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Consideration of Moving Oscillator Problem in Dynamic Responses of Bridge Cranes

The dynamics of an overhead crane system with the suspended payload on the trolley moving at a specified constant speed is considered in this paper. The beam is discretizied by 10 elements, while the trolley is modeled as particle along with payload suspended with rope system modeled as spring. The overall mass, damping and stiffness matrix is calculated at each time interval, along with finite element formulation of equivalent force vector. Equations of motion of MDOF system are given for oscillator moving on beam structure. Dynamic responses in the vertical direction for all DOFS are obtained by solving the governing equations with direct integration method. For validation purposes, the technique is first applied to a simple beam subjected to a force moving along the beam with constant velocity. The influence of moving velocity and spring stiffness are investigated.

Keywords: moving oscillator, crane structure, transverse vibrations, dynamic response, direct integration.

1. INTRODUCTION

The moving load problem is a fundamental problem in structural dynamics. In contrast to other dynamic loads these loads vary not only in magnitude but also in position. The importance of this problem is manifested in numerous applications in the field of transportation. Bridges, guideways, cranes, cableways, rails, roadways, runways and pipelines are examples of structural elements to be designed to support moving masses. Interest in analysis of moving load problems originated in civil engineering (from observation that when an elastic structure is subjected to moving loads, its dynamic displacements and stresses can be significantly higher than those due to equivalent static loads) for the design of rail-road bridges and highway structures. Applications of the moving load problem have been presented in mechanical engineering studies for the past 30 years. Its solution requires appropriate modeling of the structure and a trolley. Typical structure under a moving load (trolley) in mechanical engineering are overhead cranes, gantry cranes, unloading bridges, slewing tower cranes, cableways, guideways, shipunloaders or e.g. quayside container cranes considered in [1-4]. The application of moving load problem in cranes dynamics has obtained special attention on the engineering researchers in the last years, but unfortunately little literature on the subject is available. The following two features distinguish the moving load problem in crane industry from that in civil engineering. The first is that the structure on which the moving load moves always has traveling or rotating motion. The second is that the payload of a crane is attached via cables to a carriage moving along the

structure. Thus the dynamics of a crane includes both vibrations of the structure and dynamics of the payload pendulum.

The basic approaches in trolley modeling are: the moving force model; the moving mass model and the trolley suspension model – moving oscillator model existing in some special structures of gantry cranes and unloading bridges. The simplest dynamic trolley models are moving force models. A review of moving force models is given in [5]. The consequences of neglecting the structure-trolley interaction in these models may sometimes be minor. In most moving force models the magnitudes of the contact forces are constant in time. A constant force magnitude implies that the inertia forces of the trolley are much smaller than the dead weight of the structure. Thus the structure is affected dynamically through the moving character of the trolley only. All common features of all moving force models are that the forces are known in advance. Thus structure-trolley interaction cannot be considered. On the other hand the moving force models are very simple to use and yield reasonable structural results in some cases [2]. Moving mass as suspension model is an interactive model. Moving mass model, as well as moving force model, is the simplification of suspension model, but it includes transverse inertia effects between the beam and the mass, see e.g. [6]. Interaction force between the moving mass and the structure during the time the mass travels along the structure considers contribution from the inertia of the mass, the centrifugal force, the Coriolis force and the time-varying velocity-dependent forces. These inertia effects are mainly caused by structural deformations (structure-trolley interaction) and structural irregularities. Factors that contribute in creating trolley inertia effects include: high trolley speed, flexible structure, large vehicle mass, small structural mass, stiff trolley suspension system and large structural irregularities. Finally, the trolley speed is assumed to be known in advance and thus was not

depend on structural deformations. For moving mass models the entire trolley mass is in direct contact with the structure. In general, the dynamic structure-trolley interaction predicted by such models is very strong [2]. The detailed review of previous researches, including comprehensive references list, can be found in a dedicated excellent monograph written by Frýba [7] on the subject of moving load problems, where most of the analytical methods previously used are described. The trolley suspension models are representing physical reality of the system more closely, because, often, the vehicle mass is suspended by means of springs and dampers in such models – moving oscillator problem. An overview of vibration analysis of bridges including the moving oscillator problem between the moving vehicles and the bridge structures is given in [8].

Since the mid 1980s many authors have investigated the application of finite element method (FEM) for solving the moving load problem as an alternative to analytical approaches, e.g. [9-12], etc.

