SMEFT as a slice of HEFT's parameter space

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Abstract. The Standard Model Effective Field Theory (SMEFT) is the parametrization chosen to interpret many modern measurements. We have recently discussed, building on the work of other groups, that its overall framework can be experimentally tested, beyond simply constraining its parameters. This is because the Higgs Effective Field Theory (HEFT) is somewhat more general, as it does not assume that the Higgs boson *h* needs to be embedded in a complex doublet *H* on which the Standard Model (SM) and SMEFT are built. As a result, the HEFT parameter spaces for the various relevant channels contains hypersurfaces over which one may use SMEFT to describe data. If experimental measurements of HEFT's parameters in any of those various channels yield a point outside of any of the hypersurfaces, SMEFT is falsified; meanwhile, its framework remains appropriate (in particular, as long as the SM remains compatible with data). A common necessity of the various possible tests is that processes involving different number of Higgs bosons (maintaining the number and nature of other particles unchanged) need to be contrasted.

1 Introduction: two EFTs for the electroweak sector

Beyond the Standard Model (BSM) searches in experiments at the LHC and other machines have come to be used to constrain the well-known SMEFT expansion,

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_{i} \frac{c_i^{(n)}}{\Lambda^{n-4}} O_i^{(n)}(H) \,. \tag{1}$$

There, the various operators are organized by canonical (field and derivative) dimension as exposed by the powers of Λ made explicit, and their c_i coefficients bound by experiment [1–4]. The particle content of the electroweak symmetry breaking sector is packaged in a Higgs doublet field, just as in the SM,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_0 + i\varphi_3 \end{pmatrix} = U(\omega) \begin{pmatrix} 0 \\ (v + h_{\text{SMEFT}})/\sqrt{2} \end{pmatrix}. \tag{2}$$

There, the Cartesian coordinates φ_a can be rearranged into the polar decomposition in terms of the ω_i Goldstone bosons (which set the orientation of *H* through the unitary matrix $U(\omega)$)

and the radial coordinate h_{SMEFT} (with $|H| = (v + h_{SMEFT})/\sqrt{2}$). If, alternatively, no assumption about the Higgs-boson h being part of a doublet H is made, the appropriate effective theory is rather HEFT, directly written in terms of h (see [5] for a recent review)

$$\mathcal{L}_{HEFT} = \frac{1}{2}\partial_\mu h_{HEFT}\partial^\mu h_{HEFT} - V(h_{HEFT}) + \frac{1}{2}\mathcal{F}(h_{HEFT})\partial_\mu\omega^i\partial^\mu\omega^j\left(\delta_{ij} + \frac{\omega^i\omega^j}{v^2 - \omega^2}\right).\tag{3}$$

The h_{HEFT} Higgs (or for simplicity, h) is exposed in multiplicative functions rescaling the traditional operators of the old electroweak chiral Lagrangian; for an alternative form see [6].

It was believed that one could indistinctly convert them into one another, as between Cartesian and polar coordinates, by wrapping or unwrapping the H field to expose h and the Goldstone bosons ω_i, or to reconstruct it from them. But several works led by the San Diego group [7–10] have clarified that while HEFT can always be cast as a SMEFT, the converse is not always true, there are some requirements on the "flare" function of the Higgs field $\mathcal{F}(h_{HEFT}) = 1 + \sum_{n=1}^{\infty} a_n\left(\frac{h_{HEFT}}{v}\right)^n$, that we will shortly expose; saliently, $\mathcal{F}$ needs to have a zero h^* and satisfy certain analyticity properties.

In our recent contributions [11, 12], we have translated these criteria into practical conditions that amount to finding certain correlations among the HEFT parameters that, if violated by experimental data (also interpretable in terms of HEFT coefficients [13, 14]), would falsify SMEFT. Here we quickly overview these developments.

