#### **ORIGINAL ARTICLE**



# Boosting ensembles for estimation of discharge coefficient and through flow discharge in broad-crested gabion weirs

Aliasghar Azma $^1$  · Mohammad Tavakol Sadrabadi $^2$  · Yakun Liu $^1$  · Masoumeh Azma $^3$  · Di Zhang $^1$  · Ze Cao $^1$  · Zhuoyue Li $^1$ 

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#### **Abstract**

Gabion weirs are environment-friendly structures widely used for irrigation and drainage network purposes. These structures' hydraulic performance is fundamentally different from solid weirs' due to their porosity and the existence of a through-flow discharge. This paper investigates the reliability and suitability of a number of Machine learning models for estimation of hydraulic performance of gabion weirs. Generally, three different Boosting ensemble models, including Gradient Boosting, XGBoost, and CatBoost, are compared to the well-known Random Forest and a Stacked Regression model, with respect to their accuracy in prediction of the discharge coefficient and through-flow discharge ratio of gabion weirs in free flow conditions. The Bayesian optimization approach is used to fine-tune model hyper-parameters automatically. Recursive feature elimination analysis is also performed to find optimum combination of features for each model. Results indicate that the CatBoost model has outperformed other models in terms of estimating the through flow discharge ratio ( $Q_{in}/Q_t$ ) with  $R^2 = 0.982$ , while both XGBoost and CatBoost models have shown close performance in terms of estimating the discharge coefficient ( $C_d$ ) with  $R^2$  of CatBoost equal to 0.994 and  $R^2$  of XGBoost equal to 0.992. Weakest results were also produced by Decision tree regressor with  $R^2 = 0.821$  and 0.865 for estimation of  $C_d$  and Qin/Qt values.

Keywords Gabion weirs · Discharge coefficient · XGBOOst · CatBoost

#### Introduction

Gabion weirs are porous structures made of boulders with minimal construction cost and negative environmental effects (Yue et al. 2021, Zhang et al. 2019a,b). Generally, these structures benefit from higher stability, durability, and proper hydraulic performance (Zhang et al. 2019a,b, Fang et al. 2021, Liu et al. 2022). Besides, physical particles such

- ☐ Yakun Liu liuyakun@dlut.edu.cn
- Masoumeh Azma w2021089@njxzc.edu.cn

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- School of Hydraulic Engineering, Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian 116024, China
- Autonomous Vehicles and Artificial Intelligence Laboratory (AVAILab), Centre for Future Transport and Cities, Coventry University, Coventry, UK
- <sup>3</sup> School of Foreign Languages, University Lecturer, Nanjing Xiaozhuang University, Nanjing 211171, China

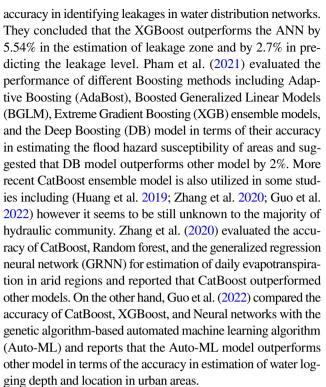
as sediments and chemical substances can pass through the porous body of these structures, which reduces sedimentation and increases flow aeration by increasing the downstream flow turbulence (Mohamed 2010; Fathi-moghaddam et al. 2018; Rahmanshahi and Bejestan 2020). The hydraulic performance of porous and solid broad-crested weirs has been studied experimentally (Azimi et al. 2013; Wüthrich and Chanson 2014; Zhang and Chanson 2016; Pirzad et al. 2021; Salmasi et al. 2021) and numerically (Jiang et al. 2018; Safarzadeh and Mohajeri 2018; Nourani et al. 2021, Yin et al. 2022a, b).

Even though solid and gabion weirs have similar geometrical shapes, the presence of the through flow highly affects their hydraulic performance (Shen et al. 2017, Chen et al. 2022). The relative diameter of filling particles is a dominant factor controlling their hydraulic performance by affecting the through flow discharge (Fathi-moghaddam et al. 2018). As a result, some researchers have concentrated on investigating how through and overflow interact under various submerged and free flow situations, and even some (Fathi-moghaddam et al. 2018; Shariq et al. 2020) have offered



formulae to calculate through flow discharge. The effect of the broad-crested weir's geometrical shape is also extensively studied. Sargison and Percy (2009) state that Solid weirs' discharge coefficient decreases as the upstream slope increases. Madadi et al. (2014) experimentally investigated the effect of upstream slope on the discharge coefficient of broad crested weirs. They concluded that by increasing the slope of the upstream ramp, the separation zone dimensions increase, which reduces the discharge coefficient. According to (Fathimoghaddam et al. 2018) the material size significantly affects the weir behaviour. For larger filling particles, decreasing the slope of side ramps increases the upstream water head and subsequently lowers the discharge coefficient of gabion weirs, whereas, for smaller filling particles, a downstream slope enhances weir performance. The use of soft computing techniques in the field of hydraulic engineering has expanded to include a number of issues in recent years including estimation of sediment scour in streams (Pandey et al. 2020, 2022; Tao et al. 2021), hydraulic performance of solid weirs such as piano-key weirs (Olyaie et al. 2019; Zounemat-Kermani and Mahdavi-Meymand 2019), side-weirs (Dursun et al. 2012), labyrinth weirs (Norouzi et al. 2019; Wang et al. 2022), cylindrical weirs (Ismael et al. 2021), solid broad crested weirs (Hameed et al. 2021) and the hydraulic performance and energy dissipation of gabion weirs (Salmasi and Sattari 2017). However, two of the most utilized methods seem to be support vector machines (Azimi et al. 2019) and Feedforward neural networks (Khatibi et al. 2014). Utilizing an automated evolutionary hyperparameter optimization approach, Tao et al. (Tao et al. 2021) investigates the accuracy of proposed XGBoost-Ga method against some machine learning methods such as decision trees, support vector machines and linear regression and concludes that the proposed XGBoost-Ga method is the superior. Azimi et al. (Azimi et al. 2019) studied the ability of six different support vector machines to predict the discharge coefficient of side weirs. They suggested that the superior model is a function of the Froude number. Norouzi et al. (Norouzi et al. 2019) compared the performance of multilayer perceptron (MLP), support vector machines (SVM), and radial basis networks for estimation of the discharge coefficient of labyrinth weirs. They concluded that the MLP model outperforms other models.

Considering that the standalone models usually face the problem of overfitting, ensemble learning models have become widely used in different applications to reduce overfitting and increase the accuracy by combining diverse base learners (Yin et al. 2022a, b, Dai et al. 2022). Osman et al. (2021) compared the performance of artificial neural networks, SVMs and the XGBoost algorithm in terms of their accuracy for estimating the groundwater level in Malaysia. Their results indicated that for every considered combination of input models, the XGBoost outperforms other models. Jiajai et al. (Wu et al. 2022) evaluated the performance of ANNs and XGBoost in terms of their



So far, different machine learning models have been utilized as surrogate models for estimation of the discharge coefficient of different solid weirs, yet only a few studies have focused on gabion weirs and no studies has considered the through flow of porous weirs due to the practical problems with their calculation and measurement resulted in the lack of a reliable dataset for such measurements. Besides, the application of ensemble models and specifically more recent and advanced models such as XGBoost and CatBoost models are also limited to few studies. Hence, this paper utilizes some base models including SVM, and Decision trees, along with some ensemble models such as Random Forest as a bagging ensemble, a Stacking ensemble model, and three different Boosting ensembles including Gradient Boosting Decision trees (GBDT), Extreme gradient Boosting model (XGB), and the more recent CatBoost model. Recursive feature elimination analysis is performed to find the optimal combination of features for each model. All models are then fine-tuned using the Bayesian hyper-parameter optimization method in order to improve their performance in estimation of the discharge coefficient and the through flow discharge of broad-crested gabion weirs.

