

Numerical modeling in determining deformation characteristics of fractured rock massifs

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Abstract. The article proposes a method for determining the deformation modulus of fractured-block rock masses, weakened by mutually-orthogonal systems of cracks within the linear section (after the closure of cracks) of the deformation curve $\sigma = f(\varepsilon)$. The values of normal - k_n and tangential - k_s stiffness are used as elastic characteristics of joints. It is shown that in such rocky structures it is impossible to apply the laws of mechanics of a solid elastic body to calculate their deformation characteristics, since the opening of interblock joints is observed when performing numerical experiments. The discrepancy between the results of numerical and analytical calculations is up to 40%. For these purposes, it is proposed to use the regression equation obtained on the basis of regression analysis based on the results of numerical modeling and the method of experiment planning. The presented equation allows, depending on three independent factors: the modulus of elasticity of the rock block - E_0 , the rock quality designation index - RQD and the dip angles of systems of mutually orthogonal joints - α , to determine the modulus of deformation of the rock mass.

1 Introduction

In the previously published work of the authors [1], the results of numerical modeling were considered when determining the deformation characteristics of fractured rock massifs. Fragments with a size of $1 \times 1 \text{ m}^2$, weakened by systems of plane-parallel and mutually orthogonal cracks (layered fractured and block fragments respectively) were investigated under the action of a load normal to the direction of one of the systems of cracks.

The accuracy of numerical calculations was estimated by comparing the results obtained with the results of the analytical calculation method proposed in [2-4]. The method has a strict mathematical justification and allows one to determine the deformation modulus of a structurally heterogeneous medium weakened by a system of plane-parallel cracks according to the formula:

$$E_{\perp} = \frac{k_n E E_0}{k_n + E_0} \quad (1)$$

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where E_{\perp} = the modulus of deformation of the medium normal to the plane of the cracks, l = the distance between cracks, k_n = the normal stiffness of the crack, E_0 = the modulus of elasticity of the structural element of the medium (rock separation).

Numerical modeling of layered-fractured fragments weakened by a plane-parallel system of fractures [1,5] showed an increase in the convergence of the results of analytical and numerical solutions with an increase in the number of structural elements (layers) of a fragment. So, with an increase in the number of layers with a modulus of elasticity $E_0 = 100000$ MPa, from 10 to 27, the accuracy of numerical calculations increased from 9.8% to 3.8%, and with $E_0 = 1000$ MPa, from 7% to 3.4%. This convergence was confirmed by the quasi-discontinuity criterion proposed in Ukhov, S.B. 1975, which formulates the conditions for using solid mechanics to solve the problem of deformation of structurally heterogeneous and fractured media. The convergence of the results obtained from dependence (1) and using numerical calculations also confirmed the accuracy of the analytical solution and the possibility of using this dependence in the practical determination of the deformation characteristics of transversely isotropic media, i.e. a medium weakened by a system of plane-parallel cracks. At the same time, studies [1] showed that this dependence, taking into account the accuracy of solving engineering problems, can be used to study the deformation of media with a different character of anisotropy. Thus, a medium weakened by a system of mutually orthogonal cracks is orthotropic, and the proposed analytical solution cannot formally be used to determine its deformation characteristics. At the same time, the results of numerical calculations of the deformation of fragments, layered-fractured and block, in which one of the systems of mutually orthogonal cracks was parallel to the action of the applied load, showed a convergence of 3.4 – 9.8% [1], which makes it possible to use dependence (1) in engineering calculations.

2 Materials and methods

The purpose of these studies is to study the possibility of using analytical solutions to determine the deformation characteristics of block fragments, the direction of the crack system of which does not coincide with the direction of the load acting on the fragment. For this, two more series of numerical experiments were carried out to study fragments weakened by systems of mutually orthogonal cracks oriented at angles $22^{\circ}30'$ and 45° to the direction of the load (Fig. 1).

