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Analogical Reasoning in Mathematical Theorems

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ABSTRACT

Analogical reasoning is one of the most powerful tools of mathematical thinking. For example, to prove a theorem it is necessary to see similarities with the previous theorem. This study aims to classify analogies in mathematics courses and examples. This classification is based on research results. The research was conducted use qualitative research. The research subjects are 12 lecturers who teach mathematics courses and study program managers. Analogical reasoning instruments are unstructured interview guidelines and observation sheets. Interview guides and observation sheets were made to be able to reveal mathematics analogical reasoning in the Mathematics Education Study Program course. The results of the research show that there are 3 types of analogy classifications in mathematics courses, namely definition analogy, theorem-defining analogy, and theorem analogy. First, the definition of similarity in the same or different courses. Second, the similarities between definitions and theorems in the same or different courses. Third, the theorem similarities in the same or different subjects. Our classification is related to theorems and analogical properties in several courses in the curriculum of the Mathematics Education Study Program. The analogy can be applied to certain mathematical topics related to real life. Meanwhile, to analyze other aspects of reasoning through analogy needs to be studied further.

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A. INTRODUCTION

We often do transfers in life, because transfers are the process of applying what one knows or can do in one situation to a novel situation or varied task (Dinsmore et al., 2014). The basis of transfer is similarities and analogies (Voiculescu, 2013). The transfer has two types near transfer and far transfer. Near transfer have a similar type of problem and the same domain. Therefore, far transfers have different types of problems and different domains (Dinsmore et al., 2014; Stevenson et al., 2013; Voiculescu, 2013). The condition that supports transfer are (1) Making sure students have relevant background knowledge and activate their knowledge; (2) Prompting students to consider whether they already know anything that might be relevant (remain them to try to transfer their knowledge); (3) Helping students abstract general principles (not just surface features); (4) Providing sufficient time and motivation (transfer can be hard); and (5) Providing multiple cases and contexts (if you only teach about fractions as they relate to pieces of pie, students will have trouble transferring what they know about fractions to non-pie problems) (Dinsmore et al., 2014; Holyoak & Thagard, 1989). Polya said to analogy is a kind of similarity (Polya, 2004). Objects that are the

same in certain aspects or analogous objects that have an analogous relationship from certain parts. Analogous reasoning is thinking that relies on similarities or analogies (Holyoak & Hummel, 1997). Then, analogy argument is a form of analogical reasoning that cites accepted similarities between two systems to support the conclusion that there is some further similarity based on explicit representations.

The analogy can be said to be the basis of human intelligence (Gentner, 2010, 2016; Hofstadter & Sander, 2013; Penn et al., 2008). This means that we compare the two domains and identify similarities in their structure so that useful conclusions can be drawn and can develop new abstractions (Lovett & Forbus, 2017). Analogies can drive scientific discoveries, such as when Rutherford famously stated that electrons orbit the nucleus like planets orbiting the sun. And allows us to apply what we have learned in past experiences to the present, such as when someone is solving a physics problem, choosing a movie to watch, or considering buying a new car.

Analogical reasoning is used in mathematics and also in everyday life (Magdas, 2015). Analogical reasoning can be the center of abstract learning (Gentner, 2016; Richland et al., 2004). The analogical comparison process is very important in obtaining new relational concepts (Christie & Gentner, 2010; Doumas & Hummel, 2013). But this can raise the question of how to compare things. For young children, spontaneous comparisons are mostly limited to very similar pairs overall. This means that the spontaneous comparison process is too limited in scope to account for the amount of relational learning we see even in young children.

Analogical reasoning is one type of thinking process that we can use to transfer knowledge from one context/situation/domain to another (Gentner, 2016). Mapping relational similarity is seeing relations among relations is at the core of analogical reasoning (Doumas & Hummel, 2013; Gentner & Maravilla, 2017; Gentner, 1983; Holyoak & Hummel, 1997). Analogical reasoning is not just about seeing similarities. The use of analogies is rarely used in education. But any analogies economy must be compensated by an additional memory effort". This means that the role of the mathematics teacher needs to encourage students to identify and use analogical reasoning as much as possible in a variety of contexts. Six supports for analogical reasoning are used familiar sources; presenting sources visually; keeping the source visible; using spatial cues to highlight alignment; gesturing; and using imagery/visualization.

