

## INFLUENCE OF TEMPERATURE ON BEHAVIOR OF THE INTERFACIAL CRACK BETWEEN THE TWO LAYERS

by

**Jelena M. DJOKOVIĆ<sup>a</sup>, Ružica R. NIKOLIĆ<sup>b\*</sup>, and Srdjan S. TADIĆ<sup>c</sup>**

<sup>a</sup>Technical faculty, University of Bor, Bor, Serbia

<sup>b</sup>Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia

<sup>c</sup>Innovation center, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

Original scientific paper

UDC: 620.178.38

DOI: 10.2298/TSCI100603013D

*In this paper is considered a problem of the semi-infinite crack at the interface between the two elastic isotropic layers in conditions of the environmental temperature change. The energy release rate needed for the crack growth along the interface was determined, for the case when the two-layered sample is cooled from the temperature of the layers joining down to the room temperature. It was noticed that the energy release rate increases with the temperature difference increase. In the paper is also presented the distribution of stresses in layers as a function of the temperature and the layers' thickness variations. Analysis is limited to the case when the bimaterial sample is exposed to uniform temperature.*

Key words: *interfacial crack, thermal stresses, two-layered sample.*

### Introduction

Thin films, coatings or multi-layer samples, made of different materials, are used for various purposes. The most common examples of application are the ceramic coatings on the metal substrate, metal layers on the polymer substrate, where the temperature at which these layers are applied is significantly higher than the working temperature; the thermo-insulating coatings like Al<sub>2</sub>O<sub>3</sub> on Ni-Cr-Al and Fe-Cr-Al alloys, hard transparent coatings on optic polymers, metal fibers on the polymer substrate in electronic modules or the photo-electric actuators.

When brittle coatings function in the presence of thermal gradients and high heat flux, they are susceptible to delamination and spalling. The most widely investigated examples are thermal barrier coatings used in turbines for power generation. Articles that analyze the mechanisms capable of providing sufficient energy release rate to drive

---

\* Corresponding author; e-mail: [ruzicarnikolic@yahoo.com](mailto:ruzicarnikolic@yahoo.com)

delamination have been presented in [1, 2]. Thermal barrier coating systems are susceptible to delamination failures in the presence of a large thermal gradient. Three possible causes of internal delamination are analyzed in [3]. Delamination of coatings initiated by small cracks paralleling the free surface is investigated in [4] under conditions of high thermal flux associated with a through-thickness temperature gradient. Certain aspects of residual life estimates of the high pressure turbine housing case, which is a thermal power plant component, were considered in [5]. The damages there appear in form of a dominant crack on the housing surface.

In the layers made of different materials, during the environmental temperature change, as a result of the difference in the thermal expansion coefficients, appear thermal stresses. Those stresses are causing the appearance of an interfacial crack. When such a crack is formed, the energy release rate needed for the crack propagation depends on stresses' intensities in both layers. If one assumes that the layers are made of the elastic isotropic materials, the stresses will depend upon the elastic and thermal characteristics of the layers' materials, as well as on the temperature variations. The driving force of the interfacial fracture in this case is the energy release rate  $G$ .

### Problem formulation

In order to solve the problem, the semi-infinite crack, at the interface between the two layers, under general loading conditions is considered. Each of the layers is homogeneous, isotropic and linearly elastic. The crack lies along the negative portion of the  $x$ -axis. Thicknesses of layers 1 and 2 are  $h_1$  and  $h_2$ , respectively. The two-layered sample is homogeneously loaded along three edges by forces and moments per unit length, as shown in figure 1. This case of the loaded sample was first analyzed by Suo and Hutchinson, [6]. Their solution can be used for interpreting behavior of the interfacial crack between the two layers in conditions of the variable environmental temperature.

