

## FINITE ELEMENT MODELLING OF CREEP PROCESS – STEADY STATE STRESSES AND STRAINS

by

**Aleksandar S. SEDMAK<sup>a\*</sup>, Ljubica P. MILOVIĆ<sup>b</sup>, Mirko N. PAVIŠIĆ<sup>a</sup>  
and Pejo I. KONJATIĆ<sup>c</sup>**

<sup>a</sup> Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

<sup>b</sup> Faculty of Technology and Metallurgy, University of Belgrade, Belgrade, Serbia

<sup>c</sup> Mechanical Engineering Faculty in Slavonski Brod, University of Osijek, Slavonski Brod, Croatia

Original Scientific Paper

DOI: 10.2298/TSCI130304182S

*Finite element modelling of steady state creep process has been described. Using an analogy of visco-plastic problem with a described procedure, the finite element method has been used to calculate steady state stresses and strains in 2D problems. An example of application of such a procedure have been presented, using real life problem - cylindrical pipe with longitudinal crack at high temperature, under internal pressure, and estimating its residual life, based on the C\* integral evaluation.*

Key words: *steady state creep, cylindrical pipe, crack, C\* integral*

### Introduction

Creep is the general term for long time process, well known for slow degradation of material and increasing strain, leading to premature failure, [1,2]. Such a process can be described mathematically if basic relation between stress, strain, temperature and time is known.

The general mathematical model of a creep process in one dimension can be expressed as follows, [3]:

$$\varepsilon^c = F(\sigma, T, t) \quad (1)$$

where  $\dot{\varepsilon}$  is the creep strain,  $F$  function of the stress, temperature and time. If one considers the crack tip fields in an elastic-secondary creeping material, as being analogous to an elastic-plastic behavior, the equation of state is:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + B\sigma^n \quad (2)$$

where the total strain rate  $\dot{\varepsilon}$  is sum of the elastic component,  $\dot{\sigma}/E$  and the nonlinear secondary creep component  $B\sigma^n$  with the creep exponent  $n$  and temperature-dependent coefficient  $B$  being material parameters. The material described by the power law (2) is referred to as an elastic-power law creep material, describing a creeping with negligible primary and tertiary stage.

---

\* Corresponding author, e-mail: asedmak@mas.bg.ac.rs

In the case of the creep under multiaxial stress conditions, a mathematical model should satisfy several requirements for correct modeling [3]:

-the material law should reduce to the correct uniaxial formulation when appropriate;

-the model should express the constancy of volume, experimentally observed;

-the equation of state should embody the lack of influence of the hydrostatic state of stress, also experimentally observed;

-for isotropic materials the principal directions of stress and strain should coincide.

Having these requirements in mind, equation (1) can be generalized to the multiaxial stress state as follows:

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E} \dot{s}_{ij} + \frac{1-2\nu}{3E} \dot{\sigma}_{kk} \delta_{ij} + \frac{3}{2} B \bar{\sigma}^{n-1} s_{ij} \quad (3)$$

where  $s_{ij} = \sigma_{ij} - 1/3 \sigma_{kk} \delta_{ij}$  and  $\bar{\sigma} = (3/2 s_{ij} s_{ij})^{1/2}$  stand for deviator stress components and effective stress, respectively,  $\delta_{ij}$  is the Kronecker delta symbol,  $\nu$  Poisson ratio. The equation (3) should be supplemented by the equilibrium equation, which can be written in the form:

$$\sigma_{\alpha\beta,\beta} = 0 \quad (4)$$

In the absence of inertial and volume forces, and by the nontrivial compatibility condition

$$\dot{\varepsilon}_{\alpha\beta,\alpha\beta} - \dot{\varepsilon}_{\alpha\alpha,\beta\beta} = 0 \quad (5)$$

for the small strain and the plane problem ( $\alpha, \beta = 1, 2$ ). The equilibrium equation (4) will be identically satisfied if the stress components are expressed in terms of the Airy stress function  $\psi(X_\alpha)$ , such that:

$$\sigma_{\alpha\beta} = \psi_{,\alpha\beta} + \psi_{,xx} \delta_{\alpha\beta} \quad (6)$$