The results of the researchers who researched related to analogical reasoning, some of which focused on: (1) analogy transfer (Trench et al., 2009), (2) similarities (Holyoak & Hummel, 2001), (3) Errors that occur in analogical reasoning in solving analogy problems (Kristayulita et al., 2020, 2017; Pang & Dindyal, 2009; Saleh et al., 2017), and (4) analogical reasoning in geometry education (Magdas, 2015). Analogical reasoning has taken on many roles in the process of thinking mathematically.

Analogical reasoning is one of the most powerful tools of mathematical thinking. But it is still used in a very low measure in education. But any minimal analogy needs to be compensated by using additional memory. This means that the role of the mathematics teacher is crucial for students to identify and use analogous reasoning as much as possible in various contexts. In this article, we approach analogical reasoning in theorems of mathematics.

This research is based on Magdas (Magdas, 2015) which discuss analogical reasoning in geometry education. Magdas produces analogical reasoning found in geometric concepts. Therefore, this research discusses analogical reasoning in mathematics courses. So, this study aims to classify analogies in mathematics courses and examples. This classification is based on research results. Our classification is related to theorems and analogical properties in several courses in the curriculum of the Mathematics Education Study Program.

Most students assume that mathematics has a set of concepts and formulas that are different. Mathematics is a complex tool in which each concept is interconnected between one concept with another concept or with other sciences and with elements of everyday life. How can we learn this complex concept? The question is difficult to give a complete answer to, we know that "the problem is not transferring knowledge, but how to acquire a way of thinking". In this case, analogical reasoning makes an important contribution to mathematical thinking. On the one hand, there is an analogy between elements of everyday life and Mathematics, and on the other hand, the analogy relates to elements of mathematical content that lead to an understanding of mathematics as a whole.

Lectures at universities, the similarity of the mathematical concepts taught can be found in the courses held. There are also similarities in the mathematical concepts taught in other courses held by other lecturers. Mathematical concepts cannot be taught separately. Like a teacher wants to teach about how to prove the law of sines. Before the teacher provides an understanding of the concept of the sine rule, the teacher must teach the concept of trigonometry comparisons. Proof of the sine rule requires mastering the concept of trigonometry comparisons. The concept of trigonometry comparisons and the sine rule is found in trigonometry courses. This shows that in mathematics courses in the learning process analogical reasoning will appear. Maybe an analogy will appear in the same or a different subject in mathematical learning.

The analogy in learning mathematics is that teachers need to support students in transferring knowledge. A teacher must know the conditions in providing support to students in transferring. As a teacher, you can support students' transfer by: (1) making sure students have relevant background knowledge and activate their knowledge; (2) prompting students to consider whether they already know anything that might be relevant (remain them to try to transfer their knowledge); (3) helping students abstract general principles (not just surface features); (4) providing sufficient time and motivation (transfer can be hard); and (5) providing multiple cases and contexts (if you only teach about fractions as they relate to pieces of pie, students will have trouble transferring what they know about fractions to non-pie problems)(Dinsmore et al., 2014; Holyoak & Thagard, 1989).

B. METHODS

The research was conducted using qualitative research (Cohen et al., 2018). The research results that will be obtained are in the form of analogical reasoning (similarity of material between courses) that appear in mathematics-based. The research subjects are 12 lecturers who teach mathematics courses and study program managers. The primary data for this research are reference books from course lecturers and course lists from study program

managers. The secondary data of this research is the semester lecture plans compiled by the mathematics course lecturers.

Analogical reasoning instruments are unstructured interview guidelines and observation sheets. The interview guide is used to reveal the mathematical concepts taught by the course lecturer. Observation sheets are used to list mathematical concepts that have similarities or mathematical analogy reasoning in the Mathematics Education Study Program course. The steps in taking the following research data: (1) Collecting data related to the list of subjects in the Mathematics Education Study Program Curriculum; (2) Collecting data related to lecturers in Mathematics Education Study Program; (3) Collecting data related to Reference Books used by lecturers and Semester Lecture Plans (SLP) in Mathematics Education Study Program; (4) Processing the research data, it was identified that there was analogical reasoning obtained in the form of Semester Lecture Plans (SLP) and Reference Books used by lecturers in charge of the Mathematics Education Study Program; and (5) Write down the research data about the obtained analogical reasoning. Then, analysis of data using a qualitative approach.

C. RESULT AND DISCUSSION

Based on the results of the analysis of textbooks or reference books used by lecturers, the research mapped analogous reasoning in mathematics course materials. Some of the results of the analogical reasoning mapping that emerged are described as follows.

