DETERMINING THE THEORETICAL RELIABILITY FUNCTION OF THERMAL POWER SYSTEM USING SIMPLE AND COMPLEX WEIBULL DISTRIBUTION

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The main subject of this paper is the representation of the probabilistic technique for thermal power system reliability assessment. Exploitation research of the reliability of the fossil fuel power plant system has defined the function, or the probabilistic law, according to which the random variable behaves (occurrence of complete unplanned standstill). Based on these data, and by applying the reliability theory to this particular system, using simple and complex Weibull distribution, a hypothesis has been confirmed that the distribution of the observed random variable fully describes the behaviour of such a system in terms of reliability. Establishing a comprehensive insight in the field of probabilistic power system reliability assessment technique could serve as an input for further research and development in the area of power system planning and operation.

Keywords: *Fossil fuel power plant system, reliability, Weibull distribution*

1. **Introduction**

Exploitation research of the reliability of fossil fuel power plant system in "Nikola Tesla, Block A4" (TENT-A4), in the period from 1996 to 2008, should define the function, or the probabilistic law, according to which the complete unplanned standstill occurs.

The thermal power system is represented as a set of three subsystems: fossil fuel boiler, steam turbines and three-phase alternator. We adopted control limits in order to determine the transmission limits of the thermal power subsystems within the thermal scheme [1]. A simplified scheme of the thermal power plant with control limits represented in an enclosed border line is given in Fig. 1. The control limit that encloses the thermal power system does not encompass: systems for storage and delivery of fuel, systems for collecting and treatment of cooling water, the block transformer and the ash dump. The number of unplanned outages of subsystems in the reported period is shown in Table 1.

Based on the presented system and relevant exploitation data of unplanned outages, the following three tasks related to determining the characteristics of random variables are solved [2]:

- creating a hypothesis of the class of distribution function to which the random value belongs, on the basis of analysing the statistical material,
- validation of the hypothesis,
- determining the unknown parameters of the distribution and evaluation of their accuracy.

Having in mind the probabilistic nature of problem analysed, the Weibull model has been used, as the most common solution to engineering problems of this kind, [3-7]. The paper discusses the development of Weibull model for reliability evaluation of thermal power system of a thermal power plant using the probabilistic approach and scrutinizes the possibilities and limitations of the suggested model [8-11]. In a broader scope, this approach can be treated as a part of the risk based analysis in structural integrity assessment [12-13].

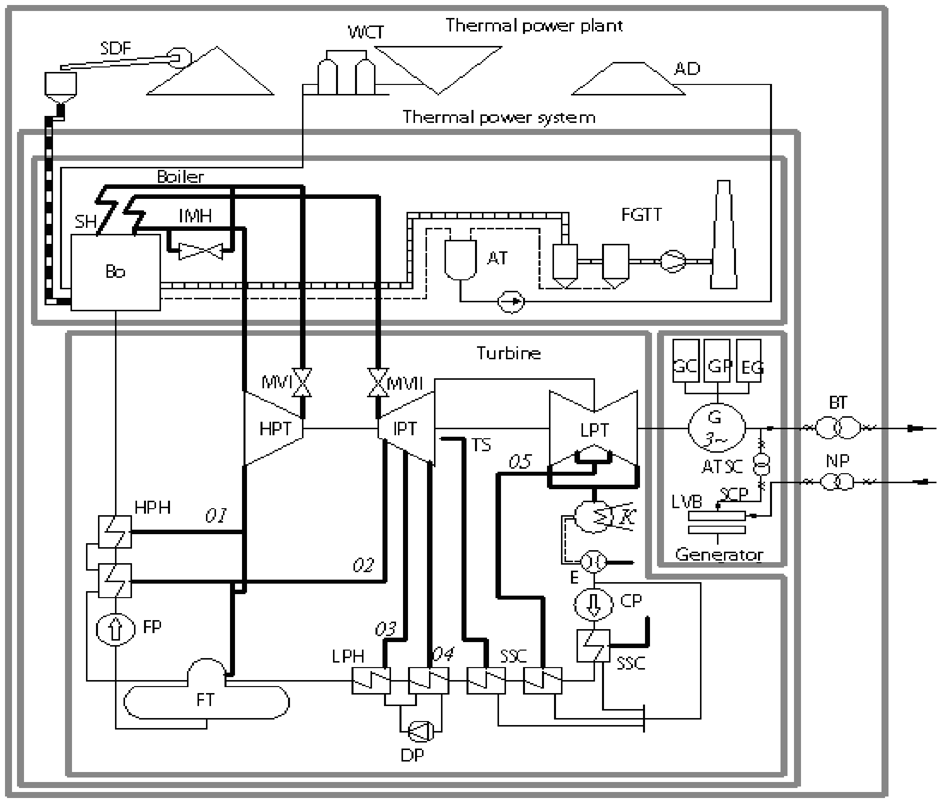

Figure 1. Scheme of thermal power plant system

Table 1. Unplanned delays of subsystems

i	Tr_i	Nn_i		
		Boiler	Turbine	Generator
1	1996.	14	1	0
2	1997.	10	1	2
3	1998.	12	1	1
4	1999.	7	1	1
5	2000.	12	3	0
6	2001.	23	1	2
7	2002.	16	0	1
8	2003.	10	3	1
9	2004.	12	1	1
10	2005.	12	2	1
11	2006.	16	0	1
12	2007.	2	4	0
13	2008.	10	1	5

