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MEASUREMENT OF STRAIN USING STEREOMETRY

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Subject review

This paper presents measurement of strain using stereometry method. It describes the use of digital stereometry to measure microstrains on the surface. This can be achieved by comparison of the images of point distribution on faced surface at different load levels. The examples present the possibility to measure strain concentration in the vicinity of crack tip on metallic sample. Applying two video cameras and corresponding software the image of strain distribution can be obtained in different forms, in 2D and 3D formats.

Keywords: *stereometric measurement, stress, strains, structural integrity, fitness-for-purpose*

Mjerenje deformacije uporabom stereometrije

Pregledni članak

Ovaj radi opisuje primjenu digitalne stereometrije za mjerenje deformacija na površini. To se postiže usporabom slika rasporeda točaka na označenoj površini uzoraka pri različitoj razini opterećenja i s time izazvanim deformacijama, što je pogodno za mjerenje deformacija na makro razini. Na primjerima je prikazana i mogućnost mjerenja koncentriranih deformacija na mikro razini u okolini vrha pukotine na metalnom uzorku. Primjenom dvije video kamere i odgovarajućeg softvera, slika raspodjele deformacija i preračunatog naprezanja može se dobiti u različitim oblicima, u 3D ili 2D formatu.

Ključne riječi: *stereometrijsko mjerenje, naprezanje, deformacije, cjelovitost konstrukcije, pogodnost-za-uspjeh*

1
Uvod
Introduction

Statistical data revealed that service failures are the consequence of static overloading (32 %), plastic collapse (14 %), fatigue (26 %), corrosion (17 %), and creep (11 %) [1, 6]. Timely detection of damage prior to a failure can be significantly reduced, e.g. by continuous monitoring of structure behavior. This can be achieved by stereometric measurement of strains during operation, followed by the structural integrity analysis and residual life assessment.

Concept of strain gages is elongation measurement of small initial measuring length. Basic concept of stereometric method is contactless measurement of point displacement of selected segment on structure surface exposed to loading and determination of induced strains and stresses [1].

2
Selecting correct measurement volume
Odabir ispravnog opsega mjerenja

The measuring volume depends on the size of the measuring object or on the size of the area to be analyzed. One should choose a measuring volume in which the measuring object or the measuring area fills the entire image as best as possible and ensure that the measuring object or the measuring area remains within the measuring volume in all deformation stages.

ARAMIS distinguishes between 2D and 3D measuring projects. If a sensor unit has two cameras, normally only a 3D project is reasonable. For the special case of very small specimens (< 3 mm), perhaps a 2D measuring project may be suitable. In this case, these specimen dimensions may result in large camera angles. A 3D point based on image data can only be computed if this point is visible to both cameras.

This is not always the case for distinct specimen geometries. In order to get information about displacements in a 2D project, it is necessary to define in the 2D parameters the 2D scaling (line distance) as a substitute for the scale bar information.

Local strains and stresses of selected points on a surface segment can be calculated based on measured relative displacement. One segment of the structural element surface with a points grid in a non-loaded state and their positions when loaded is shown in Figure 1. Point position is continuously monitored in space by measuring instantaneous coordinates, [2, 3]. This measuring system model with two video cameras is given in Figure 2. The *National Instruments* video camera is presented in Figure 3, [4], and the "Aramis system" [5], in Figure 4, with a tested specimen at the centre.

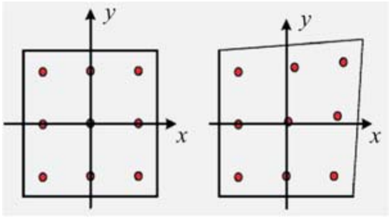

Figure 1 *The position of selected and labelled points on surface segment in non-loaded (left) and loaded state (right).*
Slika 1. *Položaj odabranih i obilježenih točaka na segmentu površine u ne-opterećenom (lijevo) i opterećenom stanju (desno)*

