AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(2): 4126–4140.
DOI: 10.3934/math.2023205
Received: 25 September 2022
Revised: 07 November 2022
Accepted: 15 November 2022
Published: 01 December 2022

Research article

New and effective solitary applications in Schrödinger equation via Brownian motion process with physical coefficients of fiber optics

Yousef F. Alharbi[1], **E. K. El-Shewy**[2,3] **and Mahmoud A. E. Abdelrahman**[1,4,*]

[1] Department of Mathematics, College of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia
[2] Department of Physics, College of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia
[3] Theoretical Physics Group, Faculty of Science, Mansoura University, Mansoura, Egypt
[4] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

* **Correspondence:** Email: mahmoud.abdelrahman@mans.edu.eg.

Abstract: Using the unified solver technique, the rigorous and effective new novel optical progressive and stationary structures are established in the aspects of hyperbolic, trigonometric, rational, periodical and explosive types. These types are concrete in the stochastic nonlinear Schrödinger equations (NLSEs) with operative physical parameters. The obtained stochastic solutions with random parameters that are founded in the form of rational, dissipative, explosive, envelope, periodic, and localized soliton can be utilized in fiber applications. The stochastic modulations of structures' amplitude and frequency caused by dramatic instantaneous influences of both fibers nonlinear, dispersive, losing and noise term effects maybe very important in new fiber communications.

Keywords: stochastic NLSEs; soliton waves; explosive; envelope; solver technique; fiber communications
Mathematics Subject Classification: 78A10, 35C07, 60H15, 60H30, 60H40, 35Q55, 35Q60

1. Introduction

Nonlinear complex phenomena in a superfluid, optical fiber communications, solid-state physics, epidemiology, plasma physics can be expressed in deterministic or stochastic nonlinear partial differential equations (SNPDEs) [1–8]. There are several advantages for using stochastic rather than deterministic equations, including in finance, biology, chemistry, mechanics, microelectronics and economics [9–12]. A complete comprehension of SNPDEs theory requires expertise in advanced probability and stochastic processes. Recently, many researchers have proposed and developed various numerical and analytical methods for solving NPDEs [13–16].

The NLSE depicts how waves move across mediums with dispersive and nonlinear effects. This equation turns into the basic ingredient for describing the wave behaviors in so many vital applications of applied science such as Bose-Einstein condensations (BEC), bimolecular dynamical modes, deep water, coastal water motions and semiconductors [17–22]. The NLSE has localized solutions (soliton), which are especially sturdy and propagate without changing form. The solitonic features in the molecular chain model that presented by NLSE with saturated nonlinear coefficient and discrete coupled NLSE have been investigated [23, 24]. Alkhidhr et al. [9] investigated the stochastic unstable NLSE and higher-order dispersive NLSE. Alharbi et al. [3] studied the perturbed NLSE with Kerr law nonlinearity in the presence of random dispersion and nonlinear effects. Kumar et al. [25] presented some innovative solutions to the generalized Schrödinger-Boussinesq equation, which illustrates the interaction of complex short-wave and real long-wave envelopes. Houwe et al. investigated the effects of modulated self-Kerr nonlinearities on discrete solitons propagated in optomechanical arrays. It was noted that nonlinear terms minimize the transient regimes of temporal solitonic structures [26]. The effect of noise on the propagation of these soliton solutions has received increased attention in recent years [9, 25, 27]. On the other hand, propagating solitons in materials having saturation impacts cannot be modeled by the NLS equation [28, 29]. To study these effects, the WKI equation must be taken into account [28, 29]. Li et al. investigated long time asymptotic solutions for finite density WKI equation [29]. Riemann–Hilbert (RH) problems is developed to study the focusing NLS equation with multiple high-order poles under nonzero boundary conditions. The solutions behaviours in various discrete spectra have been obtained [30].

Recently, the development of fractal solitary structures from a large number of nonlinear equations using fractal variational principles in various media turns into an effective technique to describe certain new occurrences in our universe [31–36]. Abdelrahman et al. [27] extracted new stochastic solutions for the conformable fractional nonlinear NLSE. The Adomian decomposition technique was defined and used to solve the newly constructed nonlinear Schrödinger equation with spatiotemporal dispersion [37]. Also, various fractional formulations have been presented to extract exact optical soliton solutions for modified NLSE with spatiotemporal dispersion [38, 39].

The Wiener process is often called Brownian motion (BM) due to its connection with the physical process called Brownian motion. Actually, this stochastic process serves as a foundation for a variety of models in applied science [40, 41]. Since a BM process is non-differentiable, Itô's formula is often utilized to get the explicit solution of SNPDEs with a BM process [42, 43]. A 1D NLSE with an additive space-time white noise is investigated in [44]. This paper considers the NLSE with various physical optical fiber coefficients forced by multiplicative noise in Itô sense [45]:

$$i\psi_t + \alpha\psi_{xx} - \delta \mid \psi \mid^2 \psi + \lambda\psi - i\sigma W_t \psi = 0, \quad i = \sqrt{-1}, \tag{1.1}$$

$\psi = \psi(x, t)$ depicts the slowly pulse amplitude and σ is the noise strength. Indeed, α, δ & λ denote fiber dispersion, nonlinear and fiber loss effects. The noise W_t denotes derivative of the Brownian motion process $W(t)$ in time [46]. This noise term will properly manage the increasing, damping, and conversion affects on the amplitudes and frequencies of the bright/dark envelope and shock forced oscillatory wave.

In the ongoing research, we aim to extract some new stochastic solutions of Eq (1.1) utilizing the unified solver approach [47]. This solver is straightforward, sturdy, burly and averts tedious computations. Moreover, this solver produces various effective solutions for describing physical phenomena. This solver can simply be utilized as a box solver. The presented solver is applicable to a wide range of NSPDE classes. Furthermore, it is simple to expand for solutions of stochastic fractional NPDEs. We also explain how the noise term affects the solutions that have been provided. Some suggested stochastic solutions' nonlinear dynamical behavior is shown. To the best of our knowledge, the proposed solver for resolving the stochastic NLSE with various physical optical fiber coefficients has never been used before.

This article is organized as follows. Section 2 presents the notions of the Brownian motion process and its prosperities. Section 3 offers closed-form solutions for a variety of NPDEs. Section 4 presents the new stochastic solutions for Eq (1.1) via Itô sense. Section 5 introduces the interpretation of the presented solutions. We also show that the influence of the noise parameter plays an essential role in changing solution trajectories. Conclusions are summarized in Section 6.

2. Brownian motion and normal distribution

Brownian motion is used to describe the random movement that happened by small particles that are suspended in liquid or gas. The Brownian motion prohibits the particles from stabilizing in fluids because it affects on them to be in constant motion. This leads to the stability of colloidal solutions. Therefore, it assists to differentiate between true and colloid solutions. Furthermore, we note that the size of particles and the viscosity of the fluid are inversely proportional to the speed of the motion.