#### **Materials and method**

#### **Experimental data**

In this study, experimental data of broad crested gabion weirs (Moradi and Fathi-Moghaddam 2014) and Salmasi et al. (2017) in free flow conditions are used to provide sufficient



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Table 1 Geometrical and hydraulic characteristics of gabion weirs from experimental studies

Data source	Lc(m)	P(m)	Q(l/s)	n (%)	$d_m(mm)$	upstream slope $(\alpha)$	Downstream slope $(\beta)$
Salmasi et al. (2017) Moradi and Fathi- moghaddam (2014)	0.15 0.15	0.15 0.15	- 3–30	39,41,45,50 43,44,46,47,50,51	5,10,15,30 7,11,14,17,22,31	90 30,45,60,90	90 30,45,60,90

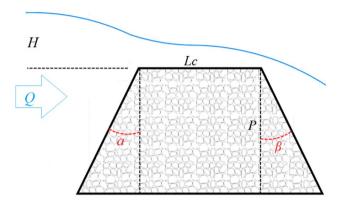


Fig. 1 Schematic view of the gabion weir parameters

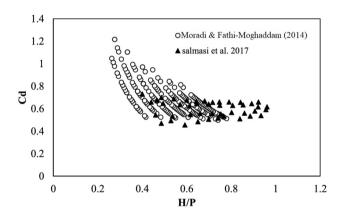


Fig. 2 Experimental  $C_d$  extracted from (Moradi and Fathi-Moghaddam 2014) and Salmasi et al. (2017)

data to train and test the models. The specifications of the experimental models are presented in Table 1 and Fig. 1. Experimental conditions include square weirs with  $15 \times 15$  dimensions with different combinations of upstream and downstream slopes filled with particles with median diameters of 5–31 mm and a porosity of 39–51%. The discharge coefficients for the abovementioned models are presented in Fig. 2.

To calculate the discharge ratio through the porous body of gabion weirs, the multivariate regression equation suggested by Fathi-moghaddam et al. (2018) is utilized as follows:

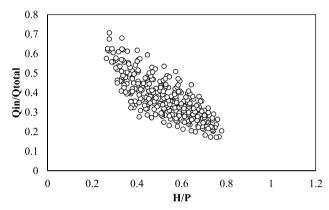


Fig. 3 Calculated through-flow discharge ratio

Calculated through flow ratio  $(Q_{in}/Q_t)$  of the abovementioned data using Eq. (1) is presented in Fig. 3.

#### **Dimensional analysis**

In free-flow conditions, discharge and discharge coefficient could be related as in Eq. (2). Hence, the effects of other hydraulic factors (i.e., weir geometry and fluid characteristics) must be taken into account in the calculation of  $C_d$ .

$$Q = \frac{2}{3} C_d B \sqrt{2g} H^{1.5} \tag{2}$$

where g is the gravitational acceleration, H is the total upstream head  $(H = h + v^2/2g)$ , h is the head over the weir, and v is the approach velocity.

For a PE weir, the discharge coefficient,  $C_d$ , can be expressed by the following functional relationship:

$$f(C_d, h, L_c, P, Q, g, \sigma, \rho, \mu, d_{50}, \alpha, \beta, n) = 0$$
 (3)

in which f is a functional symbol, P is the weir height,  $\rho$  is water density,  $\mu$  is water viscosity, and  $\sigma$  is surface tension.

$$Q_{\rm in}/Q_{\rm total} = 1.26 + 0.79 \text{Log}(Re) - 4.82 \left(\frac{H}{L_{\rm crest}}\right)^{0.149} + 0.417 \left(\frac{d_m}{p}\right) - 0.043 \tan{(\alpha)} + 0.013 \tan{(\beta)}$$
 (1)



Using the  $\Pi$  theorem of dimensional analysis (Barenblatt 1987) for the parameters in Eq. (2), the discharge coefficient for the free-flow condition can be expressed by the following dimensionless groups;

$$Cd = f\left(\frac{H}{P}, \frac{H}{Lc}, \frac{dm}{P}, \text{Re}, W, n, \tan(a), \tan(b)\right)$$
 (4)

where Re is the Reynolds number; and W represents the Weber number.

#### **Dataset preparation**

In this paper, values of H lower than 4 cm over the weir are neglected to eliminate the effect of surface tension and hence the weber number. Values of H/Lc are also neglected as they are equivalent to H/P values. Consequently, the training dataset includes 6 features as input, including Re, H/P,  $d_m/P$ , n, tan(a), and tan(b), with all values Standardized using mean and standard deviation of values. Discharge coefficient data extracted from (Moradi and Fathi-Moghaddam 2014) includes 324 values for trapezoidal weirs, and those from Salmasi et al. (2017) include 44 data for rectangular weirs without upstream and downstream slopes. Hence, the whole dataset used to train and evaluate models includes 368 data points, of which 70% are used as training and 30% are used to evaluate the model performance. Correlation coefficients between different input and output features are presented in Fig. 4. It is clear that H/P values are highly correlated with the  $C_d$  and  $Q_{in}$  values. In addition,  $d_n/P$ , porosity (n), and upstream slope show a moderate correlation with the  $Q_{\rm in}$  values.

#### Single and ensemble learning algorithms

#### Support vector machines (SVM)

SVMs (Cortes and Vapnik 1995) are a type of supervised machine learning algorithms with both applications in classification and regression which guarantees obtaining a global optimum solution. The core idea behind SVM is

	dm/P	n	H/P	tan(a)	tan(b)	Cd	Qin
dm/P	1.00	-0.52	-0.17	0.02	0.01	0.51	0.55
n	-0.52	1.00	0.01	0.12	0.12	-0.26	-0.53
H/P	-0.17	0.01	1.00	-0.13	-0.12	-0.65	-0.78
tan(a)	0.02	0.12	-0.13	1.00	0.04	-0.10	-0.45
tan(b)	0.01	0.12	-0.12	0.04	1.00	-0.19	-0.09
$\mathbf{C}d$	0.51	-0.26	-0.65	-0.10	-0.19	1.00	0.92
Qin	0.55	-0.53	-0.78	-0.45	-0.09	0.92	1.00

Fig. 4 Correlation Matrix for Different features in the Dataset



that it assigns training samples to spatial coordinates in order to maximize the distance between the two categories. Then, depending on which side of the gap they fall, new samples are projected into that same area and predicted to belong to a category. Real-world issues are often more complicated, necessitating more accurate hypotheses than those offered by existing linear learning machines, which also have certain computing drawbacks. Thus, using different kernels (Aizerman and Control 1964) available such as Radial Basis Function (Azimi et al. 2019) and polynomial kernels would be useful in examining correlations among real-world situations in order to create a specific model. Support vector regression (SVR) is a supervised-learning technique that trains using a loss function that penalizes both high and low misestimates equally. Vapnik's -insensitive approach ignores absolute values of errors above and below the estimate that are fewer than a predefined threshold by constructing a flexible tube with a small radius symmetrically around the estimated function. In this fashion, although points outside the tube are penalized, points within the tube that are above or below the function are not. One of the main advantages of SVR is that its computational cost is independent of the size of the input space. Additionally, it has a significant generalization ability and decent prediction accuracy (Awad and Khanna 2015). This research evaluates RBF and polynomial Kernels in terms of their accuracy for predicting discharge coefficient and through flow discharge.

