ANALYTICAL AND EXPERIMENTAL DETERMINATION OF THE TEMPERATURE FIELD ON THE SURFACE OF WALL HEATING PANELS

by

Ružica I. TODOROVIĆ [*]***, Miloš J. BANJAC, and Bogosav M. VASILJEVIĆ***
Department of Thermodynamics, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

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This paper presents experimental verification of the accuracy and acceptability of Faxen-Rydberg-Huber analytical expression for determining thermal characteristics of the heating panels. The verification of accuracy of the Faxen-Rydberg--Huber expression, which describes the 2-D temperature field in the wall with series of embedded heated pipes having uniform temperature, was performed by comparing the temperatures on the surface of three types of heating wall panels, differing in structure and geometric characteristics. The analysis of results has shown high accuracy of Faxen-Rydberg-Huber expression in describing the temperature field. Also, it was noted that small changes in heating fluid temperature, occurring along the pipe in the heated panel, have a negligible influence on the accuracy of prediction by the expression. This confirmed that the Faxen-Rydberg-Huber expression can be used to describe the temperature field in the wall heating panels. At the same time, this expression has proven to be extremely sensitive primarily to thermophysical characteristics of the panel layers, as well as to the geometric parameters of the panels.

Key words: *temperature field, wall heating panels, temperature measurement, Faxen-Rydberg-Huber expression*

Introduction

Requirements for more energy efficient systems for heating buildings have led to an increasing use of panel heating systems, as the most energy efficient subsystems for delivering heat to heated space. Large areas of these heat transmitters, as well as their coverage of the heated space, enable heating panels to achieve the same comfort conditions with much lower surface temperatures than other heat delivery subsystems. In addition, because of their low-temperature regime, these subsystems are typically coupled with highly efficient heat pump subsystems. High energy efficiency of both subsystems, along with the use of renewable energy sources, have made these heating and cooling systems preferable in 30% to 50% of new residential buildings in Europe, primarily in Germany, Austria, and Denmark, while in Korea this system is applied even in 90% of buildings [1].

* Corresponding author; e-mail: rtodorovic@mas.bg.ac.rs

Such widespread use of panel heating systems imposed the need of accurate prediction of thermal properties of these heat transmitters already in the design phase, primarily of the size of the heat flow, *i. e.* temperature field, formed at steady conditions in the heating panel. Actuality of demand, and the number of assumptions introduced when deriving the simplest form of analytical expression that can determine the temperature field in the wall with a built-in series of heated pipes of uniform temperature, known as the Faxen-Rydberg-Huber expression, imposed the need of experimental verification of the expression accuracy. At the same time the intention was to explore the possibility of applying this expression to predict thermal characteristics of wall heating panels, with slightly non-isothermal surfaces of heated pipes. For this purpose, upon detailed analysis of Faxen-Rydberg-Huber expression, an experimental installation was constructed, and the verification of accuracy of both hypotheses was carried out by direct measurements in the Laboratory of Thermodynamics at the Faculty of Mechanical Engineering, University of Belgrade.

Theoretical analysis and governing equations

Analytical solutions of Fourier's partial differential equation for steady 2-D heat conduction through an infinite plane wall in which the uniformly heated pipes are evenly spaced, have so far been released by only a few authors. In the literature, one can find it very complex and difficult to apply the solution of the equation obtained by using Green's functions [2], or by using the Schwarz-Christoffel integral transformations [2, 3]. The least complex, and the simplest solution to this problem by form is the Faxen-Rydberg-Huber expression. The development of this expression was initiated by Faxen, who first established the analytical expressions for the steady temperature field in the concrete layer in which the uniformly heated pipes were evenly spaced [3]. Later, Rydberg and Huber [4, 5] made supplements to this solution and adapted it to the multilayered wall structure (fig. 1).