Before we speak about the Browning motion, we display a brief description of the normal distribution. One of the main properties of the normal distribution is that it is a symmetric and continuous distribution. Also, the data around its mean is more occurrence than when they are far from it. Furthermore, its mean, median and mode are equal and it is symmetric around the mean. So, half of the values are to the right of the mean and the other half on the left with the total area under the curve equals one.

The normal distribution's probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty,$$

μ and σ denote the mean and standard deviation, which controls the spread of the data from the mean. One of the important special cases of the normal distribution is the standard normal distribution. It has zero mean and unit variance. The corresponding probability distribution function of it is

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty.$$

Now, we note that the Wiener process is defined as a stochastic process, which is continuous in time. A Brownian motion process $\{W(t)\}_{t\geq 0}$ has the following properties:

(a) $W(t)$, $t \geq 0$ is a continuous function of time t and $W(t) \sim N(0, t)$.
(b) For $s < t < u < k$, $W(s) - W(t)$ & $W(k) - W(u)$ are independent.
(c) $W(t) - W(s)$ follows a normal distribution with zero mean and variance $t - s$, i.e. $W(t) - W(s) \sim \sqrt{t-s}\,N(0, 1)$, $N(0, 1)$ is a standard normal distribution.

3. Unified solver approach

Consider the following NPDEs:

$$\mathcal{H}(q, q_x, q_t, q_{xx}, q_{xt}, q_{tt}, ...) = 0. \tag{3.1}$$

Using the wave transformation:

$$q(x, t) = q(\zeta), \qquad \zeta = x - w\,t. \tag{3.2}$$

Equation (3.1) simplified to the following ODE:

$$\mathcal{G}(q, q', q'', q''', ...) = 0, \tag{3.3}$$

where w is the wave speed. In applied science, several NPDEs can be reduced to:

$$L\,q'' + M\,q^3 + N\,q = 0, \tag{3.4}$$

where L, M and N are constants rely on the proposed equation's constants and the wave transformations' speed. The solutions of Eq (3.4) are based on the unified solver approach [47], given by:

i) Rational solutions: (when $N = 0$)

$$q_{1,2}(x, t) = \left(\mp\sqrt{\frac{-M}{2L}}\,(\eta + \varsigma)\right)^{-1}. \tag{3.5}$$

ii) Trigonometric solutions: (when $\frac{N}{L} < 0$)

$$q_{3,4}(x, t) = \pm\sqrt{\frac{N}{M}}\,tan\left(\sqrt{\frac{-N}{2L}}\,(\eta + \varsigma)\right) \tag{3.6}$$

and

$$q_{5,6}(x, t) = \pm\sqrt{\frac{N}{M}}\,cot\left(\sqrt{\frac{-N}{2L}}\,(\eta + \varsigma)\right). \tag{3.7}$$

iii) Hyperbolic solutions: (when $\frac{N}{L} > 0$)

$$q_{7,8}(x, y, t) = \pm\sqrt{\frac{-N}{M}}\,tanh\left(\sqrt{\frac{N}{2L}}\,(\eta + \varsigma)\right) \tag{3.8}$$

and

$$q_{9,10}(x, y, t) = \pm\sqrt{\frac{-N}{M}}\,coth\left(\sqrt{\frac{N}{2L}}\,(\eta + \varsigma)\right). \tag{3.9}$$

Here, the constant ς is arbitrary.

4. Stochastic solutions

Using the transformation

$$\psi(x,t) = e^{i\eta(x,t)+\sigma W(t)-\sigma^2 t} q(\vartheta), \quad \vartheta = cx + \omega t, \quad \eta(x,t) = px + \mu t. \tag{4.1}$$

Equation (1.1) was transformed into the following ODE

$$\alpha c^2 q''(\vartheta) - \alpha p^2 q(\vartheta) + \lambda q(\vartheta) + \delta\, q^3(\vartheta)\left(-e^{2\sigma W(t)-2\sigma^2 t}\right) - \mu\, q(\vartheta) = 0, \tag{4.2}$$

where ω, p, c and μ are constants. Taking expectation of both sides, we have

$$\alpha c^2 q''(\vartheta) - \alpha p^2 q(\vartheta) + \lambda q(\vartheta) - \mu\, q(\vartheta) - \delta q^3(\vartheta) e^{-2\sigma^2 t} E(e^{2\sigma W(t)}) = 0. \tag{4.3}$$

Since $E(e^{2\sigma W(t)}) = e^{2\sigma^2 t}$, Eq (4.3) reduced to

$$\alpha c^2 q''(\vartheta) - \delta q^3(\vartheta) - (\alpha p^2 - \lambda + \mu) q(\vartheta) = 0, \tag{4.4}$$

with dispersion relation

$$\omega = \frac{\pm\sqrt{-\alpha\lambda c^2\sigma^4 + \alpha^2 c^2 p^2 \sigma^4 + \alpha c^2 \sigma^4 \mu} - 2\alpha^2 c p^3 + 2\alpha\lambda c p - 2\alpha c p\, \mu}{-\lambda + \alpha p^2 + \mu}. \tag{4.5}$$

Equation (4.4) depicts an energy equation of particle dynamical motion in the form

$$\frac{1}{2}\alpha c^2 q'(\vartheta)^2 - \frac{1}{4}\delta q^4(\vartheta) - (\frac{1}{2}\alpha p^2 - \frac{1}{2}\lambda + \frac{1}{2}\mu) q^2(\vartheta) = 0. \tag{4.6}$$

The model has an exact solution as follows

$$q(\vartheta) = \frac{2\sqrt{2}\left(-\lambda + \alpha p^2 + \mu\right)}{\sqrt{-\delta\left(-\lambda + \alpha p^2 + \mu\right)}} e^{\frac{(cx+t\omega)\sqrt{-\lambda+\alpha p^2+\mu}}{\sqrt{\alpha}c}} \left(e^{\frac{2(cx+t\omega)\sqrt{-\lambda+\alpha p^2+\mu}}{\sqrt{\alpha}c}} + 1\right)^{-1},$$

$$\psi(x,t) = \frac{2\sqrt{2}\left(-\lambda + \alpha p^2 + \mu\right)}{\sqrt{-\delta\left(-\lambda + \alpha p^2 + \mu\right)}} e^{\frac{(cx+t\omega)\sqrt{-\lambda+\alpha p^2+\mu}}{\sqrt{\alpha}c}} \left(e^{\frac{2(cx+t\omega)\sqrt{-\lambda+\alpha p^2+\mu}}{\sqrt{\alpha}c}} + 1\right)^{-1} e^{i\eta(x,t)+\sigma W(t)-\sigma^2 t}. \tag{4.7}$$

In light of the presented solver, Eq (4.4) has the following solutions:
First: Rational solutions are:

$$q_1(x,t) = \left(\mp\sqrt{\frac{\delta}{2\alpha c^2}}\,(cx + \omega t + \varrho)\right)^{-1}. \tag{4.8}$$