The paper [13] is according to the authors' best knowledge the first attempt to increase the understanding of the dynamics of cranes due to the moving load. The overhead crane trolley, modeled as a simply supported beam according to Euler-Bernoulli beam theory, traverses the beam at a known prescribed uniform speed and that the pendulum may be adequately modeled as a rigid massless bar. The motion of the pendulum is assumed to be planar with small angular displacements and displacement rates from the vertical. A set of coupled, non-linear equations of motion is derived via Hamilton's principle. The paper [14] presents a technique developed for using standard finite element packages for analyzing the dynamic response of structures to time-variant moving loads. Computer program has been written which calculates the time-variant external nodal forces on a whole structure, which provide the equivalent load to point forces that move around the structure. The calculation of the equivalent nodal forces to represent the moving loads has been performed by three approximate methods. Dynamic response of structures to moving loads using combined finite element and analytical methods is given in [15]. The authors have developed a technique that includes inertia effects in the analysis. Natural frequencies and mode shapes are first calculated using a standard FEM package. Forced response of the structure is calculated using separate code developed in the paper. Rotary inertia of moving bodies is ignored as unimportant for particular mobile crane problem. In the paper [16] the three-dimensional responses of a crane structure due to the moving loads is analyzed. The axial, vertical and horizontal dynamic responses of the three-dimensional framework of a tyred overhead crane under the action of a moving trolley hoisting a swinging object were calculated using the finite element method and the direct integration method. Instead of the conventional moving force problem, where only the vertical inertia effect of the moving trolley is considered, the three-dimensional inertial effects due to the masses of both the moving trolley and the swinging object have been considered in this paper. Wu [17] has developed a finite element model for a scale crane rig in the laboratory

such that the dynamic characteristics of the scale crane rig can be predicted from the relevant features of the developed finite element model. Finite element modeling and experimental modal testing for the scale crane rig are carried out. Two kinds of coupling connecting the load cell and the tested structure for achieving the better experimental outcome are proposed. Then, the finite element model is modified, according to the experimental results, using various techniques. Also, Wu has presented in [18] a technique to replace the moving load by an equivalent moving finite element so that both the transverse and the longitudinal inertial effects due to the moving mass may easily be taken into account simultaneously. The mass, damping and stiffness matrices of the moving finite element are determined by the transverse inertia force, Coriolis force and centrifugal force of the moving mass, respectively. As it is conclusive from the recent references [17,18] considerations of the moving load problem by using and developing FEM is actual because modern FEM packages are not suited for the moving load problem, especially when the structure-vehicle interaction is to be considered.

2. DYNAMICS OF BRIDGE CRANE

Three-dimensional (3D) bridge cranes are widely used to transport heavy payloads in factories of different industries and becoming larger and faster. In order to improve the efficiency of payload transportation, the trolley of a crane should move to its destination as quickly and as precisely as possible. A crane consists of a hoisting mechanism (traditionally a hoisting line and a hook) and a support mechanism (e.g. trolley-girder for overhead cranes). The support mechanism moves the suspension point around the crane workspace, while the hoisting mechanism lifts and lowers the payload. The support mechanism in a bridge crane is composed of a trolley moving over a girder. The bridge is mounted on orthogonal railings in the horizontal plane, Fig. 1.

Figure 1. Bridge crane

In the past 50 years we have seen mounting interest in research of modeling and control of cranes [19], and many standard models have been already investigated. These models can be distinguished by different complexity in modeling and by the nature of neglected parameters. However, most of the presented models are very simple ones. Most of the standard commercial FEM softwares are fully capable to simulate such models. The most common modeling approaches are the lumped-mass and distributed mass approach, as well as the combination of the first two approaches [3]. A relatively recent review on cranes dynamics, modeling and control is given in

[19], but without considering the problems of the moving load influence on the dynamic response of cranes.

Following current trends in structural dynamics, followed by strong computational/software support, one can set up a model including more dynamic effects resulting in higher accuracy of obtained results. The goal of this paper is to present such model, with FEM simulation and solution by direct integration method. Having in mind that the speed of the entire bridge in horizontal plane (transverse direction) is usually low or the cranes do not travel in the transverse direction, and, therefore it is assumed that two space dimensions are sufficient to describe the payload movement.