#### **Decision Trees (DT)**

A class of supervised learning techniques known as decision trees (Breiman et al. 1983) are a good example of a universal function approximator, despite the difficulty of achieving this universality in their fundamental structure. Classification and regression are both possible applications for them. A Decision Tree is a collection of branches that are linked by decision nodes and finish in leaf nodes (DT). The decision node of the tree includes several alternative leaf nodes that reflect the model output, with each branch representing an algorithmic option. As a label or a continuous value, this may be used in classification and regression. Decision nodes make up a large portion of the DT's structure. Using the smallest possible tree to minimize overfitting is a fundamental aim for machine learning models when it comes to correctly representing the relationships between input and output. An ensemble DT model (EDT) is often employed, in which a number of trees are utilized to generate a final model at once (i.e., bootstrap aggregation or bagging) or sequentially (i.e., boost) (Jain et al. 2020).

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#### Random forest regression (RFR)

An RFR method is a regression technique based on machine learning. Using bagging and random subspace as a foundation, RFR is a stable foundation upon which to develop. Hence, a number of learner trees are generated, which are then merged to get an overall prediction. To train the learning trees, original training data are used to generate bootstrap samples. Each bootstrap sample (Db) is created by randomly picking n instances from the original training data (D), which comprises N instances. It is possible to replace the bootstrap samples with fresh instances. Db is about twothirds the size of D and contains no duplicate instances. Using the vector of input data, x, k distinct regression trees are constructed for each bootstrap sample. Low bias and high variance are characteristics of regression trees. Random forest predictions are generated by averaging the predictions of K regression trees, hk (x) (N. et al. 2021) as follows:

$$RFR_{prediction} = \frac{1}{k} \sum_{k=1}^{k} h_k(x)$$
 (5)

#### Stacked regression

Stacking regressions first presented by (Breiman 1996) is a technique for combining many predictors linearly to enhance prediction accuracy. The algorithm mainly consists of 2 steps, including (1) specifying a list of base learners and training each on the dataset and (2) using the predictions of the base learners as input to train the value of the metalearner and predict new values with the meta-learner. This study's defined structure of the stacked regression includes support vector machines with polynomial and RBF kernel along with Ridge regression as level-1 models and the Decision Tree Regressor as the Meta model, as depicted in Fig. 5.

#### **Gradient boosting**

Gradient Boosted Decision Trees (GBDT) (Friedman 2001) a supervised learning technique first introduced by Jerome H. Friedman starts with a set of  $\{x_i, y_i\}$  values where  $x_i$  represent the input values and  $y_i$  are corresponding target values. The Gradient Boosting technique then involves iteratively creating a set of functions of  $F^0, F^1, \dots, F^t, \dots, F^m$  which are then used to form the corresponding loss function  $L(y_i, F^t)$  that estimates  $y_i$ , to improve the estimations, another function of  $F^{t+1} = F^t + h^{t+1}(x)$  is created so that  $h^{t+1}$  is as follows:

$$h^{t+1} = \operatorname{argmin}_{h \in H} EL(y, F^t)$$
 (6)

where H Where H is the collection of candidate Decision Trees being considered for inclusion in the ensemble. Hence, the expected loss function could be defined as (Hancock and Khoshgoftaar 2020):

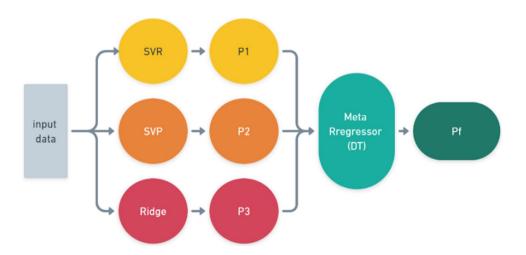
$$EL(y, F^{t+1}) = EL(y, F^t + h^{t+1})$$
(7)

#### XGBoost

First introduced by Chen et al. (Chen and Guestrin 2016), extreme gradient boosting (XGB) is a variant of gradient boosting decision trees in which each base learner learns from the previous one to reduce its error. In general, the XGBoost technique builds a more robust aggregated model by combining many base learners (decision trees). Trees' weights are used in an XGBoost algorithm to approximate the final outcome. For each dataset with m characteristics, Eq. (8) is being used to forecast the outcome:

$$\widehat{y}_i = \sum_{k=1}^K f_k(x_i), f_k \in \left\{ f(x) = \omega_q(x) \right\} \left( q : R^m \to T, \omega \in R^T \right)$$
(8)

Fig. 5 Determined structure for the Stacked Regression model





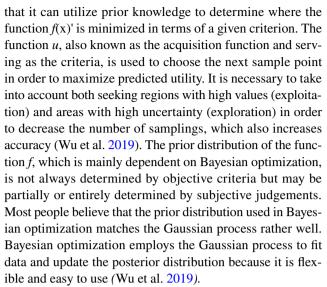
Incorrectly defining the parameters of decision trees, such as their depth or number of iterations, may lead to overfitting. The XGBoost algorithm's excellent regularisation capabilities are what make it adequate. XGBoost penalizes models for overfitting by including and regularising approaches. XGBoost model optimization needs the tweaking of many hyperparameters. As the XGBoost method is a sort of ensemble and gradient boosting technique, its hyperparameters may be divided into four categories: ensemble hyper-parameters, tree hyper-parameters, sub-sampling hyper-parameters, and regularisation hyper-parameters (Tao et al. 2021).

#### CatBoost

Propsed by Prokhorenkova et al. (2018) is an improved version of the GBDTs. First, CatBoost handles high-cardinality categorical data better than Gradient Boosting. CatBoost employs one-hot encoding for low-cardinality categories. Besides, CatBoost benefits from the Ordered Boosting method. By assuming D as the set of all available data for training the GBDT model, and having in mind that the decision tree  $h^{t+1}$  is the tree that minimizes the loss function (L), Ordered Boosting could be expressed as using the same examples used for computation of Ordered Target Statistics for computation of  $h^{t+1}$ . Oblivious Decision Trees (ODTs) are a crucial component in CatBoost's method for constructing Decision Trees. CatBoost builds a collection of ODTs. ODTs are complete binary trees, hence if there are n levels, there will be 2n nodes. In addition, all non-leaf nodes in the ODT will use the same criterion for splitting. CatBoost extends GBDT's abilities so that it could account for feature interactions so that believe CatBoost will pick the most effective feature combinations throughout training (Prokhorenkova et al. 2018; Hancock and Khoshgoftaar 2020). CatBoost is highly sensitive to the definition of hyperparameters so a proper tuning of hyper-parameters would be essential.

#### **Hyperparameter optimization**

Optimization of hyper-parameters is an important task when it comes to automated machine learning (AutoML). AutoML automates complex operations, including model parameter optimization, without the need for human knowledge and so plays a crucial role in improving the performance of machine learning models (Tao et al. 2021). Bayesian optimization is a powerful method for figuring out the extreme values of computationally challenging functions. (Brochu et al. 2016). Additionally, it may be used to compute difficult-to-calculate, difficult-to-analyze derivatives, or nonconvex functions. By integrating the prior distribution of the function f(x) with the sample data, Bayesian optimization makes the assumption



In this study, Bayesian optimization is utilized to automatically tune the hyper-parameters of all models to estimate the discharge coefficient and through flow discharge. Best models are then reconstructed and used to calculate accuracy metrics. Figure 6 visualizes the relationship between hyper-parameters and the obtained accuracy for the XGBoost regression model during the performed optimization process. The hyper-parameters obtained from the optimization process for each model are presented in Tables 2 and 3.

