공학박사학위논문

Machine Learning-based Asset Allocation Strategy and Digital Asset Investment for Portfolio Management

포트폴리오 관리를 위한 기계학습 기반 자산 배분 전략 및 디지털 자산 투자

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이 논문을 공학박사 학위논문으로 제출함

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Abstract

Machine Learning-based Asset Allocation Strategy and Digital Asset Investment for Portfolio Management

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The core of portfolio management is asset diversification and risk management. Asset diversification is to maximize the diversification effect for a multi-asset portfolio based on asset allocation by estimating the correlation between assets. Risk management is to minimize the downside risk for a given portfolio based on asset allocation by estimating the potential risk and volatility of an asset. The essential portfolio management procedure is twofold; (i) model improvement and implementation for appropriate model specifications and portfolio construction and (ii) asset class selection for investment. The first part is necessary to implement the strategy adequately to achieve the aim of that model, such as robust multi-asset portfolio management via asset diversification and single asset risk management via robust protection level maintenance. The second part is vital because a new asset class uncorrelated to the traditional asset class has potential opportunities for efficient portfolio construction. Accordingly, this dissertation focuses on research from two

perspectives dealing with the above two essential procedures. Regarding the perspective of asset diversification and risk management, the first is a study on addressing and improving the existing portfolio strategy models' limitations in model construction and estimation of input parameters for appropriate model specification. The second is a portfolio analysis of new emerging asset markets.

The first aim of this dissertation is to improve the existing portfolio management strategy in model construction for the Black–Litterman framework and input parameter estimation for the synthetic put strategy for the appropriate model specification. The second aim is to investigate the empirical results using portfolio analysis in the emerging digital asset markets, including Non-Fungible Tokens (NFTs) and the cryptocurrency market, based on the mean-variance framework or portfolio insurance framework. For the first aim, we propose to use machine learning-based models to extract the meaningful pattern of external financial data for the Black–Litterman model using firm characteristics. Furthermore, we propose to use machine learning-based forecasting models to estimate the input parameters required for portfolio insurance strategy to mitigate the difficulty of addressing complex financial data. For the second aim, we examine the economic value of NFT in terms of diversification effect on traditional asset-based portfolios and portfolio insurance strategy results regarding various risk measures and investor's utility in the cryptocurrency market.

The main findings in this dissertation are summarized as follows. First, our empirical results show that combining characteristics into view improves the distribution of portfolio returns in the Black–Litterman approach. Furthermore, prediction via machine learning affects improvement in the out-of-sample performance compared to using past information. Our study suggests that using the proposed model can result in a more efficient and diversified portfolio of the Black–Litterman framework.

Second, our empirical results of portfolio analysis in the NFT market show evidence of the hedge, safe haven, and diversification properties of NFTs, confirming two main findings: (i) NFTs act as a hedge and safe haven for several country's stock markets and oil, bond, and USD indices and these effects in stock markets fade as frequency changes, especially showing stronger safe haven benefits for bond and USD indices during the COVID-19 periods, and (ii) NFTs are distinct from traditional assets, potentially resulting in portfolio diversification which is confirmed by preliminary analysis including correlation, co-movement, and volatility spillover and portfolio analysis based on Markowitz's mean–variance framework, improving the performance of equally weighted and tangency portfolio strategies in terms of Sharpe ratio.

Third, our findings indicate that the adverse effect of volatility misestimation exists in terms of protection level error in the synthetic put strategy. We surprisingly find the protection error of insured portfolios directly linked to the precision of volatility forecasting, implying that this misestimation issue can be mitigated by employing more accurate volatility forecasting models. Another finding is that all methodologies, including traditional and machine learning-type, are better than the naive approach. Moreover, machine learning-type models, especially XGB, are the best in terms of protection and forecasting error in implementing the synthetic put strategy. This tendency supports the evidence that machine learning is better than traditional models in capturing the complex fluctuation pattern of realized volatility in highly volatile market conditions.

Finally, our findings demonstrate the outperformance of portfolio insurance strategies in terms of skewness and downside risks in the cryptocurrency market. It reveals the lower-risk feature of these strategies compared to buy-and-hold. Moreover, we surprisingly find that, in terms of curvature, the portfolio choice of prospect theory investors is opposite to the expected utility theory investors. It implies the greater impact of losses than gains on the prospect theory investors. The larger loss-aversion propensity reinforces investors' preference for portfolio insurance strategies. As the most shocking result, we find portfolio insurance, when compared to the buy-and-hold strategy, provides a better opportunity to offer a higher utility in the cryptocurrency market than the traditional stock market, regardless of the investor's utility. It implies that portfolio insurance strategies can provide greater economic value in terms of risk management for a larger number of cryptocurrency investors.

By improving the portfolio management models in terms of asset diversification of the multi-asset portfolio of the Black–Litterman model and risk management of a given portfolio or a single asset of synthetic put strategy, and by examining the portfolio analysis in new digital asset markets such as NFT and cryptocurrency market based on mean-variance and portfolio insurance framework, this dissertation's overall findings can help investors achieve an improved portfolio strategy and obtain a more efficient and well-diversified portfolio for the improved portfolio management.