For analyzing the responses of the crane model, the general approach is to divide the whole structure into two sections: fixed structure and the moving sub-structure. In the simplest model shown in Figure 2, we have moving force approach where uniform simply supported beam-framework is subjected to a constant vertical force-substructure moving with speed v. Analytical solutions for such a problem is given in [7], while FEM can be introduced for e.g. non-uniform beams by applying a technique which considers moving loads as functions of time [20]. This approach has two important roles. At first, for initial design process it gives satisfactory data for describing dynamic behavior of cranes. Moreover, it can be used for verification of advanced models and algorithms for obtaining desired responses.
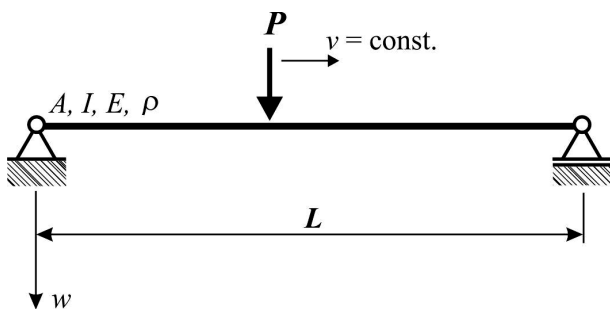

Figure 2. Uniform, simply supported beam subjected to a constant vertical force P moving at constant speed v

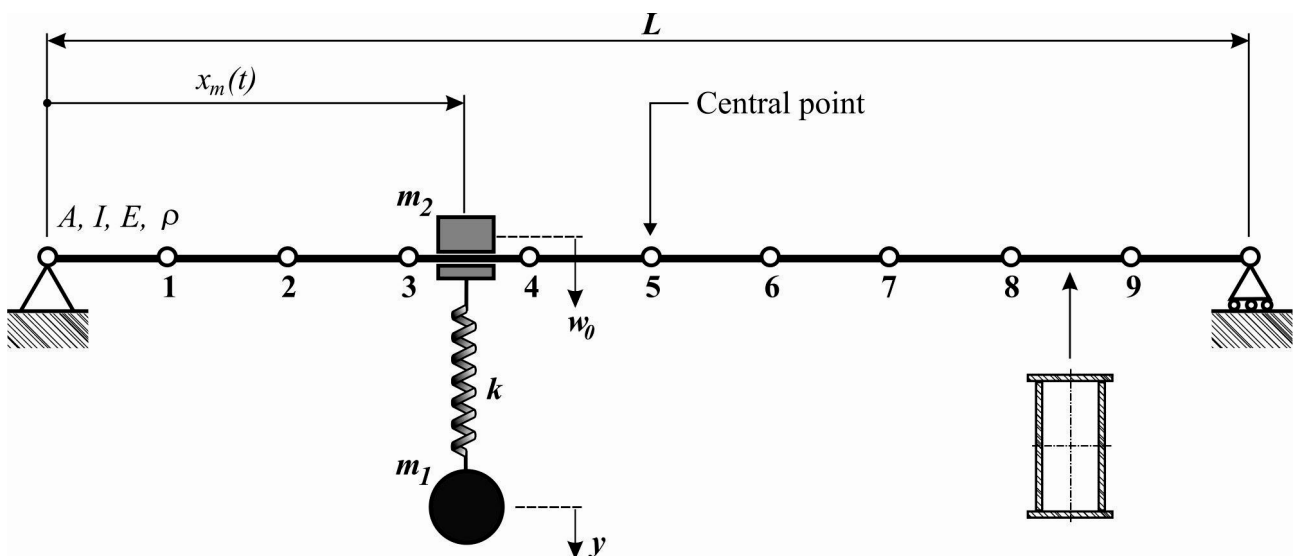Generally, the moving mass includes in itself the hoisting object and trolley and is often larger than the mass of the bridge crane structure, which implies the moving mass approach. This also implies structure-trolley interaction, thus forces between the structure and the substructure depend on the motions of both of them.

3. MODEL FORMULATION

Schematic presentation of the dynamic model is shown in Figure 3. The entire crane is first divided into two parts: the stationary beam structure and the moving substructure. The relation between the beam and the moving substructure is simplified into one moving load due to the assumption that a loading is symmetrically distributed on the bridge rail(s). The global position of the moving substructure on the beam is defined by coordinate $x_m(t)$.

The mass of the moving substructure (m_s) is composed of the mass of trolley m_2 and the mass of payload m_1, which are modeled as lumped masses. They are connected by a rope system of equivalent spring stiffness k. The trolley is assumed to be always in contact with the crane structure. Variable w_0 denotes the vertical deflection of m_2 and is equal to the dynamic deflection of the support beam at the position of contact. Vertical displacement of the suspended payload is denoted as y.

The beam structure is simply a supported uniform beam and its properties are Young's modulus E, volume density ρ, cross-sectional area A (box section), length L and sectional moment of inertia I. The model is done with discretization in 10 identical beam elements connected with nodes. The nodes between the supports of the beam structure represent places for obtaining the dynamic responses of the structure. Nodal displacements are u_i ($i = 1, 2, 3 \ldots 9$). The whole system has 10 DOF's.