#### Model development and accuracy assessment

All models are developed and implemented using Scikitlearn (DT, AdaB,RF) (Pedregosa et al. 2011), Mlxtend (Stacked reg.) (Raschka 2018), XGBoost library (Chen 2016–2022), and the CatBoost (LLC 2017–2022) library. Three accuracy measures are used to evaluate the accuracy of the utilized ensemble models, including Mean absolute error, Mean squared error and the correlation coefficient which are frequently used in similar research for regression tasks, i.e. see (Azimi et al. 2019) (Hameed et al. 2021).

Mean Absolute error (MAE):

MAE = 
$$\frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}$$
 (9)

Mean-squared error (MSE):

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$
 (10)

and the correlation coefficient ( $R^2$ ):

$$R^{2} = 1 - \frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\sum_{i=1}^{n} (y_{i} - \overline{y}_{i})^{2}}$$
(11)



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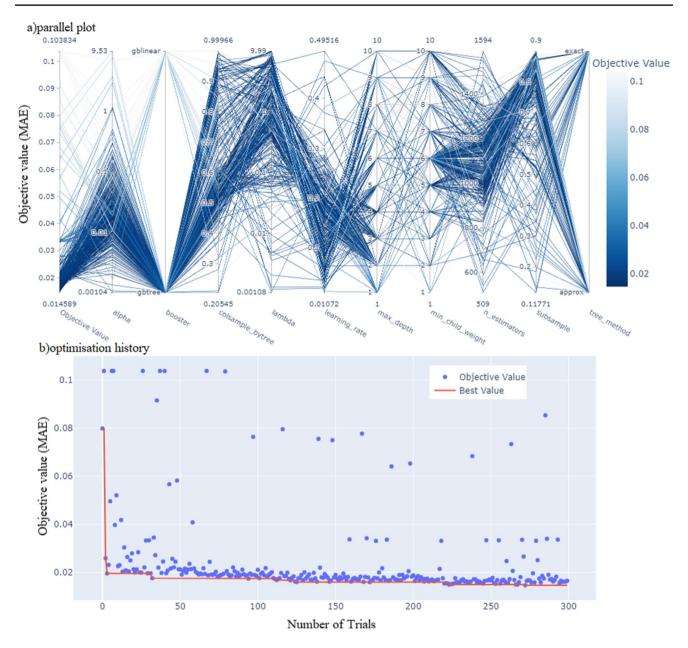


Fig. 6 Visualization of the hyper parameter optimization for the XGBoost regressor

Table 2 Utilized Hyper-parameters for base models

Model	SVR	SVP	DT
Hyper parameters	Kernel = RBF Coef0 = 0.44 tol = 0.01 $\varepsilon = 0.028$ C = 9	Kernel = poly Degree = 3 Coef0 = 0.51 tol = 0.051 $\varepsilon = 0.025$ C = 1	Splitter = best Max_depth = 25 Loss = MAE Min_leaf = 2 Min split = 2

The target value from experimental data  $\hat{y}_i$  is the model predictions and  $\bar{y}$  represents the mean values. Figure 7 presents the general workflow of the current paper.

### **Results and discussion**

# Feature importance and recursive feature elimination

Considering the sensitivity of models to the input data and features, the relative importance of input features for different algorithms to estimate the discharge coefficient  $C_d$  and the through-flow discharge ratio  $Q_{in}/Q_t$  are presented in Figs. 8 and 9, respectively. It could be concluded that the porosity (n) with the weak correlation of (-0.26) with the  $C_d$ , had a higher correlation than upstream and downstream slopes. Yet, its relative importance for all



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 Table 3
 Utilized Hyper-parameters for ensembles

Model	GBDT	XGB	CatB	RF
Hyper parameters	Loss = Huber Criterion = MSE learning_rate = 0.1 subsample = 0.35 min split = 2 min_leaf = 1 weight_fraction = 0.001 max_depth = 4 N_estimators = 300	n_estimators = 1200 max_depth = 2 subsample = 0.8759 colsample_bytree = 0.5094 alpha = .029 tree_method = approx booster = 'gbtree' min_child_weight = 6 lambda = 1.0794 learning_rate = 0.2	$N_{\text{estimators}} = 2000$ learning_rate = 0.042 depth = 5 $12\_\text{leaf\_reg} = 0.0045$ loss = RMSE max_bin = 353 min_data_in_leaf = 223	Splitter = best Max_depth = 25 Loss = MAE Min_leaf = 2 Min split = 2 N <sub>estimators</sub> = 2300

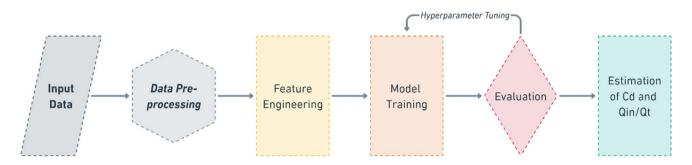


Fig. 7 model training workflow

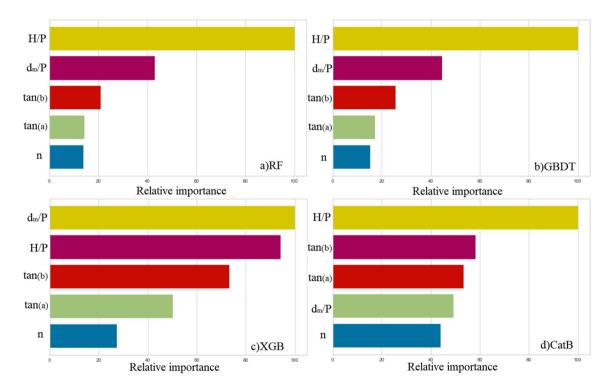
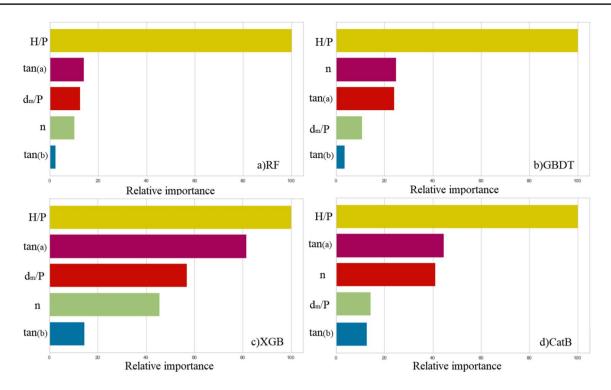


Fig. 8 Feature relative importance for estimation of  $C_d$ 



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**Fig. 9** Feature relative importance for estimation of  $Q_{in}/Q_t$ 

regression models is the lowest among all the features. Besides, the relative flow head (H/P) and the relative particle size  $(d_m/P)$  have the highest relative importance for estimating  $C_d$  in all models except the CatBoost, in which the importance if downstream and upstream slopes are higher than that of  $d_m/P$ . Regarding models trained for estimation of  $Q_{\rm in}/Qt$ , the lowest importance of features belongs to the downstream slope  $\tan(b)$  while the importance of porosity (n) is increased especially for CatBoost and GBDT models. Yet the most important feature is the

relative flow head (*H/P*). In order to investigate the effect of different feature combination on the models accuracy, a recursive feature elimination (RFE) test is performed for all models. Generally results indicate that models perform best when all features are included yet the only exception is GBDT which had higher accuracy with the 4 best features. Results of the RFE test for XGB are depicted in Fig. 10. Best models are then recreated and fitted on the training dataset combining the results of hyper parameter optimization and RFE test.