Digital stereometric measurement is based on photogrammetry allowing to obtain displacements of material surface points by comparing images in two successive stress states [1-5]. Hence, an array of points in the initial state is imposed on a micrograph – a grid of rows and columns with a position defined by expression:

$$r(u,v)=\frac{N\cdot\sum_i I_i\cdot M_i-\left(\sum_i I_i\right)\cdot\left(\sum_i M_i\right)}{\sqrt{N\cdot\sum_i I_i^2-\left(\sum_i I_i\right)^2-N\cdot\sum_i M_i^2\cdot\left(\sum_i M_i\right)^2}}\qquad(1)$$

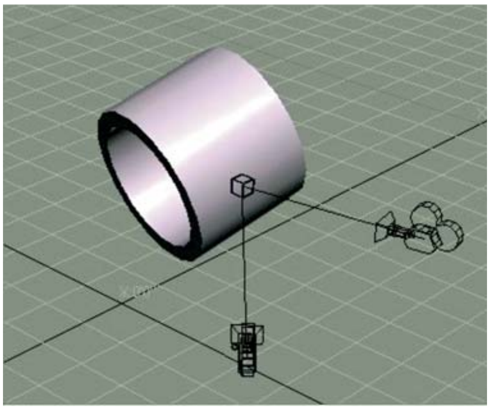

Figure 2 *Recording of strains on structural surface with two video cameras (scheme)* [5]
Slika 2. *Snimanje deformacija na površini konstrukcije s dvije video kamere (shema)* [5]

Figure 3 *Video cameras National Instruments* [4]
Slika 3. *Video kamere National Instruments* [4]

Figure 4 *Video cameras "Aramis" with the tested sample at the centre* [5]
Slika 4. *Video kamere "Aramis" s ispitivanim uzorkom u sredini* [5]

Displacement change monitoring requires a procedure with an algorithm for normalized image control, where $r(u,v)$ defines the correlation degree between image zones, I is the stressed zone of image region, M is the model of pixel location (x_i, y_i), I_i is the pixel location zone $(u+x_i, v+y_i)$, N is the number of pixels in the image model, i ranges from 1 to N, [6]. The trained image region is a square array (matrix) of pixels with sides N around each grid measurement point. A correlation value $r = 1$ indicates a perfect matching, correlation is nil at $r = 0$, and for $r = -1$ complete mismatch is given. This algorithm determines the surface points position on deformed material relative to initial image. Rotation is not accounted. Image processing, alignment, and correlation of measurement functions is done with the software by LabView, [7] – a graphic programming language adapted for Pentium PC. The software uses the correlation index (r) within the search region to interpolate between pixel locations and estimated subpixel displacement. After displacement components are determined in each measurement point, a two-dimensional stress tensor is calculated by smoothing and differentiating the displacement field, [6, 8].

2.1
Position of coordinate system
Položaj koordinatnog sustava

The position of the coordinate system depends on the calibration of the cameras and usually has no logical relation to the specimen. The coordinate system allows for unambiguously describing the position of points in the 3D space by stating three numerical values (X, Y, Z coordinates). The point where all numerical values are 0 is also called the origin of the coordinate system. Depending on the measurement task, the strain and displacement data of a measuring project sometimes should be transformed into a defined coordinate system in order to be interpreted correctly.

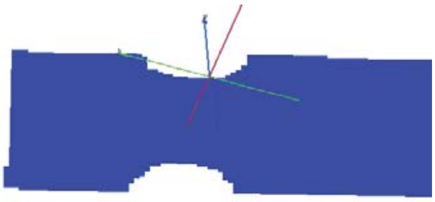

Figure 5 *Specimen in stage 0*
Slika 5. *Uzorak u stanju 0*

Often it is necessary to define the coordinate system based on the geometry of the specimen. So, for example, edges of the specimen or drilled holes are important to define axes or the origin, Fig. 5. For ARAMIS projects which observe a 3D deformation simultaneously by several ARAMIS sensor units, a transformation by reference points (circular markers) is required. The 3D coordinates of these reference points were previously recorded by the photogrammetric system TRITOP.