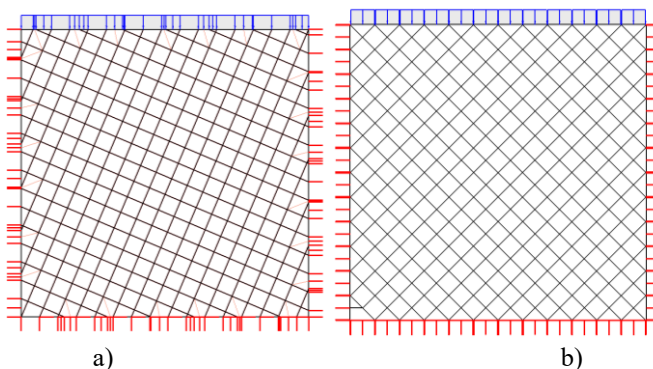


Fig. 1. Fragment with an inclined system of cracks: a) $22^{\circ}30'$; b) 45° .

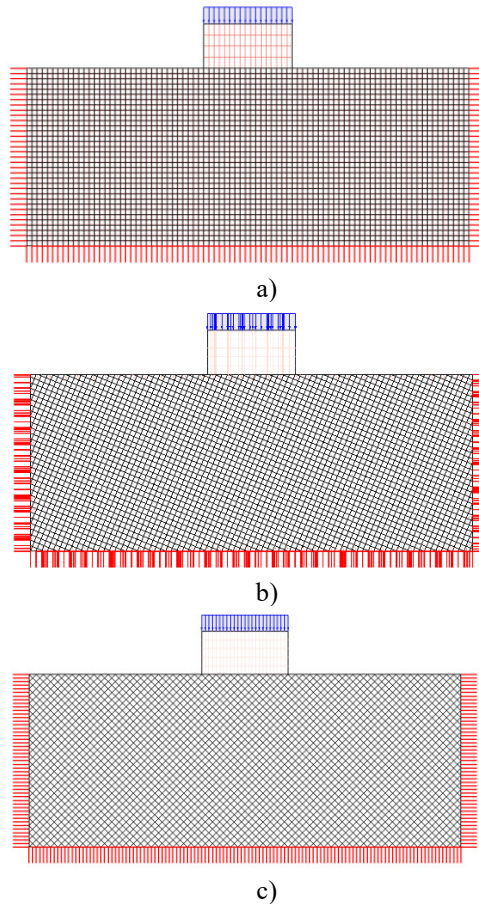


Fig. 2. Stamp tests: a) 0°; b) 23°; c) 45°.

However, preliminary calculations have shown that tests of block fragments containing inclined systems of cracks, under the condition of zero displacements of the side walls of the fragment (odometer conditions), give incorrect results, since the indicated limitation on horizontal displacements prevents the layers (blocks) from displacing along the planes of the cracks.

Taking this into account, a series of methodical numerical experiments was carried out on models of stamps that transfer the load to the block base. We investigated the foundations weakened by two systems of mutually orthogonal cracks oriented to the direction of the force action at angles: 0° 22°30' and 45° (Fig. 2).

The width of the stamp was $B = 1$ m, the dimensions of the block model of the rock mass were 5×4 m². The stamp load was increased in steps. All calculations were performed for cases in which the frequency of cracks (the number of cracks per 1 m) increased: $\lambda=7$; $\lambda=9$; $\lambda=16$ и $\lambda=26$. The dependence proposed in Priest, S. & Hudson J. 1976. (Fig. 2) shows that the accepted values of the frequency of cracks λ correspond to the following values of the quality index of the considered area of the rock mass – RQD: 85%; 75%; 50%; 25% [6]. All numerical calculations (FEM) were performed using the Z-Soil software package. In numerical modeling, the modulus of deformation of the base of the stamp was calculated from the average value of the settlement of five points of contact of the stamp with the base.

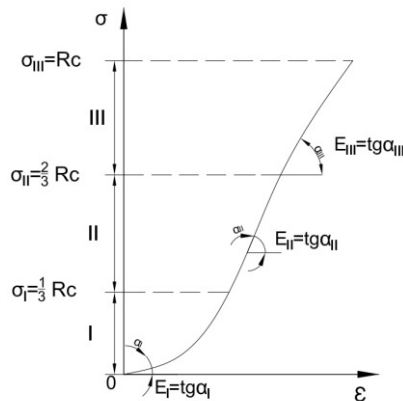


Fig. 3. Deformation curve of block fragments.

