displaystyle_i \frac{u_1}{u_{si}}} \quad 19 \qquad (8)$$

Figure 1 depicts the corresponding Student's distribution.

As a rule, the end result is expressed with a statistical certainty of 95 %. In the tables of the Student's distribution a value $t_{\rm use,19,95\,\%} = 2.093$ can be seen. The extended measurement uncertainty is

$$U_{\rm c} \quad t_{u_{se},95\%} u_{\rm c} \quad t_{19.95\,\%} u_{\rm c} \quad 2.093 \; 100.2 \quad 210$$

so the mean value of the background radiation can be written as

$$N \quad 7077 \quad 210$$

regarding that the number of counted impulses must be an integer.

In the second experiment, the background radiation was measured 8 times (for one hour) using the first GM counter at the place where it was located. The results obtained are listed in tab. 4.

The mean value of the measurement using the GM counter 1 is 2065, the standard deviation (standard uncertainty tip A) is $u_1 = 141$, and the number of degrees of freedom is 7. The equivalent mean value is $F_{\rm m1} = 2065$.

In the case of measurement using the second GM counter, $F_{\rm m2} = 1000$, and the standard deviation is equal to the standard uncertainty $\sigma_2 = u_2 = 50$. The number of degrees of freedom is $u_{s_2} \quad \infty$.

The mean value of the background radiation when both sources are in the room is

$$F_{\rm m} \quad F_{\rm m1} \quad F_{\rm m2} \quad 2065 \quad 1000 \quad 3065 \qquad (9)$$

and the equivalent measurement uncertainty is

$$u_{\rm c} \quad \sqrt{u_1^2 \quad u_2^2} \quad \sqrt{141^2 \quad 50^2} \quad 150 \qquad (10)$$

In this case, the distribution of the number of counted impulses by the GM counters 1 and 2 due to background radiation has a Student's distribution whose number of degrees of freedom is given by Welch-Satterthwaite equation

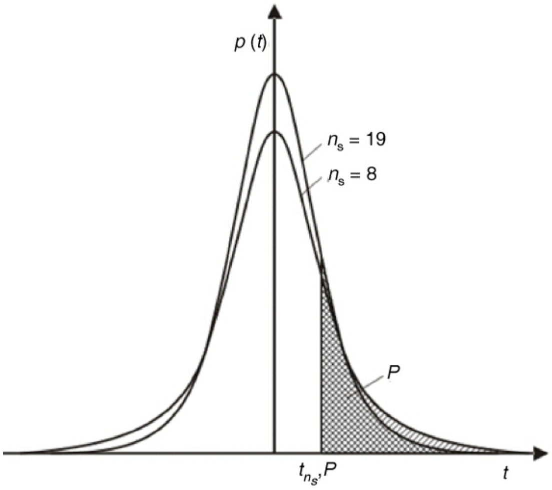

Figure 1. Student's distributions for 19 and 8 degrees of freedom, where *P* is the probability that certain probability value will be surpassed

$$u_{se} = \frac{u_c^4}{\sum_i \frac{u_i^4}{u_{si}}} = 8 \tag{11}$$

Figure 1 shows the corresponding Student's distribution.

In order to express the result with statistical certainty of 95 %, there is a value $t_{u_{se},95\%} = t_{8,95\%} = 2.306$ in the Student's distribution table. The extended measurement uncertainty is then

$$U_c = t_{n_{se},95\%} u_c = t_{8,95\%} u_c = 2.306 \cdot 150 = 346$$

so the result of counting can be written as

$$N = 3065 \pm 346$$

CONCLUSION

This paper points out that it is possible to determine the mean value of the non-homogenous background radiation and express the measurement uncertainty in a relatively short time. The experimental procedure described in this paper is based on the measurement using two and three GM counters. However, there is no problem for the same procedure to be applied with greater number of counters in shorter time of data collection. The expressed measurement uncertainty is a confirmation of the correctness of the procedure and the acceptance of the obtained results.

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AUTHORS' CONTRIBUTIONS

All authors performed the theoretical analysis. The experiments were carried out by N. M. Kartalović. The literature research was carried out and manuscript was written by all authors. The figure and tables were prepared by T. M. Stojić. All authors analyzed and discussed the results.

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ОДРЕЂИВАЊЕ СРЕДЊЕ ВРЕДНОСТИ НЕХОМОГЕНОГ ПОЗАДИНСКОГ ЗРАЧЕЊА И МЕРНЕ НЕСИГУРНОСТИ ПРИМЕНОМ WELCH-SATTERTHWAITE-ОВЕ ЈЕДНАЧИНЕ

У раду се разматра поступак одређивања средње вредности нехомогеног позадинског зрачења и изражавања мерне несигурности. Позадинско зрачење описује се помоћу Student-ове расподеле, а мерна несигурност коришћењем Welch-Satterthwaite-ове једначине. Предложени алгоритам је експериментално верификован у добро контролисаним лабораторијским условима и добијени су задовољавајући резултати.

Кључне речи: Гајгер-Милеров бројач, нехомогено позадинско зрачење, мерна несигурност