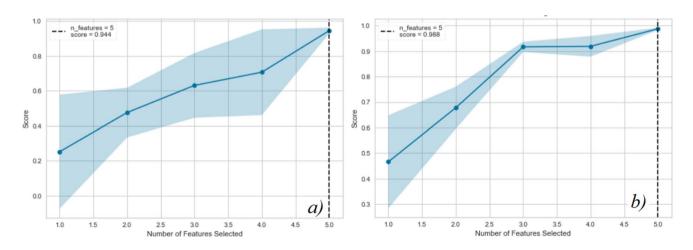


Fig. 10 Recursive Feature elimination analysis for XGB model for a  $C_d$  and b  $Q_{inf}/Q_t$ 



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**Table 4** Accuracy metrics of different algorithms in estimation of  $C_d$ 

Model	Train		Test	Test		
	$R^2$	$R^2$	MAE	MSE		
SVR	0.977	0.962	0.0188	0.00067		
SVP	0.968	0.931	0.0221	0.00122		
DT	0.921	0.821	0.0432	0.00373		
RF	0.940	0.912	0.0245	0.00171		
Stacked	0.993	0.9643	0.0175	0.00062		
GBDT	0.996	0.962	0.0187	0.00069		
XGB	0.999	0.974	0.0147	0.00048		
CatB	0.993	0.982	0.126	0.00033		

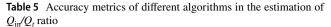
## Estimation of the discharge coefficient ( $C_d$ )

Based on the optimization results, the best models were then rebuilt and trained over the train-test dataset.

The results obtained from single and ensemble models in terms of their accuracy in predicting the dischargecoefficient are presented in Table 4 and Fig. 11. It is indicated that the base DT model performs poorly in estimation of the discharge coefficient. Standalone support vector machine with RBF kernel (SVR) and the support vector model with a third degree polynomial kernel (SVP) models have shown proper performance of  $R^2 = 0.96$  and 0.931, respectively. Among ensemble models, the well-known Random forest (RF) model as a representative of bagging techniques has improved the accuracy of the based DT model by 9% showing an accuracy of  $R^2 = 0.921$  and RMSE and MSE equal to 0.0245 and 0.00171, respectively, while it is still lower than that of support vector machines. The stacking regression method composed of SVMs and the DT as meta-model however shows a higher  $R^2 = 0.964$  with MAE = 0.175 and MSE = 0.00061 compared to the optimized GBDT model with  $R^2 = 0.962$  and MAE = 0.161, and MSE = 0.0061. Consequently, the variance and the standard deviation of the residuals in the stacked model is higher than that of GBDT model despite its higher R-squared value. XGB and CatBoost offer higher accuracy among other models with  $R^2 = 0.974$  and 0.982, respectively. CatBoost outperforming the XGB in terms of all accuracy measures by 0.8% margin in  $\mathbb{R}^2$  value. Consequently, it could be concluded that even though the XGB model outperformed all other models, Cat-Boost has outperformed XGB considerably by increasing 1.2% the  $R^2$  value and reducing MSE value for almost 31%.

# Estimation of the through-flow discharge ratio $(Q_{in})$

The results obtained from single and ensemble models in terms of their accuracy in predicting the  $Q_{in}/Q_t$  ratio are presented in Table 5 and Fig. 12. Clearly, the poorest results obtained from Decision Tree model with  $R^2 = 0.865$  and



Model	Train		Test		
	$R^2$	$R^2$	MAE	MSE	
SVR	0.977	0.976	0.0141	0.00028	
SVP	0.971	0.968	0.0154	0.00036	
DT	0.983	0.865	0.0321	0.00159	
RF	0.991	0.947	0.0219	0.00074	
Stacked	0.997	0.978	0.0132	0.00025	
GBDT	0.996	0.989	0.0087	0.00012	
XGB	0.999	0.992	0.0079	0.000091	
CatB	0.998	0.992	0.0071	0.000094	

Random Forest with and  $R^2 = 0.947$  and MAE = 0.0219. The SVP and SVR models have shown proper accuracy of  $R^2 = 0.968$  and  $R^2 = 0.976$ , respectively, while the Stacking regression ensemble model has resulted in an accuracy of  $R^2 = 0.978$  which could be considered a slight improvement of the performance of the base SVR model. *GBDT* model shows a significant performance improvement compared to other models with  $R^2 = 0.989$ , MAE = 0.0087 and MSE = 0.00012. Utilizing the XGBoost and CatBoost algorithms, which produced MAE = 0.0079 and 0.0071 and MSE of 0.000093 and 0.000094, respectively, and may be deemed to have almost comparable performance, model accuracy is improved up to  $R^2 = 0.992$ .

#### Residuals and feature importance analysis

Residuals are the difference between the observed value of the dependent variable (y) and the expected value ( $\hat{y}$ ). The residuals plot depicts the difference between residuals on the vertical axis and the dependent variable on the horizontal axis, aiding in the identification of error-prone areas of the target. The residuals of the different algorithms by means of estimation of the discharge coefficient  $C_d$  and the through-flow discharge ratio  $Q_{in}/Q_t$  are presented in Figs. 13 and 14, respectively. A comparison of the residuals shows that the distribution of residuals around the centreline for all models is fairly random, suggesting that the models have generalized well, yet the CatBoost model shows almost zero residual for the training phase, which could be considered as overfitting. It is also worth noting that the majority of errors arise from  $C_d$  values close or above 1 which represents the experiments with lower discharge and limited training data.

# Model sensitivity analysis

Considering that the sensitivity analysis is an important step in each ML project, this paper studies the sensitivity



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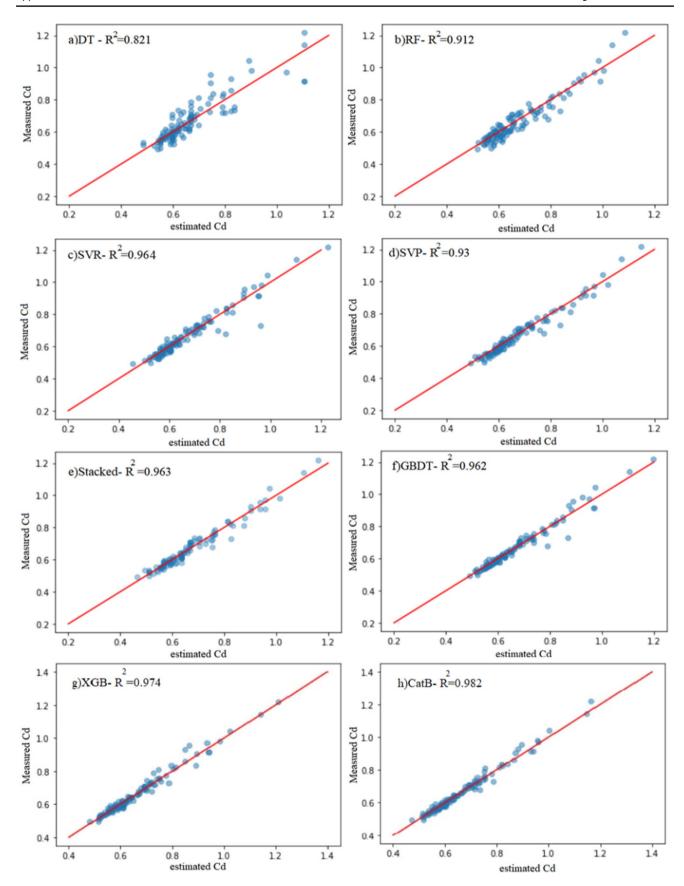