Based on analysis by Suo and Hutchinson, [6], far away from the crack tip, the two-layered sample can be considered as the composite beam. The neutral axis lies at a distance  $\Delta$  from the bottom of layer 2, where

$$\Delta = \frac{\delta}{h_1} = \frac{1 + 2\Sigma\eta + \Sigma\eta^2}{2\eta(1 + \Sigma\eta)} \quad (1)$$

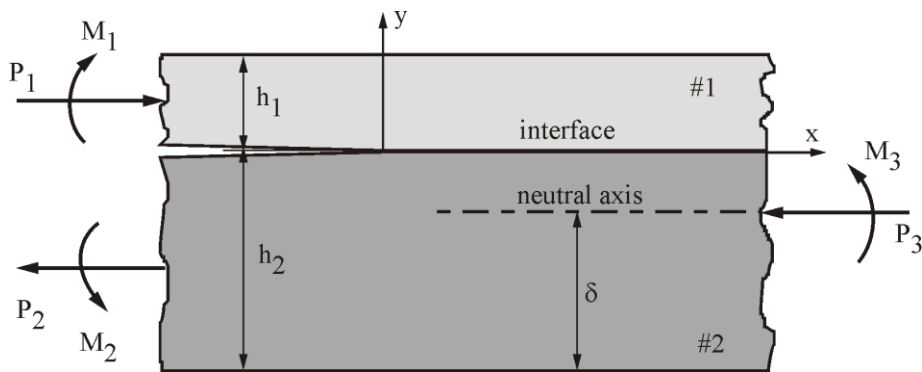
where  $\eta = h_1 / h_2$  – is the relative layer thickness and  $\Sigma = \bar{E}_1 / \bar{E}_2$  – is the ratio of the reduced elasticity moduli, with  $\bar{E}_1 = E_1 / (1 - \nu_1^2)$  and  $\bar{E}_2 = E_2 / (1 - \nu_2^2)$  for the plain strain conditions. Variables  $E_1$  and  $E_2$  represent the Young modules of layers 1 and 2, while  $\nu_1$  and  $\nu_2$  are their Poisson's ratios, respectively.

The two-layered sample is in the pure tension conditions, combined with the pure bending. The only stress component which is non zero is the normal stress  $\sigma_{xx}$ . The corresponding strain component is the linear function of a distance from the neutral axis, *i. e.*:

$$\varepsilon_{xx} = -\frac{1}{E_2} \left( \frac{P_3}{h_1 A} + \frac{M_3}{h_1^3 I} y \right) \quad (2)$$

where the non-dimensional variables of the cross section and the second area moment are defined as:

$$A = \frac{1}{\eta} + \Sigma, \quad I = \Sigma \left[ \left( \Delta - \frac{1}{\eta} \right)^2 - \left( \Delta - \frac{1}{\eta} \right) + \frac{1}{3} \right] + \frac{\Delta}{\eta} \left( \Delta - \frac{1}{\eta} \right) + \frac{1}{3\eta^3} \quad (3)$$



**Figure 1. Two-layer sample with an interfacial crack along the negative direction of the x-axis in general loading conditions**

By application of the superposition principle, the problem shown in figure 1 is reduced to the problem presented in figure 2, where the number of loading parameters, which control the crack behavior, is reduced to two, which represent the linear combination of the edge loads:

$$P = P_1 - C_1 P_3 - C_2 \frac{M_3}{h_1}, \quad M = M_1 - C_3 M_3 \quad (4)$$

where:

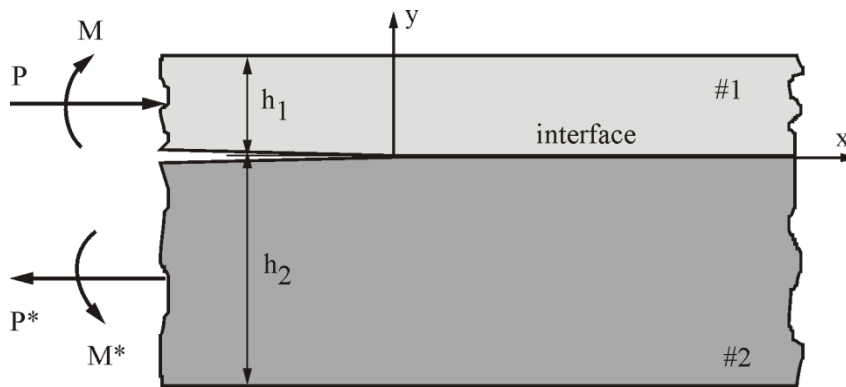
$$C_1 = \frac{\Sigma}{A}, \quad C_2 = \frac{\Sigma}{I} \left( \frac{1}{\eta} + \frac{1}{2} - \Delta \right) \quad \text{and} \quad C_3 = \frac{\Sigma}{12I} \quad (5)$$

are the geometric factors.