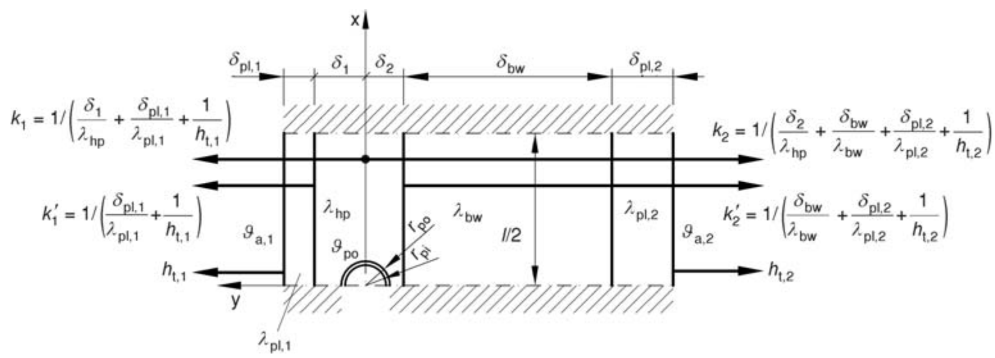


Figure 1. Single element of wall heating panel, with display of the ostensible overall heat transfer coefficients

Under the assumption that the material from which the individual layers of the panel are made has a homogeneous and isotropic structure, that the distance between the pipes is much larger than their diameter, that the temperature on the outer surface of the pipe is constant, and that the air temperatures in the rooms on both sides of the panel are equal to each other, Faxen-Rydberg-Huber expression for determining the steady temperature field in the cross-section of the panel reads:

$$\frac{\theta l}{A\pi} = -G_1 y - |y| - G_2 + \\ + \frac{l}{\pi}\sum_{j=1}^{\infty}\frac{1}{j}\left[e^{-2\pi j\frac{|y|}{l}} + g_1(j)e^{-2\pi j\frac{y}{l}} + g_2(j)e^{2\pi j\frac{y}{l}}\right]\cos\left(2\pi j\frac{x}{l}\right) \tag{1}$$

where $\theta = \vartheta - \vartheta_{a1}$ is the temperature difference, while G_1 and G_2 mark the relations:

$$G_1 = \frac{k_1 - k_2}{k_1 + k_2} \quad \text{and} \quad G_2 = -\frac{2\lambda_{hp}}{k_1 - k_2} \tag{2}$$

The system of two equations, on the basis of which the variables $g_1(j)$ and $g_2(j)$ are determined, is described as:

$$\left(\frac{k_1'}{\lambda_{hp}} - \frac{2\pi j}{l}\right)[1 + g_1(j)]e^{-4\pi j\frac{\delta_1}{l}} + \left(\frac{k_1'}{\lambda_{hp}} + \frac{2\pi j}{l}\right)g_2(j) = 0 \tag{3a}$$

$$\left(\frac{k_2'}{\lambda_{hp}} - \frac{2\pi j}{l}\right)[1 + g_2(j)]e^{-4\pi j\frac{\delta_2}{l}} + \left(\frac{k_2'}{\lambda_{hp}} + \frac{2\pi j}{l}\right)g_1(j) = 0 \tag{3b}$$

The dependence used to determine the variable A has the following form:

$$\frac{\theta_{po,m}}{A} = \ln\frac{l}{d_{po}\pi} - G_2\frac{\pi}{l} + \sum_{j=1}^{\infty}\frac{g_1(j) + g_2(j)}{j} \tag{4}$$

Based on the equality of heat flow that, in steady conditions, crosses from the front surface of the panel to the air in the heated room, and heat flow that passes from the panel "core" to the air in the same room is determined by the temperature difference, *i. e.* average temperature on the front surface of the panel. Analogue to this, the average temperature at the back surface of the panel is determined:

$$\theta_{sp1,m} = \theta_{y=0,m}\frac{k_1}{h_{t,1}} \quad \text{and} \quad \theta_{sp2,m} = \theta_{y=0,m}\frac{k_2}{h_{t,2}} \tag{5}$$

Experimental testing of the accuracy of Faxen-Rydberg-Huber expression

Experimental instalation

Experimental testing of the accuracy of Faxen-Rydberg-Huber expression (1), was done by measuring the temperature field on three different types of wall heating panels. The heating panels are part of the geothermal heat pump installation (fig. 2). Because of significant inhomogeneous and non-isentropic structure of heating panels, and the resulting uncertainty in measurements which would determine the temperature of the panel cross-section, only boundary surfaces of the panel *i. e.* of the supporting wall had been examined. All measurements were performed in several series. Two types of data were obtained by the measurements. First, the ones defining independent variables that appear in Faxen-Rydberg-Huber analytical expression, and the other measurements, where

Figure 2. Measuring installation

the measured values were temperatures, which, in pre-defined and measured conditions, stabilize on the panel surfaces.