1. Functions Definition and Homomorfism Definition

The Real Analysis course discusses functions and the Algebraic Structure course discusses homomorfisms. The concepts of function and homomorphism have conceptual similarities. Students in understanding the concept of homomorphism need to remember the concept of function in Basic Mathematics, Discrete Mathematics, and Real Analysis courses. This means that there is a mapping process between the function concept and the homomorphism concept. In detail the definition of function in real analysis and the definition of homomorphism in algebraic structure can be seen in Table 1.

Table 1. Functions Definition and Homomorfism Definition

Source Definition	Target Definition
Functions Definition (Bartle & Sherbert, 2011) Let A and B sets. Then a function from A to B is a set f of ordered pairs in $A \times B$ such that for each $a \in A$ there exists a unique $b \in B$ with $(a, b) \in f$. (In other words, if $(a, b) \in f$ and $(a, b') \in f$, then $b = b'$).	Homomorphism Definition (Malik et al., 1997) Let $(G,*)$ and $(G_1,*_1)$ be group and f a function from G into G_1. Then f is called a homomorfism of G into G_1 if for all $a, b \in G$, $f(a * b) = f(a) *_1 f(b)$

2. Natural Logarithmic Functions and Improper Integrals

If we study improper integrals, we need to start by studying the original logarithm. The original logarithmic form can be converted to integral form with a lower bound of 1 and an upper bound of x where $x > 0$. While the original logarithm can be written in the form of the limit of the integral of a function with a lower bound $-\infty$ and an upper bound ∞. In detail, it can be seen in Table 2.

Table 2. Natural Logarithmic Functions and Improper Integrals

Source Definition	Target Definition
Natural Logarithmic Functions (Purcell et al., 2010) The natural logarithm function, denoted by $\ln x = \int_1^x \frac{1}{t}\,dt, \quad x > 0$ The domain of the natural logarithm function is the set of positive real numbers.	Improper Integrals (Purcell et al., 2010) $\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$ $\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$ If the limit on the right exists and has finite values, then we say that the corresponding improper integrals converge and have those values. Otherwise, the integrals are said to diverge.

3. The Pythagorean Theorem and The Law of Cosine and Trigonometry Comparison and the Rule of Sine

In trigonometry, courses have the same theorem. The Pythagorean Theorem has similarities to the Law of Cosine and Comparative Trigonometry with the Rule of Sine. The proof of the law of cosine uses the Pythagorean theorems, while the proof of the sine rule uses trigonometric comparisons. In detail can be seen in Table 3 and Table 4.

Table 3. The Pythagorean Theorem and the Law of Cosine

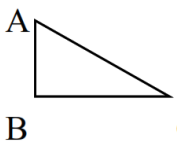
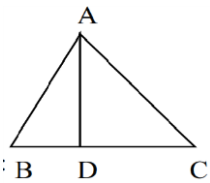
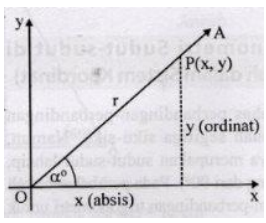
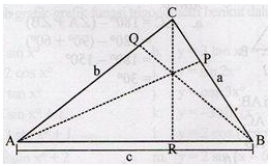
Source Theorem	Target Theorem
The Pythagorean Theorem (Kariadinata, 2013)  $AB^2 + BC^2 = AC^2$	The Law of Cosine (Kariadinata, 2013)  $BD^2 + AD^2 = AB^2$ .....(1) $AD^2 + DC^2 = AC^2$ .....(2) From (1) and (2) we obtain: $AB^2 = BD^2 + AC^2 - DC^2$ $= BD^2 + AC^2 - (BC - BD)^2$ $= AC^2 - BC^2 + 2BC \cdot BD$ But because of $BD = AD \cdot \cos(B)$. We obtain: $BC^2 = AB^2 + AC^2 - AB \cdot AC \cdot \cos(B)$

Table 4. Trigonometry Comparison and The Rule of Sine

Source Definition	Target Theorem
Trigonometry Comparison (Kariadinata, 2013)  Trigonometric comparisons	**The Rule of Sine** (Kariadinata, 2013)  Let triangle ABC acute. Lines AP, BQ, and CR are the height of sides a, b, and c. Let Δ ACR:

based on analytical geometry are defined as:

$$\sin \alpha^o = \frac{\text{ordinat}}{\text{distance}} = \frac{y}{r}$$
$$\cos \alpha^o = \frac{\text{absis}}{\text{distance}} = \frac{x}{r}$$
$$\tan \alpha^o = \frac{\text{ordinat}}{\text{absis}} = \frac{y}{x}$$