The energy release rate can be computed, within the plain strain conditions concept, as a difference between energies within the bulk far ahead and behind the crack tip. The result is the positive square form of  $P$  and  $M$ , which can be written as:

$$G = \frac{1}{2\bar{E}_1} \cdot \left[ \frac{P^2}{Uh_1} + \frac{M^2}{Vh_1^3} + 2 \frac{PM}{h_1^2 \sqrt{UV}} \cdot \sin \gamma \right] \quad (6)$$

where:  $U = \frac{1}{1 + \Sigma(4\eta + 6\eta^2 + 3\eta^3)}$ ,  $V = \frac{1}{12(1 + \Sigma\eta^3)}$ ,  $\gamma = \arcsin(6\Sigma\eta^2(1 + \eta)\sqrt{UV})$



**Figure 2. Reduced problem of the two-layered sample with the crack along the negative direction of the x-axis in general loading conditions and with  $M^* = M + P(h_1 + h_2)/2$**

The energy release rate determines intensity of a singularity in the crack tip vicinity, but it does not determine the mixed mode. This can be determined based on the complex stress intensity factor  $K$ , which in accordance with linearity and dimensional analysis can be written as:

$$K = K_1 + iK_2 = h_1^{-i\varepsilon} \sqrt{\frac{1-\alpha}{1-\beta^2}} \cdot \left( \frac{P}{\sqrt{2Uh_1}} - ie^{i\gamma} \frac{M}{\sqrt{2Vh_1^3}} \right) e^{i\omega} \quad (7)$$

where parameters  $\alpha$  and  $\beta$  represent the Dundurs' parameters, which are defined as, [7]:

$$\alpha = \frac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2}, \quad \beta = \frac{[\bar{E}_1(1-\nu_1)(1-2\nu_2) - \bar{E}_2(1-\nu_2)(1-2\nu_1)]}{2[\bar{E}_1(1-\nu_1)(1-2\nu_2) + \bar{E}_2(1-\nu_2)(1-2\nu_1)]} \quad (8)$$

while the parameter  $\varepsilon$  is called the bielastic constant or the oscillatory index and it is a characteristic of the interfacial crack. It is determined as, [8]:

$$\varepsilon = \frac{1}{2\pi} \ln \left( \frac{1-\beta}{1+\beta} \right) \quad (9)$$

Angle  $\omega \equiv \omega(\alpha, \beta, \eta)$  is a function of Dundurs parameters  $\alpha$  and  $\beta$  and relative layer thickness  $\eta$ . This function is defined in [9] based on solving the elastic problem and processing the tabular results of Suo and Hutchinson, [6].

If  $h$  is the referent length for the real and imaginary part of  $K$ , based on equation (7) one can write:

$$\begin{aligned} K_1 = \operatorname{Re}(Kh^{i\varepsilon}) &= \sqrt{\frac{1-\alpha}{1-\beta^2}} \cdot \left[ \frac{P}{\sqrt{2Uh}} \cos \omega + \frac{M}{\sqrt{2Vh^3}} \sin(\omega + \gamma) \right] \\ K_2 = \operatorname{Im}(Kh^{i\varepsilon}) &= \sqrt{\frac{1-\alpha}{1-\beta^2}} \cdot \left[ \frac{P}{\sqrt{2Uh}} \sin \omega - \frac{M}{\sqrt{2Vh^3}} \cos(\omega + \gamma) \right] \end{aligned} \quad (10)$$

The phase load angle, as the measure of relative value of Mode 2 with respect to Mode 1, in accordance with [10], for the referent length  $h$  ahead of the crack tip, will be:

$$\psi = \operatorname{arctg} \frac{K_2}{K_1} = \operatorname{arctg} \left[ \frac{\xi \sin \omega - \cos(\omega + \gamma)}{\cos \omega + \sin(\omega + \gamma)} \right] \quad (11)$$

where:  $\xi = \frac{Ph}{M} \sqrt{\frac{U}{V}}$ .