Fig. 11 Scatter plot of observed vs estimated values of  $\mathcal{C}_d$  for different methods



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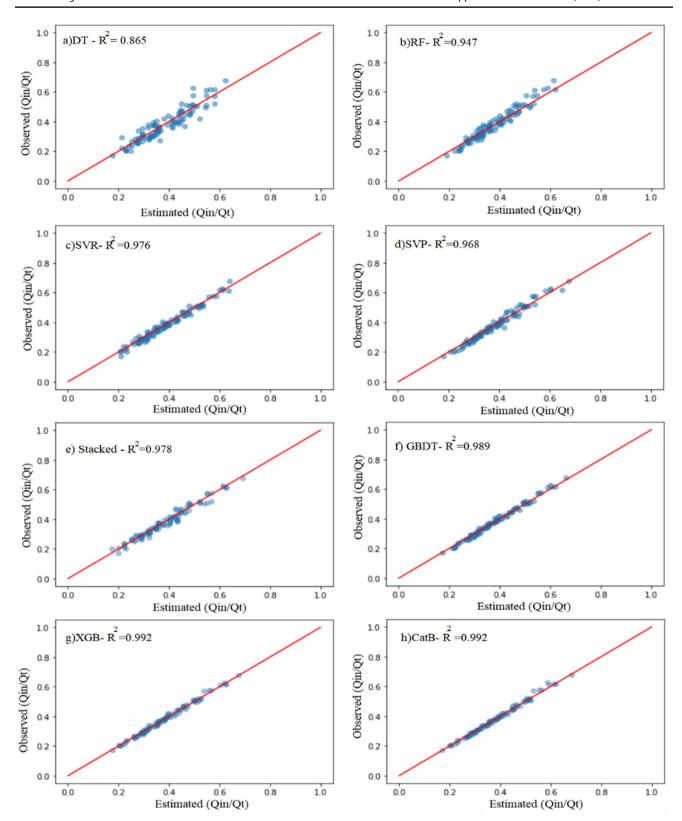


Fig. 12 Scatter plot of observed vs estimated values of  $Q_{\rm in}/Q_t$  for different methods



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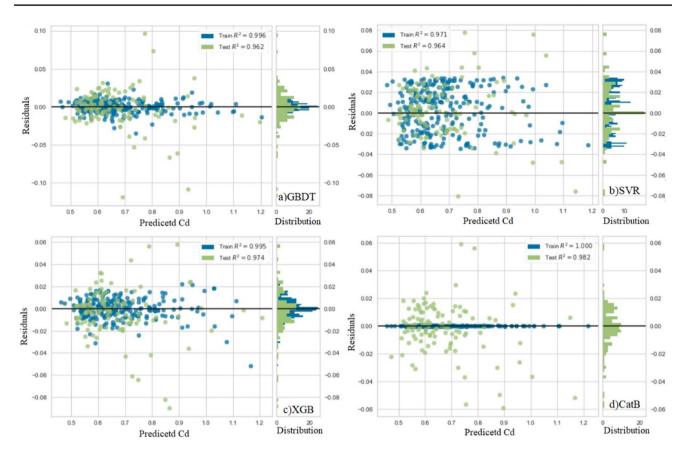


Fig. 13 Models residuals for estimation of  $C_d$ 

of the top two models against the elimination of different features. Hence, at each step, one feature is eliminated and models are trained on the rest of the features. Obtained values of  $R^2$  score are presented in Table 6. Results indicate that the most important features affecting the results are H/P, upstream slope and downstream slope. It should be noted that elimination of Reynolds number has increased the accuracy of CatB model to  $R^2 = 0.994$  while it has reduced  $C_d$  values.

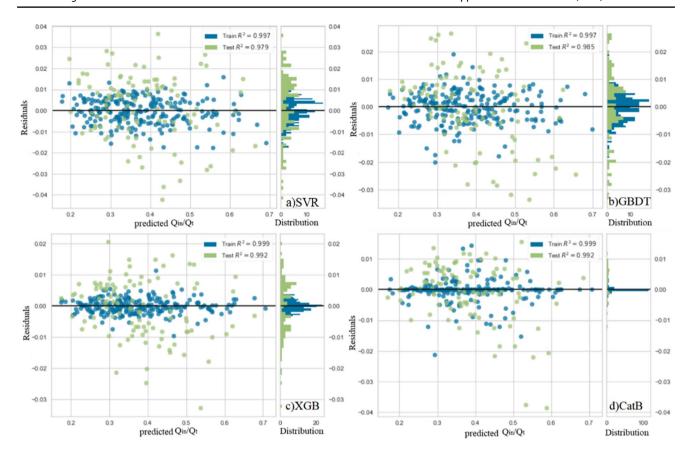
#### **Conclusion**

The current study makes use of some the well-known machine learning techniques such as Support vector machines, Decision trees(DT) and Random Forest (RF) models, as well as the stacking ensemble technique and three different Boosting ensemble models, including Gradient Boosting Decision Trees (GBDT), Extreme gradient Boosting model (XGB), and the more recent CatBoost model in order to evaluate the effectiveness of these models in estimation of the discharge coefficient ( $C_d$ ) and the through-flow discharge ratio( $Q_{in}/Q_t$ ) of broad-crested gabion weirs. A

total of 368 data points from literature were extracted and used to train and evaluate the model's accuracy. The Bayesian optimization framework as well as the recursive feature elimination analysis are combined in order to improve the accuracy of the utilized ML models. Results indicate that the weakest performance with  $R^2 = 0.82$  and 0.865 was obtained using the DT model to estimate the discharge coefficient and through flow discharge. The use of the stacking model improved the accuracy of base models to  $R^2 = 0.9643$  and 0.978 for estimation of  $C_d$  and  $Q_{in}/Q_p$  respectively. The maximum accuracy for estimating the discharge coefficient was achieved from the CatBoost model with  $R^2 = 0.982$ , MAE = 0.0126, and MSE = 0.00033. All boosting strategies produced accuracies higher than  $R^2 = 0.962$  in terms of both the Cd and  $Q_{in}/Q_t$  estimation. Utilizing a sensitivity analysis for the top two models of XGBoost and CatBoost, a slight increase in performance of models was identified in terms of estimation of  $Q_{in}/Q_t$  ratio. While estimating the  $Q_{in}/Q_t$ values, CatBoost has outperformed other models including XGBOOst slightly with  $R^2 = 0.994$  against 0.922 of XGboost model. Yet, the distribution of error residuals obtained from XGBoost is slightly better distributed, showing a slightly better performance in the model's generalisation.



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**Fig. 14** Models residuals for estimation of  $Q_{in}/Q_t$ 

**Table 6**  $R^2$  values resulted from sensitivity analysis of model results to features

	Eliminated feature	Tan(α)	Tan(β)	d <sub>50</sub> /p	n	H/P	Re	d <sub>50</sub> /p & n
$\overline{C_d}$	CatB	0.781	0.707	0.981	0.975	0.652	0.980	0.764
	XGB	0.805	0.659	0.976	0.971	0.776	0.964	0.791
Q	CatB	0.702	0.938	0.992	0.990	0.702	0.994	0.761
	XGB	0.756	0.934	0.989	0.989	0.756	0.992	0.745

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**Data availability** The data that support the findings of this study are available from the corresponding authors upon reasonable request.

#### **Declarations**

**Conflict of interest** There is no conflict of interest in this manuscript.

#### Ethical approval

The author consents to participate in the works under the Ethical Approval and Compliance with Ethical Standards.