4. OVERALL PROPERTY MATRICES AND FORMULATION OF PROBLEM

The governing equation of motion of multi-degree-of-freedom (MDOF) structural system is written as [21]:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F(t)\} \qquad (1)$$

where $[M]$, $[C]$, $[K]$ are the mass, damping and stiffness

Figure 3. Dynamic model of the system

matrices of the system, respectively; $\{\ddot{q}\}, \{\dot{q}\}, \{q\}$ are the acceleration, velocity and displacement vectors for the whole system, respectively, while $\{F(t)\}$ is the external force vector.

4.1 Overall stiffness and mass matrices of the beam

As it was mentioned above, in order to take the inertial effects of moving load into consideration, one must add the contribution of the equivalent moving mass matrix to the overall mass matrix of the beam itself $[M_{\text{st}}]$. It is assumed that the entire structure mass is concentrated at the beam nodes, Fig. 4a. So, the mass matrix of the beam is formulated here throughout the lumped system model, and is given by

$$[M_{\text{st}}]_{9 \times 9} = m_{\text{bi}}[I] \qquad (2)$$

where point mass is $m_{\text{bi}} = m_{\text{st}}/10$ $(i = 1 - 9)$, m_{st} is mass of the beam structure calculated from beam parameters and $[I]$ is the square identity matrix with size of 9.
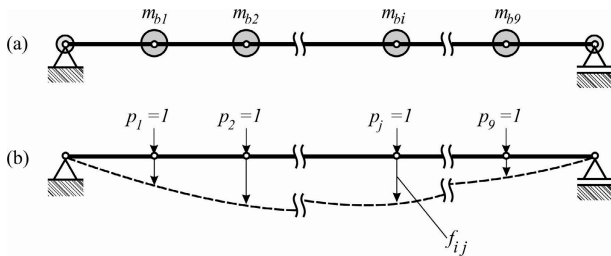

Figure 4. (a) lumping of the mass at nodes and (b) influence coefficients of the beam

The procedure for defining the stiffness properties of the beam structure is done by flexibility influence coefficient f_{ij}, noting the deflection of coordinate i due to unit load applied at coordinate j, Fig. 4b. Here, for the beam divided into 10 identical segments, where nodal translational displacements are being considered, the flexibility coefficients f_{ij} $(i, j = 1 - 9)$ are given by

- for $i \leq j$

$$f_{ij} = \frac{1L^3}{6EI} \left\{ \left(1 - \frac{j}{10}\right) \left(\frac{i}{10}\right) \cdot \right.$$

$$\left. \cdot \left[1 - \left(1 - \frac{j}{10}\right)^2 - \left(\frac{i}{10}\right)^2\right] \right\}, \qquad (3)$$

- for $i > j$

$$f_{ij} = \frac{1L^3}{6EI} \left\{ \left(1 - \frac{j}{10}\right) \left(\frac{i}{10}\right) \cdot \right.$$

$$\left. \cdot \left[1 - \left(1 - \frac{j}{10}\right)^2 - \left(\frac{i}{10}\right)^2\right] + \left(\frac{i-j}{10}\right)^3 \right\}. \qquad (4)$$

The flexibility matrix of the structure, $[f]_{9 \times 9}$ is obtained by using (3) and (4). With inversion of the flexibility matrix, one can obtain the stiffness matrix of the beam as

$$[K_{\text{st}}]_{9 \times 9} = [f]^{-1}. \qquad (5)$$

It is obvious that this formulation can be used for beams with other types of end supports like clamped-clamped, pin-clamped etc.

Overall damping matrix of beam structure is neglected here, because it is usually unobtainable. It can be only determined with known damping ratios [22].

4.2 Equivalent nodal forces and external force vector

The beam is subjected to a concentrated vertical force P, thus external forces on all the nodes are equal to zero except on the nodes of element s, Fig. 5, which is subjected to the concentrated force. According to [23], the external force vector takes the following form:

$$\{F(t)\} = \{0 \ 0 \ 0 .... f_1^{(s)} f_2^{(s)} f_3^{(s)} f_4^{(s)} .... 0 \ 0 \ 0\} \qquad (6)$$