2. Determining reliability function of the thermal power plant TENT-A4 using the simple Weibull distribution

The two-parameter Weibull distribution is often used for determining the theoretical reliability function of thermal power plants, whose density function is given in the form of [14]:

$$f\left(t,\beta,\eta\right)=\begin{cases}\dfrac{\beta}{\eta}\cdot\left(\dfrac{t}{\eta}\right)\cdot\exp\left(-\left(\dfrac{t}{\eta}\right)^{\beta}\right) & t>0\\ 0 & t<0\end{cases} \tag{1}$$

In order to determine the statistical indicators we shall use a graphical procedure, or Weibull probability plotting graph paper, and since we observe a two-parameter Weibull distribution, it is possible to draw the curve *F(t)* as a straight line on the aforementioned paper [15].

Based on exploitation studies of thermal power plant system reliability, each data point $\left[t_i, F\left(t_i\right)_{50\%}\right]$ can be plotted in a Weibull probability plotting graph paper. After that, in most cases we draw one best fitting straight line through those points, although, as can be easily noted that some points do not lie on this line. For this reason, this paper aims to analyse the empirical and theoretical reliability functions and their parameters obtained by the exploitation research of reliability of the mentioned thermal power system for cases when the entered data are approximated by simple and complex distributions [16].

Operating time intervals that include all data required for reliability processing of the system are defined in terms of one year, or 8760 working hours, for the period from 1996 until 2008 (Table 2).

By plotting points from Table 2 on a Weibull paper (Fig. 2) one can obtain parameter values as:

$$\eta=7,0731;\ \beta=1,5133$$

The listed functions are analytical expressions that represent distribution laws of the observed random variable [17]:

$$\text{reliability } R\left(t\right)=\exp\left(-t^{1,5133}/19,307\right) \tag{2}$$

$$\text{unreliability } F\left(t\right)=1-\exp\left(-t^{1,5133}/19,307\right) \tag{3}$$

$$\text{failure density } f\left(t\right)=0,0784\cdot t^{0,5133}\cdot\exp\left(-t^{1,5133}/19,307\right) \tag{4}$$

$$\text{failure rate } \lambda\left(t\right)=0,0784\cdot t^{0,5133} \tag{5}$$

Each data point from Table 1 could be plotted in a similar manner on other probabilistic papers, such as: three-parameter Weibull, log-normal, normal, exponential, and two-parameter exponential probability plotting paper. According to the obtained parameters of listed distributions, one can draw curves of theoretical functions such as: reliability (Fig. 3), density of delay (Fig. 4) and intensity of delay (Fig. 5).

Table 2. The value of exploitation reliability indicators and reliability components of the system

Observation period				Time					Reliability							Convenience of maintenance
	Tk_i	T_{i-1}	T_i	Ta_i	Tpz_i	Tnz_i	$Trez_i$	Tr_i	Nn_i	$\sum_{i=1}^{n} Nn_i$	Nt_i	f_i	F_i	R_i	λ_i	MR
	[year]	[h]		[h]	[h]	[h]	[h]	[h]	[-]	[-]	[-]	[h^{-1}]	[-]	[-]	[h^{-1}]	[-]
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1996	0-8760		5456	2199,25	617,24	487,11	5943,11	15	15	176	0,08	0,08	0,92	0,0852	7,70
2	1997	8760-17520		6962	1078,38	529,38	189,44	7151,44	13	28	163	0,07	0,15	0,85	0,0800	14,47
3	1998	17520-26280		6756	1062,25	590,54	350,41	7106,41	14	42	149	0,07	0,22	0,78	0,0940	21,80
4	1999	26280-35040		5894	2126,36	372,38	366,46	6260,46	9	51	140	0,05	0,27	0,73	0,0643	26,50
5	2000	35040-43800		6584	1345,1	477,25	353,25	6937,25	15	66	125	0,08	0,35	0,65	0,1200	34,32
6	2001	43800-52560		6533	733,54	1212,57	280,09	6813,09	26	92	99	0,14	0,49	0,51	0,2626	47,9
7	2002	52560-61320		7176	746,01	597,56	240,03	7416,03	17	109	82	0,09	0,58	0,42	0,2073	56,8
8	2003	61320-70080		7234	862,17	405,09	258,34	7492,34	14	123	68	0,07	0,65	0,35	0,2059	64,11
9	2004	70080-78840		7035	1169,57	547,12	7,51	7042,51	14	137	54	0,07	0,72	0,28	0,2593	71,42
10	2005	78840-87600		7172	908,45	424,35	254,40	7426,40	15	152	39	0,08	0,80	0,20	0,3846	79,26
11	2006	87600-96360		7113	508,39	993,49	144,32	7257,32	17	169	22	0,09	0,89	0,11	0,7727	88,14
12	2007	96360-105120		1878	6430,39	239,27	211,54	2089,54	6	175	16	0,03	0,92	0,08	0,375	91,27
13	2008	105120-113880		8443	0	272,57	44,03	8487,03	16	191	0	0,08	1,00	0	$+\infty$	99,63