### Stationary crack-tip fields

For plane problem the stress and displacement fields are functions of  $x_1$  and  $x_2$  only. The stress component and strain component vanish, as well as  $\sigma_{\alpha 3}$  and the strain component  $\varepsilon_{\alpha 3}$ , and  $\sigma_{33}$  and  $\varepsilon_{33}$  vanish for the plane stress and strain problems, respectively. An investigation of crack-tip stress and displacement fields is important because these fields typically govern the fracture process occurring at the crack tip.

We start considering the crack-tip fields for stationary crack in time-dependent material undergoing either plane stress or plane strain in mode I loading. For further investigations it is convenient to rewrite Eqn (3), in terms of stress tensor  $\sigma_{ij}$

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E} \dot{\sigma}_{ij} - \frac{\nu}{E} \dot{\sigma}_{kk} \delta_{ij} + \frac{3}{2} B \bar{\sigma}^{n-1} s_{ij} \quad (7)$$

$$\dot{\varepsilon}_{\alpha\beta} = \frac{1+\nu}{E} \dot{\sigma}_{\alpha\beta} - \frac{\nu}{E} \dot{\sigma}_{kk} \delta_{\alpha\beta} + \frac{3}{2} B \bar{\sigma}^{n-1} s_{\alpha\beta} \quad (8)$$

$$\dot{\varepsilon}_{33} = \frac{1}{E} \dot{\sigma}_{33} - \frac{\nu}{E} \dot{\sigma}_{kk} + \frac{3}{2} B \bar{\sigma}^{n-1} s_{33} \quad (9)$$

From (3) and (6) it follows that  $\sigma_{\alpha\alpha} = \psi_{,\alpha\alpha}$ ;  $\sigma_{33} = 3/2 s_{33} + 1/2 \psi_{,\alpha\alpha}$  and

$$s_{\alpha\beta} = -\psi_{\alpha\beta} + \frac{1}{2}(\psi_{\gamma\gamma} - s_{33})\delta_{\alpha\beta}; \quad s_{33} = -s_{\alpha\alpha} \quad (10)$$

Then the effective stress can be written in the form:

$$\bar{\sigma} = \frac{\sqrt{3}}{2}(2\psi_{\alpha\beta}\psi_{\alpha\beta} - \psi_{\gamma\gamma}\psi_{\delta\delta} + 3s_{33}^2)^{1/2} \quad (11)$$

Also, from (9) and (10), it follows

$$\frac{2}{3}\dot{\epsilon}_{33} = \frac{1-2\nu}{3E}\psi_{\alpha\alpha}\dot{\sigma}_{33} + \frac{1}{E}\dot{s}_{33} + B\bar{\sigma}^{n-1}s_{33} \quad (12)$$

Finally, combining (5), (6), (8) and (10), one gets:

$$\frac{2-\nu}{3E}\nabla^4\dot{\psi} - \frac{\nu}{E}\dot{s}_{33,\alpha\alpha} - \frac{1}{2}B\left\{\left[(\psi_{\gamma\gamma} + s_{33})\delta_{\alpha\beta} - 2\psi_{\alpha\beta}\right]\bar{\sigma}^{n-1}\right\}_{\alpha\beta} = 0 \quad (13)$$

where  $\nabla^4 = (\ )_{\alpha\alpha\beta\beta}$  is the bi-harmonic operator. For the plane stress problem,  $\sigma_{33} = \sigma_{33} = 0$  and  $S_{33} = -1/3 \psi_{\alpha\alpha}$ ; according to (10<sub>2</sub>). Then

$$\frac{2}{E}\nabla^4\dot{\psi} - B\left[(\psi_{\gamma\gamma}\delta_{\alpha\beta} - 3\psi_{\alpha\beta})\bar{\sigma}^{n-1}\right]_{\alpha\beta} = 0 \quad (14)$$

$$\bar{\sigma} = \left[\frac{3}{2}\psi_{\alpha\beta}\psi_{\alpha\beta} - \frac{1}{2}\psi_{\alpha\alpha}\psi_{\beta\beta}\right]^{1/2} \quad (15)$$

represent the governing equations for the Airy stress function. The governing equation for  $\epsilon_{33}$  can be written in the form:

$$\dot{\epsilon}_{33} = -\frac{\nu}{E}\dot{\psi}_{\alpha\alpha} - \frac{1}{2}B^{n-1}\psi_{\alpha\alpha} \quad (16)$$