When the environmental temperature changes from the initial value  $T_0$  to  $T$ , the thermal stress will appear in layer 1, defined by the following expression, [1]:

$$\sigma^T = \frac{E_1}{1-\nu_1} (\alpha_2 - \alpha_1)(T - T_0) \quad (12)$$

where  $\alpha_1$  and  $\alpha_2$  are the thermal expansion coefficients of layers 1 and 2, respectively.

The normal stress distribution in the  $x$ -axis direction in both layers for problem shown in figure 2, far ahead of the crack tip is:

$$\begin{aligned} (\sigma_{xx})_1 &= -\frac{P}{h_1} + \frac{M}{I_1} \left( y - \frac{h_1}{2} \right) \quad za \quad y > 0 \\ (\sigma_{xx})_2 &= \frac{P}{h_2} + \frac{M}{I_2} \left( y + \frac{h_2}{2} \right) \quad za \quad y < 0 \end{aligned} \quad (13)$$

where the second area moments (moments of inertia) of layers 1 and 2 are:  $I_1 = h_1^3 / 12$  and  $I_2 = h_2^3 / 12$ , respectively.

The strain distribution in layers, as a function of stress and temperature is:

$$\begin{aligned}(\varepsilon_{xx})_1 &= \frac{(\sigma_{xx})_1}{E_1} + \alpha_1(T - T_0) \quad za \quad y > 0 \\(\varepsilon_{xx})_2 &= \frac{(\sigma_{xx})_2}{E_2} + \alpha_2(T - T_0) \quad za \quad y < 0\end{aligned}\quad (14)$$

The unknown values of equivalent loads  $P$  and  $M$ , in equation (13) are being determined from the boundary conditions for the problem shown in figure 2, which are: equality of layers 1 and 2 curvatures, *i. e.*  $\kappa_1 = \kappa_2$  and equality of the strain component  $\varepsilon_{xx}$  on the crack surfaces, *i. e.*  $(\varepsilon_{xx})_1 = (\varepsilon_{xx})_2$ . By applying these two conditions, one obtains from equation (13):

$$\begin{aligned}P &= \frac{\eta(1+\eta^3)}{(1+\eta)^5} \bar{E}_1 (\alpha_2 - \alpha_1)(T - T_0)(h_1 + h_2) \\M &= \frac{\eta^3}{2(1+\eta^3)} P(h_1 + h_2).\end{aligned}\quad (15)$$

By substituting equations (15) in the first of equations (13), after some algebraic rearrangements, the stress distribution in layer 1 is obtained as:

$$\bar{\sigma} = \frac{1 + \Sigma\eta^3}{(\Sigma\eta^2 - 1)^2 + 4\Sigma\eta(\eta + 1)^2} \sigma^T \quad (16)$$

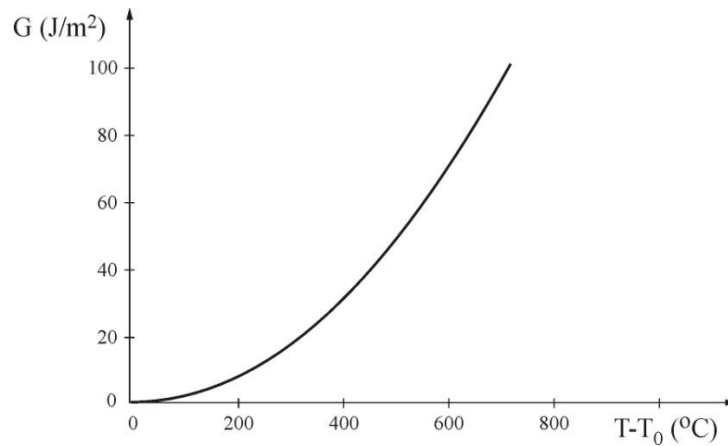
while the stress distribution at the surface of layer 1 is:

$$\sigma = \frac{1 - 3\Sigma\eta^2 - 2\Sigma\eta^3}{(\Sigma\eta^2 - 1)^2 + 4\Sigma\eta(\eta + 1)^2} \sigma^T \quad (17)$$

## Results and discussion

In figure 3 is presented the variation of the energy release in terms of the environmental temperature changes, for an arbitrary value of the relative layer thickness,  $(h_1/h_2)$ . The diagram is obtained based on equations (6) and (15) by application of the programming package *Mathematica*<sup>®</sup>.