Hence, the stochastic solutions of Eq (1.1) are

$$\psi_1(x,t) = \left(\mp\sqrt{\frac{\delta}{2\alpha c^2}}\,(cx + \omega t + \varrho)\right)^{-1} e^{i(px+\mu t)+\sigma W(t)-\sigma^2 t}. \tag{4.9}$$

Second: Trigonometric solutions are

$$q_{2,3}(x,t) = \pm\sqrt{\frac{\alpha p^2+\mu-\lambda}{\delta}}\, tan\left(\sqrt{\frac{\alpha p^2+\mu-\lambda}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right) \tag{4.10}$$

and

$$q_{4,5}(x,t) = u_{2,3}(x,t) = \pm\sqrt{\frac{\alpha p^2+\mu-\lambda}{\delta}}\, cot\left(\sqrt{\frac{\alpha p^2+\mu-\lambda}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right). \tag{4.11}$$

Hence, the stochastic solutions of Eq (1.1) are

$$\psi_{2,3}(x,t) = \pm\sqrt{\frac{\alpha p^2+\mu-\lambda}{\delta}}\ \ tan\left(\sqrt{\frac{\alpha p^2+\mu-\lambda}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right)e^{i(px+\mu t)+\sigma W(t)-\sigma^2 t} \tag{4.12}$$

and

$$\psi_{4,5}(x,t) = \pm\sqrt{\frac{\alpha p^2+\mu-\lambda}{\delta}}\ \ cot\left(\sqrt{\frac{\alpha p^2+\mu-\lambda}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right)e^{i(px+\mu t)+\sigma W(t)-\sigma^2 t}. \tag{4.13}$$

Third: Hyperbolic solutions are:

$$q_{6,7}(x,t) = \pm\sqrt{\frac{\lambda-\alpha p^2-\mu}{\delta}}\, tanh\left(\sqrt{\frac{\lambda-\alpha p^2-\mu}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right) \tag{4.14}$$

and

$$q_{8,9}(x,t) = \pm\sqrt{\frac{\lambda-\alpha p^2-\mu}{\delta}}\, coth\left(\sqrt{\frac{\lambda-\alpha p^2-\mu}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right). \tag{4.15}$$

Thus, the stochastic solutions for Eq (1.1) are

$$\psi_{6,7}(x,t) = \pm\sqrt{\frac{\lambda-\alpha p^2-\mu}{\delta}}\ \ tanh\left(\sqrt{\frac{\lambda-\alpha p^2-\mu}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right)e^{i(px+\mu t)+\sigma W(t)-\sigma^2 t} \tag{4.16}$$

and

$$\psi_{8,9}(x,t) = \pm\sqrt{\frac{\lambda-\alpha p^2-\mu}{\delta}}\ \ coth\left(\sqrt{\frac{\lambda-\alpha p^2-\mu}{2\alpha c^2}}\,(cx+\omega t+\varrho)\right)e^{i(px+\mu t)+\sigma W(t)-\sigma^2 t}. \tag{4.17}$$

Here, $p, c, \mu, \varrho, \lambda, \alpha$ are constants.

5. Results and discussion

Mathematical treatments of the stochastic nonlinear Eq (1.1), which contains nonlinear, dispersive, losing and noise terms, produce a dynamical energy Eq (4.6) with restricted Eq (4.5) and various structurally important solutions that can be as characteristics of wave behaviors in optical communications. Using Matlab release 18 and Mathematica release 13, we introduce some 2D and 3D graphs for some selected solutions of Eq (1.1) for appropriate parametric choices. Eq (4.5) gives all physical relations between Schrödinger equation coefficients and the transformation parameters as depicted in Figure 1. In this figure, we show the relationship between ω, α, λ and σ, which is a conditional constraint on the energy equation. It was found that ω increased by increasing both λ and σ. Moreover, it was concluded that ω increases with the increase of α until a specific value and then begins to gradually rebate. By examining Eq (4.6) with potential phase portrait variations as shown in Figures 2 and 3. Both potential and phase portraits represent the regions of stability (instability) of a particle in a potential well with a change in δ parameter. Phase portrait is a relation plotted between $dq/d\vartheta$ and q at different values of energy constant for $p = 0.3, \lambda = 0.3,\ \alpha = -3, c = 0.5$ and $\mu = 1$. It was noted that there are two stable and one unstable point for $\delta = -2$ as in Figure 2 and an unstable state for $\delta = 2$ as in Figure 3. To discuss some of the obtained solutions in this work for $\sigma = 0$, Equation (4.7) described bright solitary solutions in the form of breather and bell-shaped solitons as in Figures 4 and 5. The rational leading form Eq (4.9) is an effective solution for the production of periodic blow-up solution and explosive solitary behavior as given in Figures 6 and 7.

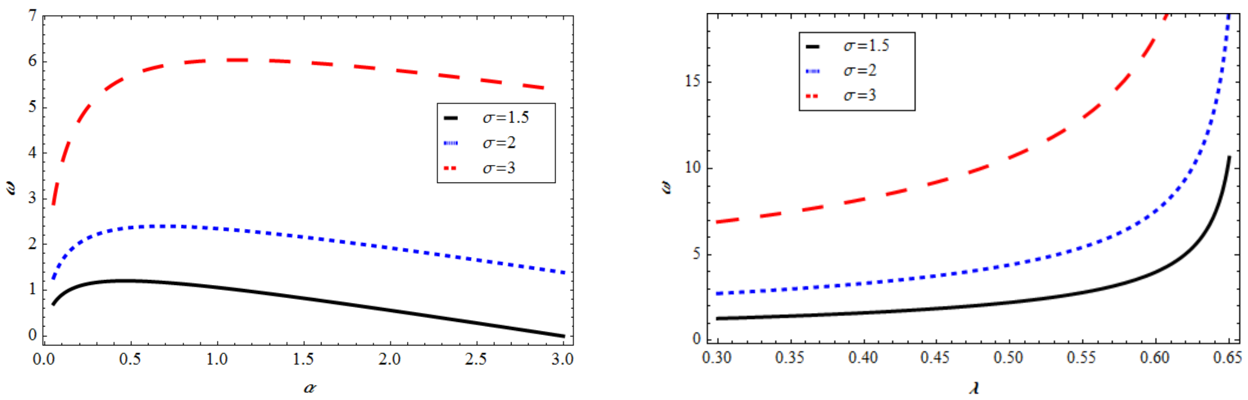

Figure 1. Plot of ω with α, λ and σ for $p = 0.6, c = 0.3, \delta = 6, k = 0.5, \mu = 1$.