The ARAMIS sensor units partly recorded the same reference points. In a last step, the measuring data from the different views of the ARAMIS sensor units are transformed into the coordinate system of the TRITOP reference points by means of the best-fit method. This method is also suitable for static deformation projects in

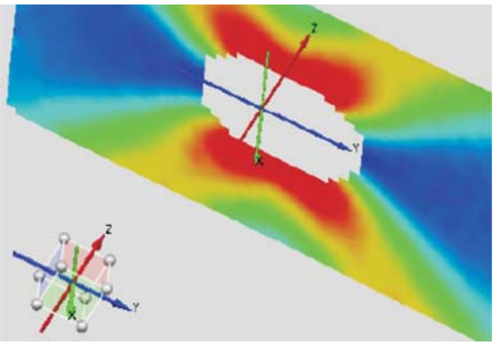

Figure 6 *Possibilities to display the coordinate system in the 3D view*
Slika 6. *Mogućnost prikazivnja koordinatnog sustava u 3D pogledu*

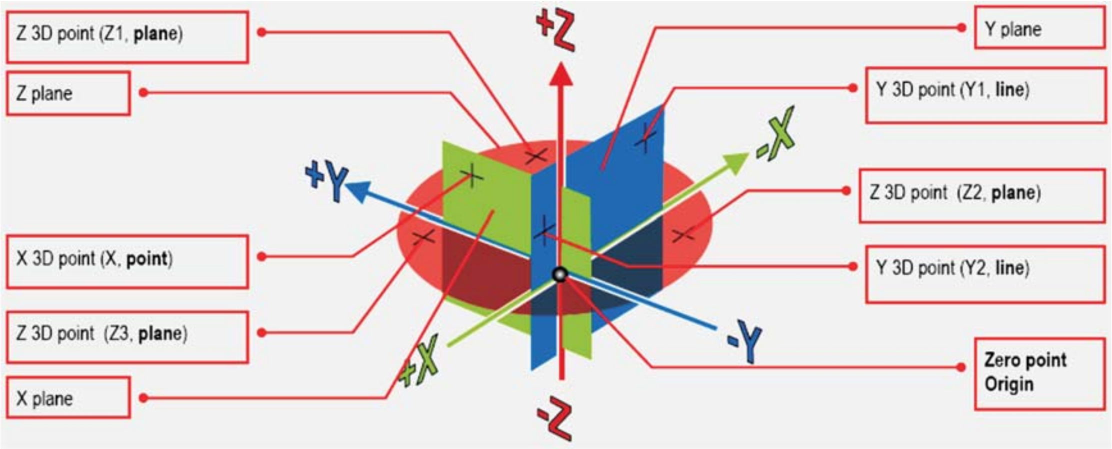

***Figure** 7 3-2-1 Transformation of coordinate system* [5]
***Slika 7.** 3-2-1 transformacija koordinatnog sustava* [5]

which the static deformations are captured with just one ARAMIS sensor from different positions and are then transformed into the same coordinate system using the common reference points.

2.2
Visualization of the coordinate system
Vizualizacija koordinatnog sustava

ARAMIS can show the coordinate system in the left bottom corner of the screen, Fig 6. It is displayed as a dice and serves as guide for easy rotating the measuring object. By clicking on the axes or the corner points you may rotate the measuring object into different views. In addition, you may display the coordinate system in its origin or hide it completely.

2.3
3-2-1 Transformation of coordinate system
3-2-1 Transformacija koordinatnog sustava

3-2-1 Transformation of coordinate system is one of the mostly used methods. 3-2-1 means that three 3D points (Z1, Z2, Z3, located as far as possible from each other and not in a line) describe a plane, two additional 3D points describe a line (Y1, Y2, located as far as possible from each other in the X-axis) and one 3D point describes a point (X).