2 Regularity conditions for the applicability of SMEFT

Alonso, Jenkins and Manohar [15] formulated HEFT in geometrical terms in the space of Higgs-fields and detailed the regularity conditions that the HEFT Lagrangian needs to satisfy to support a SMEFT, employing an elegant covariant formalism. We will proceed in a more pedestrian way, converting SMEFT into HEFT and viceversa with

$$|\partial H|^2 + \frac{1}{2}B(|H|^2)(\partial(|H|^2))^2 \longleftrightarrow \frac{v^2}{4}\mathcal{F}(h_{HEFT})\operatorname{Tr}\{\partial_\mu U^\dagger\partial^\mu U\} + \frac{1}{2}(\partial h_{HEFT})^2,\tag{4}$$

The change from SMEFT to HEFT is straightforward and always possible, with the canonical, nonlinear change of variables given in differential form as

$$dh_{HEFT} = \sqrt{1 + (v + h_{SMEFT})^2 B(h_{SMEFT})}\, dh_{SMEFT},\tag{5}$$

where the flare-function is provided by $\mathcal{F}(h_{HEFT}) = (1 + h_{SMEFT}/v)^2$.

However, the reciprocal conversion $h_{HEFT} = \mathcal{F}^{-1}\left((1 + h_{SMEFT}/v)^2\right)$ from HEFT to SMEFT is not always possible, because of the need to reconstruct squared operators of the Higgs doublet field H necessary for SMEFT, such as

$$|H|^2 = \frac{(v + h_{SMEFT})^2}{2},$$
$$(\partial|H|^2)^2 = (v + h_{SMEFT})^2 (\partial h_{SMEFT})^2 = 2|H|^2 (\partial h_{SMEFT})^2.$$

The extra $|H|^2$ on the right hand side of the second equation appears then in a denominator

$$\mathcal{L}_{HEFT} = \underbrace{|\partial H|^2}_{=\mathcal{L}_{SM}} + \underbrace{\frac{1}{2}\left[\frac{8|H|^2}{v^2}\left((\mathcal{F}^{-1})'\left(2|H|^2/v^2\right)\right)^2 - 1\right]\frac{(\partial|H|^2)^2}{2|H|^2}}_{=\Delta\mathcal{L}_{BSM}}.\tag{6}$$

As SMEFT requires a Taylor expansion as in Eq. (1), this singularity precludes its existence and needs to be cancelled by the preceding bracket in the second line of Eq. (6).

The existence of that zero $h^* \equiv \mathcal{F}^{-1}(0)$ of $\mathcal{F}$, and the analyticity required for a power series expansion for $\mathcal{F}$, the Higgs potential V, etc., become necessary requirements for a HEFT Lagrangian density to be expressible as a SMEFT. Our findings, in agreement with [10], can be abstracted as follows.

1. The $\mathcal{F}$ function in the HEFT Lagrangian density of Eq. (3) $\mathcal{F}(h^*) = 0$ must have a double zero; $SU(2) \times SU(2)$ is a good global symmetry there.

2. There, its second derivative must satisfy $\mathcal{F}''(h^*) = \frac{2}{v^2}$.

3. All odd derivatives vanish, $\mathcal{F}^{(2\ell+1)}(h^*) = 0$.

The restrictions over $\mathcal{F}$ at the symmetric zero h^* ($h = -v$ in the SM) that guarantee the existence of a SMEFT are theoretically illuminating but not too practical, since our measurements are taken around the vacuum $h = 0$.

3 Translating the geometrical conditions into constraints over the accessible HEFT parameter space

A key contribution of our earlier work has been to translate those conditions into restrictions over the a_i at the physical vacuum $h = 0$. To do it, we match two series expansions, the one around $h = 0$ (setting $a_0 := 1$ and taking h normalized to v, so that $v = 1$) and the one around h^*, expanding in terms of $a_j^* = \mathcal{F}^{(j)}(h^*)/j!$,

$$\mathcal{F}(h) = \sum_{i=0}^{n} a_i h^i , \quad \mathcal{F}(h) = \sum_{j=0}^{n} a_j^* (h - h^*)^j . \tag{7}$$

By matching the two expansions around the two different points it is easy to read off the coefficients a_j^* that encode the conditions over $\mathcal{F}$ in terms of the a_i, accessible to experiment. The resulting correlations among the HEFT conditions are given in Table 1. The relation between the three lowest-order coefficients involving one, two and three h bosons, (respectively a_1, a_2, a_3) is shown in Figure 1. Finally, the numerical intervals for these HEFT parameters a_i currently consistent with experimental data are given in Table 2.