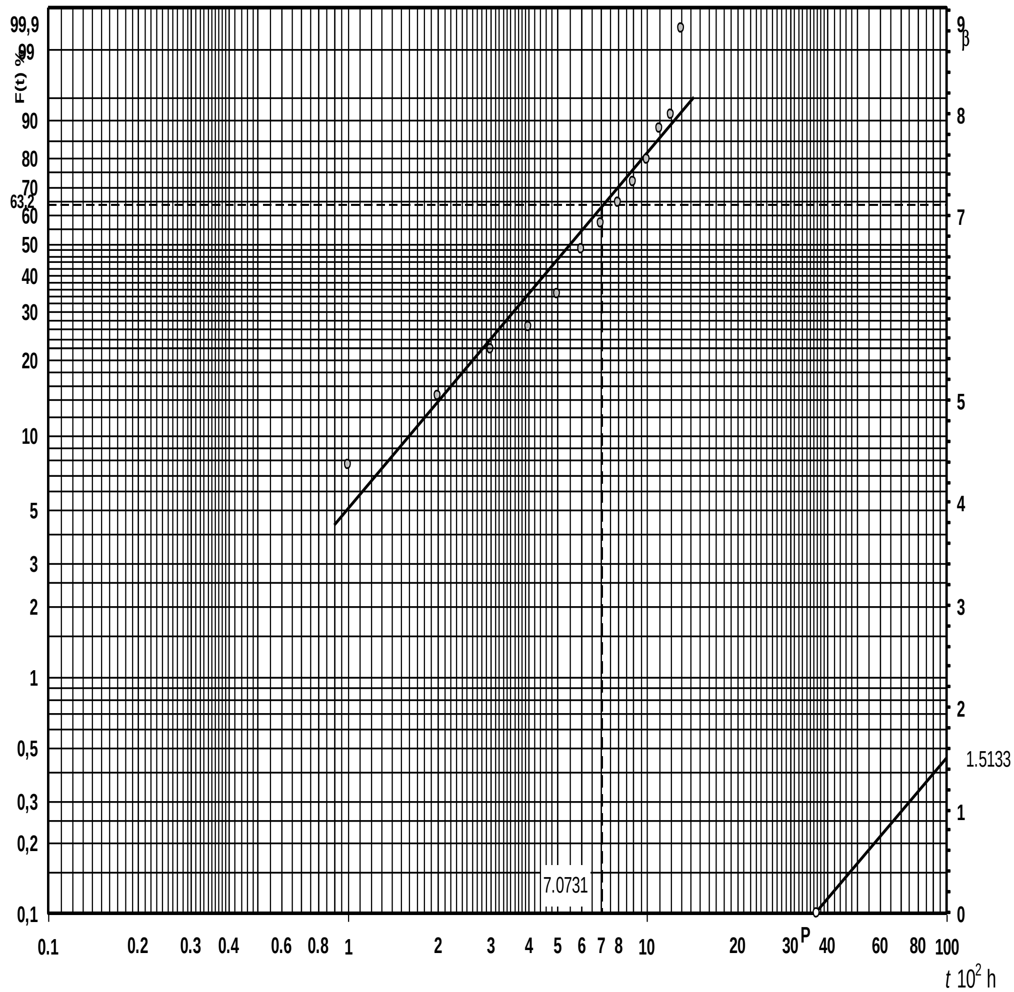

Figure 2. Weibull probability paper for simple distribution (TENT-A4)

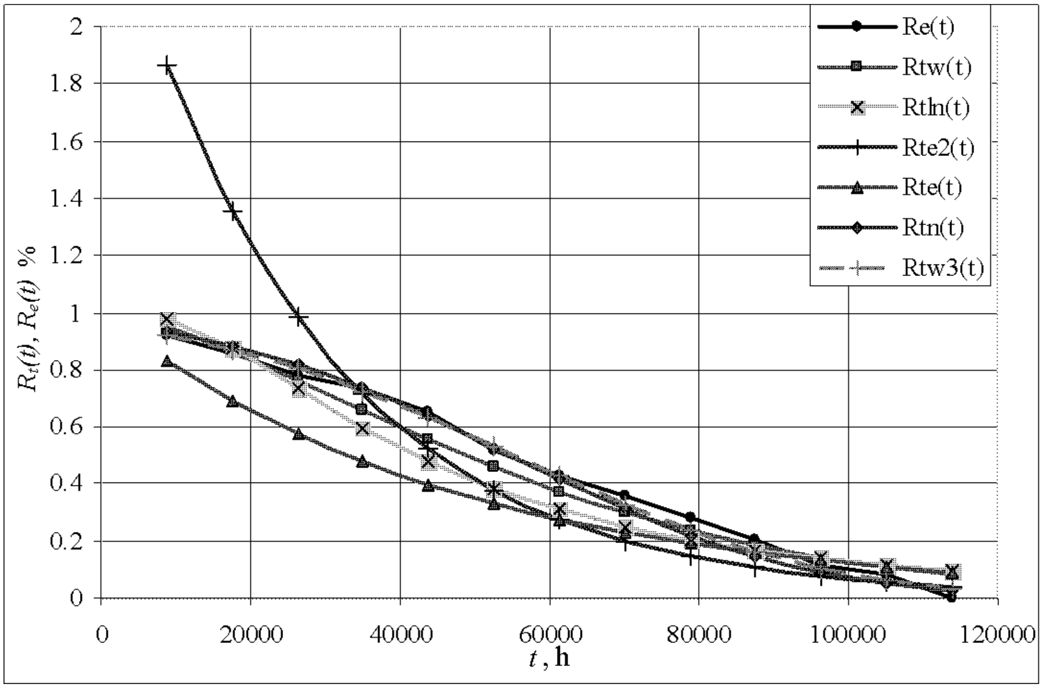

Figure 3. Exploitation and theoretical forms of reliability function for a simple distribution