From figure 3 one can notice, for the interfacial crack between the two layers, a tendency of the energy release rate increase with increase of the environmental temperature change. The interface destruction will occur when the energy release rate exceeds the value of the interface fracture toughness.



**Figure 3. Variation of the energy release rate with increase of the environmental temperature change for a single value of the layer's relative thickness**

In figure 4 is depicted the variation of the normalized stress inside layer 1 as a function of the relative layer thickness ( $h_1/h_2$ ), for three different bimaterial combinations, *i. e.* three different values of parameter  $\Sigma$ , which represents ratio of the reduced elasticity modules of layers 1 and 2. Diagrams are obtained based on equation (16), by application of *Mathematica*®.

In figure 5 is shown the variation of the normalized stress at the surface of layer 1 as a function of the relative layer thickness ( $h_1/h_2$ ), for three different bimaterial combinations. Diagrams are obtained based on equation (17).

Thermal stress,  $\sigma^T$  that is defined by equation (12) is a stress in layer 1, which will exist if layer 2 has the infinite thickness, *i. e.*  $\eta = h_1/h_2 \rightarrow 0$ . This means that the stress in layer 1 is basically thermal stress  $\sigma^T$  when the layer is thin ( $\eta \ll 1$ ), but it is reduced only to a portion of  $\sigma^T$  when layers have approximately same thicknesses, as can be seen from figure 4.

From figure 5 can be seen that the normalized stress on the surface of layer 1 has the negative value and that its values are higher than those of the stress in the layer.

From figures 4 and 5 can be concluded that, when layer 1 is significantly thinner than layer 2, ( $\eta \ll 1$ ), the normalized stresses at the surface and in the layer are equal to thermal stress  $\sigma^T$ . On the contrary, when the layer 1 is significantly thicker than layer 2, ( $\eta \rightarrow \infty$ ), the stress inside layer 1 is approaching zero. In the area between those two boundary cases, not only that the stress at the surface of layer 1 has the negative value, but its intensity is significantly higher than that of the stress inside the layer. This means that at the layer surface will appear the significant tensile stress, though it is exposed to compressive load.

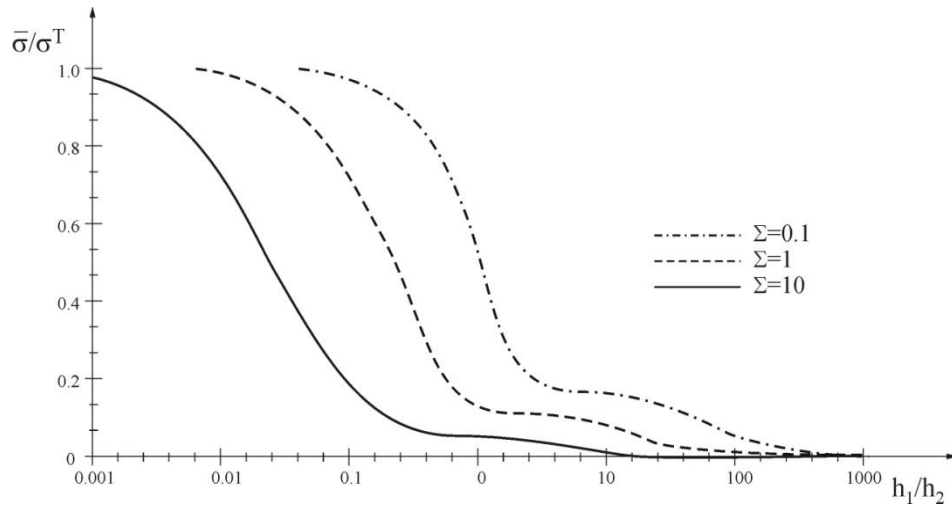


Figure 4. Normalized stress inside layer 1 as a function of the relative layer thickness ( $h_1/h_2$ )