$\sin A = \frac{CR}{AC} \Leftrightarrow CR = AC \cdot \sin A$ .....(1)

Let Δ BCR:

$\sin B = \frac{CR}{CB} \Leftrightarrow CR = CB \cdot \sin B$ .....(2)

Equation (1) = (2), obtained:

$$AC \cdot \sin A = CB \cdot \sin B$$
$$\Leftrightarrow \frac{AC}{\sin B} = \frac{CB}{\sin A} \Leftrightarrow \frac{b}{\sin B} = \frac{a}{\sin A} \text{ .....(3)}$$

Let Δ BAP:

$\sin B = \frac{AP}{AB} \Leftrightarrow AP = AB \cdot \sin B$ .....(4)

Let Δ CAP:

$\sin C = \frac{AP}{AC} \Leftrightarrow AP = AC \cdot \sin C$ .....(5)

Equation (4) = (5), obtained:

$$AB \cdot \sin B = AC \cdot \sin C$$
$$\Leftrightarrow \frac{AB}{\sin C} = \frac{AC}{\sin B} \Leftrightarrow \frac{c}{\sin C} = \frac{b}{\sin B} \text{ .....(6)}$$

Equation (3) = (6), obtained:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

4. Quadratic Equation and Linear Homogeneous Equation of order 2

The value of the determinant in the quadratic equation can be used to solve the second-order linear homogeneous equation. This means that to determine the solution of the second-order linear homogeneous equation with the auxiliary equation in the form of a quadratic equation, it is necessary to pay attention to the requirements of the determinant value to get the roots of the specified auxiliary quadratic equation. The solution to solving homogeneous linear equations of order 2 must remember the concept of quadratic equations. In detail, the solution of quadratic equations in the Basic Mathematics course and the solution of second-order homogeneous linear equations in the Differential Equations course can be seen in Table 5.

Table 5. Quadratic Equation and Linear Homogeneous Equation of order 2

Source Theorem	Target Theorem
Quadratic Equation (Purcell et al., 2010) Quadratic equation form: $ax^2 + bx + c = 0$ with $a \neq 0, b\ c \in R$. Solution of roots of quadratic equation, where: $r_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ There are 3 cases in determining the solution of homogeneous differential equations with constant coefficients, namely: a. Cases $b^2 - 4ac > 0$, have the solution of real and unequal roots b. Cases $b^2 - 4ac = 0$, have	Linear Homogeneous Equation of order 2 (Hendriana et al., 2002) Homogeneous differential equation of order 2 of the form: $L[y] = ay'' + by' + cy = 0$ $= (aD^2 + bD + c)y = 0$ with $a \neq 0, b\ c \in R$ The solution to the differential equation is valid on the interval $-\infty < x < \infty$. If $y = \phi(x)$, have properties: $ay'' + by' + cy = 0$ for all x. Functions that have properties of $y = \phi(x)$ is an exponential function (e^{rx}). Then determine $y = e^{rx}$, $y' = re^{rx}$, $y'' = r^2e^{rx}$ substituted is obtained: $L[e^{rx}] = a(e^{rx})'' + b(e^{rx})' + c(e^{rx}) = 0$ $ar^2e^{rx} + bre^{rx} + ce^{rx} = 0$ $e^{rx}(ar^2 + br + c) = 0$ Because e^{rx} non zero, then must $ar^2 + br + c = 0$. Solution of differential equation $L[y] = ay'' + by' + cy = 0$ is $y = e^{rx}$, with r are the roots of quadratic equation, where: $r_{1,2} =$

real and equal roots. c. Cases $b^2 - 4ac < 0$, have complex roots or have no solution in quadratic equation.	$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ There are 3 cases in determining the solution of homogeneous differential equation with constant coefficients are a. Cases $b^2 - 4ac > 0$ Real and unequal roots are $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ b. Cases $b^2 - 4ac = 0$ Real and equal roots are $y = (c_1 + c_2)e^{rx}$ c. Cases $b^2 - 4ac < 0$ Complex roots are $(\lambda \pm i\mu)$ yaitu $y = c_1 e^{(\lambda+i\mu)x} + c_2 e^{(\lambda-i\mu)x}$ So the solution: $y = c_1 e^{\lambda x} \cos \mu x + c_2 e^{\lambda x} \sin \mu x$