#### **Ethical conduct**

The principles of ethical and professional conduct have been followed.

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#### References

- Aizerman MAJA, Control R (1964) Theoretical foundations of the potential function method in pattern recognition learning. Autom Remote Control 25:821–837
- Awad M, Khanna R (2015). Support vector regression. Efficient learning machines: theories, concepts, and applications for engineers and system designers. Berkeley, CA, Apress: 67–80
- Azimi AH, Rajaratnam N, Zhu DZ (2013) Discharge characteristics of weirs of finite crest length with upstream and downstream ramps. J Irrig Drain Eng 139(1):75–83. https://doi.org/10.1061/(ASCE) IR.1943-4774.0000519
- Azimi H, Bonakdari H, Ebtehaj I (2019) Design of radial basis function-based support vector regression in predicting the discharge coefficient of a side weir in a trapezoidal channel. Appl Water Sci 9(4):78. https://doi.org/10.1007/s13201-019-0961-5
- Breiman L (1996) Stacked regressions. Mach Learn 24(1):49-64. https://doi.org/10.1007/BF00117832
- Breiman L, Friedman JH, Olshen RA, Stone CJ (1983). Classification and Regression Trees.
- Brochu E, Cora V, de Freitas N (2016) A tutorial on bayesian optimization of expensive cost functions, with application to active user modeling and hierarchical reinforcement learning. University of British Columbia, Department of Computer Science
- Chen Z, Liu Z, Yin L, Zheng W (2022) Statistical analysis of regional air temperature characteristics before and after dam construction. Urban Clim. https://doi.org/10.1016/j.uclim.2022.101085
- Chen T, Guestrin C (2016). XGBoost: a scalable tree boosting system.
  In: Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. San Francisco, California, USA, Association for Computing Machinery: 785–794
  Chen T (2016–2022). "https://github.com/dmlc/xgboost."
- Cortes C, Vapnik V (1995) Support-vector networks. Mach Learn 20(3):273–297. https://doi.org/10.1007/BF00994018
- Dai L, Wang Z, Guo T, Hu L, Chen Y, Chen C, Chen J (2022) Pollution characteristics and source analysis of microplastics in the qiantang river in southeastern China. Chemosphere (oxford) 293:133576. https://doi.org/10.1016/j.chemosphere.2022.133576
- Dursun OF, Kaya N, Firat M (2012) Estimating discharge coefficient of semi-elliptical side weir using ANFIS. J Hydrol 426–427:55–62. https://doi.org/10.1016/j.jhydrol.2012.01.010
- Fang X, Wang Q, Wang J, Xiang Y, Wu Y, Zhang Y (2021) Employing extreme value theory to establish nutrient criteria in bay waters: a case study of Xiangshan Bay. J Hydrol. https://doi.org/10.1016/j. jhydrol.2021.127146
- Fathi-moghaddam M, Sadrabadi MT, Rahmanshahi M (2018) Numerical simulation of the hydraulic performance of triangular and trapezoidal gabion weirs in free flow condition. Flow Meas Instrum 62:93–104. https://doi.org/10.1016/j.flowmeasinst.2018.05.005
- Friedman JH (2001) Greedy function approximation: a gradient boosting machine. Ann Stat 29(5):1189–1232
- Ganesh N, Jain P, Choudhury A, Dutta P, Kalita K, Barsocchi P (2021) Random forest regression-based machine learning model for accurate estimation of fluid flow in curved pipes. Processes 9(11):2095
- Guo Y, Quan L, Song L, Liang H (2022) Construction of rapid early warning and comprehensive analysis models for urban waterlogging based on AutoML and comparison of the other three machine learning algorithms. J Hydrol 605:127367. https://doi.org/10. 1016/j.jhydrol.2021.127367
- Hameed MM, AlOmar MK, Khaleel F, Al-Ansari N (2021) An extra tree regression model for discharge coefficient prediction: novel, practical applications in the hydraulic sector and future research directions. Math Probl Eng 2021:7001710. https://doi.org/10.1155/2021/7001710

- Hancock JT, Khoshgoftaar TM (2020) CatBoost for big data: an interdisciplinary review. J Big Data 7(1):94. https://doi.org/10.1186/ s40537-020-00369-8
- Huang G, Wu L, Ma X, Zhang W, Fan J, Yu X, Zeng W, Zhou H (2019) Evaluation of CatBoost method for prediction of reference evapotranspiration in humid regions. J Hydrol 574:1029–1041. https://doi.org/10.1016/j.jhydrol.2019.04.085
- Ibrahem Ahmed Osman A, Najah Ahmed A, Chow MF, Feng Huang Y, El-Shafie A (2021) Extreme gradient boosting (Xgboost) model to predict the groundwater levels in Selangor Malaysia. Ain Shams Eng. J. 12(2):1545–1556. https://doi.org/10.1016/j.asej.2020.11.
- Ismael AA, Suleiman SJ, Al-Nima RRO, Al-Ansari N (2021) Predicting the discharge coefficient of oblique cylindrical weir using neural network techniques. Arab J Geosci 14(16):1670. https://doi.org/10.1007/s12517-021-07911-9
- Jain P, Coogan SCP, Subramanian SG, Crowley M, Taylor S, Flannigan MD (2020) A review of machine learning applications in wildfire science and management. Environ Rev 28(4):478–505. https://doi. org/10.1139/er-2020-0019
- Jiang L, Diao M, Sun H, Ren Y (2018) Numerical modeling of flow over a rectangular broad-crested weir with a sloped upstream face. Water 10(11):1663
- Khatibi R, Salmasi F, Ghorbani MA, Asadi H (2014) Modelling energy dissipation over stepped-gabion weirs by artificial intelligence. Water Resour Manage 28(7):1807–1821. https://doi.org/10.1007/s11269-014-0545-y
- Liu E, Chen S, Yan D, Deng Y, Wang H, Jing Z, Pan S (2022) Detrital zircon geochronology and heavy mineral composition constraints on provenance evolution in the western pearl river mouth basin, northern south China sea: a source to sink approach. Mar Pet Geol 145:105884. https://doi.org/10.1016/j.marpetgeo.2022.105884
- LLC, Y. (2017-2022). "catboost.ai."
- Madadi MR, HosseinzadehDalir A, Farsadizadeh D (2014) Investigation of flow characteristics above trapezoidal broad-crested weirs. Flow Meas Instrum 38:139–148. https://doi.org/10.1016/j.flowmeasinst.2014.05.014
- Mohamed HI (2010) Flow over gabion weirs. J Irrig Drain Eng 136(8):573–577. https://doi.org/10.1061/(ASCE)IR.1943-4774. 0000215
- Moradi, M. and M. Fathi-Moghaddam (2014). Investigation of the effect of upstream and downstream slopes of broad-crested gabion weirs on discharge coefficient. In: 10th International River Engineering Conference Ahwaz,Iran.
- Norouzi R, Daneshfaraz R, Ghaderi A (2019) Investigation of discharge coefficient of trapezoidal labyrinth weirs using artificial neural networks and support vector machines. Appl Water Sci 9(7):148. https://doi.org/10.1007/s13201-019-1026-5
- Nourani B, Arvanaghi H, Salmasi F (2021) Effects of different configurations of sloping crests and upstream and downstream ramps on the discharge coefficient for broad-crested weirs. J Hydrol 603:126940. https://doi.org/10.1016/j.jhydrol.2021. 126940
- Olyaie E, Banejad H, Heydari M (2019) Estimating discharge coefficient of PK-weir under subcritical conditions based on high-accuracy machine learning approaches. Iran J Sci Technol Trans Civ Eng 43(1):89–101. https://doi.org/10.1007/s40996-018-0150-z
- Pandey M, Zakwan M, Khan MA, Bhave S (2020) Development of scour around a circular pier and its modelling using genetic algorithm. Water Supply 20(8):3358–3367. https://doi.org/10.2166/ws.2020.244%JWaterSupply
- Pandey M, Jamei M, Ahmadianfar I, Karbasi M, Lodhi AS, Chu X (2022) Assessment of scouring around spur dike in cohesive sediment mixtures: a comparative study on three rigorous