For the transformation method ZZZ-YY-X means the following: Three Z points (Z1, Z2, Z3, red plane) define the Z plane. The additional two Y points (Y1, Y2, blue plane) define the Y plane. The X point (X, green plane) now defines the X plane. At the intersection of the planes is the zero point of the coordinate system. Figure 7 illustrates these connections. Of course, other transformations like XXX-YY-Z are possible as well.

The example shows the factual relations using the minimum number of points required for this transformation method. You may use reference points, pixel points or 3D points. In this case, the points define the coordinate system directly. It is important that the points reliably describe the required coordinate system.

3
Definition of *x-y* strain values in 2D coordinate system
Definicija vrijednosti deformacija *x-y* u 2D koordinatnom sustavu

Values ε_x, ε_y and ε_{xy} can directly be read from the symmetric stretch tensor **U** with with the following form:

$$\mathbf{U} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} = \begin{pmatrix} 1+\varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{xy} & 1+\varepsilon_y \end{pmatrix} \tag{2}$$

The strain values ε_x, ε_y and ε_{xy} have the disadvantage of being defined as dependent on the coordinate system. The geometrical interpretation of strain values is described with the following example values: ε_x=40 %, ε_y=0 % and ε_{xy}=0,2.

So the stretch tensor is given by:

$$\mathbf{U} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} = \begin{pmatrix} 1{,}4 & 0{,}2 \\ 0{,}2 & 1 \end{pmatrix}$$

Regarding a unit square in the 2D space (points (0/0), (0/1), (1/0), (1/1)) the deformation introduced by this stretch tensor is shown in Figure 8.

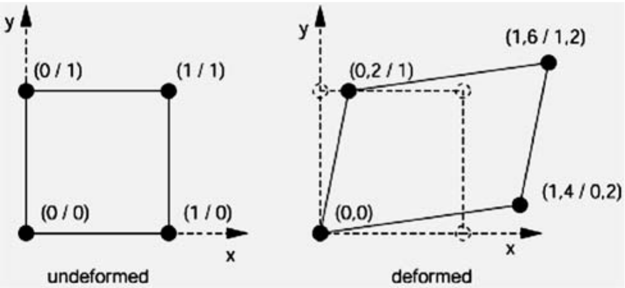

***Figure 8** Example for the deformation of a unit square*
***Slika 8.** Primjer deformacije jediničnog kvadrata*

For the geometrical interpretation of the value ε_{xy} the shear angle γ_{xy} is used. This angle describes the change of an angle of 90° in the undeformed state to a new angle in the deformed state. For large strain values and angles as used in this example the assumption for small strains from the elastic strain theories must NOT be used:

$$\gamma_{xy} \neq 2\varepsilon_{xy} \tag{3}$$

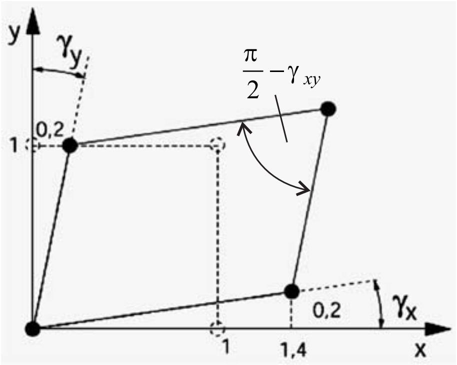

Figure 9 *Shear angle definition*
Slika 9. *Određivanje kuta smika*

Based on the used example values, the definition of the shear angle can be separated generally from Figure 9 as follows:

$$\gamma_{xy} = \gamma_x + \gamma_y = 0{,}339\ \text{rad, where are:} \tag{4}$$

$$\gamma_x = \arctan\left(\frac{\varepsilon_{xy}}{1+\varepsilon_x}\right) = \arctan\left(\frac{0{,}2}{1{,}4}\right) = 0{,}142\ \text{rad}$$

$$\gamma_y = \arctan\left(\frac{\varepsilon_{xy}}{1+\varepsilon_y}\right) = \arctan\left(\frac{0{,}2}{1}\right) = 0{,}197\ \text{rad}$$

As given by this example, the values for γ_x and γ_y can be different. With a symmetric stretch tensor only a parallelogram can be realized for the local deformation field. The fixed values for γ_x and γ_y show that the orientation of the parallelogram to the coordinate system is fixed. The stretch tensor cannot describe rotations. This coordinate system is defined as *x*''-*y*'' system.