5. Exact Equation Theorem and Cauchy-Riemann Equation Theorem

The Cauchy-Riemann equation theorem has similar concepts with exact equations. Both concepts have the same function $(x,y)dx + N(x,y) = 0$ $(f(z) = u(x,y) + iv(x,y))$. The difference is that there is an imaginary number $i = \sqrt{-1}$ in the function $f(z)$. If presented in a mathematical problem, the two theorems have relatively the same completion steps. This means that in understanding the theorem of the Cauchy-Reimann equation, it is necessary to first understand the theorem of exact equations. Furthermore, the steps for solving problems related to the Cauchy-Reimann equation need to use exact equation problem solving steps. In detail, the exact equation theorem in the Differential Equations course and the Cauchy-Riemann equation theorem in the Complex Variable Functions course can be seen in Table 6.

Table 6. Exact Equation Theorem and Cauchy-Riemann Equation Theorem

Source Theorem	Target Theorem
Exact Equation Theorem (Hendriana et al., 2002) If the functions M, N, M_y, and N_x are continuous in a region in the plane xy: $\alpha < x < \beta$ and $\gamma < y < \delta$, then the differential equation: $M(x,y)dx + N(x,y) = 0$ is an exact differential equation in the plane xy, if and only if: $M_y(x,y) = N_x(x,y)$	Cauchy-Riemann Equation Theorem (Brown & Churcill, 2009) A complex function $f(z) = u(x,y) + iv(x,y)$ then f is analytic in the domain D, if and only if the first partial derivatives of u and v satisfy the Cauchy-Riemann equation: $u_x(x,y) = v_y(x,y)$ and $u_y(x,y) = -v_x(x,y)$ Or it can be written: $\frac{\partial u}{\partial x}(x,y) = \frac{\partial v}{\partial y}(x,y)$ and $\frac{\partial u}{\partial y}(x,y) = -\frac{\partial v}{\partial x}(x,y)$ In the polar form on the complex number $z = r(\cos\theta + i\sin\theta)$ and the function $w = f(z) = u(r,\theta) + iv(r,\theta)$, then Cauchy-Riemann equation becomes: $u_r(r,\theta) = \frac{1}{r}v_\theta(r,\theta)$ and $v_r(r,\theta) = \frac{1}{r}u_\theta(r,\theta)$ Or it can be written: $\frac{\partial u}{\partial r}(r,\theta) = \frac{1}{r}\frac{\partial v}{\partial \theta}(r,\theta)$ dan $\frac{\partial v}{\partial r}(r,\theta) = -\frac{1}{r}\frac{\partial u}{\partial \theta}(r,\theta)$.

6. L'Hopital's Rule for forms of type 0/0 Theorem and L'Hopital's Rule for forms of type ∞/∞ Theorem

L'Hopital's Theorem of type 0/0 has similarities with the L'Hopital Line Theorem of type ∞/∞. However, there is a difference between the two theorems is that there is an absolute value in the function that is solved. The two L'Hopital theorems in detail can be seen in Table 7.

Table 7. L'Hopital's Rule for forms of type 0/0 Theorem and L'Hopital's Rule for forms of type ∞/∞ Theorem

Source Theorem	Target Theorem
Theorem L'Hopital's Rule for forms of type 0/0 (Purcell et al., 2010) Suppose that $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$. If $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists in either the finite or infinite sense (i.e., if this limit is a finite number or $-\infty$ or $+\infty$), then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.	**Theorem L'Hopital's Rule for forms of type ∞/∞** (Purcell et al., 2010) Suppose that $\lim_{x\to a}\lvert f(x)\rvert = \lim_{x\to a}\lvert g(x)\rvert = \infty$. If $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists in either the finite or infinite sense, then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$. Here u may stand for any of the symbols, $a, a'', a''', -\infty, +\infty$.

7. Theorem 2.1.9 and Theorem 2.2.8

Theorem 2.1.9 is a source theorem (Bartle & Sherbert, 2011), this theorem needs to be known by students before studying Theorem 2.2.8 (Bartle & Sherbert, 2011) which is the target theorem. Students to prove theorem 2.2.8 need to use theorem 2.1.9. This means that there is a process of mapping (mapping) carried out by students from Theorem 2.2.8 to Theorem 2.1.9. In detail the theorem 2.1.9 and theorem 2.2.8 can be seen in Table 8.