45 Page 16 of 16 Applied Water Science (2023) 13:45

- machine learning models. J Hydrol 606:127330. https://doi.org/10.1016/j.jhydrol.2021.127330
- Pedregosa F, Varoquaux G, Gramfort A, Michel V, Thirion B, Grisel O, Blondel M, Louppe G, Prettenhofer P, Weiss R, Weiss RJ, Vanderplas J, Passos A, Cournapeau D, Brucher M, Perrot M, Duchesnay E (2011) Scikit-learn: machine learning in python. J Mach Learn Res 12:2825–2830
- Pham QB, Pal SC, Chakrabortty R, Norouzi A, Golshan M, Ogunrinde AT, Janizadeh S, Khedher KM, Anh DT (2021) Evaluation of various boosting ensemble algorithms for predicting flood hazard susceptibility areas. Geomat Nat Haz Risk 12(1):2607–2628. https://doi.org/10.1080/19475705.2021.1968510
- Pirzad M, Pourmohammadi MH, GhorbanizadehKharazi H, SolimaniBabarsad M, Derikvand E (2021) Experimental study on flow over arced-plan porous weirs. Water Supply 22(3):2659–2672. https://doi.org/10.2166/ws.2021.446%JWaterSupply
- Prokhorenkova L, Gusev G, Vorobev A, Dorogush AV, Gulin A (2018). CatBoost: unbiased boosting with categorical features. In: Proceedings of the 32nd International Conference on Neural Information Processing Systems. Montréal, Canada, Curran Associates Inc.: 6639–6649
- Rahmanshahi M, Bejestan MS (2020) Gene-expression programming approach for development of a mathematical model of energy dissipation on block ramps. J Irrig Drain Eng 146(2):04019033. https://doi.org/10.1061/(ASCE)IR.1943-4774.0001442
- Raschka S (2018) MLxtend: providing machine learning and data science utilities and extensions to Python's scientific computing stack. J Open Source Softw 3:638
- Safarzadeh A, Mohajeri SH (2018) Hydrodynamics of rectangular broad-crested porous weirs. J Irrig Drain Eng. https://doi.org/10. 1061/(ASCE)IR.1943-4774.0001338
- Salmasi F, Sattari MT (2017) Predicting discharge coefficient of rectangular broad-crested gabion weir using M5 tree model. Iran J Sci Technol Trans Civ Eng 41(2):205–212. https://doi.org/10.1007/s40996-017-0052-5
- Salmasi F, Sabahi N, Abraham J (2021) Discharge coefficients for rectangular broad-crested gabion weirs: experimental study. J Irrig Drain Eng 147(3):04021001. https://doi.org/10.1061/(ASCE)IR. 1943-4774.0001535
- Sargison JE, Percy A (2009) Hydraulics of broad-crested weirs with varying side slopes. J Irrig Drain Eng 135(1):115–118. https://doi. org/10.1061/(ASCE)0733-9437
- Shariq A, Hussain A, Ahmad Z (2020) Discharge equation for the gabion weir under through flow condition. Flow Meas Instrum 74:101769. https://doi.org/10.1016/j.flowmeasinst.2020.101769
- Shen X, Hong Y, Zhang K, Hao Z (2017) Refining a distributed linear reservoir routing method to improve performance of the CREST model. J Hydrol Eng 22(3):4016061. https://doi.org/10.1061/ (ASCE)HE.1943-5584.0001442
- Tao H, Habib M, Aljarah I, Faris H, Afan HA, Yaseen ZM (2021) An intelligent evolutionary extreme gradient boosting algorithm development for modeling scour depths under submerged weir. Inf Sci 570:172–184. https://doi.org/10.1016/j.ins.2021.04.063

- Wang F, Zheng S, Ren Y, Liu W, Wu C (2022) Application of hybrid neural network in discharge coefficient prediction of triangular labyrinth weir. Flow Meas Instrum 83:102108. https://doi.org/10.1016/j.flowmeasinst.2021.102108
- Wu J, Chen XY, Zhang H, Xiong LD, Lei H, Deng SH (2019) Hyperparameter optimization for machine learning models based on bayesian optimizationb. J Electron Sci Technol 17(1):26–40. https://doi.org/10.11989/JEST.1674-862X.80904120
- Wu J, Ma D, Wang W (2022) Leakage identification in water distribution networks based on XGBoost algorithm. J Water Resour Plan Manage 148(3):04021107. https://doi.org/10.1061/(ASCE)WR. 1943-5452.0001523
- Wüthrich D, Chanson H (2014) Hydraulics, Air Entrainment, and Energy Dissipation on a Gabion Stepped Weir. J. Hydr. Eng 140(9):04014046. https://doi.org/10.1061/(ASCE)HY.1943-7900. 0000919
- Yin L, Wang L, Keim BD, Konsoer K, Zheng W (2022a) Wavelet Analysis of Dam Injection and Discharge in Three Gorges Dam and Reservoir with Precipitation and River Discharge. Water 14(4):567. https://doi.org/10.3390/w14040567
- Yin L, Wang L, Zheng W, Ge L, Tian J, Liu Y, Liu S (2022b) Evaluation of empirical atmospheric models using swarm-C satellite data. Atmosphere 13(2):294. https://doi.org/10.3390/atmos13020
- Yue Z, Zhou W, Li T (2021) Impact of the indian ocean dipole on evolution of the subsequent ENSO: relative roles of dynamic and thermodynamic processes. J Clim 34(9):3591–3607. https://doi. org/10.1175/JCLI-D-20-0487.1
- Zhang G, Chanson H (2016) Gabion stepped spillway: interactions between free-surface, cavity, and seepage flows. J Hydr Eng 142(5):06016002. https://doi.org/10.1061/(ASCE)HY.1943-7900. 0001120
- Zhang K, Ali A, Antonarakis A, Moghaddam M, Saatchi S, Tabatabae-enejad A, Moorcroft P (2019a) The sensitivity of north american terrestrial carbon fluxes to spatial and temporal variation in soil moisture: an analysis using radar-derived estimates of root-zone soil moisture. J Geophys Res Biogeosci 124(11):3208–3231. https://doi.org/10.1029/2018JG004589
- Zhang K, Wang S, Bao H, Zhao X (2019b) Characteristics and influencing factors of rainfall-induced landslide and debris flow hazards in Shaanxi Province, China. Nat Hazard 19(1):93–105. https://doi.org/10.5194/nhess-19-93-2019
- Zhang Y, Zhao Z, Zheng J (2020) CatBoost: a new approach for estimating daily reference crop evapotranspiration in arid and semi-arid regions of Northern China. J Hydrol 588:125087. https://doi.org/10.1016/j.jhydrol.2020.125087
- Zounemat-Kermani M, Mahdavi-Meymand A (2019) Hybrid metaheuristics artificial intelligence models in simulating discharge passing the piano key weirs. J Hydrol 569:12–21. https://doi.org/ 10.1016/j.jhydrol.2018.11.052