As shown by the results of field and laboratory experimental studies obtained by Bieniawski Z.T. 1978, Shiryaev R.A., Karpov N.M. and Pridorogina, I.V. 1976, Ukhov S.B., Gaziev E.G. and Lykoshi A.G. 1981, Jaeger Ch. 1983, as well as the results of calculations of Zertsalov M.G. and Sakania B.E. 1994, the graph of deformation of fractured rock massifs under compression $\sigma = f(\varepsilon)$ is a curve (Fig. 3), which can be divided into three sections [7].

The nonlinearity of the first section (0A) is due to the process of closing the cracks in the rock mass. The end of this process corresponds to the beginning of the second section (AB), within which linear deformation of the massif is observed, since rocky parts (blocks) are deformed almost elastically. In addition, as experimental studies show, the blocks begin to shift along the planes of interblock cracks. However, shear deformations, within the second section, increase slowly and proportionally to the load, which does not affect the linear nature of the deformation of the rock mass. Deformation nonlinearity begins to manifest itself in the third section (BC). The reason for this lies in the beginning increase in the relative displacements of adjacent rock blocks, as well as their destruction. When studying the interaction of structures with a rock mass, it is necessary to know the boundaries of each of the sections. Based on the analysis of the research results of Sakania B.E. 1997, Vitke V. 1990, Urastembekov B.A. 1996, it was found that the boundary of the first section of the deformation curve is approximately determined by the value of stresses equal to 1/3 of the compressive strength of the rock joint $\sigma_1 = 1/3R_{comp}$, and the end of the second section corresponds to a stress equal to approximately $\sigma_1 = 2/3R_{comp}$. At the same time, the analysis of the interaction of engineering structures with rock mass shows that the stress level, even at the base of high concrete dams, as a rule, does not exceed the values corresponding to the second linear section of the deformation curve. Since, as mentioned above, the influence of the angle of inclination of the system of cracks with respect to the direction of the load begins to manifest itself after the complete closure of the cracks and the appearance of the possibility of shear blocks along them, the interaction of the stamp with the block mass was studied only within the limits of the second (linear) section of deformation of the curve $\sigma = f(\varepsilon)$.

3 Results

The results of numerical modeling of a methodological series of experiments were compared with the results of analytical calculations according to the dependence proposed

[8]. The dependence allows calculating the deformation modulus of the medium, weakened by the system of mutually intersecting cracks:

$$E_m = \left[\frac{1}{E_0} + \frac{\cos^2 \theta_1}{L_1} \left(\frac{\sin^2 \theta_1}{k_{s1}} + \frac{\cos^2 \theta_1}{k_{n1}} \right) + \frac{\cos^2 \theta_2}{L_2} \left(\frac{\sin^2 \theta_2}{k_{s2}} + \frac{\cos^2 \theta_2}{k_{n2}} \right) \right]^{-1} \quad (2)$$

where E_m = the modulus of deformation of the block medium, l = the distance between the cracks, k_n = the normal stiffness of the crack, k_s = the tangential stiffness of the crack [9,10], E_0 = the modulus of elasticity of the rock block.

Comparison of the results of a methodological series of numerical calculations with the results calculated by Formula 2 showed their discrepancy (within 15 - 40%) in more than half of the cases studied. Analysis of the deformation of the base of the stamps made it possible to establish the reason for this difference in the results. With an increase in the load on the stamps, the interblock seams began to open, disrupting the continuity of the base, which indicated that it was incorrect to use the dependences of the mechanics of a continuous elastic medium when determining the deformation modulus of the massif.