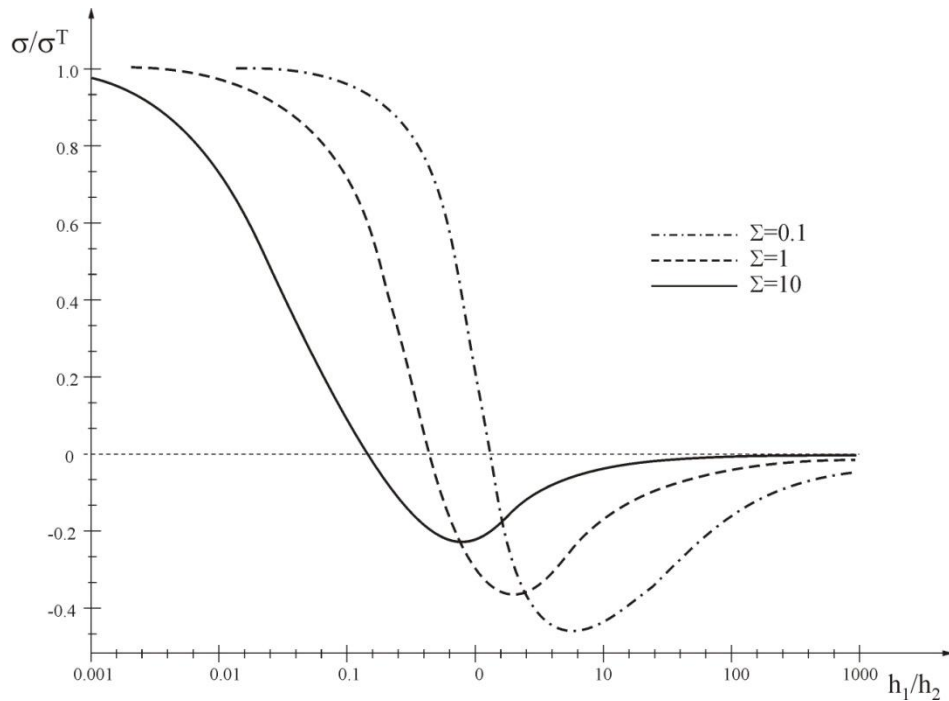


Figure 5. Normalized stress at the surface of layer 1 as a function of the relative layer thickness ( $h_1/h_2$ )

Values for  $\eta$ , which correspond to such tensile stresses, are responsible for appearance of the so-called "crazing" mode of the layer's destruction. From equation (17) can



be seen that the tensile stresses exist at the surface of layer 1 for all the values of the layer relative thickness ( $\eta = h_1 / h_2$ ), that are higher than the values that are satisfying the equation  $3\Sigma\eta^2 + 2\Sigma\eta^3 = 1$ . For the small values of  $\Sigma$ , *i. e.* when  $E_1 \ll E_2$ , the minimal value for relative layer thickness, for which still exist normal stresses is  $\eta = \sqrt[3]{(1/2\Sigma)}$ . For  $\Sigma=1$ , *i. e.* when layers' materials are the same, the minimal value is  $\eta=1/2$ . For the case when  $E_1 \gg E_2$ , *i. e.* for large values of  $\Sigma$ , the minimal value of  $\eta$  is  $\eta = \sqrt[2]{(1/3\Sigma)}$ .

## Conclusions

In the paper are presented the theoretical fundamentals for determination of the driving force for the interfacial fracture in the two layer sample in conditions of the environmental temperature variations. The energy release rate is determined in terms of the environmental temperature change increase. It was noticed that the energy release rate has the tendency of increase with increase of the temperature difference. Also presented is the variation of the stress inside and at the surface of the layer in terms of the layer relative thickness. It was found that the stress at the surface layer has the negative value with respect to the thermal stress, as well as that its intensity significantly exceeds the values of stresses inside the layer. This means that at the surface layer will appear the tensile stresses, which can lead to the crazing mode layer destruction.