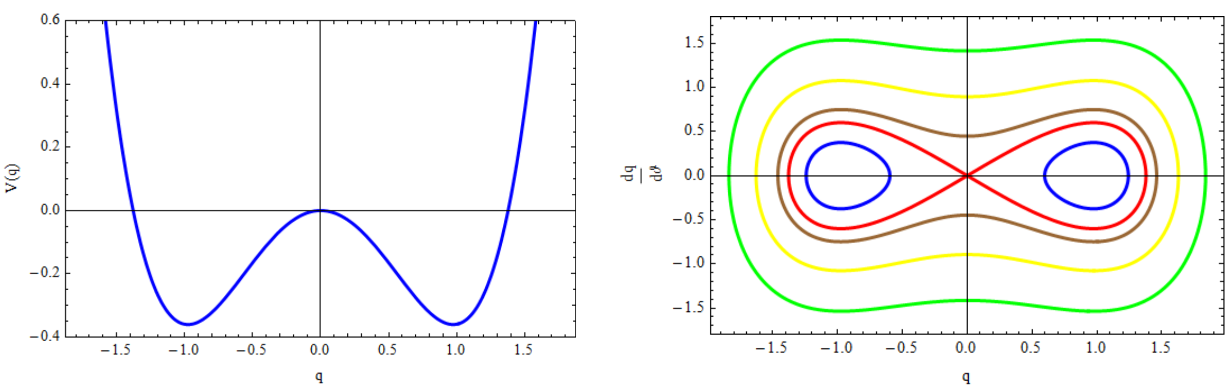

Figure 2. Plot of **V** and $\frac{dq}{dv}$ with q for $\lambda = 0.2, \alpha = 5,\ p = 0.6, c = 0.5, \delta = -2, \mu = 1$.

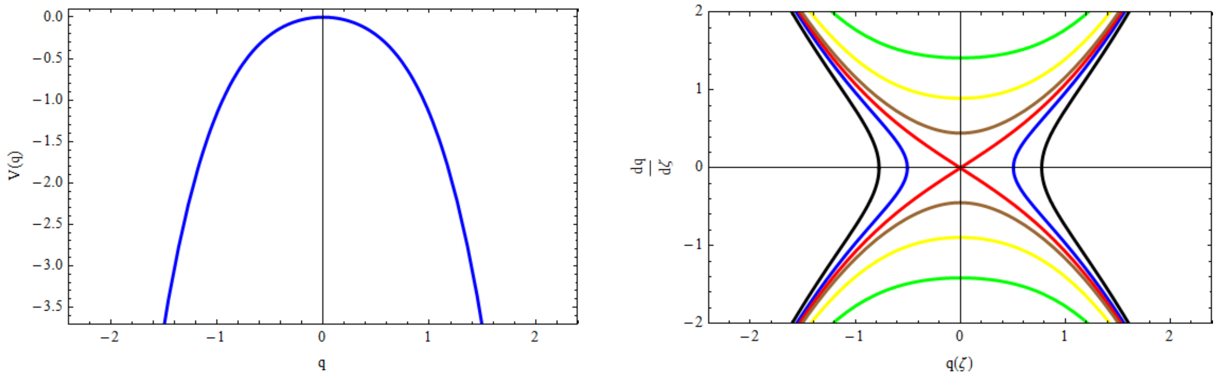

Figure 3. Plot of **V** and $\frac{dq}{dv}$ with q for $\lambda = 0.2, \alpha = 5,\ p = 0.6, c = 0.5, \delta = 2, \mu = 1$.

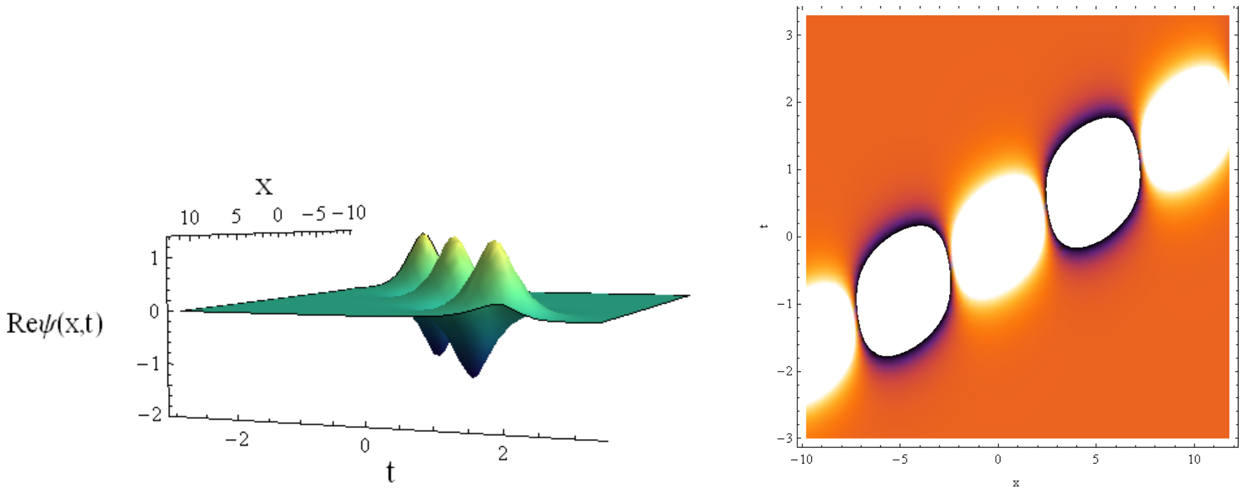

Figure 4. Plot of $Re\,\psi(x,t)$ in (4.7) with x, t for $\lambda = 0.2, \alpha = 5,\ p = 0.3, c = 0.5,\ \delta = -2,\ \mu = 1$.

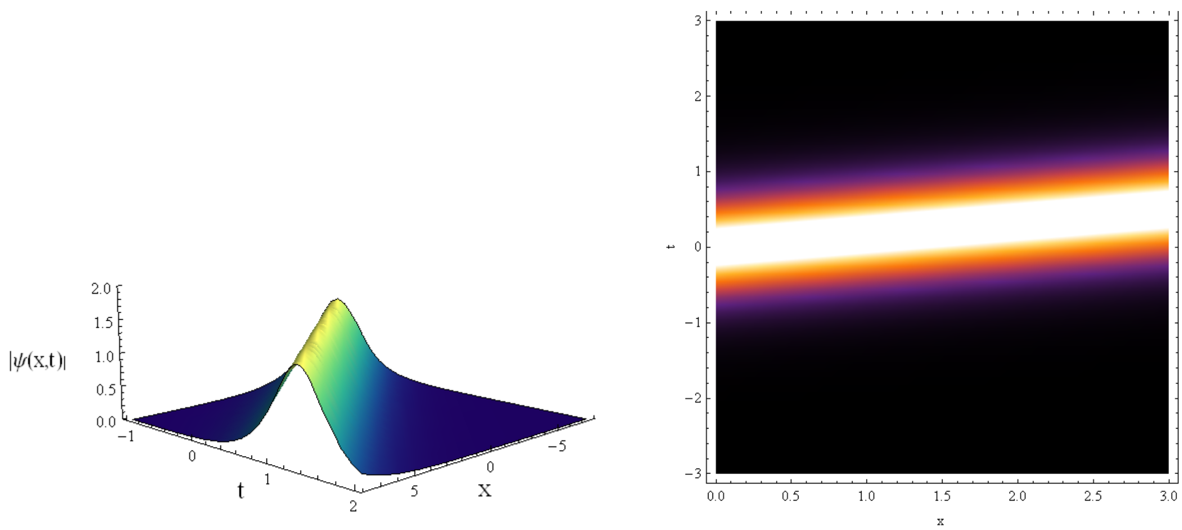

Figure 5. Plot of $|\psi(x,t)|$ in (4.7) with x, t for $\delta = -2,\ \lambda = 0.2, \alpha = 5,\ p = 0.3, c = 0.5, \mu = 1$.