Since the Faxen-Rydberg-Huber expression was derived under the assumption of uniform temperature on the wall of built-in pipes, which is not the case with the heating panel, the value of temperatures on the inner surface of pipes was determined by calculation. The logarithmic mean temperature difference of water was presumed as the temperature of the heating fluid. The transitional regime flow was established based on the measured flow rate and pipe diameter. Therefore, the Mikheev's criterion equation for forced convection heat transfer in the transition regime flow of water through the pipe [6] was used for determination of the heat transfer coefficient from heated water in the wall panel pipes.

Table 1 provides a comparative overview of the basic geometrical characteristics of the three types of wall panels shown in fig. 3.

Table 1. Geometric characteristics of wall panels

Panel no.	a_{sp}, [m]	b_{sp}, [m]	a_p, [m]	b_p, [m]	$\phi d \times \delta$, [mm]	l, [cm]	l_p, [m]
1	1.55	1.95	1.40	1.70	$\phi 16 \times 2$	10	27.3
2	1.85	1.95	1.70	1.90	$\phi 16 \times 2$	10	35.8
3	1.25	2.0	1.00	1.85	$\phi 11.6 \times 1.5$	7.5	29.5

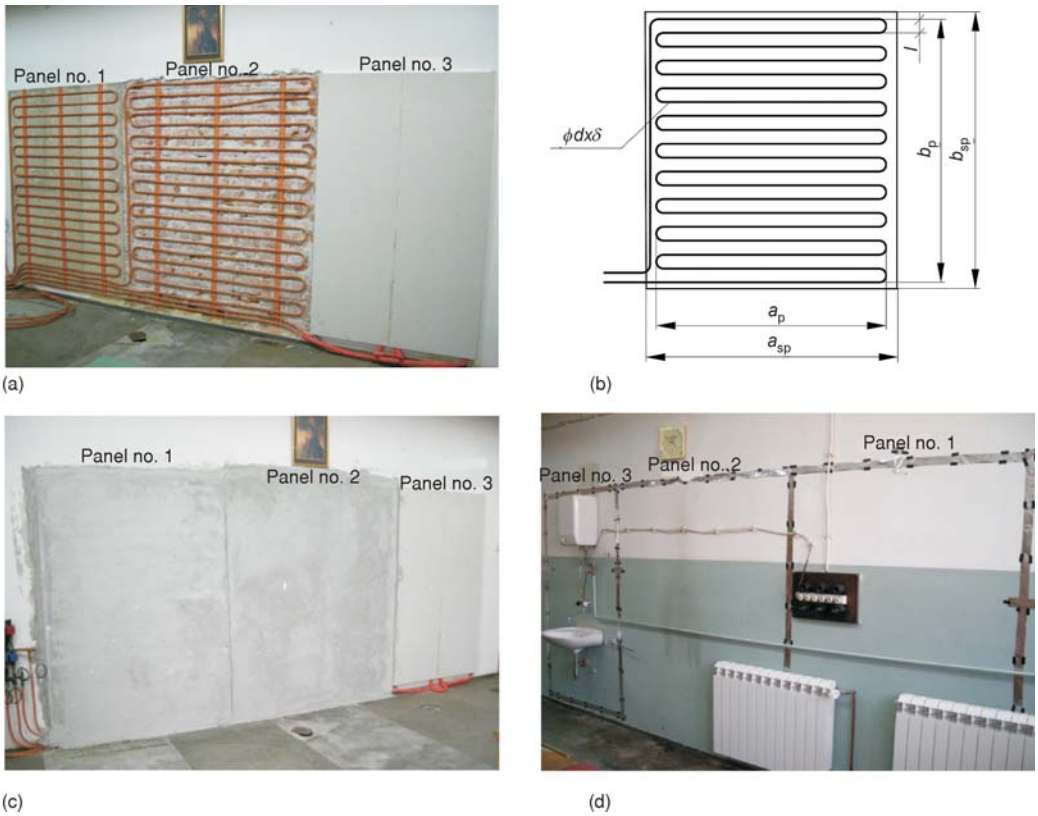


Figure 3. Wall heating panels: (a) prior to placing the final layer of plaster, (b) with geometrical characteristics, (c) at the end of the set-up process, (d) the back of the panels

Panel no. 1 – wall panel with base plate, is made with the standard structure for this type of panel, shown in fig. 4. The base plate (2) is made of wood wool, with thermal conductivity λ_{bp} = = 0.09 W/mK. Thermal conductivity of plastic pipes (1) is λ_p = 0.5 W/mK, and thermal conductivity of ecological heating plaster (3), also known as thermal plaster, which accumulates well thermal energy and delivers it uniformly, is λ_{hp} = = 0.91 W/mK.