We have next examined a couple of other HEFT multiplicative form-factor functions that incorporate powers of h to further operators of the electroweak Lagrangian; our second example here is the nonderivative $V(H)$ Higgs-potential, accessible at "low" $\sqrt{s}$ as the absence of derivatives minimizes the number of momentum powers. The traditional SM form

$$\mathcal{L}_{\mathrm{SM}} = |\partial H|^2 - \underbrace{\left(\mu^2 |H|^2 + \lambda |H|^4 \right)}_{V(H)}, \tag{8}$$

written in terms of h acquires additional non-renormalizable couplings in HEFT, organized in a power-series expansion

$$V_{\mathrm{HEFT}} = \frac{m_h^2 v^2}{2} \left[\left(\frac{h_{\mathrm{HEFT}}}{v} \right)^2 + v_3 \left(\frac{h_{\mathrm{HEFT}}}{v} \right)^3 + v_4 \left(\frac{h_{\mathrm{HEFT}}}{v} \right)^4 + \dots \right], \tag{9}$$

with $v_3 = 1$, $v_4 = 1/4$ and $v_{n \geq 5} = 0$ in the SM. For the applicability of SMEFT, the coefficients of this series must satisfy the constraints listed in Table 3 and Figure 2. From the ATLAS [17]

bound $\Delta v_3 \in [-2.5, 5.7]$, $O(1/\Lambda^2)$ SMEFT predicts the coefficient intervals in the last column of Table 3, which can be tested in few-Higgs final states. A coupling $c_{H\Box} \neq 0$ (stemming from the SMEFT operator relevant at the TeV scale for the electroweak symmetry breaking sector $O_{H\Box} = |H|^2 \Box |H|^2$ [11]) introduces the correction $\Delta a_1 \propto c_{H\Box}$ that appears in the first row

Table 1. Correlations among the a_i HEFT coefficients necessary for SMEFT to exist, at the orders Λ^{-2} and Λ^{-4}, given in terms of $\Delta a_1 := a_1 - 2 = 2a - 2$ and $\Delta a_2 := a_2 - 1 = b - 1$ (so that all entries in the table vanish in the SM, with all equalities becoming $0 = 0$). Notice that the r.h.s. of each identity in the second column shows the $O(\Lambda^{-4})$ corrections to the relations of the first column. The third one assumes the perturbativity of the SMEFT expansion.

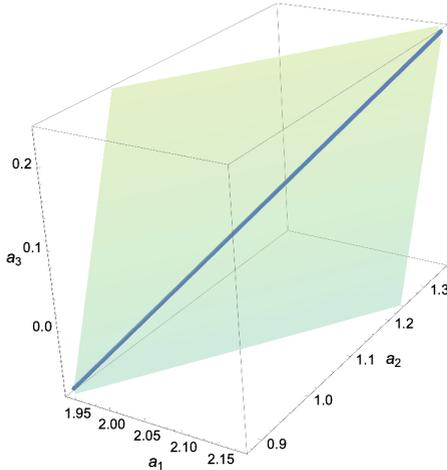| Correlations accurate at order Λ^{-2}	Correlations accurate at order Λ^{-4}	Λ^{-4} Assuming SMEFT perturbativity
$\Delta a_2 = 2\Delta a_1$		$|\Delta a_2| \leq 5|\Delta a_1|$
$a_3 = \frac{4}{3}\Delta a_1$	$\left(a_3 - \frac{4}{3}\Delta a_1\right) = \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}(\Delta a_1)^2$	
$a_4 = \frac{1}{3}\Delta a_1$	$\left(a_4 - \frac{1}{3}\Delta a_1\right) = \frac{5}{3}\Delta a_1 - 2\Delta a_2 + \frac{7}{4}a_3 =$	those for a_3, a_4, a_5, a_6
	$= \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{7}{12}(\Delta a_1)^2$	
$a_5 = 0$	$a_5 = \frac{8}{5}\Delta a_1 - \frac{22}{15}\Delta a_2 + a_3 =$	all the same
	$= \frac{6}{5}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}(\Delta a_1)^2$	
$a_6 = 0$	$a_6 = \frac{1}{6}a_5$	