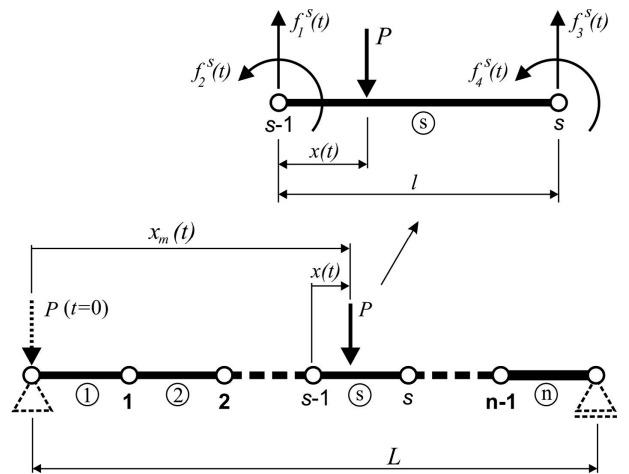where $f_i^{(s)}(t), (i = 1, 2, 3, 4)$, represent the equivalent nodal forces.

Figure 5. Nodal forces of the element s for a beam subjected to a concentrated vertical force

Nodal forces are determined with expression

$$\{f^{(s)}(t)\} = P\{N\} \qquad (7)$$

where P is the magnitude of the vertical force acting upon structure, and

$$\{N\} = [N_1 \ N_2 \ N_3 \ N_4]^T \qquad (8)$$

represent shape functions [21,23]

$$N_1 = 1 - 3\xi^2 + 2\xi^3 \qquad (9)$$

$$N_2 = l(\xi - 2\xi^2 + \xi^3) \qquad (10)$$

$$N_3 = 3\xi^2 - 2\xi^3 \qquad (11)$$

$$N_4 = l(-\xi^2 + \xi^3) \qquad (12)$$

$$\xi = \frac{x}{l} \qquad (13)$$

noting that l is the element length and x is distance along the element to the point of application of P, Fig.5.

Considering m time steps and choosing a time interval Δt, the total time is then given by

$$\tau = m \cdot \Delta t \qquad (14)$$

At any time $t = r \, \Delta t$ ($r = 1$ to m), the position of the moving force, relative to the left end of the beam, is given by

$$x_m(t) = v \cdot r \cdot \Delta t . \qquad (15)$$

One can find the element number s, which the moving mass is applied to at any time t, as

$$s = \text{Int}\left[\frac{x_m(t)}{l}\right] + 1 . \qquad (16)$$

The two nodes of the sth beam elements are $s-1$ and s. Therefore, the following equations for nodal forces and moments are formed when the moving force P is on the sth beam element ($s = 1$ to n) at any time $t = r \, \Delta t$ ($r = 1$ to m):

$$F_{s-1} = P \, N_1 = f_1^{(s)} \qquad (17)$$

$$F_s = P \, N_3 = f_3^{(s)} \qquad (18)$$

$$F_i = 0 \ (i = 1 \text{ to } n-1, \text{ except } s-1 \text{ and } s) \qquad (19)$$

$$M_{s-1} = P \, N_2 = f_2^{(s)} \qquad (20)$$

$$M_s = P \, N_4 = f_4^{(s)} \qquad (21)$$

$$M_i = 0 \ (i = 1 \text{ to } n-1, \text{ except } s-1 \text{ and } s) \qquad (22)$$

where N_1, N_2, N_3, N_4 are given by (9) – (12).

Equation (13) can be rewritten in terms of the global $x_m(t)$ instead of the local $x(t)$:

$$\xi = \frac{x_m(t) - (s-1)l}{l} . \qquad (23)$$

So, $[N]^T$ is a vector with zero entries except for those corresponding to the nodal displacement of the element on which the load is acting, i.e. for beam element with 4 DOF, the number of non-zero entries within vector will be four. This sub-vector is time dependent as the load moves from one position to another. The study brought in this paper uses the "simple/no moment" method [14], thus $N_2 = N_4 = 0$, which agrees with the accepted translational displacement of the beam. The procedure described above is done for a beam with 10 elements and 9 nodes ($i = 1 - 9$).

4.3 Moving mass formulation

As postulated, moving substructure consists of two masses, sprung mass m_1 and the unsprung mass m_2, which is assumed to be always in contact with the support beam structure. Following the procedure presented in [10] and by using FEM, it can be obtained the equations governing the dynamic systems as

$$m_1 \ddot{y} + k(y - w_0) = 0 \qquad (24)$$

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [N]^T P \qquad (25)$$

where interaction force is given by

$$P = m_1(g - \ddot{y}) + m_2(g - \ddot{w}_0) . \qquad (26)$$

The function $w(x,t)$ can be obtained from the shape functions and nodal displacement of the beam as

$$w = [N]\{u\} . \qquad (27)$$

The variable w_0 denotes the vertical dynamic deflection of m_2 and is equal to the dynamic deflection of the support beam in the contact position.