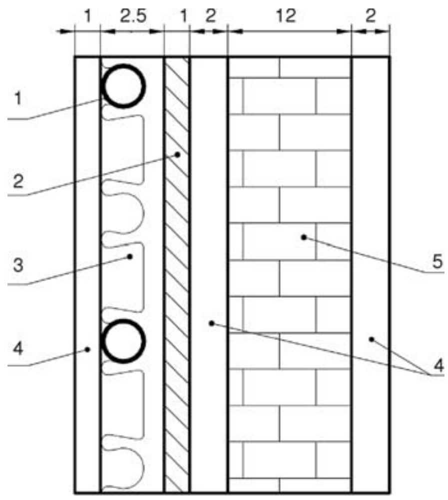


Figure 4. Structure of the inner wall with built-in panel no. 1 (dimensions in cm)

Panel no. 2 – wall panel without base plate, was made almost exactly in the same way as the wall panel with the base plate, except that the rail bearings for fastening pipes were placed directly on the wall of bricks (5), considering that the final layer of lime-cement plaster from the inner wall of the room was previously removed (4).

Panel no. 3 – wall panel for drywall construction, was obtained by connecting two gypsum fiberboards, of standard size $a_{gb} \times b_{gb} \times \delta_{gb}$ = = 0.625 × 2 × 0.018 m, in which pre-embedded pipes were placed in the form of vertical serpentine. Before attaching the fiberboards on plastered wall with fastening screws and joining them with joint adhesive, a layer of styro-foam insulation, 2 cm thick, was placed on the wall.

Measurement procedure and results

In order to experimentally verify accuracy of eq. (1), *i. e.* to verify temperature values on panel surfaces calculated by eq. (5), the following measurements were carried out:
– the volume flow rate of water in the pipe of each wall panel,
– water temperature at the pipe's entrance and at the pipe's exit of each wall panel,
– air temperature in the heated and in the adjacent room,
– emissivity of the panel surfaces,
– average temperature of the surfaces in heated and adjacent rooms that surround the observed panel – temperature of the surrounding surfaces, and
– average temperature of the front and back surface of each wall panel.

Average temperature of the panel surfaces was determined by measuring the temperature in three ways, using: (a) contact probe TESTO 435, (b) NTC thermistors TTF103, and (c) thermal imaging camera IK21. Temperature measurements were carried out immediately one after another, so that the environment impact on the results was minimal.

During the experimental determination of the temperature on the panel surfaces, wall panels were allowed to function independently from each other. In addition, the water temperature at the entrance of the heat pump, which operated in the heating mode, was set at 40 °C in all three cases, while the volume flow rate through each of the observed panels amounted to 2 l/min. Measurements were executed when the temperature at the observed wall panel surfaces was stabilized, meaning that the panel was in steady state operation.

The water temperature at the pipe entrance and at the pipe exit of the wall panels was measured using the NTC Honeywell T7425A 1005 thermometer, fitted on the supply and return utility lines of all three panel types. Due to the discontinuous operation of the heat pump, these temperatures in apparent steady operating mode oscillate within fixed temperature intervals.

Therefore, first digital multimeter measured minimum and maximum values of resistance shown by the thermometers. Based on them, mean values of resistance were determined, which, knowing the characteristics of the temperature-resistance of used temperature sensors, also determined the mean water temperature at the pipe entrance and at the pipe exit of the heating panel (tab. 2). Due to the large system inertia, heating water temperatures determined in this way have been adopted as relevant.

Table 2. Mean values of water temperature and air temperatures

Panel no.	$\vartheta_{w,in}$, [°C]	$\vartheta_{w,out}$, [°C]	$\vartheta_{w,m}$, [°C]	$\vartheta_{a,1}$, [°C]	$\vartheta_{a,2}$, [°C]
1	43.61	40.97	42.29	26.4	26.1
2	41.75	39.12	40.44	26.0	25.6
3	41.26	39.58	40.42	26.1	25.6

Air temperatures in the heated and in the adjacent room were measured by standard procedure [7], using a calibrated dilatation mercury thermometer, with a measuring range from 0 to 50 °C and the scale division of 0.1 °C. Measurements were taken on the central vertical axis of the rooms at three specific points set at the following heights: – 0.05 m above the floor, – 1.50 m above the floor, and – 0.05 m from the ceiling of the room. Accepted measured mean values of the air temperature in the heated and adjacent room are shown in the tab. 2.