4
Definition of the *x-y* strain values and the strain direction in 3D
Određivanje vrijednosti deformacije *x-y* i smjera deformacije u 3D

The description so far showed in detail the calculation of strain in 2D. But the measurement data in general consists of 3D points of the specimen's surface. To be able to use the above defined 2D models of calculation, an extended definition for the local directions is needed. The local strain coordinate systems must be tangential to the local surface and for the strain calculation the 3D data must be transformed into 2D space.

4.1
Definition of strain directions in 3D
Određivanje smjera deformacije u 3D

In Figure 10 the definition of the local strain directions is shown. The global coordinate system *x-y-z* can not be used in general for the local strain values. The *x-y-z* coordinate system in general is not parallel to the local tangential directions. For the local strain calculation in ARAMIS and ARGUS an *x*'-*y*' coordinate system is defined for the undeformed state as follows:

For each point (e.g. the point P_1 in Figure 10)

- the local strain direction *x*' is:
  - tangential to the surface of the local point
  - parallel to the *x-z* plane
- the local direction *y*' is:
  - tangential to the surface of the local point
  - perpendicular to the local *x*'.

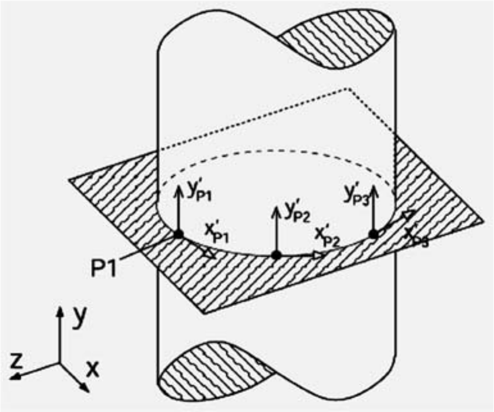

Figure 10 *Definition of the undeformed local surface strain coordinate system in 3D based on a plane parallel to x-z*
Slika 10. *Određivanje nedeformirane lokalne površinske deformacije koordinatnog sustava u 3D zasnovane na ravnini paralelnoj s x-z*

In Figure 10 the local coordinate system ($x'_{P1}-y'_{P1}$; $x'_{P2}-y'_{P2}$; $x'_{P3}-y'_{P3}$) is shown for a cylindrical specimen for three different points (P_1, P_2, P_3). In this case the global y direction is parallel to the axis of the cylinder. For this special case the y' directions for all surface points are parallel to the global y direction.

In general, both directions (*x*' and *y*') are not parallel to the global coordinate system. This is shown in Figure 11. The dashed ellipses are parallel to the *x-z* plane and the different local *y*' directions are tangential to the surface. Equal to Figure 11a for the deformed state again an *x*''-*y*'' coordinate system must be introduced as shown in Figure 11b. In the deformed state the *x*''-*y*'' strain directions are still tangential to the surface in the local 3D points and are defined by the stretch tensor in the same way as in the 2D situation.

The unit square is deformed to a parallelogram. The geometry of the parallelogram together with the stretch tensor (γ_x and γ_y) define the local strain directions (*x*'' and *y*'') in the deformed state.

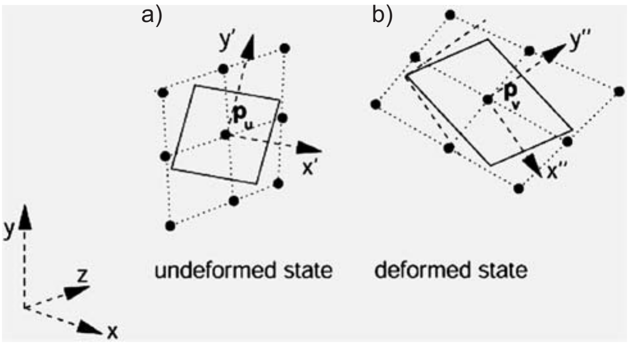

Figure 11 *Definition of the local surface strain coordinate system in 3D*
Slika 11. *Definicija određene površine deformacije koordinatnog sustava u 3D*

Parallel to the definition of the directions the 3D data must be transformed into the 2D space. For this, two different models can be used. These models are based on planes or splines.