The time derivates of w_0 are

$$\dot{w}_0 = \frac{\partial w}{\partial x}\dot{x} + \frac{\partial w}{\partial t} \qquad (28)$$

$$\ddot{w}_0 = \frac{\partial^2 w}{\partial^2 x}\dot{x}^2 + 2\frac{\partial^2 w}{\partial x \partial t}\dot{x} + \frac{\partial w}{\partial x}\ddot{x} + \frac{\partial^2 w}{\partial^2 t} . \qquad (29)$$

Substituting (28) and (29) into (27), yields the following

$$\frac{\partial^2 w}{\partial^2 x} = [N]_x'' \{u\} \qquad \frac{\partial w}{\partial x} = [N]_x' \{u\}$$

$$\frac{\partial^2 w}{\partial x \partial t} = [N]_x' \{\dot{u}\} \qquad \frac{\partial^2 w}{\partial^2 t} = [N]_x \{\ddot{u}\} . \qquad (30)$$

Substituting (26) – (29) into (24) and (25) yields, in matrix form,

$$\begin{bmatrix} [M_{\text{st}}] + [M_1] & 0 \\ 0 & m_1 \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} [C_1] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \dot{y} \end{Bmatrix} +$$

$$+ \begin{bmatrix} [K_{\text{st}}] + [K_1] + [K_2] & -k[N]^T \\ -k[N] & k \end{bmatrix} \begin{Bmatrix} \{u\} \\ y \end{Bmatrix} =$$

$$= \begin{Bmatrix} [N]^T (m_1 + m_2)g \\ 0 \end{Bmatrix} \qquad (31)$$

where

$$[M_1] = m_2 [N]^T [N]$$

$$[C_1] = 2m_2 \dot{x} [N]^T [N]_x'$$

$$[K_1] = k [N]^T [N]$$

$$[K_2] = m_2 \dot{x}^2 [N]^T [N]_x'' + m_2 \ddot{x} [N]^T [N]_x' . \qquad (32)$$

One can see that symbols $[M]$, $[C]$, $[K]$ in (1) are called *instantaneous* matrices because they are time-dependent matrices composed of constant matrices due to the structure itself and time-dependent matrices due to the moving oscillator.

5. NUMERICAL RESULTS

The equation (31) is used for studying the dynamic response of a crane structure due to a moving oscillator and is solved by means of the direct step-by-step-integration method based on Newmark algorithm [24].

The characteristics of the steel-made structure of bridge crane are mass density $\rho = 7850$ kg/m^3 and Young modulus $E = 2.1 \cdot 10^{11}$ N/m^2. It is used push-to-

limit span of girder $L = 40$ m, with box tube cross-section with area $A = 0.04$ m^2 and sectional moment of inertia $I = 0.00667$ m^4. The beam is subjected to an overall nominal mass of 10000 kg which includes the mass of payload and trolley, moving with constant velocity.

Dynamic responses are investigated within this range of mass, with variation of hoist speed v and spring parameter k. Original in-house software MovMass is created to investigate this dynamic problem. The maximum interval for direct integration is $\Delta t = 0.05$ s. Structural damping is not included in the model. Dynamic deflections of all DOF are gained, but here are only presented the middle span deflection as main parameter in crane design.

5.1 Case 1: Moving force effect

At first, the problem is investigated by a moving force approach which is a common practice for verification of the given algorithm. The software is adjusted to find the responses (q_i, $i = 1 - 9$) for moving force approach by neglecting instantaneous matrices, with defined external force vector. The beam parameters are defined above, and force is 100000 N.

The dynamic responses at the mid-span of the beam are evaluated for various speed values, i.e. for various values of ratio T_1/τ. The symbol T_1 denotes the fundamental period of the beam and $\tau = L/v$ represents the travel time of the force from left end to the right end of the beam. The dynamic deflections for $q_i = u_i$, ($i = 1 - 9$) are obtained. The results for mid-span of the beam are shown in Figure 6 and are presented as the ratio of maximum dynamic deflection and maximum static deflection at the beam mid-span through nondimensional time.
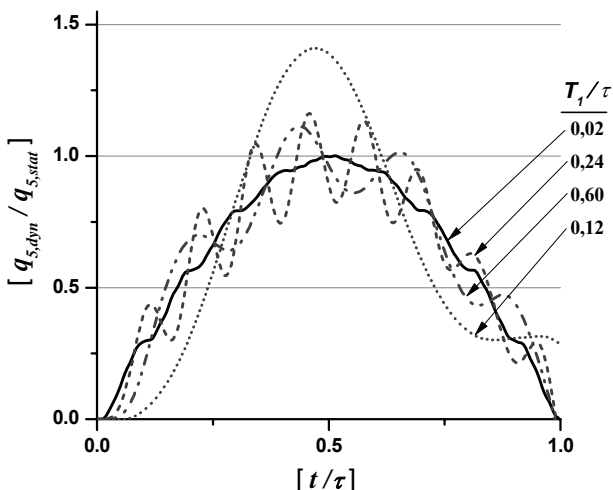