Table 8. Theorem 2.1.9 and Theorem 2.2.8 in Analysis Real Book

Source Theorem	Target Theorem
Theorem 2.1.9 (Bartle & Sherbert, 2011) If $a \in R$ is such that $0 \le a \le \varepsilon$ for every $\varepsilon > 0$, then $a = 0$. Proof Suppose to the contrary that $a > 0$. Then if we take $\varepsilon_0 = \frac{1}{2}a$, we have $0 \le \varepsilon_0 \le \varepsilon$. Therefore, it is false that $a < \varepsilon$ for every $\varepsilon > 0$ and we conclude that $a = 0$.	**Theorem 2.2.8** (Bartle & Sherbert, 2011) Let $a \in R$. If x belongs to the neighborhood $V_\varepsilon(a) = a - \varepsilon \le x \le a + \varepsilon$ for every $\varepsilon > 0$, then $x = a$. Proof If a particular x satisfies $\lvert x - a\rvert < \varepsilon$ for every $\varepsilon > 0$, then it follows from 2.19 that $\lvert x - a\rvert = 0$, and hence $x = a$

Based on the results of a study of mathematics books which are used as references for mathematics lecturers in lectures, they have the same concept in both the same and different subjects. The similarity of concepts between courses raises analogical reasoning in students. The similarity that appears can be in the form of similarity of definition, similarity of definition with theorems, or similarity of theorems.

The similarity of definitions can appear in the same or different courses. The similarity of definitions in the same courses is Calculus. For example, natural logarithmic functions and improper integrals (Calculus). The similarity of definitions in different courses are Calculus and Algebra Structure. For Example, the functions definition and homomorphism definition. That is, students can understand the concept the definition of homomorphism needs to have

an initial knowledge of the definition of a function. The similarity of definition with the theorem is Trigonometry. For example, the definition of trigonometry comparisons and the sine rule theorem. That is, students can understand the concept the sine rule theorem needs to have an initial knowledge of of trigonometry comparisons.

Theorem similarities can appear in the same or different courses. The similarity theorem in the course is trigonometry, Calculus, and Real Analysis. For example, the pythagorean theorem and the law of cosine (Trigonometry); L'Hopital's Rule for forms of type 0/0 Theorem and L'Hopital's Rule for forms of type ∞/∞ Theorem (Calculus); theorem 2.1.9 and theorem 2.2.8 (Real Analysis). Theorem similarities in different courses such as Calculus with Differential Equation and Differential Equation with Complex Variables. For example, Quadratic Equation and Linear Homogeneous Equation of order 2 (Calculus and differential Equation); Exact Equation Theorem and Cauchy-Riemann Equation Theorem (Differential Equation and Complex Variables). That is, students need to know previous theorems that have similarities so they can prove the theorem being solved. Therefore, students need to do analogical reasoning in order to be able to understand mathematical concepts in each mathematics course.

A powerful learning mechanism is an analogy. An analogy can project information from one analogy to another analogy. Although the role of projection inference is widely studied and is at the core of the mapping process, it is not the only process that supports learning. Analogies can add and expand knowledge in at least three other ways: schema abstraction (generalization), difference detection (contrast), and re-representation (Cambria et al., 2015; English, 2004; D Gentner & Maravilla, 2017).

Overall, analogical reasoning can help students use analogies to relate mathematical material. Amir-Mofidi et al. to said that facilitating students to analogical reasoning can help students to connect new mathematical knowledge to existing knowledge, learn more in-depth math, and math concepts can be stored in long-term memory (Amir-Mofidi et al., 2012; Vendetti et al., 2015).

D. CONCLUSION AND SUGGESTIONS

The conclusion that can be made is that all students who study mathematics do not all become mathematicians but mathematics is needed in life. Based on the basic concepts of mathematics, a mathematical way of thinking is needed. One of mathematical thinking is analogical reasoning. Based on the studies that have been done, there is analogical reasoning in proving the target theorem using the previous theorem (source theorem). The proof sees that there is a similarity between the source definition and the target definition; the definition and the theorem; or the source theorem and the target theorem. Then, types of analogy classifications in mathematics courses, namely definition analogy, theorem-defining analogy, and theorem analogy.

Taking into account the considerations presented in this article, we advise lecturers: (1) to always develop analogical reasoning in learning mathematics; (2) to highlight the relationship between concepts, theorems, properties, and similar problems; (3) lead students to see analogies between math, other sciences, or real-life topics; (4) encourage students to make generalizations through analogies; and (5) show students that not all analogies prove

true. In this article, we approach analogical reasoning in mathematical theorems. The analogy can be applied to certain mathematical topics related to real life. Meanwhile, to analyze other aspects of reasoning through analogy needs to be studied further.

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