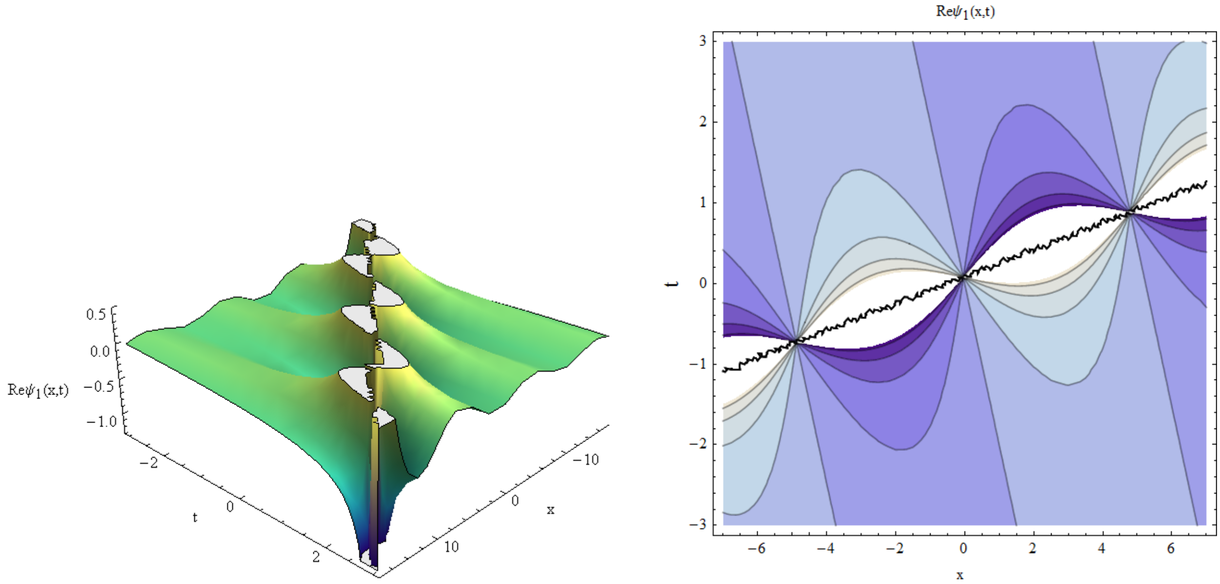

Figure 6. Plot of $Re\,\psi_1(x,t)$ with x, t for $\lambda = 0.2, \alpha = 5,\ p = 0.3, c = 0.5,\ \delta = -2,\ \mu = 1.$

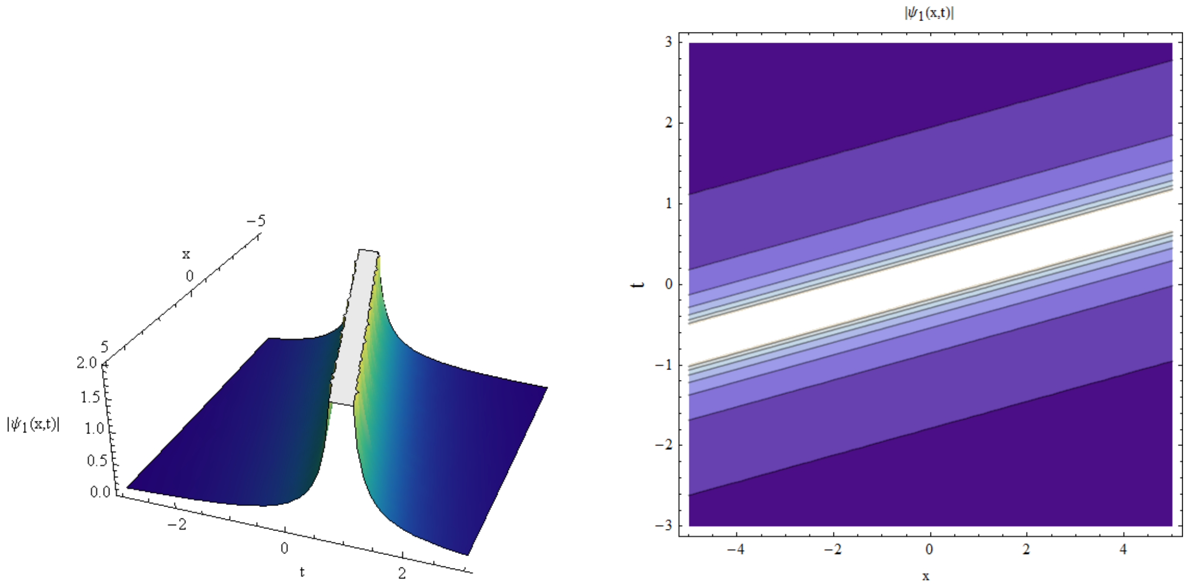

Figure 7. Plot of $|\psi_1(x,t)|$ with x, t for $\delta = -2,\ \lambda = 0.2, \alpha = 5,\ p = 0.3, c = 0.5, \mu = 1.$

On the other hand, many trigonometric and hyperbolic dissipative solutions have been illustrated in Eqs (4.12), (4.13), (4.16) and (4.17). In Figure 8, a blow-up dissipative structure has been obtained. One of the priorities of this work is to explore the effects of the random pulses on the state and characteristics of the resulting waves through the change of stochastic time and the consequent changes of wave shapes and phase differences. Figure 9 shows the variation of Eq (4.7) with time and σ. We found that the increase of σ reduces the amplitude and affected the soliton tails until the soliton shape converts to soliton-like dissipative stochastic wave. Variations of Eq (4.16) with time and σ is depicted in Figures 10 and 11. It was determined to that the shock wave tails were deformed with σ in form of damping in positive tail and forcing in negative tail. Furthermore, the random parameter damped the envelope soliton in the negative t-axis and forced the envelope in the positive t-axis.