For the plane strain  $\epsilon_{33} = \dot{\epsilon}_{33} = 0$  so that eqn (12) becomes

$$\frac{1-2\nu}{3E}\dot{\psi}_{\alpha\alpha} + \frac{1}{E}s_{33} + B\bar{\sigma}^{n-1}s_{33} = 0 \quad (17)$$

The crack-tip fields are anticipated to be singular at the crack tip for the assumed material law [3]. If it is also assumed that the creep exponent  $n$  is greater than unity ( $n > 1$ ), then the creep strain rates will dominate around the crack tip, so that the linear (elastic) terms in (17) and (21) can be neglected. The resulting equations have the same forms as the equations governing the asymptotic behavior in a rate-insensitive, power law strain hardening material. Hence, the stress and strain singularities are of the HRR type:

$$\sigma_{ij} = \left[\frac{C(t)}{BI_n r}\right]^{1/(n+1)} \bar{\sigma}_{ij}(\theta) \quad \dot{\epsilon}_{ij} = \left[\frac{C(t)}{BI_n r}\right]^{n/(n+1)} \bar{\epsilon}_{ij}(\theta) \quad (18)$$

where  $\bar{\sigma}_{ij}(\theta)$  and  $\bar{\varepsilon}_{ij}(\theta)$  represent the angular distribution of stresses and strains, respectively, around crack tip,  $r$  and  $\theta$  are the polar coordinates around the crack tip (Fig. 1), and  $C(t)$  is the time-dependent loading parameter, defined as:

$$C(t) = \lim_{\varepsilon \rightarrow 0} \int_{\partial D_\varepsilon} \left( \frac{n}{n+1} \sigma_{ij} \dot{\varepsilon}_{ij} dx_2 - \sigma_{ij} n_j \frac{\partial \dot{u}_i}{\partial x_1} ds \right) \quad (19)$$

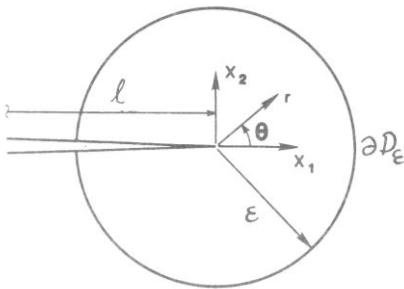


Figure 1. Infinitesimal loop around crack tip

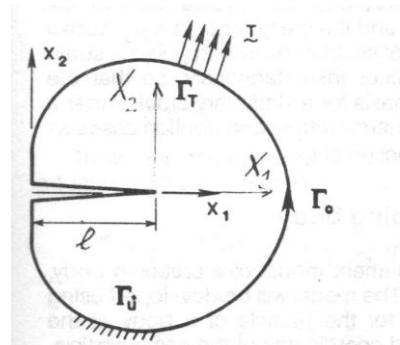


Figure 2. Contour around crack tip

The contour  $\partial D_\varepsilon$  is a small loop with the radius  $\varepsilon$ , centered at the crack tip, shrinking to the crack tip as  $\varepsilon \rightarrow 0$  (Fig. 1). From Eqns (18) it is clear that  $C(t)$  is the loading parameter defining the strength of the crack-tip singularity fields. If the applied load remains fixed  $n$  time, then Eqn (3) implies that the stress field becomes time-independent as  $t \rightarrow \infty$ . The elastic strain rates vanish and secondary (steady-state) creep extends throughout the body. Such a behavior can be recognized as nonlinear viscous flow, leading to the conclusion that  $C(t) \rightarrow C^*$ , where  $C^*$  is the path independent integral:

$$C^* = \int_{\Gamma} \left[ \dot{W} \dot{\varepsilon}_{ij} dx_2 - \sigma_{ij} n_j \frac{\partial \dot{u}_i}{\partial x_1} ds \right] \quad (20)$$

where  $W(\dot{\varepsilon}_{ij})$  is the strain energy rate density, defined by:

$$W(\dot{\varepsilon}_{ij}) = \int_0^{\dot{\varepsilon}_{ij}} \sigma_{ij} d\dot{\varepsilon}_{ij} \quad (21)$$

Therefore, the near-tip fields for the stationary crack in a steady-state creeping body are:

$$\sigma_{ij} = \left[ \frac{C^*}{B I_n r} \right]^{\frac{1}{n+1}} \bar{\sigma}_{ij}(\theta) \quad \dot{\varepsilon}_{ij} = \left[ \frac{C^*}{B I_n r} \right]^{\frac{n}{n+1}} \bar{\varepsilon}_{ij}(\theta) \quad (22)$$

It follows that the  $C^*$  integral is the loading parameter that determines the strength of the singular stationary crack tip in a body undergoing steady-state creep. Under the assumptions introduced,  $C^*$  integral is an analogous parameter to the well-known  $J$  integral. This enables application of any handbook of  $J$  integral expressions, as shown in [4,5],

providing that  $u_i$  and  $\varepsilon_{ij}$  are replaced by their rates, and material constant  $\alpha\varepsilon_y / \sigma_y^n$  by  $B$ , where  $\varepsilon_y$  and  $\sigma_y$  are yield strain and strength, respectively.

### Finite element model of creeping body

Here we present the general finite element model of a creeping body, including elastic and plastic deformation. The model will be developed using the global Cartesian coordinates  $z^i(\xi^a, t)$  for the particle of a body at the moment  $t$ , where  $\xi^a$  denote the convected coordinates of the same particle. Using the convected coordinates, one can rewrite the expression (11) in a form:

$$\dot{\varepsilon}^{ab} = \frac{1+\nu}{E} \sigma^{ab} - \frac{\nu}{E} \dot{\sigma}^{cc} g^{ab} + \frac{3}{2} B \bar{\sigma}^{n-1} s^{ab} \quad (23)$$

For the further considerations, it will be useful to separate the creep-dependent part of the strain rate:

$$\dot{\varepsilon}_{cd}^c = \frac{3}{2} B \bar{\sigma}^{n-1} s_{cd} \quad (24)$$

If, in addition, the plastic strain rate should be considered, one can write the following expression:

$$\dot{\varepsilon}_{cd}^p = \frac{3}{2\bar{\sigma}} \frac{\beta}{E_p} \dot{\bar{\sigma}} s_{cd} \quad \beta = \begin{cases} 1 & \text{for } \bar{\sigma} = \sigma_y \text{ and } \dot{\bar{\sigma}} > 0 \\ 0 & \text{for } \bar{\sigma} < \sigma_y \text{ and } \dot{\bar{\sigma}} > 0 \end{cases} \quad (25)$$

or, in the case of the temperature-dependent plasticity:

$$\dot{\varepsilon}_{cd}^p = \frac{3}{2\bar{\sigma}} \frac{\beta}{E_p} \left( \dot{\bar{\sigma}} + \frac{\partial f}{\partial \theta} \dot{\theta} \right) s_{cd} \quad (26)$$

In these expressions  $g^{ab}$  is the covariant metric tensor,  $\theta$  is the relative temperature,  $E_p = E \cdot E_T / (E - E_T)$  is so-called ‘‘plastic’’ modulus,  $E_T$  is the tangent modulus, and  $f$  is the yield function:

$$f = \bar{\sigma} - \sigma_y(\varepsilon_{cd}^p, \theta) \quad (27)$$

It should be noticed that one can combine (23) and (25) or (26), to get the creep plastic strain rate:

$$\dot{\varepsilon}_{cd}^{cp} = \frac{3}{2\bar{\sigma}} \left[ \frac{1}{E_p} \left( \dot{\bar{\sigma}} + \frac{\partial f}{\partial \theta} \dot{\theta} \right) + B \bar{\sigma}^n \right] s_{cd} \quad (28)$$