Measurement of temperature field on the wall panel surfaces using contact probe

Measurement of temperature field on the panel surfaces was performed by using Testo 435 contact probe, containing K-type thermocouple. Display of chessboard arrangement of measuring points on the wall panel surfaces, determined by steps s_1 and s_2, is given in fig. 5.

In the case of panel no. 3, due to its smaller width the number of measuring points at the horizontal level was 3 instead of 4 measuring points, with respect to panel no. 1 and 2 surfaces. Temperature field on the surfaces of panels is given in the fig. 6.

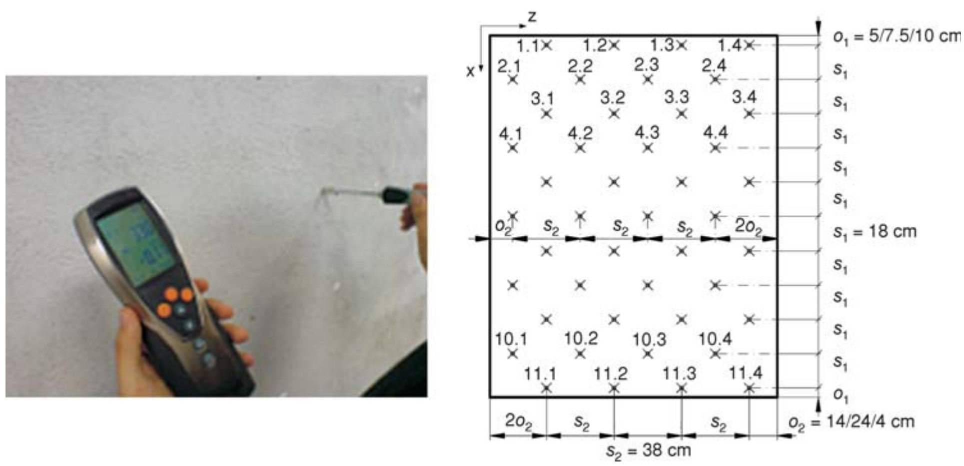
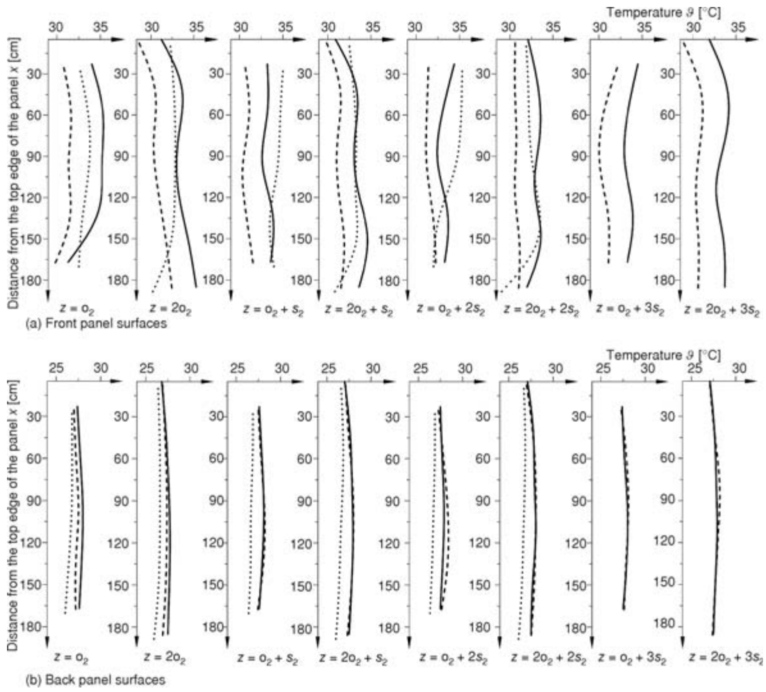


Figure 5. Measurement of temperature field on the panel surfaces by contact probe

Figure 6. Temperature field at the panel surfaces determined by contact probe;
—— Panel no. 1
------ Panel no. 2
········ Panel no. 3



Measurement of temperature field on the wall panel surfaces using NTC thermistors

Control measurements of temperature on the wall panel surfaces, at diagonal measurement points, were carried out using NTC thermistors TTF103. Positions of measuring points on the wall panel surfaces, determined by steps s_1 and s_2, are shown in fig. 7.