Keywords: Portfolio management, Machine learning, Asset allocation, Risk management, Digital asset, Cryptocurrency, Non-Fungible Tokens, NFTs

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Chapter 1

Introduction

1.1 Background and motivation

Since Markowitz (1952) 's seminal study on the Modern Portfolio Theory (MPT) explains how an investor selects an optimal portfolio, portfolio theory-related studies, such as asset pricing and portfolio management, have developed as the main topic in finance. In particular, based on Markowitz's portfolio theory, asset pricing, which is the most important topic in financial market research, has been studied. Above all, as the first study to explain the price of an asset, the capital asset pricing model (CAPM) was proposed by Lintner (1965), Mossin (1966), Sharpe (1964), and Treynor (1961a,b) based on the Markowitz model. CAMP is an equilibrium asset pricing model that seeks to answer how prices are determined so that markets are clear in equilibrium. The model explains that the risk premium is determined by a security's risk as measured by beta on the market portfolio. Furthermore, as extensions of CAPM theory, Merton (1973) proposed Intertemporal CAPM, and Lucas Jr (1978) proposed Conditional CAPM.

On the other hand, some researchers criticized the drawback of CAPM. Specifically, Roll (1977) pointed out the limitation of not being testable, caused by the unobservability of CAPM's market portfolio. Accordingly, Ross (1976) proposed the

Arbitrage pricing theory (APT) to overcome this limitation based on the law of one price. The APT is a factor model for asset pricing, conceptualizing that the pricing of assets is related to numerous macroeconomic risk variables. Based on this APT model, other researchers have introduced many factor models, such as Fama & French (1992, 1993) 's three-factor model and Fama & French (2015) 's five-factor model. As such, the portfolio theory of Markowitz (1952) has formed the basis of modern finance. In other words, portfolio theory is the heart of financial assets and financial market analysis.

Portfolio management is another vital topic that portfolio theory directly addresses. The core of portfolio management consists of two components; asset diversification and risk management. *Asset diversification* can be defined as maximizing the diversification effect of a multi-asset portfolio. It is achieved through asset allocation based on precise estimation of the correlation among assets and the expected return of each asset. *Risk management* can be defined as minimizing the downside risk for a given portfolio. It is also attained through asset allocation by accurately quantifying and estimating the potential risk and volatility of a risky asset.

Accordingly, including Markowitz (1952)'s modern portfolio theory, numerous portfolio strategies have been researched and developed as asset diversification strategies for a multi-asset portfolio and a single asset risk management. First, for multiple assets portfolio management for diversification, there is the mean–variance portfolio framework introduced in Markowitz's portfolio theory. In the mean–variance framework, the expected value of the asset return is calculated as the expected return. The volatility, the squared root of the variance for the asset return, is also calculated as the risk. By utilizing these, the mean–variance model provides an optimal portfolio having the lowest risk level under the given fixed level of expected return or the maximum return level under the given fixed level of risk. It has the advantage of flexibility in performing optimal asset allocation based on mathematical optimization, explaining the investor's asset allocation problem in terms of a trade-off between return and risk. Another representative portfolio management model is the Black–Litterman framework proposed by Black & Litterman (1991). The Black–Litterman model is a Bayesian update-based methodology that calculates the optimal weight of a portfolio by using an investor's view. Using a new updated return distribution based on the investor's view is more explanatory than the portfolio of only using original return information in estimating expected return and correlation matrix.

Second, researchers have proposed numerous portfolio strategies for risk management for a given portfolio or single asset from an asset allocation perspective. In carrying out investment, after proper asset selection is performed, it should be possible to properly manage risk for a risky position of the investor's portfolio based on the market condition. The portfolio insurance strategy is the most representative model directly aiming at this concept. The portfolio insurance strategy is a hedging strategy through appropriate asset allocation based on market risk estimation. Precisely, it is a methodology to protect an investor's portfolio value by not falling below a specific protection level through an algorithm that appropriately adjusts the investment proportion between risk-free and risky assets. A large number of methodologies to implement this portfolio insurance, such as stop-loss (Bird et al., 1988; Rubinstein, 1985), synthetic put (Leland & Rubinstein, 1988; Rubinstein & Leland, 1981), and constant proportion (Black & Jones, 1987, 1988; Black & Perold, 1992; Perold & Sharpe, 1988) portfolio insurance strategy, have been proposed. In this dissertation, we conduct strategy improvement and portfolio analysis studies regarding asset diversification for multi-asset portfolios and risk management for a given portfolio or a single asset, which are the two perspectives introduced above. Therefore, we focus on Markowitz's mean–variance framework, Black–Litterman framework, and portfolio insurance framework in terms of two perspectives.