4.2
The plane model
Ravninski model

The first model assumes that the local neighborhood of a point can be well approximated by a tangential plane. Due to the arbitrary deformation of the surface, the tangential plane needs to be calculated separately for the deformed and undeformed state. The points in the local neighborhood are then projected perpendicularly onto the tangential plane. The result is two sets of points, for the deformed and undeformed state, in the two-dimensional space in which the strain now can be calculated. Summarized, this process consists of the following tasks:

- Calculation of the tangential plane.
- Transformation of the 3D neighborhoods into the tangential planes.
- Coordinate transformation of the tangential plane into the 2D space (x'-y' and x''-y'' coordinate system).
- Calculation of the deformation gradient tensor from the 2D sets of points.

4.3
The spline model
Zakrivljeni model

The tangential model described above provides good results as long as the assumption of the linearization of a local neighborhood of points is valid. In deep drawing, the deformed materials have in part strong locally curved planes. The problem now is to apply the characteristics to be measured to the respective object in such a frequency that the assumption of local linearity is still given. However, this characteristic can hardly be provided in reality. Therefore, it is better to use other models which are more accurate in modeling the true shape of the surface. Splines are a good model for continuously curved lines. In order to calculate the side length not only according to a linear model, it is necessary to have more information than two points on a side. This means that the adjacent points of a four-sided facet have to be included in the calculations. Figure 12 shows the adjacent points of the hatched four-sided facet. In the facet, the side lengths are calculated using the formed splines. The resulting lengths can be used to construct a quadrangle in the two-dimensional space. Now, the strain calculations described above can be used.

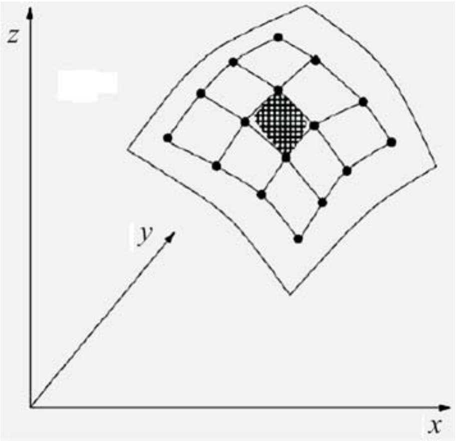

Figure 12 *Four-sided facet with adjacent points*
Slika 12. *Četverostrana ploha sa susjednim točkama*

5
Application of stereometric measurement at macro scale
Primjena stereometrijskog mjerenja u makro mjerilu

Continuous monitoring of a real structure substantially differs from strain state monitoring in laboratory conditions. The reason is that in laboratory conditions, sample strains are continuously monitored by stereometric measurement and the experiment runs until fracture of a sample, whereas real structure is monitored during predetermined time interval and the obtained images are used structural integrity and life assessment.

As an illustration of testing by stereometric measurement in the laboratory, Figure 13 shows the disposition of an experimental research with two "Aramis" video cameras directed to the fracture mechanics' specimen, [5]. In the scope of MOSTIS project, [10], the tensile specimen of M(T) type with two symmetrical cracks emanating from the central circular hole had been tested according to ASTM standard, [11,12]. By applying the corresponding software, [9], it is possible to determine the stress field in the specimen that has fractured in the experiment, with considerable plastic deformation, Figure 14.