Figure 6. Response in the mid-span of a simply supported beam subjected to a concentrated force moving with a constant velocity

These results can be compared with analytical solution [7,9]. Moreover, first curve corresponding to $T_1/\tau = 0.02$, which refers to the speed of trolley 2 m/s and structural input data, is compared and validated by the exact solution given in [20]. For this case the dynamic response is close to the static deflection. It is obvious that for further increase of the speed the

dynamic deflection is also increasing. For $T_1/\tau = 0.12$, it is obtained that dynamic deflection is 1.45 times larger than the static one. However, such a large dynamic amplification factor (DAF) applies for speeds of crane trolleys that are not likely achievable. But, this fact can be used for problems in transportation engineering.

5.2 Case 2: Moving mass approach

In further examples are investigated two characteristic boundary velocities: 2 m/s, which is a real parameter and speed of 5 m/s which is a maximum speed for bridge cranes. Here, the masses are $m_2 = 10$ t, $m_1 = 0$. In this example the dynamic deflections for each of 9 adopted DOFs are obtained (q_i, $i = 1 - 9$). The inertial effects of the moving load are taken into account.

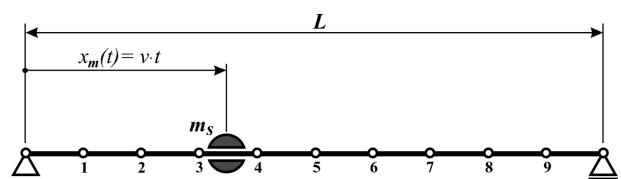The typical presentation of moving mass on simple beam structure is shown in Figure 7.

Figure 7. A simple beam subjected to moving mass

The dynamic responses for given travel speeds in the beam mid-span are shown in Figure 8, as the ratio of dynamic and static deflection. It is interesting to note that when the travel speed is relatively low – 2 m/s, the dynamic response resembles a static case. However, for the speed of 5 m/s we have DAF of 1.085 which comes from inertial effect of moving mass. Thus, vertical central displacement increases with the increase of the moving speed.
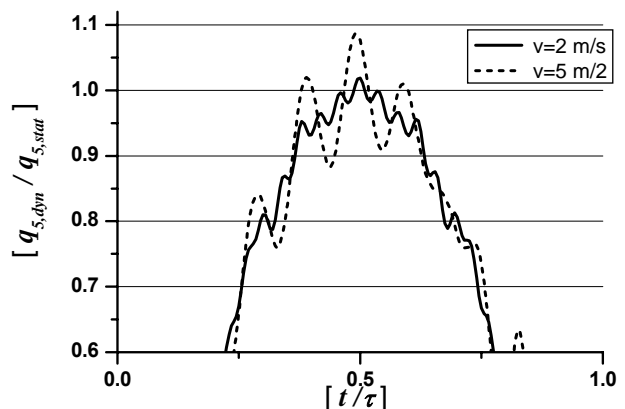

Figure 8. Central response due to moving mass

5.3 Effect of sprung mass

For this case a complete postulated algorithm is used, (30), which requires calculation of all the instantaneous matrices. Here, the masses of the moving substructure are $m_2 = 0$ and $m_1 = 10$ t. The substructure is moving with real maximum speed of 5 m/s.

Different coefficients are studied in order to investigate the influence of reeving system on the dynamic responses of the top beam.