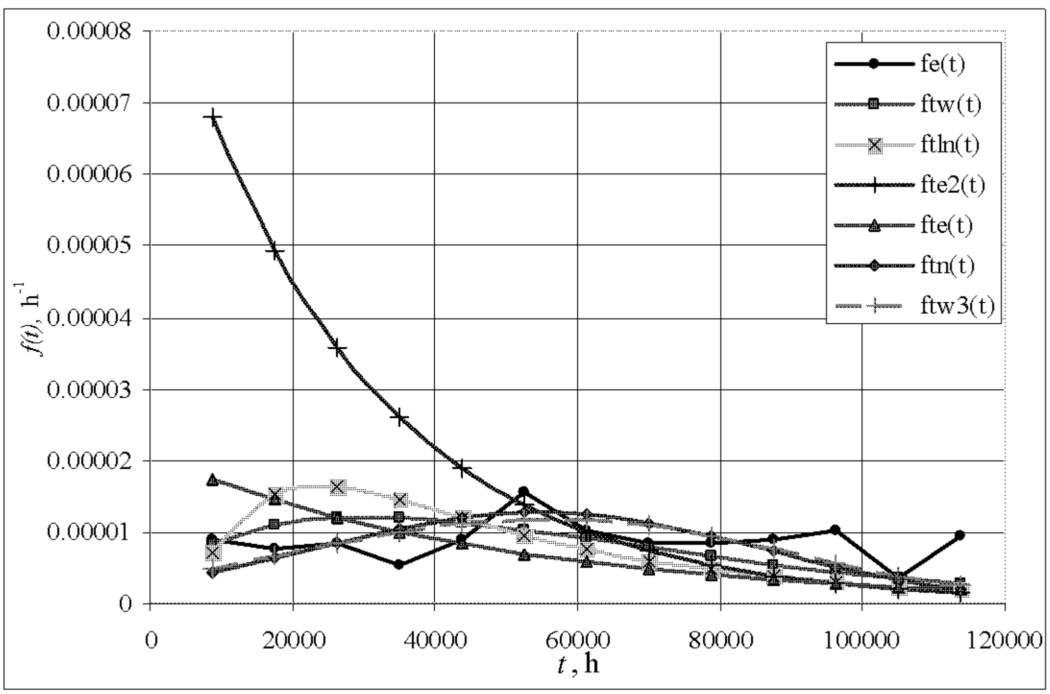

Figure 4. Exploitation and theoretical forms of failure density function for a simple distribution

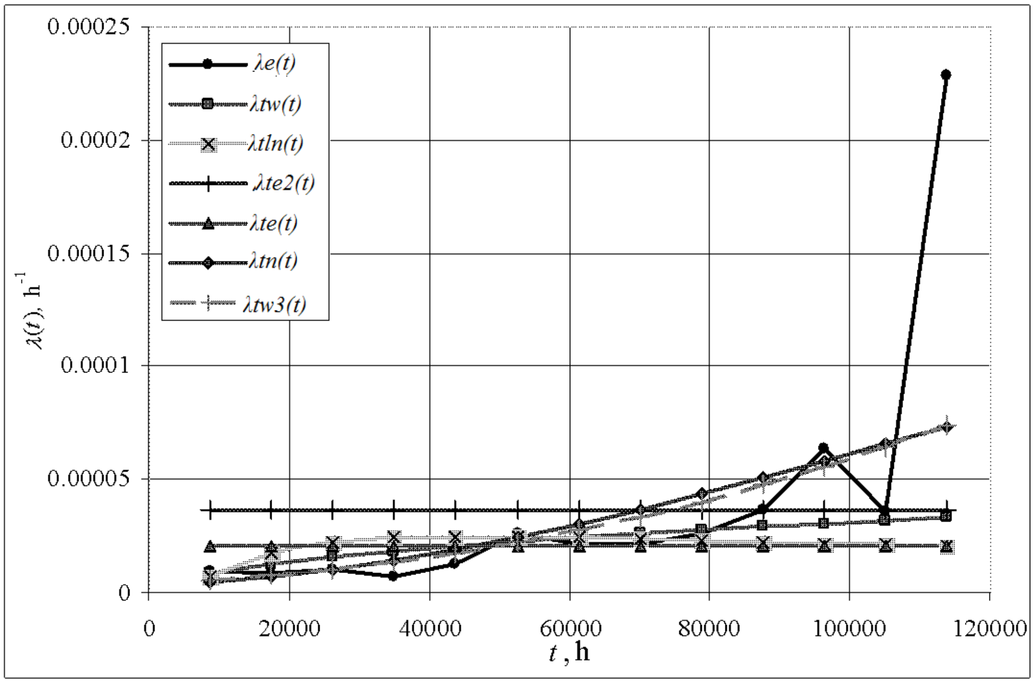

Figure 4. Exploitation and theoretical forms of failure rate function for a simple distribution

3. Determining the reliability function of thermal power plant TENT-A4 using the complex Weibull distribution

After calculating failure probabilities (Table 1) and plotting times and their corresponding rank values in a Weibull probabilistic paper, it could be noted that two straight lines better fit those points than does one line (Fig. 6).