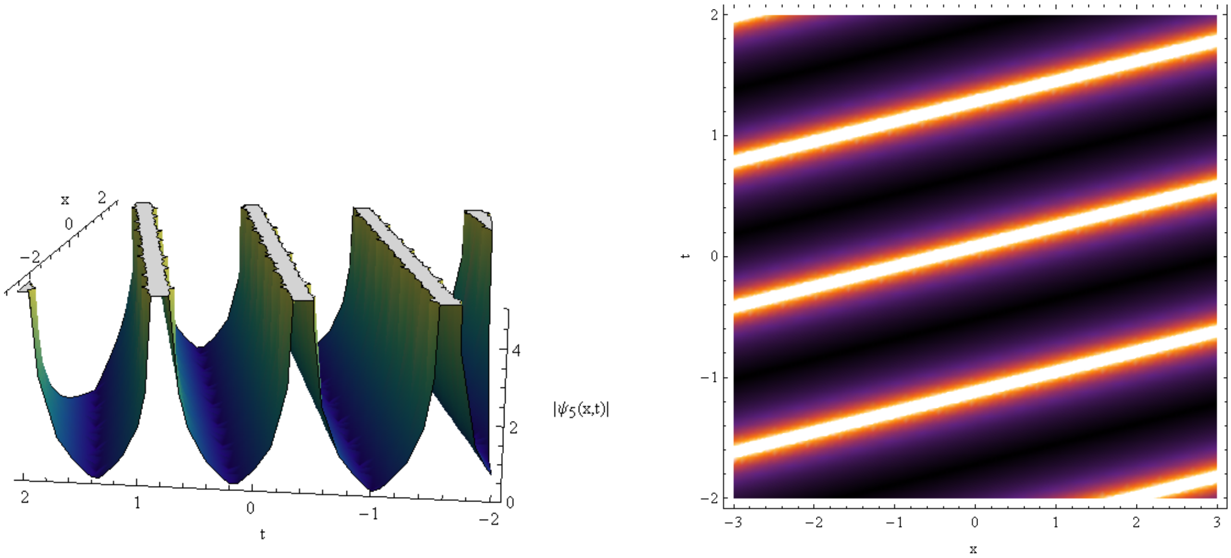

Figure 8. Plot of $|\psi_5(x,t)|$ with x, t for $\delta = -2$, $\lambda = 0.2$, $\alpha = 5$, $p = 0.3$, $c = 0.5$, $\mu = 1$.

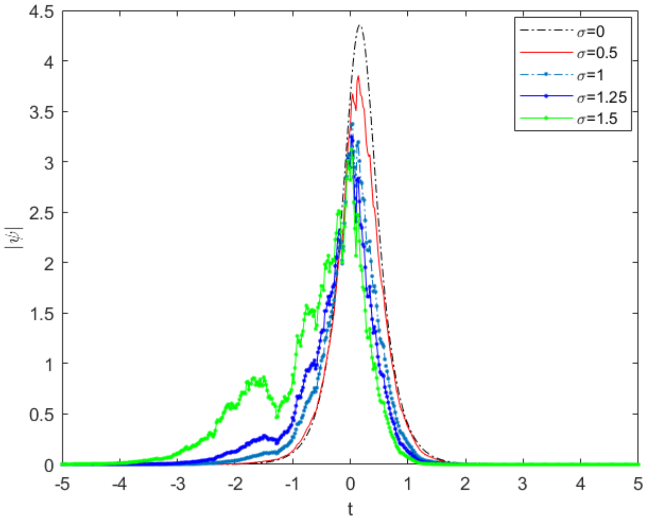

Figure 9. Stochastic plot of $|\psi(x,t)|$ in (4.7) with σ, t for $\delta = -2$, $\lambda = 0.2$, $\alpha = 5$, $p = 0.6$, $c = 0.5$, $\mu = 0.3$.

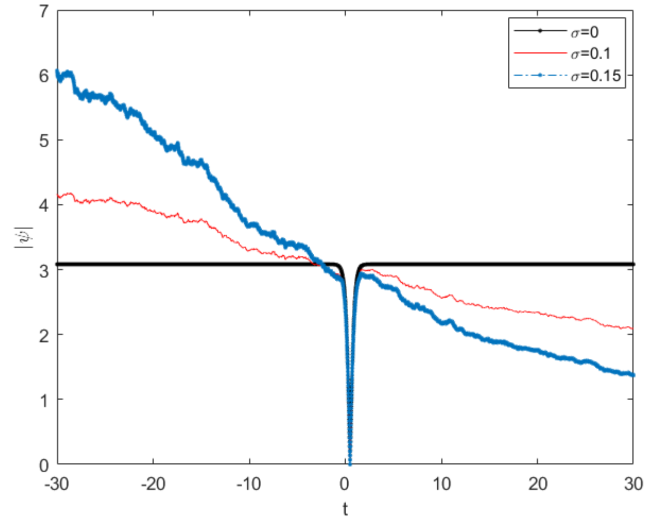

Figure 10. Stochastic plot of $|\psi_6(x,t)|$ with σ, t for $\delta = -2$, $\lambda = 0.2, \alpha = 5$, $p = 0.6, c = 0.5$, $\mu = 0.3$.

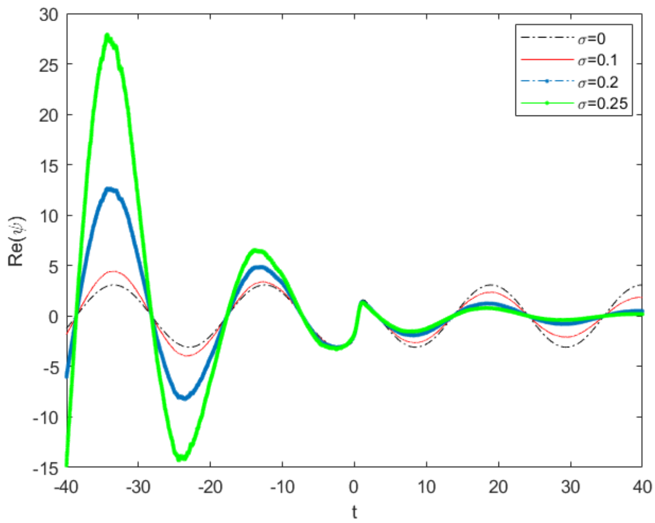

Figure 11. Stochastic plot of $Re\,\psi_6(x,t)$ with σ, t for $\delta = -2$, $\lambda = 0.2, \alpha = 5$, $p = 0.6, c = 0.5$, $\mu = 0.3$.

In summary, the stochastic random effects control the properties of solitary and other forms in this model in the form of damping and forcing wave tails.

6. Conclusions

The random NLSE equation with losing and noise terms has been solved by a mathematical solver. The stability of the dynamical system corresponding to the random NLSE equation has been examined by phase plane to identify the solutions appropriate for this model. Several wave profiles have been derived in the form of blow-up, periodical, soliton, breather and shock-like structures. The noise effects in stochastic behaviors play an improvements controller for wave features. The parameter of random noise modified the shape and features of the model's solutions. It causes some physical effects as damping, fluctuations and forcing for amplitudes and wave tails. The results obtained here may be profitable in physical models with losing terms in stochastic dispersive models.