Figure 1. Correlations for the HEFT coefficients from Table 1 that need to be satisfied for SMEFT to be a valid EFT of the electroweak symmetry breaking sector. The solid diagonal is the correlation of order Λ^{-2}, that becomes broadened as the indicated band (greenish online) at order Λ^{-4}.

Table 2. Bounds on the a_i HEFT coefficients that need to be satisfied for SMEFT to exist. The first column being consistent at $O(\Lambda^{-2})$ with 95% confidence-level [16]) experimental bounds $a_1/2 \in [0.97, 1.09]$. The second column corresponds to the next order [11].

$O(1/\Lambda^2)$	$O(1/\Lambda^4)$
$\Delta a_2 \in [-0.12, 0.36]$	
$a_3 \in [-0.08, 0.24]$	$a_3 \in [-3.1, 1.7]$
$a_4 \in [-0.02, 0.06]$	$a_4 \in [-3.3, 1.5]$
	$a_5 \in [-1.5, 0.6]$
	$a_6 = a_5$

(and affects the flare function already discussed), but in the potential is for now numerically negligible since v_3 experimental uncertainties are way larger than the allowed Δa_1.
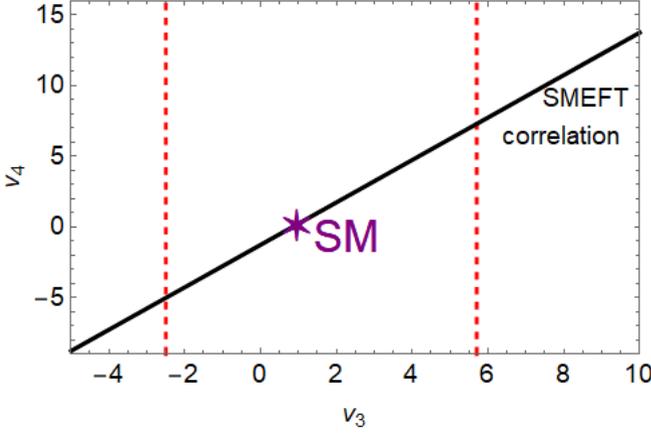

Figure 2. The correlation $v_4 = \frac{3}{2}v_3 - \frac{5}{4} - \frac{1}{6}\Delta a_1$ that SMEFT predicts at $O(1/\Lambda^2)$, from the 95% confidence interval for $v_3 \in [-2.5, 5.7]$ [17]. The experimental a_1 uncertainty [16, 19], $\frac{a_1}{2} \in [0.97, 1.09]$, is numerically negligible and allows to predict the (solid, diagonal) SMEFT band. An experimental measurement of v_4 is necessary to test this, by adding a horizontal band to the plot.

As a third relevant example, we proceed from the SM coupling the top quark to the Higgs boson. This has been extended into HEFT [20] by a multiplicative function $\mathcal{G}(h)$

$$\mathcal{L}_Y = -\mathcal{G}(h)M_t \bar{t}t \sqrt{1 - \frac{\omega^2}{v^2}}, \tag{10}$$

that can be Taylor-expanded around the physical $h = 0$ vacuum according to

$$\mathcal{G}(h_{\text{HEFT}}) = 1 + c_1 \frac{h_{\text{HEFT}}}{v} + c_2 \left(\frac{h_{\text{HEFT}}}{v}\right)^2 + \dots \tag{11}$$

(in the SM $c_1 = 1$ and $c_{i\geq 2} = 0$ simplify). The correlations among these coefficients induced by SMEFT at order $1/\Lambda^2$ are also given, in the last row of Table 3 and in Figure 3.