We assume the data are inhomogeneous, i.e. they do not have the same character, and that they can be approximated with a complex distribution. The sample of failure probabilities is divided into two parts [18], after which the median rank is calculated for both (Tables 2 and 3).

By drawing the best possible straight lines through the plotted points (as shown in Fig. 7) we obtain the Weibull distribution parameters for both lines:

$$\eta_I = 2.9212\,,\ \beta_I = 1{,}3922$$
$$\eta_{II} = 9.3955\,;\ \beta_{II} = 3.1877$$

Theoretical reliability functions for both samples are:

$$R_{tI}(t) = \exp\left(-0{,}2248 \cdot t^{1{,}3922}\right);\ R_{tII}(t) = \exp\left(-0{,}00079 \cdot t^{3{,}1877}\right) \qquad (6)$$

Listed functions are analytical expressions that represent distribution laws of the observed random variable for the complex distribution of the whole set:

- reliability $R_t(t) = \frac{n_I}{n} \cdot R_{tI}(t) + \frac{n_{II}}{n} \cdot R_{tII}(t)$

$$R_t(t) = \left[0.3455 \cdot \exp\left(-0{,}2248 \cdot t^{1{,}3922}\right) + 0.6544 \cdot \exp\left(-0{,}00079 \cdot t^{3{,}1877}\right)\right] \cdot 100\% \qquad (7)$$

- failure density

$$f_t(t) = \frac{dF}{dt} = \frac{n_I}{n} \cdot \frac{\beta_I}{\eta_I} \cdot \left(\frac{t}{\eta_I}\right)^{\beta_I - 1} \cdot \exp\left(-\left(\frac{t}{\eta_I}\right)^{\beta_I}\right) + \frac{n_{II}}{n} \cdot \frac{\beta_{II}}{\eta_{II}} \cdot \left(\frac{t}{\eta_{II}}\right)^{\beta_{II} - 1} \cdot \exp\left(-\left(\frac{t}{\eta_{II}}\right)^{\beta_{II}}\right)$$

$$f_t(t) = 0{,}1082 \cdot t^{0{,}3922} \cdot \exp\left(-0{,}2248 \cdot t^{1{,}3922}\right) + 0{,}0016 \cdot t^{2{,}1877} \cdot \exp\left(-0{,}00079 \cdot t^{3{,}1877}\right) \tag{8}$$

- failure rate

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\frac{n_I}{n} \cdot \frac{\beta_I}{\eta_I} \cdot \left(\frac{t}{\eta_I}\right)^{\beta_I - 1} \cdot e^{-\left(\frac{t}{\eta_I}\right)^{\beta_I}} + \frac{n_{II}}{n} \cdot \frac{\beta_{II}}{\eta_{II}} \cdot \left(\frac{t}{\eta_{II}}\right)^{\beta_{II} - 1} \cdot e^{-\left(\frac{t}{\eta_{II}}\right)^{\beta_{II}}}}{\frac{n_I}{n} \cdot R_{tI}(t) + \frac{n_{II}}{n} \cdot R_{tII}(t)}$$

$$\lambda(t) = \frac{0{,}1082 \cdot t^{0{,}3922} \cdot \exp\left(-0{,}2248 \cdot t^{1{,}3922}\right) + 0{,}0016 \cdot t^{2{,}1877} \cdot \exp\left(-0{,}00079 \cdot t^{3{,}1877}\right)}{0.3455 \cdot \exp\left(-0{,}2248 \cdot t^{1{,}3922}\right) + 0.6544 \cdot \exp\left(-0{,}00079 \cdot t^{3{,}1877}\right)} \tag{9}$$

By using the obtained values of functions for simple and complex two-parameter Weibull distribution, we show the graphical comparison of exploitation and theoretical functions of reliability (Fig. 8), failure density (Fig. 9) and failure rate (Fig. 10) for the whole system. The required values for plotting functions of exploitation reliability, density and failure rate are given in Table 1.