Taking this into account, according to the results of numerical modeling using the method of experiment planning, a parametric regression analysis was performed and a regression equation was obtained to determine the modulus of deformation of a block medium depending on three independent factors. These factors include: the modulus of elasticity of the rock unit (block) E_0 , the quality index of the rock mass RQD and the angle of inclination of the system of mutually orthogonal cracks to the direction of action of the load applied to the stamp α . Experimental studies, as well as the results of numerical modeling have shown that the selected independent factors have the greatest effect on the deformation characteristics of a block medium within the second deformation section. Since, by the beginning of the specified section, the cracks in the mass are closed, and the block medium begins to deform linearly, the values of the normal stiffness k_n within the second section remain constant and equal to the value corresponding to the beginning of this section, as a result of which the value of k_n can be determined by the formula:

$$k_n = \frac{\sigma_1}{v_{max}} b \quad (3)$$

where according to Bandis S., Lumsden A. and Barton N. 1983. σ_1 = stress at the end of the first (beginning of the second) deformation sections, $\sigma_1 = \frac{1}{3} R_{comp}$, v_{max} = the maximum value of crack closure.

The values of independent parameters varied within the following limits: $E_0 = 1000 - 100000$ MPa, RQD = 25 - 85%, angle $\alpha = 0^\circ - 45^\circ$. Taking these factors into account, a matrix for planning numerical experiments was made. For each numerical calculation determined by the planning matrix, the value of k_n was determined by Formula 3. Since the problem is solved in an elastic formulation, when modeling the shear of blocks along the interblock planes of cracks, the values of the tangential stiffness k_s were calculated using the formula of the theory of elasticity:

$$k_s = \frac{k_n}{2(1+\nu)} \quad (4)$$

Based on the results of calculations performed in accordance with the experiment planning matrix, a regression equation was obtained to determine the deformation modulus of a block medium for any combination of values of the above independent parameters, within the range of their variation.

$$E_m = 11227 + 4107 \times \frac{RQD-55}{30} + 10703.3 \times \frac{E_0-50500}{49500} - 2056.8 \times \frac{\alpha-22.5}{22.5} + 3860.1 \times \frac{RQD-55}{30} \times \frac{E_0-50500}{49500} - 1418.2 \times \frac{RQD-55}{30} \times \frac{\alpha-22.5}{22.5} - 2008.7 \times \frac{E_0-50500}{49500} \times \frac{\alpha-22.5}{22.5} - 1382.4 \times \frac{RQD-55}{30} \times \frac{E_0-50500}{49500} \times \frac{\alpha-22.5}{22.5} \quad (5)$$

The performed verification of the adequacy of the equation showed the value of the coefficient of determination $r^2 = 0,85$, which is a good indicator ($r^2 \geq 0,8$) and allows the equation to be used in solving engineering problems.

Next, three main series of numerical experiments were carried out. Their results were compared, both with the results determined by the regression equation and by Formula 2.

In the first series of experiments (Fig. 2a), in which one of the systems of cracks at the base of the stamp had a direction parallel to the direction of the load ($\alpha = 0^\circ$), the results of numerical simulation were very close to the results of analytical calculations (about 4%) obtained by Formulas 1 and 2.

In the second series of experiments, the deformation of a block fragment was investigated, which had an inclination of one of the systems of cracks to the line of action of the applied load at an angle of $22^\circ 30'$ (Fig. 2b). In the third series of experiments, the angle of inclination was 45° , respectively (Fig. 2c).

1 4 Discussion

The size of the article does not allow considering all the research results of interest. Therefore, Figure 4 shows, as an example, two cases of deformation of the block mass of the base of the stamp, reflecting the general trend, at randomly chosen combinations of the values of independent factors ($E_0=50500$ MPa, $RQD=50\%$, $\alpha = 22^\circ 30'$ and $E_0=50500$ MPa, $RQD=50\%$, $\alpha = 45^\circ$). In both cases, one can clearly see the boundaries of the opening of interblock seams and, determined by them, the areas of deformation of the array. The research results make it possible to evaluate both the effect on the deformation modulus of the stamp base E_m , angle α , and the accuracy of the calculation methods used.