Table 3. First two rows: correlations among the coefficients $\Delta v_3 := v_3 - 1$, $\Delta v_4 := v_4 - 1/4$, v_5 and v_6 of the HEFT Higgs potential expansion in Eq. (9) that need to hold, at $O(1/\Lambda^2)$, if SMEFT is a valid description of the electroweak sector. The third row includes the leading correlations for the Yukawa $\mathcal{G}(h)$ function of Eq. (11), constraining c_2 and c_3 by c_1 and a_1 (from the correction to the value of the symmetric point h^*). We make use of current 95% confidence interval for the top Yukawa coupling $c_1 \in [0.84, 1.22]$ [18].

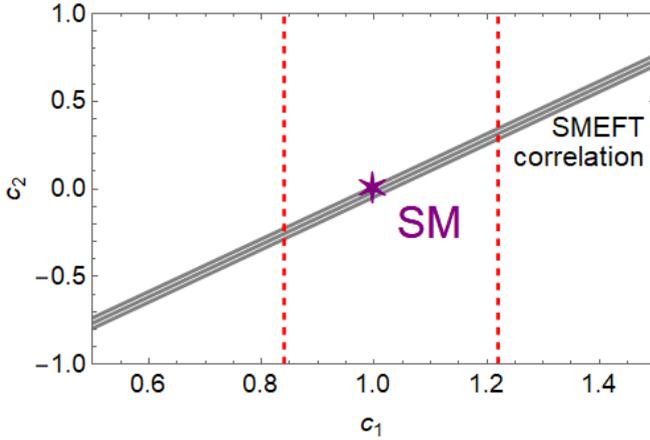|	
$\Delta v_4 = \frac{3}{2}\Delta v_3 - \frac{1}{6}\Delta a_1$	$\Delta v_4 \in [-3.8, 8.6]$
$v_5 = 6v_6 = \frac{3}{4}\Delta v_3 - \frac{1}{8}\Delta a_1$	$v_5 = 6v_6 \in [-1.9, 4.3]$
$c_2 = 3c_3 = \frac{3}{2}(c_1 - 1) - \frac{1}{4}\Delta a_1$	$c_2 = 3c_3 \in [-0.27, 0.35]$

Figure 3. The correlation $c_2 = \frac{3}{2}(c_1 - 1) - \frac{1}{4}\Delta a_1$ predicted by SMEFT at $O(1/\Lambda^2)$ in the top Yukawa sector of HEFT. The 95% confidence interval for the coupling $c_1 \in [0.84, 1.22]$ is taken from [18] (red dashed lines). The experimental uncertainty [16, 19] on a_1, namely $a_1/2 \in [0.97, 1.09]$ (at 95% CL), is reflected in the width of the gray band with the SMEFT correlation.

4 Conclusion:
Testing the embedding of SMEFT into HEFT at colliders

The SM is compatible, to date, with the measurements of the electroweak sector. It is a particular case of SMEFT and as a matter of course, all correlations discussed in this article among the coefficients of HEFT (many trivially) are satisfied.

Nevertheless, if the high-luminosity LHC of a future accelerator such as a new e^-e^+ machine finds cracks in the SM, testing [21] whether both SMEFT and HEFT or only HEFT can be applied will become an issue.

The $\omega\omega \to nh$ processes with n a varying number of Higgs particles would be key to disentangling the EWSBS. They give direct access to the a_i coefficients of the flare function $\mathcal{F}$, and hence to their correlations, as listed in Table 1. Figure 4 shows the typical Feynman diagrams describing the contact part of these processes that yield direct access to the a_i coefficients.
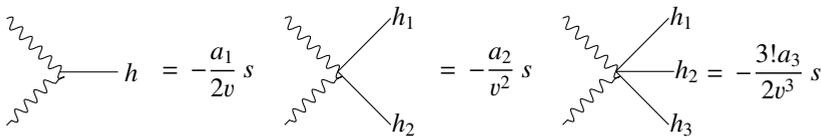

Figure 4. Contact diagrams which provide a_i; in a given physical process they are summed to t-channel exchanges between the two initial Goldstone bosons (wiggly lines) ω_i, as given in [11].