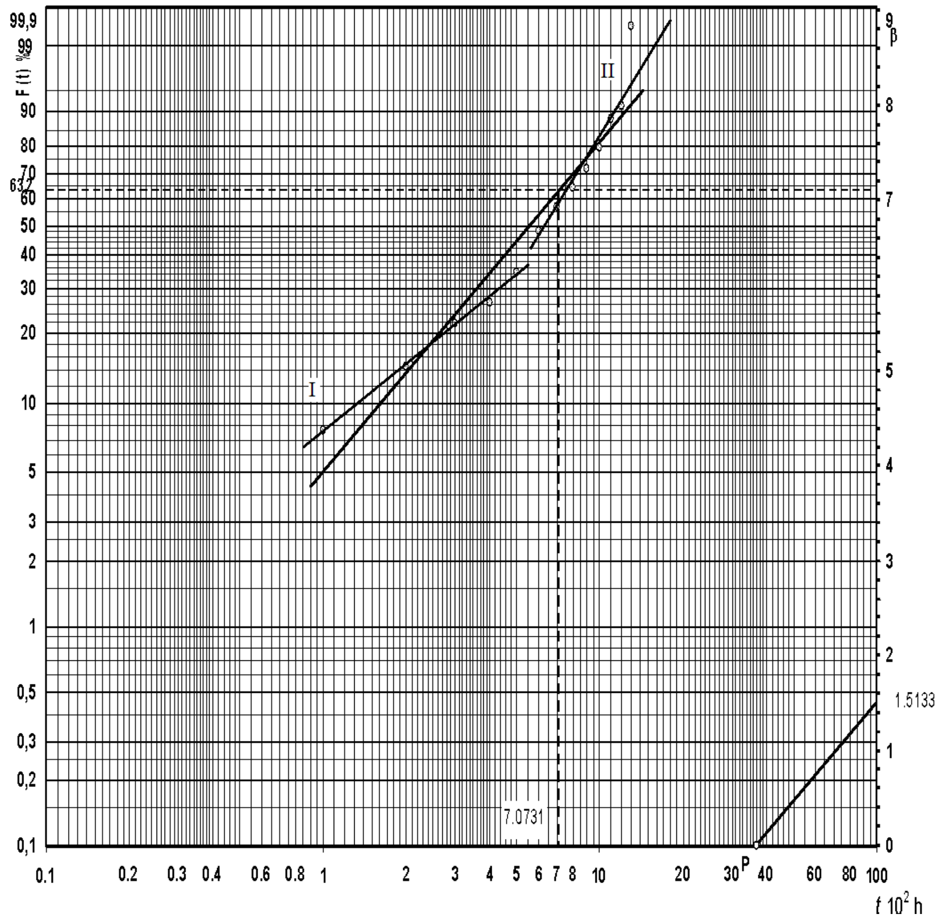

Figure 6. Weibull probability paper showing divided failure probabilities (TENT-A4)

Table 3. Values of exploitation indicators for line I

	Tk_i	Nn_i	$\sum_{i=1}^{n} Nn_i$	MR
1	1996	15	15	22,14
2	1997	13	28	41,72
3	1998	14	42	62,8
4	1999	9	51	76,36
5	2000	15	66	98,94

Table 4. Values of exploitation indicators for line II

	Tk_i	Nn_i	$\sum_{i=1}^{n} Nn_i$	MR
6	2001	26	26	20,49
7	2002	17	43	34,05
8	2003	14	57	45,22
9	2004	14	71	56,38
10	2005	15	86	68,34
11	2006	17	103	81,9
12	2007	6	109	86,68
13	2008	16	125	99,4

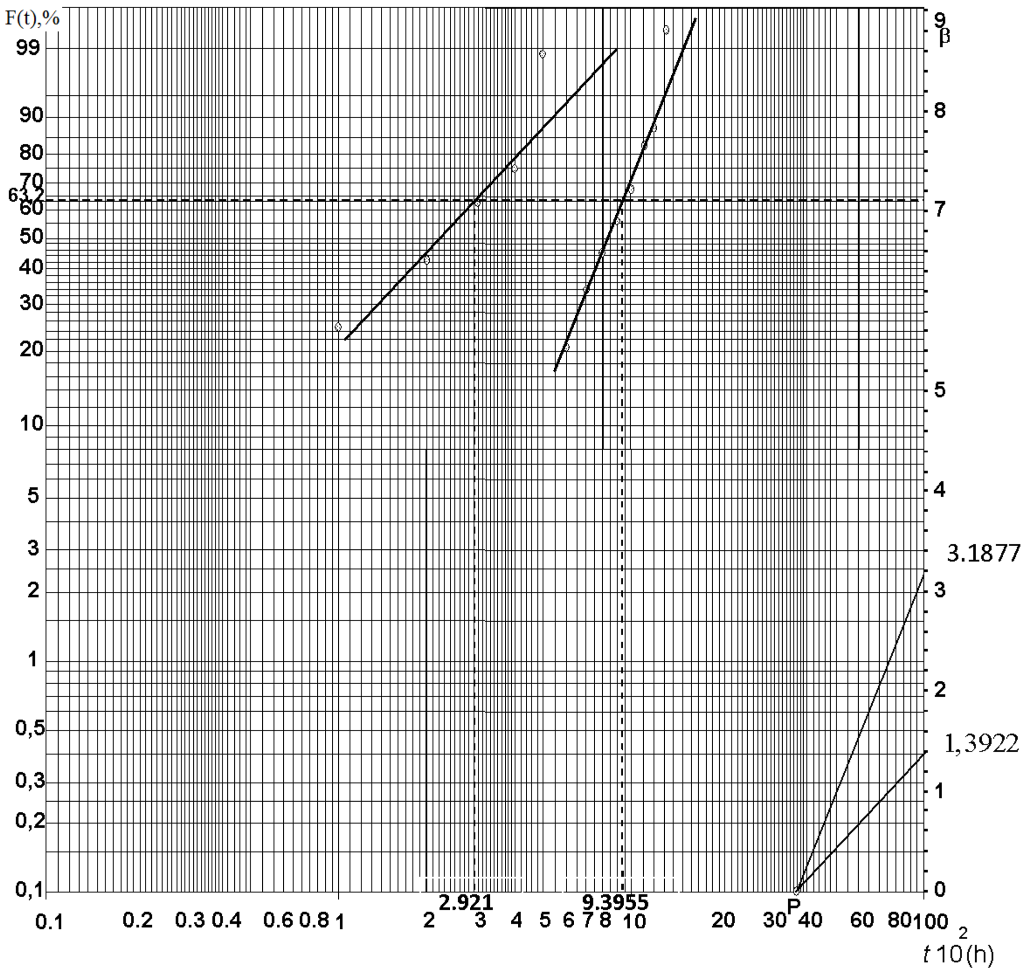

Figure 7. Weibull probability paper for complex distribution (TENT-A4)