Figure 13 *Continuous stereometric recording in laboratory conditions, testing of the tensile specimen type M(T)* [5]
Slika 13. *Kontinuirano stereometrijsko praćenje u laboratorijskim uvjetima, mjerenje rastezanja na uzorku tipa M(T)* [5]

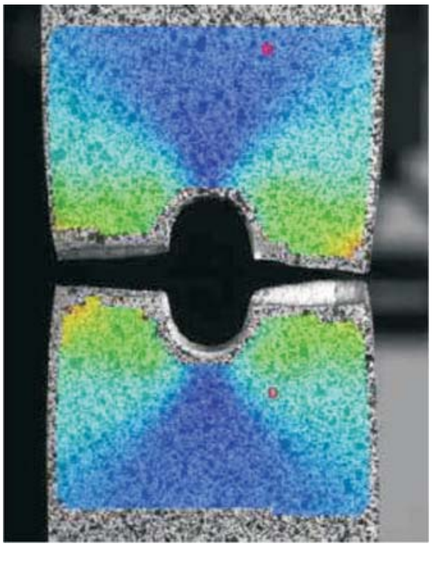

Figure 14 *Stress distribution in a fractured M(T) specimen, obtained by software analysis of stress state* [5, 10]
Slika 14. *Raspored naprezanja na prelomljenom uzorku dobiven softverskom analizom stanja naprezanja* [5, 10]

However, continuous damage monitoring during operation in case of real structure is not simple. When the procedure is applied to the real structure, stationary camera positioning is not always possible, and so then the inspection is performed at shutdowns between two operational stages. Cameras should be repositioned for each measurement and calibrated before measuring. Images are recorded in various load levels after inputting the measuring project into the software. It is to notice that stereometric measurement allows only damages visible to the naked eye to be tracked (e.g. surface fatigue cracks of proper size), or clearly expressed local strains. Successful software elaboration gives results in 3D image, [3], and other presentations in the form of typical reports or statistical data can be obtained also.

6

Analysis of a crack

Analiza pukotine

The grid of points, presented in Figure 1 is used for determining the strain field by contactless stereometric measurement of indicated point's displacement and by comparing spatial images (3D) at the initial stage and applied loads of different levels. This can be applied also to cracked samples in the research, when data on the microlevel are required. Here, one typical application for crack analysis will be considered: in the steel welded joint specimen test [1]. In the performed research, two stationary positioned video cameras were used for continuous monitoring of the material in the vicinity of the crack tip.

Thanks to data processing by computer, the principle of stereometric measurement offers an additional advantage. It is possible to magnify a considered region up to a predefined level, owing to equipment and software capacity. That means, for example, that material behaviour in the crack tip vicinity can be monitored sufficiently clear at the micro level, and stress and strain state accurately analysed. With such data, structural integrity assessment is also more accurate and the safety of the considered structure is easily controlled.

Stereometric measurement is successfully applied in continuous monitoring of cracked specimen behaviour in tests, Figure 15 [1]. Crack development through the weld metal of welded joint of heterogeneous microstructure and mechanical properties is the subject of this research.

The positioned grid is of significant size compared to crack tip singularity. The distance between measurement points A to F in both directions is larger than 1 mm. Having in mind strain continuity, the final result is presented in the form of spatial strain distribution around crack tip, Figure 16.

Regarding the accuracy of results, it is to have in mind:

- strain distribution is continuous, although non-uniform,
- the diagram in Fig. 16 is obtained based on measurements at points indicated in Fig. 15b,
- for the purpose of this research – in determining the location and size of maximal strain, the accuracy obtained by applied measurement grid is satisfactory, so neither a denser grid is necessary, nor higher magnification.