Temperature field on the surfaces of all three wall panels is shown in fig. 8.

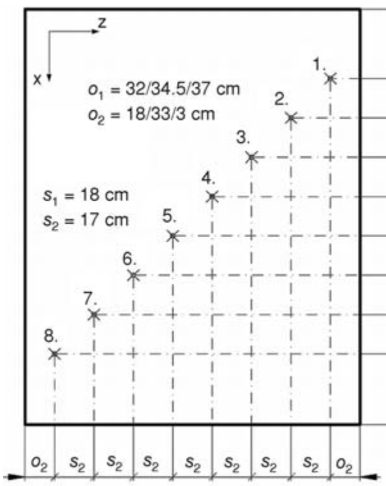


Figure 7. Measuring points on the panel surfaces using NTC thermistors

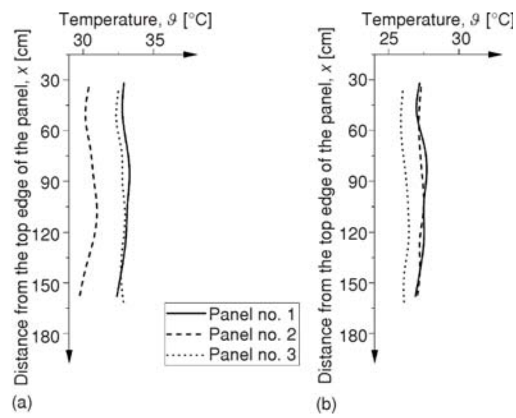


Figure 8. Temperature field at (a) the front and (b) the back panel surfaces, determined by NTC thermistors

Table 3. Input parameters for thermal imaging camera

Panel no.	Emissivity of the panel surface		Temperature of the surrounding surfaces	
	ε_{sp1} [–]	ε_{sp2} [–]	$\vartheta_{o1,m}$ [°C]	$\vartheta_{o2,m}$ [°C]
1	0.91	0.86	23.12	23.40
2	0.91		23.48	22.54
3	0.79		23.16	23.45

Measurement of temperature field on the wall panel surfaces using thermal imaging camera

Temperature fields on the panel surfaces were also measured by contactless method – using thermal imaging camera IK21. Before the final recording of panel surfaces, input parameters, tab. 3, were loaded into the camera software – panel surface emissivity and temperature of the surrounding surfaces. Determination of these parameters was performed immediately before the final recording, in accordance with the procedures prescribed by the camera manufacturer.

Temperature of the surrounding surfaces is equal to the average surface temperature of the so called diffuse reflector, measured by thermal imaging camera (tab. 3). As a diffuse reflector, an 30 cm × 30 cm aluminum foil was used, pre-creased, and then flattened, and placed between the panel surface and the camera lens. Since there was no thermal radiation on its own surface, the camera detected the reflected thermal radiation energy of the surroundings.

The method used for determining the emissivity of heating panel surfaces was the "reference emitter", according to which it is necessary to know the exact temperature of the surface the emissivity of which is being determined. In this case, the exact temperature value of the panel surface was determined by measuring temperature of the duct tape (size 10 cm × 10 cm, emissivity 0,89), which was fixed to the panel. Measurement was done by means of thermal imaging camera. With the already known value of the surface temperature, and by directly setting the required emissivity of the surface on the camera, the adjustment of known – exact temperature of the surface and the instantly measured one is performed. When these two temperatures are equal, the presumed – set emissivity value is exactly the desired surface emissivity (tab. 3). For verification, determination of the emissivity of the panel surfaces was performed in several series.