Similarly, one can add (25) or (26) to (23) to get

$$\varepsilon_{cd} = \left\{ \frac{1}{E} \left[ (1+\nu) g_{ac} g_{bd} - \nu g_{ab} g_{cd} \right] + \beta \left( \frac{3}{2} \right)^2 \left( \frac{1}{E_T} - \frac{1}{E} \right) s_{ab} s_{cd} \right\} \dot{\bar{\sigma}}^{ab} + \frac{3}{2} B \bar{\sigma}^{n-1} s_{cd} \quad (29)$$

However, for computational purposes it is convenient to resolve the equation (23) or (29), as follows:

$$\dot{\sigma}^{ab} = \frac{E}{1+\nu} \left[ \left( g^{ac} g^{bd} + \frac{\nu}{1-2\nu} g^{ab} g^{cd} - \frac{3\beta}{2} \frac{E-E_T}{3E-(1-2\nu)E_T} \frac{s^{ab} s^{cd}}{\bar{\sigma}^2} \right) \dot{\varepsilon}_{cd} + \frac{3B}{2} \bar{\sigma}^{-n-1} \left( \beta \frac{E-E_T}{3E-(1-2\nu)E_T} \right) s^{ab} \right] \quad (30)$$

Note that in the case when there is no creep ( $B=0$ ) the Eqn (30) reduces to the well-known rate constitutive equation for the plasticity with the yield surface. Similarly, if there is no plasticity ( $\beta=0$ ) Eqn (30) reduces to:

$$\dot{\sigma}^{ab} = \frac{E}{1+\nu} \left[ \left( g^{ab} g^{bd} + \frac{\nu}{1-2\nu} g^{ab} g^{cd} \right) \dot{\varepsilon}_{cd} - \frac{3}{2} B \bar{\sigma}^{-n-1} s^{ab} \right] \quad (31)$$

what can be written in a simpler form

$$\dot{\sigma}^{ab} = E^{abcd} \dot{\varepsilon}_{cd} - 3\mu B \bar{\sigma}^{-n-1} s^{ab} \quad (32)$$

where  $E^{abcd} = \frac{E}{1+\nu} \left( g^{ac} g^{bd} + \frac{\nu}{1-2\nu} g^{ab} g^{cd} \right)$  and  $\mu = \frac{E}{2(1+\nu)}$ . It is evident that (31) or (32)

can be found directly by the solution of (23) for  $\dot{\sigma}^{ab}$ .

### Constitutive equations

The constitutive equations for the stresses can be written in the form:

$$\sigma^{ab} = E^{abcd} (\varepsilon_{cd} - \varepsilon_{cd}^{cp}) - E^{ab} \theta \quad (33)$$

where  $E^{abcd} = 2\nu \frac{1+\nu}{1-2\nu} \alpha g^{ab}$  denote the tensor of thermoelastic coefficients for an isotropic material. In this expression  $\varepsilon_{cd}$  is the total strain tensor, which can be determined from the strain-displacement relation:

$$\varepsilon_{cd} = \frac{1}{2} \delta_{ij} (z_c^i u_d^i + z_d^i u_c^i + z_c^i u_d^i) \quad (34)$$

where  $z_c^i = \partial z^i / \partial \xi^c$  are the coordinates of base vectors,  $u_c^i = \partial u^i / \partial \xi^c$  are the displacement gradients. In addition, one should note that the strain rate is

$$\dot{\varepsilon}_{cd} = \frac{1}{2} \delta_{ij} (z_c^i z_d^i + z_d^i z_c^i) \quad (35)$$