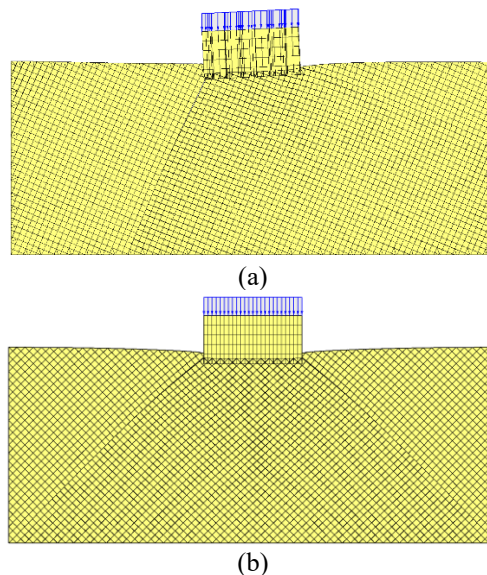


Fig. 4. Deformation of the fractured mass: a) $22^\circ 30'$; b) 45° .

In Figure 5, graphs of the dependence $\sigma = f(\varepsilon)$, constructed according to the regression Equation 5, according to the results of numerical modeling, and also according to the Formula 2 are presented. Comparison of the graphs shows that the modulus of deformation of the base of the stamp E_m decreases with increasing angle α . Moreover, in the first case ($\alpha = 22^\circ 30'$), the accuracy of numerical calculations in relation to the results of calculations according to the regression Equation 5 and Formula 2 is, respectively, 6.1% and 15.8%, and in the second ($\alpha = 45^\circ$) – 19.3% and 29.1%.

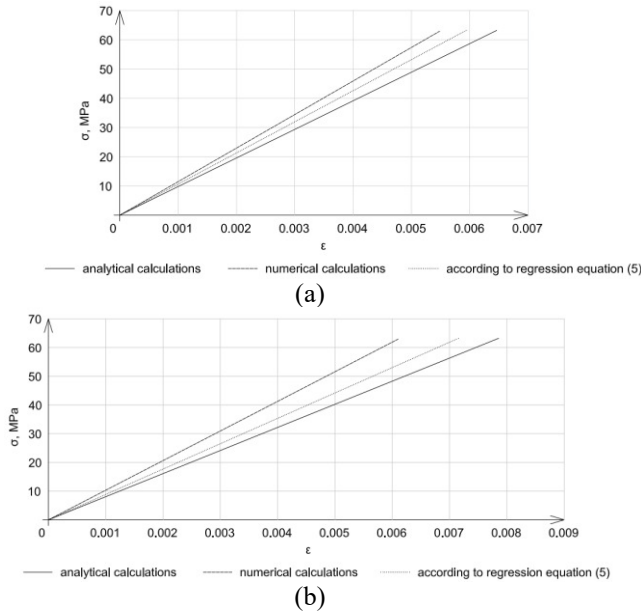


Fig. 5. Deformation graph of a block massif: a) $22^\circ 30'$; b) 45° .

The accuracy obtained by comparing the calculation methods used in these studies is typical for all the cases considered. It follows from this that the accuracy of determining the deformation modulus of the stamp according to the regression equation corresponds to the engineering accuracy of calculations (15% - 20%) and can be used for a preliminary assessment of the deformation characteristics of rock massifs. It should be especially noted that, in case of deformation of fractured-block media, as mentioned above, the opening of interblock cracks occurs. Considering this, the use of the recommended calculation methods based on solid mechanics is possible only when studying rock massifs weakened by systems of mutually orthogonal cracks. At the same time, one of the systems must be close in the direction of the load applied to the massif. In other cases, analytical methods should be used with caution, taking into account the high inaccuracy (up to 50%) of the results obtained.

2 5 Conclusions

1. The use of analytical methods of calculation based on the laws of solid mechanics is advisable in cases when a fractured block massif is weakened by systems of mutually orthogonal cracks, provided that one of the systems of cracks is close in the direction of the applied load.
2. The regression equation proposed in the work to determine the modulus of deformation of fractured-block media, allows to obtain results with an accuracy accepted in engineering

calculations (15 - 20%). This allows us to recommend it for use in a preliminary assessment of the deformation characteristics of rock massifs.

3. Taking into account the complexity of the task of determining the mechanical characteristics of rock massifs, further development of research in this direction is necessary, with special attention to the development of computational methods.

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