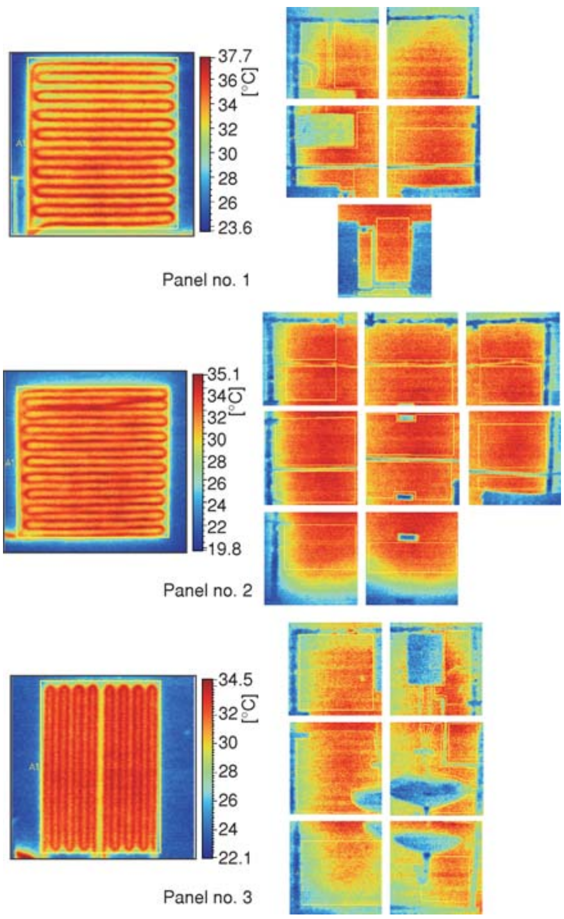


Figure 9. Thermograms of the front and back surface of wall heating panels

The recorded thermograms of the front and back surface of all three types of wall panels are shown in fig. 9.

View and comparative analysis of analytical results and results of measurement

Analytical determination of the temperature field on the wall panel surfaces

When determining the temperature field on the wall panel surfaces in an analytical way, all the geometric and process data were taken to match those achieved in the experiments. By using the expression (1), where the y-co-ordinate has the value corresponding to the front panel surface, while the value of the x-co-ordinate changes within the specified interval $x \in [l/2, 3l/2]$, the steady temperature field on the surface facing the heated room was analytically determined, for all three types of wall panels (fig. 10).

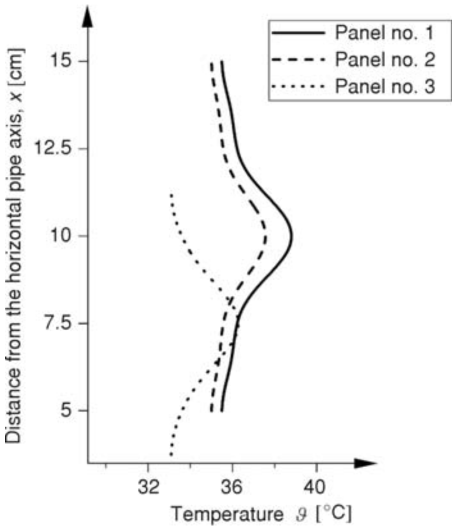


Figure 10. Steady temperature field on the front surfaces of wall panels

Table 4 shows the values of the ostensible overall heat transfer coefficients and average total heat transfer coefficients that correspond to the experiment.

Table 4. Values of ostensible overall heat transfer coefficients and average total heat transfer coefficients

Panel no.	k_1, [Wm^{-2}K^{-1}]	k_2, [Wm^{-2}K^{-1}]	k_1', [Wm^{-2}K^{-1}]	k_2',[Wm^{-2}K^{-1}]	$h_{t,1}$, [Wm^{-2}K^{-1}]	$h_{t,2}$, [Wm^{-2}K^{-1}]
1	8.70	2.17	9.88	2.24	11.25	17.92
2	8.42	3.01	9.52	3.14	10.78	15.94
3	7.80	0.95	10.08	0.96	10.08	46.64

Analysis of average temperatures on the surfaces of wall panels determined analytically and experimentally

Comparative review of average temperatures on the surfaces of panels that have been determined analytically and experimentally, in steady operating mode, is given in tab. 5.

Table 5. Display of average temperatures on the surfaces of wall panels

Panel no.	$\vartheta_{sp1,m}$, [°C]				$\vartheta_{sp2,m}$, [°C]			
	Analytically	Experimentally			Analytically	Experimentally		
		Testo423	TTF103	IK21		Testo423	TTF103	IK21
1	35.53	33.45	32.94	34.26	27.83	27.67	27.32	27.64
2	34.59	30.97	30.42	31.84	28.08	27.62	27.25	27.57
3	33.95	33.00	32.71	33.48	26.42	26.58	26.14	26.12

By comparing the experimentally measured values of average temperatures on the surfaces of panels, the following was observed:

- average temperatures of the front surfaces of the panels measured by contact measuring methods were up to 4.5% lower than those measured by contactless method – using thermal imaging camera,
- differences between the average temperatures of the panel surfaces facing the adjacent room, measured by contact and non-contact measuring methods were less than 1.8%, and
- the smallest discrepancies between the average temperatures of the panel surfaces facing the heated room, measured by contact and non-contact measuring methods, which were less than 2.3%, were recorded on the front surface of the panel no. 3, consisting of smooth gypsum fiberboards.