### Discretized equations of motions

In this paper, a slow motion of a creeping body are considered, and hence the inertial forces can be neglected. Consequently, the discretized equations of motion are of the form:

$$R^j - F^j = 0 \quad (36)$$

Where  $R^j$  are external and  $F^j$  internal forces, both in column matrix form. The entries in these columns are

$$R^{ij} = \sum_e \int_{\varepsilon} \rho f^j P^j dV + \sum_e \int_{\partial\varepsilon} \sigma^j P^j dA \quad (37)$$

$$F^{ji} = \sum_e \int_{\varepsilon} z_a^i P_b^j \sigma^{ab} dV \quad (38)$$

where  $f^j$  are the body forces,  $\sigma^j$  the boundary tractions,  $P^j$  the interpolation functions, ( $P_b^j = \partial P^j / \partial \xi^b$ ). The summation is performed over all finite elements, and integration over the body  $\varepsilon$  or the boundary  $\partial\varepsilon$  of each element. However, in order to be solved, the equation (33) should be linearized as follows:

$$\left[ K^{ij} + S^{ij} \right] u^i = \sum_e \int_{\varepsilon} R_h^j + R^i - 2F^i - h\delta R^i + P^i \quad (39)$$

In this expression,  $K^{ij}$  is the stiffness matrix

$$K^{iij} = \sum_e \int_{\varepsilon} \left( z_a^j z_c^i P_b^j P_d^i E^{abcd} \right) dV \quad (40)$$

$S^{ij}$  is the geometric stiffness matrix

$$S^{iij} = \sum_e \int_{\varepsilon} \delta^{ij} P_b^j P_a^i E^{ab} dV \quad (41)$$

$U^i$  is the column matrix of the increments of nodal displacement,  $h\delta R^j$  is the column matrix of the external forces at the moment  $t + h$  (at the end of a time step under consideration). If the temperature changes ( $\Delta T$ ) are considered (during time step), the column matrix  $P^i$  can be expressed as follows

$$P^{iij} = \sum_e \int_{\varepsilon} z_a^j P_b^i \Delta T E^{ab} dV \quad (42)$$

Finally, the most crucial part of the expression (54) is the dissipation term  $h\delta R^i$ , where the entries are

$$\delta R^{ji} = -\sum_e \int_{\varepsilon} z_a^j P_b^i \dot{\varepsilon}_{cd}^{cp} E^{abcd} dV \quad (43)$$

At the beginning of the current time step, it is obvious that the values at the end of a previous step i.e.  $z^i$ ,  $\sigma^{ab}$  and  $\dot{\varepsilon}_{cd}^{cp}$  are known. Hence, one can compute  $K^{ij}$ ,  $S^{ij}$ ,  $R^i$ ,  $R_h^j$ ,  $F^i$  and  $\delta R^{ji}$  by the use of Eqns (40), (41), (37), (38) and (43), respectively. At the stage it is necessary to choose the length of a time step,  $h$ . The recommended length of a time step [5] is

$$h < N\bar{\varepsilon} / \dot{\varepsilon}^{cp} : N \ll 1$$

where  $\bar{\varepsilon}$  is the total effective strain, and  $\dot{\varepsilon}^{cp}$  is the effective creep-plastic strain rate. However, it is believed that the integration scheme proposed in this paper is more stable, so that  $N < 1$  would be sufficient. Anyhow, some possibilities for automatic or interactive step length adjustment during the programme execution is certainly beneficial. Having all these values known, one can compute  $u^i$ , the displacement increment, from (39).

This algorithm has been presented in [5], but not used until recently. Its use is presented in this paper for the first time, providing good results with relatively small calculation effort, as shown in the following example.

### Example

Example to illustrate the procedure introduced here, has been taken from [4], where the cylindrical pipe with internal longitudinal or circumferential crack has been subjected to internal pressure or axial loading, respectively, and high temperature in long time period, as shown in Fig. 3. In paper [4], EPRI procedure has been applied to obtain  $C^*$  integral values, then used to calculate the residual life.

$$\frac{da}{dt} = B \cdot (C^*)^m \quad (44)$$

where  $B$  and  $m$  are materials constants.