The unified solver technique in this work via stochastic sense can be applied in studying nonlinear equations using some recently proposed definitions of fractional calculus such as the generalized fractional derivative (GFD) definition [48, 49]. Actually, this definition with its special functions [50] is so powerful because it overcomes some issues associated with conformable derivatives and other fractional derivatives.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. F. Mirzaee, S. Rezaei, N. Samadyar, Solving one-dimensional nonlinear stochastic sine-Gordon equation with a new meshfree technique, *Int. J. Numer. Modell.*, **34** (2021), 2856. http://doi.org/10.1002/jnm.2856
2. Y. F. Alharbi, M. A. E. Abdelrahman, M. A. Sohaly, M. Inc, Stochastic treatment of the solutions for the resonant nonlinear Schrödinger equation with spatio-temporal dispersions and inter-modal using beta distribution, *Eur. Phys. J. Plus*, **135** (2020), 368. http://doi.org/10.1140/epjp/s13360-020-00371-2
3. Y. F. Alharbi, M. A. Sohaly, M. A. E. Abdelrahman, Fundamental solutions to the stochastic perturbed nonlinear Schrödinger's equation via gamma distribution, *Results Phys.*, **25** (2021), 104249. http://doi.org/10.1016/j.rinp.2021.104249
4. F. Mirzaee, S. Rezaei, N. Samadyar, Numerical solution of two-dimensional stochastic time-fractional sine-Gordon equation on non-rectangular domains using finite difference and meshfree methods, *Eng. Anal. Boundary Elem.*, **127** (2021), 53–63. http://doi.org/10.1016/j.enganabound.2021.03.009
5. K. Hosseini, D. Baleanu, S. Rezapour, S. Salahshour, M. Mirzazadeh, M. Samavat, Multi-complexiton and positive multi-complexiton structures to a generalized B-type Kadomtsev-Petviashvili equation, *J. Ocean Eng. Sci.*, 2022. http://doi.org/10.1016/j.joes.2022.06.020
6. K. Hosseini, M. Mirzazadeh, S. Salahshour, D. Baleanu, A. Zafar, Specific wave structures of a fifth-order nonlinear water wave equation, *J. Ocean Eng. Sci.*, **7** (2022), 462–466. http://doi.org/10.1016/j.joes.2021.09.019
7. Z. Q. Li, S. F. Tian, J. J. Yang, E. Fan, Soliton resolution for the complex short pulse equation with weighted Sobolev initial data in space-time solitonic regions, *J. Differ. Equations*, **329** (2022), 31–88. http://doi.org/10.1016/j.jde.2022.05.003

8. S. F. Tian, M. J. Xu, T. T. Zhang, A symmetry-preserving difference scheme and analytical solutions of a generalized higher-order beam equation, *Proc. R. Soc. A Math. Phys. Eng. Sci.*, **477** (2021), 455. http://doi.org/10.1098/rspa.2021.0455
9. H. A. Alkhidhr, H. G. Abdelwahed, M. A. E. Abdelrahmand, S. Alghanim, Some solutions for a stochastic NLSE in the unstable and higher order dispersive environments, *Results Phys.*, **34** (2022), 105242. http://doi.org/10.1016/j.rinp.2022.105242
10. P. A. Cioica, S. Dahlke, Spatial Besov regularity for semilinear stochastic partial differential equations on bounded Lipschitz domains, *Int. J. Comput. Math.*, **89** (2012), 2443–2459. http://doi.org/10.1080/00207160.2011.631530
11. S. Alipour, F. Mirzaee, An iterative algorithm for solving two dimensional nonlinear stochastic integral equations: a combined successive approximations method with bilinear spline interpolation, *Appl. Math. Comput.*, **371** (2020), 124947. http://doi.org/10.1016/j.amc.2019.124947
12. M. A. E. Abdelrahman, M. A. Sohaly, Solitary waves for the nonlinear Schrödinger problem with the probability distribution function in stochastic input case, *Eur. Phys. J. Plus*, **132** (2017), 339. http://doi.org/10.1140/epjp/i2017-11607-5
13. F. Mirzaee, N. Samadyar, Implicit meshless method to solve 2D fractional stochastic Tricomi-type equation defined on irregular domain occurring in fractal transonic flow, *Numer. Methods Part. Differ. Equations*, **37** (2021), 1781–1799. http://doi.org/10.1002/num.22608
14. M. H. Heydari, M. R. Hooshmandasl, A. Shakiba, C. Cattani, Legendre wavelets Galerkin method for solving nonlinear stochastic integral equations, *Nonlinear Dyn.*, **85** (2016), 1185–1202. http://doi.org/10.1007/s11071-016-2753-x
15. F. Mohammadi, Numerical treatment of nonlinear stochastic Itô-Volterra integral equations by piecewise spectral-collocation method, *J. Comput. Nonlinear Dyn.*, **14** (2019), 031007. http://doi.org/10.1115/1.4042440
16. R. A. Alomair, S. Z. Hassan, M. A. E. Abdelrahman, A. H. Amin, E. K. El-Shewy, New solitary optical solutions for the NLSE with δ-potential through Brownian process, *Results Phys.*, **40** (2022), 105814. http://doi.org/10.1016/j.rinp.2022.105814
17. F. Dalfovo, S. Giorgini, L. P. Pitaevskii, S. Stringari, Theory of Bose-Einstein condensation in trapped gases, *Rev. Mod. Phys.*, **71** (1999), 463. http://doi.org/10.1103/RevModPhys.71.463
18. H. Triki, C. Bensalem, A. Biswas, S. Khan, Q. Zhou, S. Adesanya, et al., Self-similar optical solitons with continuous-wave background in a quadratic-cubic non-centrosymmetric waveguide, *Opt. Commun.*, **437** (2019), 392–398. http://doi.org/10.1016/j.optcom.2018.12.074
19. A. S. Davydov, *Solitons in molecular systems*, Springer, 1985. http://doi.org/10.1007/978-94-017-3025-9
20. K. Nakkeeran, Bright and dark optical solitons in fiber media with higher-order effects, *Chaos Solitons Fract.*, **13** (2002), 673–679. http://doi.org/10.1016/S0960-0779(00)00278-2
21. V. N. Serkin, A. Hasegawa, Novel soliton solutions of the nonlinear Schrödinger equation model, *Phys. Rev. Lett.*, **85** (2000), 4502–4505. http://doi.org/10.1103/PhysRevLett.85.4502
22. K. Hosseini, M. Mirzazadeh, D. Baleanu, S. Salahshour, L. Akinyemi, Optical solitons of a high-order nonlinear Schrodinger equation involving nonlinear dispersions and Kerr effect, *Opt. Quantum Electron.*, **54** (2022), 177. http://doi.org/10.1007/s11082-022-03522-0

23. A. Houwe, Y. Saliou, P. Djorwe, S. Abbagari, L. Akinyemi, S. Y. Doka, Modulation instability gain and modulated wave shape incited by the acoustic longitudinal vibrations in molecular chain model, *Phys. Scr.*, **97** (2022), 085206. http://doi.org/10.1088/1402-4896/ac7a6b

24. S. Abbagari, A. Houwe, L. Akinyemi, Y. Saliou, T. B. Bouetou, Modulation instability gain and discrete soliton interaction in gyrotropic molecular chain, *Chaos Solitons Fract.*, **160** (2022), 112255. http://doi.org/10.1016/j.chaos.2022.112255