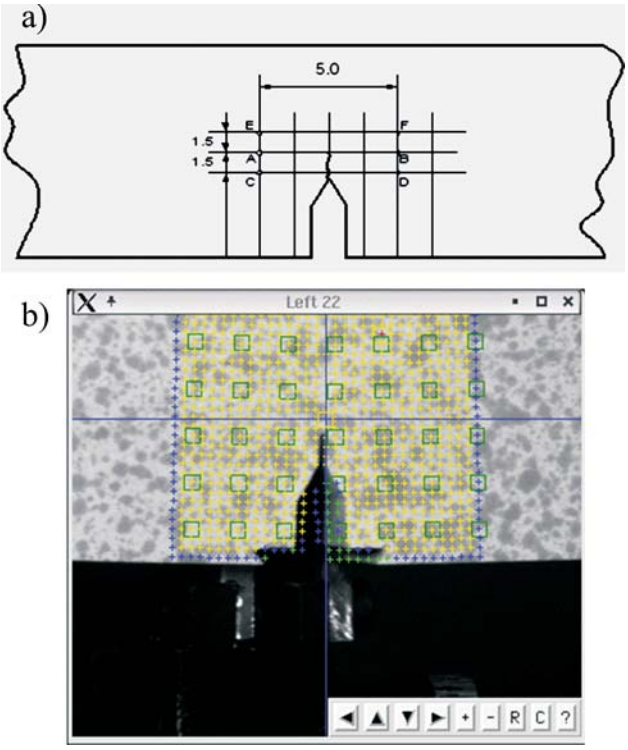

Figure 15 *Welded joint specimen pre-cracked in weld metal* [1] *(a - scheme of depicted grid, b - view of loaded specimen)*
Slika 15. *Uzorak zavarenog spoja prethodno napuknutog u materijalu zavara* [1] *(a - shema dobivene rešetke, b – izgled opterećenog uzorka)*

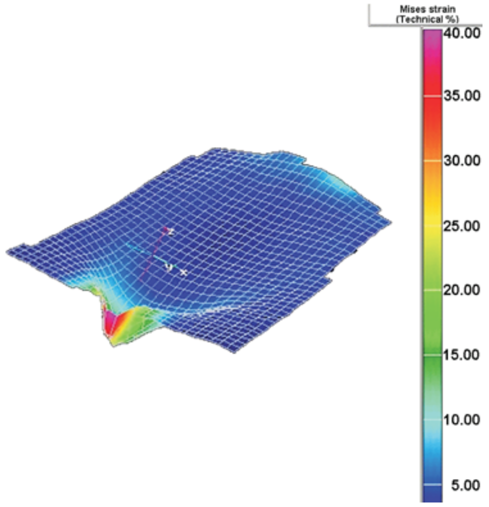

Figure 16 *Strain distribution around crack tip obtained by the stereometric measurement procedure* [1]
Slika 16. *Distribucija naprezanja oko vrha pukotine dobivena stereometrijskim mjerenjem* [1]

7

Discusion

Diskusija

Locations of each homologous point pair can be determined for all three displacements in the plane (2D) from images in the initial and loaded state, and the spatial (3D) coordinates can be determined via back projection. When distinctive 3D spatial point coordinates of each measurement of the considered surface are determined, strains can be calculated, and also stresses by corresponding formulae. The selected system calculates surface strain through transformation of 3D displacement distribution into a 2D displacement distribution, thus strain is calculated in 2D space, [14]. Initially, for each point, a tangential plane is calculated for both states. The obtained rectangular area is referred to as the object facet. Points in this object facet are then projected onto the tangential plane in the direction of its normal vector, which renders the problem as planar. Further calculation of deformation gradient is possible by using the theory of large deformations which allows large plastic deformations as well as rigid body motion, [15]. Extreme strain gradients and excessive local out-of-plane displacement can be expected in the tiny area of specimen, near the crack tip.

8
Conclusions
Zaključci

The complexity of structural integrity research may be recognised from the diversity of data necessary for fitness-for-purpose assessment. The direction of research is also shown here, consistent with needs defined through experience and practice of structural integrity assessment and fitness-for-purpose assessment procedures.

Analysis of local microstrains and stresses on the surface may lead to better understanding of damage and fracture development. The stereometric method enables insight into the stress–strain field corresponding to the microstructure of the tested material. Owing to this, one can expect extended applications of this technique for experimental research of fracture in the future.

9
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