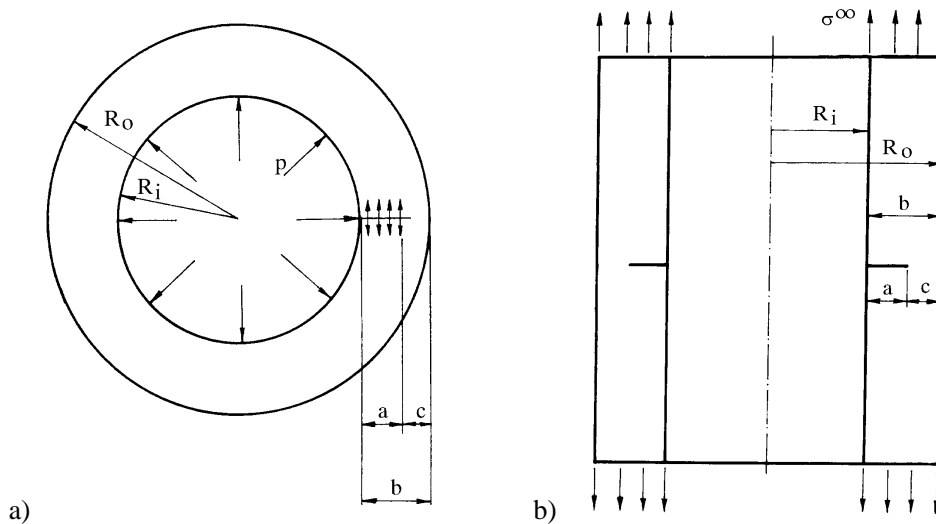


Figure 3. Cylindrical pipe: a) longitudinal crack under internal pressure, b) circumferential crack under axial loading

After substituting  $C^*$  as  $dJ/dt$  one gets:

$$\frac{da}{dt} = B \cdot \left( \frac{dJ}{dt} \right)^m \quad (45)$$

which can be written in more suitable form:

$$\frac{da}{dt} = B \cdot \left( \frac{dJ}{da} \cdot \frac{da}{dt} \right)^m \quad (46)$$

After simple transformations, one can get:



$$\frac{da}{\sqrt[1-m]{\left( B \cdot \left( \frac{dJ}{da} \right)^m \right)}} = dt \quad (47)$$

The complete procedure has been described in [4], including  $C^*$  integral evaluation and residual life calculation for four different cases (two different loading cases and two different materials). In this paper the same procedure has been applied, except that  $C^*$  integral has been calculated by using the finite element method. Time is calculated as the cumulative value needed to reach given crack length.

*Input data and results*

The initial crack length:  $a_0=0.25$  mm, crack length increment,  $\Delta a=0.5$  mm, internal diameter,  $R_i=91.5$  mm, pipe thickness  $b=18$  mm, valid for both cases, cylindrical pipe with internal longitudinal or circumferential crack, subjected to internal pressure or axial loading, respectively,

The “new” material properties:  $\alpha=1.128$ ,  $\varepsilon_0=1.685 \cdot 10^{-3}$ ,  $\sigma_0=262.8$  MPa,  $n=9.8382$ ,  $B=2.56 \cdot 10^{-3}$ ,  $m=0.75$ , providing  $da/dt$  in [mm/s], and  $C^*$  in [N/(mm·s)].

The “old” material properties:  $\alpha=2.08$ ,  $\varepsilon_0=1.66 \cdot 10^{-3}$ ,  $\sigma_0=259$  MPa,  $n=7.98$ ,  $B=1.03 \cdot 10^{-3}$ ,  $m=0.73$ .

Results for the cumulative time needed to reach given crack length are given in Tables 1-4 for combination of two different materials and two cylindrical pipes, one with longitudinal crack under internal pressure and the other one with circumferential crack subjected to axial loading, respectively.