Analysis performed in this paper is limited for the case when the two layer sample is exposed to uniform temperature. The case when the layers external surfaces temperatures are variable remains for the future analysis.

## Nomenclature

$A, I$	– non-dimensional variables of the cross section and second area moment, [–]	<i>Greek letters</i>	
$C_1, C_2, C_3$	– geometric factors	$\alpha, \beta$	– Dundur's parameters, [–]
$E_1, E_2$	– Young modules of layers 1 and 2, [Nm <sup>2</sup> ]	$\alpha_1, \alpha_2$	– thermal expansion coefficients of layers 1 and 2, [K <sup>-1</sup> ]
$\bar{E}_1, \bar{E}_2$	– reduced elasticity modules of layers 1 and 2, [Nm <sup>-2</sup> ]	$\delta$	– distance from neutral axis, [m]
$G$	– energy release rate, [Jm <sup>-2</sup> ]	$\Delta$	– relative distance, [–]
$h_1, h_2$	– thicknesses of layers 1 and 2, [m]	$\varepsilon$	– bielastic constant, [–]
$I_1, I_2$	– second area moments of layers 1 and 2, [m <sup>4</sup> ]	$\varepsilon_{xx}$	– strain component, [–]
$K$	– complex stress intensity factor, [MPam <sup>1/2</sup> ]	$\eta$	– relative layer thickness, [–]
$P, M$	– edge loads,	$\nu_1, \nu_2$	– Poisson's ratios of layers 1 and 2, [–]
$T, T_0$	– temperature, [K]	$\sigma$	– stress distribution at the surface of layer 1, [MPa]
		$\bar{\sigma}$	– stress distribution in layer 1, [MPa]
		$\sigma^T$	– thermal stress, [MPa]
		$\sigma_{xx}$	– normal stress in the x-axis direction, [MPa]
		$\Sigma$	– ratio of the reduced elasticity modules of layers 1 and 2, [–]
		$\psi$	– phase load angle, [°]

## References

- [1] Evans, A. G., Hutchinson J. W., The Thermo Mechanical Integrity of Thin Film and Multi-Layers, *Acta Metall. Mater.*, 43 (1995), 7, pp. 2507-2530
- [2] Evans, A. G., Hutchinson J. W., The Mechanics of Coating Delamination in Thermal Gradients, *Surface&Coatings Technology*, 201 (2007), 18, pp. 7905-7916
- [3] Hutchinson, J. W., Evans A. G., On The Delamination of Thermal Barrier Coatings in Thermal Gradients, *Surface&Coatings Technology*, 149 (2002), 2-3, pp. 179-184
- [4] Xue, Z., Evans, A. G., Hutchinson, J.W., Delamination Susceptibility of Coatings under High Thermal Flux, *J. Appl. Mech.*, 76 (2009), 4, pp. 1-7
- [5] Jovičić, G., *et al.*, Residual Life Estimation of a Power Plant Component – The High-Pressure Turbine Housing Case, *Thermal Science*, 13 (2009), 4, pp. 99-106
- [6] Suo, Z., Hutchinson, J. W., Interface Crack Between Two Elastic Layers, *Int. J. Fracture*, 43 (1990), 1, pp. 1-18
- [7] Dundurs, J., Elastic Interaction of Dislocations with Inhomogeneities, in: *Mathematical Theory of Dislocations* (Ed. T. Mura) ASME, New York, USA, 1969, pp. 77-114.
- [8] Rice, J. R., Elastic Fracture Mechanics Concepts for Interfacial Cracks, *J.Appl.Mech.*, 55 (1988), 1, pp. 98-103
- [9] Veljkovic, J. M., Nikolic, R. R., Application of the Interface Crack Concept to the Problem of a Crack Between a Thin Layer and a Substrate, *Fact universities*, 3 (2003), 13, pp. 573-581
- [10] Hutchinson, J. W., Suo, Z., Mixed Mode Cracking in Layered Materials, *Advances in Applied Mechanics*, 29 (1992), pp. 63-191

Paper submitted: March 28, 2010

Paper revised: June 6, 2010

Paper accepted: July 2, 2010