When comparing analytically and experimentally determined average temperatures of the panel surfaces, the following was noticed:

- differences between the average temperatures of the panel surfaces determined analytically and measured by contact methods were less than 13.7%,
- differences between the average temperatures of the panel surfaces determined analytically and measured by thermal imaging camera were less than 8.6%, and
- in all analyzed cases, analytically obtained temperature values had somewhat higher values, compared to the experimentally determined ones.

Based on the observed dependencies, it was concluded that:

- surface roughness is the main reason of the temperature discrepancies measured by the contact and non-contact methods. The least rough were the panel surfaces facing the adjacent room, slightly higher was the roughness of the gypsum fiberboard panel surface, *i. e.* surface of the panel no. 3, and the highest was the roughness of the front surfaces of panels no. 1 and 2. This order is consistent to the order in the recorded increase of differences in temperature measurements by contact and non-contact methods,
- temperatures of the panel surfaces measured by thermal imaging camera should be used as equally relevant average temperatures, free from the influence of surface roughness,
- the main reason why discrepancies between analytically calculated and experimentally determined values of average temperatures of the panel surfaces occur is probably resulting from adopted and not measured thermal conductivity values of the materials from which panel layers are made. In case of the drywall heating panel – panel no. 3, where the thermal conductivity values of materials are defined and guaranteed by the manufacturer, the lowest differences were observed, and
- use of the logarithmic mean temperature difference of the heating fluid, *i. e.* not taking into consideration the changes in the heating fluid temperature along the pipe of the heating panel in Faxen-Rydberg-Huber expression, has a negligible effect on the accuracy of predictions of average temperature on the wall surface, and can be used with high accuracy to describe the temperature field in the heating panels as well.

Conclusions

Based on previous observations it can be concluded:

- Faxen-Rydberg-Huber expression enables reliable determination of the temperature field in the wall with a series of built-in heating pipes of uniform temperature,
- by using the logarithmic mean temperature difference of the heating fluid, and the appropriate mean temperature of inner pipe surface, respectively, Faxen-Rydberg-Huber

expression can contribute to reliable determination of the average surface temperature of the wall heating panel, and

- the uncertainty of geometric characteristics and thermophysical properties of individual wall panel layers has substantial impact on the accuracy and represents the main problem when using the Faxen-Rydberg-Huber expression to determine the average temperature of the heating panel surface.

Nomenclature

A	– variable in Faxen-Rydberg-Huber expression, [°C]
a	– width, [m]
b	– height, [m]
d	– diameter, [m]
G_1, G_2	– variables in Faxen-Rydberg-Huber expression, [1]
g_1, g_2	– variables in Faxen-Rydberg-Huber expression, [1]
h_t	– average total heat transfer coefficient, [$Wm^{-2}K^{-1}$]
k, k'	– ostensible overall heat transfer coefficients, [$Wm^{-2}K^{-1}$]
j	– member of the series, [1]
l	– step between pipes, [m]
l_p	– total length of the panel pipe, [m]
o_1, o_2	– distance which defines the position of the measurement points, [m]
r	– radius, [m]
s_1, s_2	– step between measurement points, [m]
x, y, z	– position co-ordinates, [m]

Greek symbols

δ	– thickness, [m]
ε	– emissivity of surfaces, [1]
θ	– temperature difference, [°C]
λ	– thermal conductivity, [$Wm^{-1}K^{-1}$]
ϑ	– temperature, [°C]

Subscripts

a	– air
bp	– base plate
bw	– brick wall
gb	– gypsum fiberboard
hp	– eco heating plaster
in	– pipe entrance
m	– average conditions
o	– surrounding surfaces
out	– pipe exit
p	– pipe
pi	– inner pipe surface
pl	– lime-cement plaster
po	– outer pipe surface
sp	– panel surface
w	– water
1	– heated room
2	– adjacent room

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