**Table 1. The “new” material, Longitudinal crack, internal pressure, p=60MPa**

|                  |      |      |      |      |      |      |      |      |      |
|------------------|------|------|------|------|------|------|------|------|------|
| a [mm]           | 0.75 | 1.25 | 1.75 | 2.25 | 2.75 | 3.25 | 3.75 | 4.25 | 4.75 |
| t [h] from [4]   | 2154 | 2877 | 3133 | 3227 | 3263 | 3277 | 3282 | 3284 | 3285 |
| t [h] this paper | 2134 | 2857 | 3113 | 3216 | 3239 | 3250 | 3252 | 3244 | 3238 |

**Table 2. The “old” material, longitudinal crack, internal pressure, p=60 MPa**

|                  |      |      |      |      |      |      |      |
|------------------|------|------|------|------|------|------|------|
| a [mm]           | 0.75 | 1.25 | 1.75 | 2.25 | 2.75 | 3.25 | 3.75 |
| t [h] from [4]   | 763  | 937  | 980  | 992  | 995  | 996  | 996  |
| t [h] this paper | 756  | 930  | 972  | 982  | 983  | 980  | 980  |

**Table 3. The “new” material, circumferential crack, axial force, F=3.5MPa**

|                  |      |      |       |       |       |       |       |
|------------------|------|------|-------|-------|-------|-------|-------|
| a [mm]           | 0.75 | 1.75 | 2.75  | 3.75  | 4.75  | 5.25  | 5.75  |
| t [h] from [4]   | 5802 | 9638 | 10466 | 10650 | 10690 | 10695 | 10698 |
| t [h] this paper | 5762 | 9598 | 10422 | 10602 | 10638 | 10635 | 10637 |

**Table 4. . The “old” material, circumferential crack, axial force, F=3.5MPa**

|                  |      |      |      |      |      |      |      |      |      |
|------------------|------|------|------|------|------|------|------|------|------|
| a [mm]           | 0.75 | 1.25 | 1.75 | 2.25 | 2.75 | 3.25 | 3.75 | 4.25 | 4.75 |
| t [h] from [4]   | 1910 | 2562 | 2793 | 2877 | 2909 | 2921 | 2925 | 2927 | 2928 |
| t [h] this paper | 1891 | 2542 | 2770 | 2852 | 2879 | 2890 | 2892 | 2893 | 2894 |

## Conclusion

Based on the presented results, one can conclude that the procedure for the finite element method evaluation of  $C^*$  integral provides reliable results, as proved by comparison with results obtained by EPRI procedure, and can be used for residual life estimation components operating at high temperature under steady state conditions.

## Note

This paper is partly based on the lecture delivered during the Fifth International Fracture Mechanics Summer School, publish in [3].

## Acknowledgement

The research presented in this paper is supported by the Ministry for education, science and technological development, Republic of Serbia.

## References

- [1] G. Bakić, V. Šijački-Žeravčić, Estimation of Long-Term Strength of the Material Exposed to the High-Temperature Creep Using the Microstructure Dependent Parameter, *Structural Integrity and Life*, 3, 2003, 1, p. 23-30
- [2] G. Bakić, V. Šijački-Žeravčić, Determination of Time-to-Fracture of Low Alloyed Steels Under Creep Conditions as a Function of Microstructural Parameters. Second Part: Determination of time-to-fracture, *Structural Integrity and Life*, 3, 2003, 2, p. 85-92
- [3] Berkovic, M., Sedmak, A., Jaric, J.,  $C^*$  integral – theoretical basis and numerical analysis. *Proc. of IFMASS 5, The Application of Fracture Mechanics to Life Estimation of Power Plant Components*, EMAS, 1990, pp. 71-88
- [4] S. Damjanovic, A. Sedmak, H.A. Anyiam, N. Trišović, Lj. Milović,  $C^*$  integral evaluation by using EPRI procedure, *Structural Integrity and Life*, 1, 2001, 2, p. 67-73
- [5] B.Grujić, A.Sedmak, Z.Burzić, M.Pavišić: Remaining life assessment of power plant components exposed to high temperature stationary creep, *Thermal Science*, 2, 1998, 2, p. 75