Spring stiffness variation is included in parametric form as

$$k = \beta k_1,\qquad(33)$$

where β is ratio and

$$k_1 = \frac{48EI}{L^3}\qquad(34)$$

is stiffness of structure at middle span observed as single DOF (SDOF) system [25]. Three different coefficients are studied: β = 0.10, 1, and 10. Figure 9 shows the vertical central displacement of the beam for this case. For the coefficient β = 0.10, one can find that vertical central displacement resembles the static case.
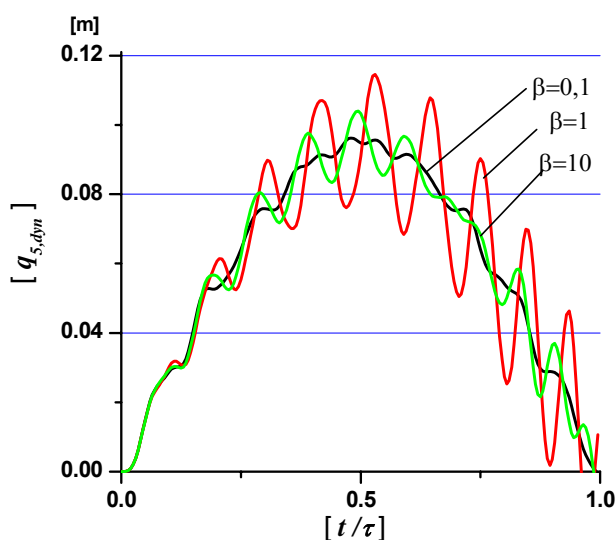

Figure 9. Mid-span displacement with variation of spring stiffness

Results for the coefficient β = 1 show higher vibration amplitude of the central point with the maximum value of deflection 11.5 cm. This increasment of 12 % from the static deflection is related to the natural frequency of the beam. The coefficient β = 10 assumes high stiffness reeving system which resembles the results from the moving mass approach.

6. CONCLUSION

This paper deals with the moving oscillator problem in dynamic responses at bridge cranes. Equations of motion for a given mathematical model are shown, and direct integration is used here with developed subroutines based on Newmark's method, because of the time variant nature of all system matrices.

The dynamic response is shown for the middle span beam. Numerical calculus indicates that deflection of the beam depends on trolley speed and masses. The beam deflection for given masses is increasing by increasing trolley speed. The influence of moving mass for trolley velocity of 5 m/s shows additional increasment of middle span displacement due to inertial effect. Moreover, the spring stiffness has influences on the vertical response of the structure. All of this, for the crane model with maximal realistic performances, can bring DAF to 1.12. However, for relatively low speeds (≤ 2 m/s) the beam behaves in a quasi-static manner with maximum beam deflection close to the middle of the beam.

The aim of this work is to emphasize the moving oscillator problem in crane structures. The algorithm can be applied to models of material handling machines with various structural types. Also, it can be applied to transportation engineering problems.

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NOMENCLATURE

$[M]$	structural mass matrix
$[C]$	structural damping matrix
$[K]$	structural stiffness matrix
$\{q\}$	displacement vector
$\{\dot{u}\}$	velocity vector
$\{\ddot{u}\}$	acceleration vector
$\{F(t)\}$	external force vector
N_i	shape functions ($i = 1 - 4$)
v	velocity of moving system
x_m	distance of the moving system from the left node of a beam structure
x	distance between the contact position of moving load and left end of beam element
l	beam element lenght
m_1	payload mass
m_2	trolley mass
$w(x,t)$	vertical displacement of a beam
w_0	vertical displacement at the contact position
$\dot{x}$	travel velocity of a moving system
$\ddot{x}$	travel acceleration of a moving system
k	spring stiffness
T_1	fundamental period of beam
τ	time required for moving system to travel from left end to right end of beam
A	cross-sectional area of a beam
I	sectional moment of inertia
E	Young's modulus
ρ	mass density of beam
L	length of a beam structure
g	gravitational constant

РАЗМАТРАЊЕ УТИЦАЈА ПОКРЕТНОГ ОСЦИЛАТОРА НА ДИНАМИЧКО ПОНАШАЊЕ МОСНЕ ДИЗАЛИЦЕ

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У раду је разматрано динамичко понашање носеће конструкције мосне дизалице услед дејства овешеног терета на колицима која се крећу детерминисаном брзином. Главни носач је моделиран као греда подељена на 10 елемената, колица као покретна маса и терет као маса, која је овешена на колица системом ужади моделираним опругом. Матрице инерције, пригушења и крутости се рачунају за сваки временски интервал, а еквивалентно спољашње оптерећење греде је формулисано кроз коначноелементни приступ проблему. Једначине кретања система са коначним бројем степена слободе су приказане за модел покретног осцилатора који се креће детерминисаном брзином по гредном носачу. Динамички одзив (угиб, брзине, убрзања) за све степене слободе је добијен решавањем једначина нумерички, методом директне интеграције. Верификација целог алгоритма је дата кроз поређење решења за динамички угиб просте греде под дејством покретне силе која се креће константном брзином. У раду је разматран утицај брзине кретања колица и крутости опруге на динамичко понашање носеће конструкције дизалице.