25. D. Kumar, K. Hosseini, M. K. A. Kaabar, M. Kaplan, S. Salahshour, On some novel solution solutions to the generalized Schrödinger-Boussinesq equations for the interaction between complex short wave and real long wave envelope, *J. Ocean Eng. Sci.*, **7** (2022), 353–362. http://doi.org/10.1016/j.joes.2021.09.008

26. A. Houwe, P. Djorwe, S. Abbagari, S. Y. Doka, S. G. Nana Engo, Discrete solitons in nonlinear optomechanical array, *Chaos Solitons Fract.*, **154** (2022), 111593. http://doi.org/10.1016/j.chaos.2021.111593

27. M. A. E. Abdelrahman, M. A. Sohaly, Y. F. Alharbi, Fundamental stochastic solutions for the conformable fractional NLSE with spatiotemporal dispersion via exponential distribution, *Phys. Scr.*, **96** (2021), 125223. http://doi.org/10.1088/1402-4896/ac119c

28. Z. Q. Li, S. F. Tian, J. J. Yang, On the soliton resolution and the asymptotic stability of N-solitonsolution for the Wadati-Konno-Ichikawa equation with finite density initial data in space-time solitonic regions, *Adv. Math.*, **409** (2022), 108639. http://doi.org/10.1016/j.aim.2022.108639

29. Z. Q. Li, S. F. Tian, J. J. Yang, Soliton resolution for the Wadati-Konno-Ichikawa equation with weighted Sobolev initial data, *Ann. Henri Poincare*, **23** (2022), 2611–2655. http://doi.org/10.1007/s00023-021-01143-z

30. J. J. Yang, S. F. Tian, Z. Q. Li, Riemann-Hilbert problem for the focusing nonlinear Schrödinger equation with multiple high-order poles under nonzero boundary conditions, *Phys. D*, **432** (2022), 133162. http://doi.org/10.1016/j.physd.2022.133162

31. W. W. Ling, P. X. Wu, A fractal variational theory of the Broer-Kaup system in shallow water waves, *Therm. Sci.*, **25** (2021), 2051–2056. http://doi.org/10.2298/TSCI180510087L

32. J. H. He, W. F. Hou, C. H. He, T. Saeed, T. Hayat, Variational approach to fractal solitary waves, *Fractals*, **29** (2021), 2150199. http://doi.org/10.1142/S0218348X21501991

33. J. H. He, N. Qie, C. H. He, T. Saeed, On a strong minimum condition of a fractal variational principle, *Appl. Math. Lett.*, **119** (2021), 107199. http://doi.org/10.1016/j.aml.2021.107199

34. J. H. He, N. Qie, C. H. He, Solitary waves travelling along an unsmooth boundary, *Results Phys.*, **24** (2021), 104104. http://doi.org/10.1016/j.rinp.2021.104104

35. A. Houwe, S. Abbagari, S. Y. Doka, M. Inc, T. B. Bouetou, Clout of fractional time order and magnetic coupling coefficients on the soliton and modulation instability gain in the Heisenberg ferromagnetic spin chain, *Chaos Solitons Fract.*, **151** (2021), 111254. http://doi.org/10.1016/j.chaos.2021.111254

36. A. Houwe, H. Rezazadeh, A. Bekir, S. Y. Doka, Traveling-wave solutions of the Klein-Gordon equations with M-fractional derivative, *Pramana*, **96** (2022), 26. http://doi.org/10.1007/s12043-021-02254-2

37. M. K. A. Kaabar, F. Martínez, J. F. Gómez-Aguilar, B. Ghanbari, M. Kaplan, H. Günerhan, New approximate analytical solutions for the nonlinear fractional Schrödinger equation with second-order spatio-temporal dispersion via double Laplace transform method, *Math. Methods Appl. Sci.*, **44** (2021), 11138–11156. http://doi.org/10.1002/mma.7476
38. B. Ghanbari, J. F. Gómez-Aguilar, New exact optical soliton solutions for nonlinear Schrödinger equation with second-order spatiotemporal dispersion involving M-derivative, *Modern Phys. Lett. B*, **33** (2019), 1950235. http://doi.org/10.1142/S021798491950235X
39. E. YaŞar, E. YaŞar, Optical solitons of conformable space-time fractional NLSE with Spatio-temporal dispersion, *New Trends Math. Sci.*, **3** (2018), 116–127. http://doi.org/10.20852/ntmsci.2018.300
40. T. Ueda, W. L. Kath, Dynamics of optical pulses in randomly birefrengent fibers, *Phys. D*, **55** (1992), 166–181. http://doi.org/10.1016/0167-2789(92)90195-S
41. H. G. Abdelwahed, E. K. El-Shewy, R. Sabry, M. A. E. Abdelrahman, Characteristics of stochastic Langmuir wave structures in presence of Itô sense, *Results Phys.*, **37** (2022), 105435. http://doi.org/10.1016/j.rinp.2022.105435
42. S. M. Ross, *Brownian motion and stationary processes*, Academic Press, 2014.
43. M. A. E. Abdelrahman, H. A. Alkhidhr, A. H. Amin, E. K. El-Shewy, A new structure of solutions to the system of ISALWs via stochastic sense, *Results Phys.*, **37** (2022), 105473. http://doi.org/10.1016/j.rinp.2022.105473
44. G. E. Falkovich, I. Kolokolov, V. Lebedev, S. K. Turitsyn, Statistics of soliton-bearing systems with additive noise, *Phys. Rev. E*, **63** (2001), 025601. http://doi.org/10.1103/PhysRevE.63.025601
45. H. G. Abdelwahed, E. K. El-Shewy, S. Alghanim, M. A. E. Abdelrahman, Modulations of some physical parameters in a nonlinear Schrödinger type equation in fiber communications, *Results Phys.*, **38** (2022), 105548. http://doi.org/10.1016/j.rinp.2022.105548
46. I. Karatzas, S. E. Shreve, *Brownian motion and stochastic calculus*, Springer, 1991. http://doi.org/10.1007/978-1-4612-0949-2
47. M. A. E. Abdelrahman, H. AlKhidhr, A robust and accurate solver for some nonlinear partial differential equations and tow applications, *Phys. Scr.*, **95** (2020), 065212. http://doi.org/10.1088/1402-4896/ab80e7
48. M. Abu-Shady, M. K. A. Kaabar, A generalized definition of the fractional derivative with applications, *Math. Probl. Eng.*, **2021** (2021), 9444803. http://doi.org/10.1155/2021/9444803
49. F. Martínez, M. K. A. Kaabar, A novel theoretical investigation of the Abu-Shady–Kaabar fractional derivative as a modeling tool for science and engineering, *Comput. Math. Methods Med.*, **2022** (2022), 4119082. http://doi.org/10.1155/2022/4119082
50. M. Abu-Shady, M. K. A. Kaabar, A novel computational tool for the fractional-order special functions arising from modeling scientific phenomena via Abu-Shady–Kaabar fractional derivative, *Comput. Math. Methods Med.*, **2022** (2022), 2138775. http://doi.org/10.1155/2022/2138775