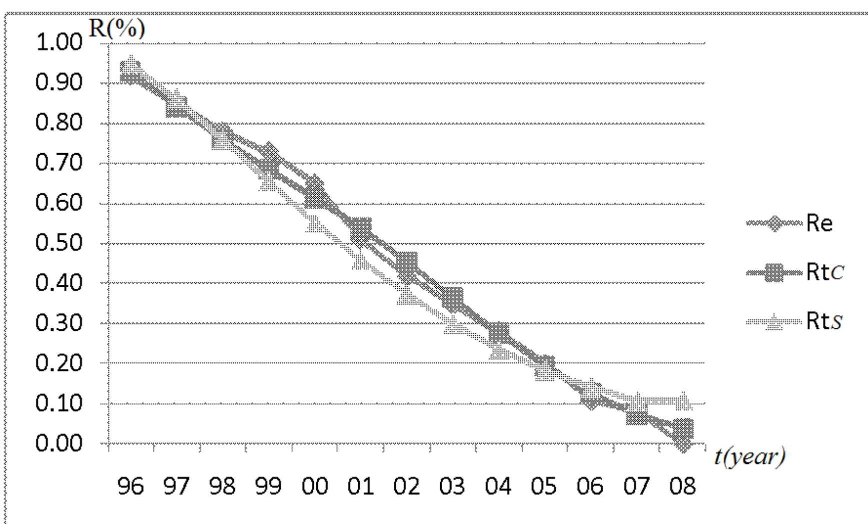

Figure 8. Exploitation and theoretical forms of reliability function for simple and complex distribution

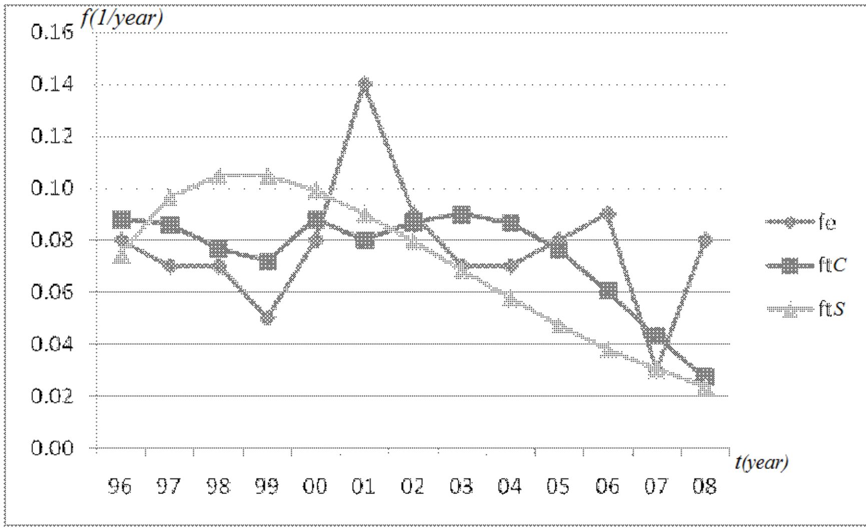

Figure 9. Exploitation and theoretical forms of failure density function for simple and complex distribution

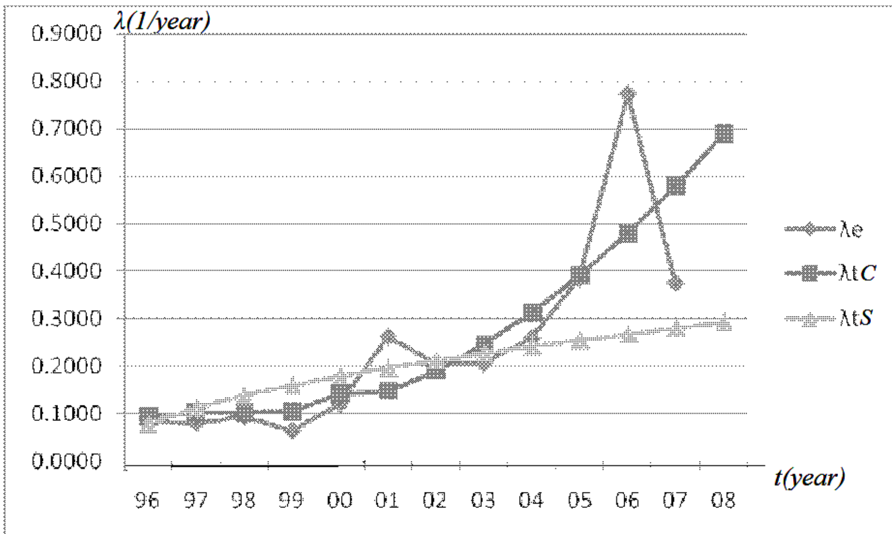

Figure 10. Exploitation and theoretical forms of failure rate function for simple and complex distribution

4. Conclusion

Exploitation research of the fossil fuel power plant system TENT-A4 and the application of the reliability theory, i.e. determination of theoretical distribution law of a random variable, has led us to the following conclusions:

- The boiler subsystem in the thermal power system is a least reliable subsystem (more than 80% of all unplanned delays took place here);
- Delays in this subsystem are mainly a consequence of material fatigue;
- By comparing exploitation values and values obtained by simple distributions for determining theoretical functions of reliability, density and failure rate, leads us to the conclusion that of all theoretical functions, the best agreement is shown with the three-parameter Weibull distribution, followed by the normal and the two-parameter Weibull distributions; larger discrepancies are seen in log-normal and exponential distributions, while the applied two-

parameter exponential distribution provides a considerable disagreement (especially at certain time intervals);

- For describing the theoretical distribution law of a random variable during the normal operation period of the thermal power system, it is more precise to use rather complex than the simple Weibull distribution, what is clear by a better matching of the systems empirical data and complex theoretical reliability functions.

Nomenclature

MR - medial rang (=(*j*-0,3)/(*n*+0,4)), [-]
n - total number of failures in the reported period, [-]
Nn - total number of failures, [-]
$\sum_{i=1}^{n} Nn_i$ - cumulative sum of failures, ($j = \sum_{i=1}^{n} Nn_i$),[-]
Nt - reverse cumulative sum of failures, [-]
Ta - engaged time, [h]
Tk - calendar time, [year]
Tnz - total time of unplanned outages, [h]
Tpz - total time of planned outages, [h]
Tr - mean time available (=*Ta*+*Trez*), [h]
Trez - total time in storage state (=*Tk*-(*Ta*+*Tpz*+*Tnz*)), [h]

Greek letters

β - shape parameter, [-]
η - scale parameter, [-]

Subscripts

c- complex distribution
i – number of operating intervals of the system
e – exploitation
s - simple
t- theoretical
te – theoretical exponential distribution
te2 - theoretical two-parameter exponential distribution
tln - theoretical lognormal distribution
tn - theoretical normal distribution
tw - theoretical two-parameter distribution
tw3 - theoretical three-parameter distribution

References

[1] Kalaba, D., Thermal power system reliability (in Serbian), University of Priština, Faculty of Tehnical Sciences Kosovska Mitrovica, Kosovska Mitrovica, Serbia, 2011
[2] Barlow, R., Clarotti, C., Spizzichino, F., Reliability and decision making, Chapman and Hall, New York, USA, 1993

[3] Nelson, Wayne (2000). "Weibull prediction of a future number of failures," Quality and Reliability Engineering International, 16, 23-26.
[4] Daniel J. Nordman and William Q. Meeker, Weibull Prediction Intervals for a Future Number of Failures *Technometrics* Vol. 44, No. 1 (Feb., 2002), pp. 15-23
[5] Matavž, A., Krope, J., Goričanec, D., Gubeljak, N. The impact of ageing on mechanical characteristics of resin bonded wheels. *Strojarstvo*, 2007, vol. 49, 4, str. 311-316.
[6] Zbyněk MARTÍNEK, Veronika KRÁLOVCOVÁ, THE RELIABILITY CHARACTERISTIC OF POWER PLANT UNIT, Acta Electrotechnica et Informatica, Vol. 10, No. 1, 2010, 47–50
[7] Chen Yu, Chen Xiao, Zhao Bing, Application of Weibull Distribution in Nuclear Power Plant Reliability Database Atomic Energy Science and Technology 2003 37 (4): ISSN: 1000-6931 CN: 11-2044/TL
[8] Barlow, R., Engineering Reliability, SIAM, Philadelphia, USA, 1998
[9] Milčić, D., Mijajlovic, M., Methods for constructing of thermal power system based on reliability (in Serbian), 13th Symposium on Thermal Science and Engineering of SCG, *Proceedings on CD*, Sokobanja, Serbia, 2007
[10] Milčić, D. *et al*., Exploitation researches of the thermo – energetic system's availability, 15th Symposium on Thermal Science and Engineering of Serbia, Sokobanja, Serbia, 2011, pp. 905-917
[11] Kalaba, D. *et al*., Determining the reliability of thermal power system in „Gacko“ thermal power Plant (in Serbian), *Elektroprivreda*, 3 (2010), pp. 222-232
[12] Đorđević, P., Kirin, S., Sedmak, A., Džindo, E. Risk analysis in Structural Integrity, Structural Integrity and Life, 11, 2(2011), pp. 135-138
[13] Kirin, S., Jovanovic, A., Stanojevic, P., Sedmak, A.; Dzindo, E., Risk management in Structural Integrity – application to a large company. Structural Integrity and Life, 11, 3(2011), pp. 205-209
[14] Murthy, P., Min, X., Renyan, J., Weibull models, Wiley, New York, USA, 2004[15] Ebeling, C., An Introduction to Reliability and Maintainability Engineering, McGraw-Hill, New York, USA, 1997
[16] Rausand, M., Arnjlot, H., System Reliability Theory; Models, Statistical Methods and Applications (Second Edition), Wiley, New York, USA, 2004
[17] Milčić, D., Reliability of mechanical systems (in Serbian), Faculty of Mechanical Engineering, University of Niš, Serbia, 2005
[18] Milčić, D., Mijajlović, M., Reliability of mechanical systems - Collection of solved tasks (in Serbian), Faculty of Mechanical Engineering, University of Niš, Serbia, 2008