For example, the BSM part of the two- to two-body amplitude is given by a single SMEFT coefficient,

$$T^{\text{SMEFT}}_{\omega\omega \to hh}(s) = -2s\frac{c_{H\Box}}{\Lambda^2} + O(\Lambda^{-4})\,. \tag{12}$$

At this same order, the two- to three-body one receives a contribution from the a_3 coefficient and also t-channel exchanges; in HEFT, these are controlled by a_1 and a_2, and are independent. However, in SMEFT all three coefficients are functions of the same $c_{H\Box}$ and appear to cancel, $T^{\text{SMEFT}}_{\omega\omega \to hhh}(s) = 0 + O(\Lambda^{-4})$.

Figure 5 displays the currently allowed experimental band (highlighted in yellow) for the first two coefficients, a_1 and a_2 (in the popular κ notation in which the experimental cross section is scaled to the SM one, these are κ_V and κ_{2V}) [1].
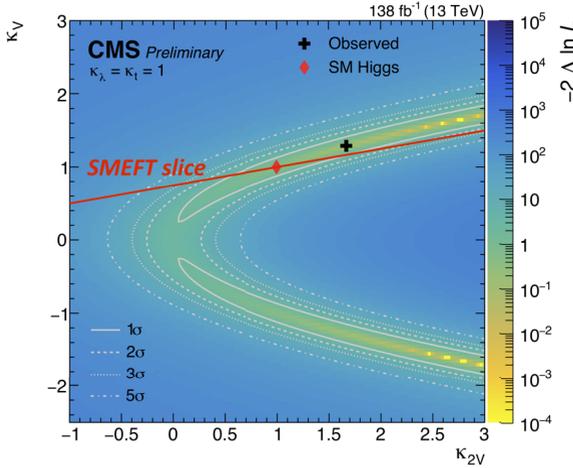

Figure 5. Experimental CMS exclusion plot in the a_1, a_2 plane (reproduced from [22] under the Creative Commons BY 4.0 license.) We have superimposed the SMEFT slice through this parameter space, that is seen to run parallel to one of the branches of the experimental exclusion band.

We have superimposed the SMEFT line (red online) which represents the $O(1/\Lambda^2)$ correlation between these two parameters.

This shows that current data is much more sensitive to a_1 than a_2, but the gentle sloping almost parallel to the SMEFT slice through that HEFT parameter space can also signal that the measurement is more sensitive to the separation from SMEFT than to the coefficient of SMEFT itself. This might be due to the fact that $c_{H\square}$ only brings an overall renormalization of h and no additional derivatives, so it does not change the shape of any kinematic distributions, it is a parameter of more difficult access. The SM (and therefore SMEFT too) is seen to comfortably sit within the $1-\sigma$ band, in good agreement with the data.

Current experimental constraints are not really significant for $n \geq 2$ and it will be challenging to obtain information on the coefficients for higher number of Higgs bosons n. Better reconstruction techniques at the high-luminosity LHC run but especially a future high-energy collider (either hadronic or muonic) will hopefully improve the situation.

Likewise, to exploit the correlation in Figure 3, an experimental determination of the $t\bar{t} \rightarrow hh$ coupling c_2 needs to be undertaken. If it would be measured, SMEFT can be tested by comparing to $c_2 \in [-0.27, 0.35]$ (where uncertainties have been added linearly). Similar measurements in the Yukawa sector, with an increasing number of Higgs bosons, would lead to further tests of the SMEFT framework. Similar considerations apply to the Higgs potential $V(h)$.

To conclude, while other authors have also proposed methods to try to distinguish SMEFT from HEFT [23], we believe that our work offers clear criteria that depend only on the availability of sufficiently precise experimental data, and eventually higher order EFT computations [24] to match that precision.

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