The procedure of portfolio management can be divided into two parts; (i) *model improvement* and *implementation* (how do you invest?) and (ii) *asset selection* (what are you investing in?). First indicates the importance of the portfolio construction model. Suppose an investor does not construct the portfolio properly using the incorrect model specification. In that case, it fails to achieve a well-diversified weight proportion of a multi-asset portfolio and maintain an appropriate level of risky position for a given portfolio or a single asset. That is, it is impossible to obtain asset diversification and accomplish a better level of risk management through appropriate asset allocation. In order to perform the first procedure, choosing an appropriate investment strategy model is crucial. Specifically, selecting a methodology that can maximize the diversification effect by asset allocation is necessary, providing the optimal weight of a multi-asset portfolio. Additionally, it is necessary to select a methodology that can perform risk management of a given portfolio or a single asset through appropriate estimation of risk measures for asset allocation. However, it is widely known to be challenging to implement these for portfolio management appropriately. As a result, model construction and input parameter estimation for the appropriate model specification is key to achieving our aim in term of portfolio management.

The second implies the importance of selecting the appropriate asset class, which can offer a maximized diversification effect to the investor's portfolio. In order to conduct the second procedure, the risk of each asset and the correlation between assets are utilized. Selecting an asset class with a low correlation between assets makes a more efficient portfolio construction possible. In other words, if new assets are uncorrelated with existing ones, they have economic value in that they are the potential to expand the efficient frontier when the investment universe includes them. In this respect, analyzing and selecting a new asset class is essential. Suppose a new asset class can diversify the asset class contained in the existing investment basket. In that case, building a more efficient portfolio will be possible, and we can expect a greater return with less risk. In this dissertation, we focus on conducting research from two perspectives dealing with the above two essential procedures. The first is a study on how to address and improve the existing portfolio strategy models' limitations in model construction and estimation of input parameters for appropriate model specification, and the second is a portfolio analysis of new asset markets.

Regarding the first procedure of model improvement and implementation, addressing the limitations inherent in the portfolio model is crucial in portfolio management. Suppose an investor performs asset diversification using a portfolio model in which the inherent problem is not mitigated. In that case, the desired objective will not be achieved, thus exposing the investor's portfolio to greater risk. In a multi-asset portfolio for the diversification effect, the estimation error of Markowitz's mean–variance model is a representative example. The problem in the mean–variance portfolio model occurs because of the parameter estimation error for the expected return of assets and the correlation matrix between assets which are input parameters required for model building (DeMiguel et al., 2009). It is about the problem that the performance is not guaranteed because the estimated expected return or the estimated correlation matrix using the in-sample are not accurate and robust estimates for the out-of-sample. Since the optimal solution responds excessively and sensitively to changes in the estimated value, the final portfolio weight changes significantly even with minor changes in input parameters. In the end, the potential problem with this misestimation is that it produces highly biased, incomprehensible, and implausible portfolios. As a result, a portfolio construction that is difficult to implement is accompanied by various intrinsic and extrinsic costs, resulting in an inefficient portfolio. Such misestimation can be classified into two. The first is the estimation error of the expected return, and the second is the estimation error of the correlation matrix. The cause of the misestimation problem of expected return is that the sample means of historical data used in the mean–variance framework is a poor estimation method for out-of-sample. The cause of the misestimation problem of the covariance matrix is that the estimated sample covariance is unstable similar to the sample mean. In particular, when the number of assets exceeds the number of observations, it may suffer from ill-conditioning. As a result, it can be a massive problem because it causes significant economic capital loss.

Researchers have proposed numerous portfolio models to overcome these limitations. Among them, the most representative successful model is the Black & Litterman (1991) model. The Black–Litterman model is a methodology that calculates the weight of a portfolio based on market equilibrium and the investor's view. It uses the external information of the investor's view to obtain accurate parameter estimation with the new updated return distribution based on the Bayesian update method. This new return distribution has more explanatory power than the original return distribution that only uses information of return alone as in the mean–variance framework. As a result, the Black–Litterman model requires structuring external information to construct the investor's view. However, the wrong view is reflected in the model if an investor does not perform this structuring correctly. For this reason, the advantage of the Black–Litterman model, which is the flexibility of the reflection of external information, can be a disadvantage because noise can be added to the meaningful signal from data. In other words, it suffers from worse performance than the market equilibrium portfolio due to incorrect estimation through incorrect data processing procedures and noisy information reflection. Therefore, proper view construction is a critical issue to be dealt with in portfolio management using the Black–Litterman framework.

Similarly, the synthetic put strategy, an asset allocation strategy for risk management, also has an estimation error problem for input parameters required for adequate implementation (Zhu & Kavee, 1988). Among them, the most critical issue is volatility misestimation. Due to the volatility estimation error, replication of the put option is not performed correctly, so the protection error problem for the insured portfolio can be fatal. In other words, it can completely fail the objective of risk hedging that the strategy seeks to pursue. In order to mitigate this problem, various methodologies such as constant proportion portfolio insurance (Black & Jones, 1987, 1988; Black & Perold, 1992; Perold & Sharpe, 1988) and time-invariant portfolio protection (Estep & Kritzman, 1988) bypass volatility estimation problem have been proposed. However, to the best of our knowledge, no study directly deals with this problem of synthetic put strategy.

In the financial field, machine learning models have attracted substantial attention from academia and practitioners due to their outstanding superiority in various tasks such as information extraction, identifying complex data patterns, time-series forecasting, and so on (Bucci, 2020; D'Ecclesia & Clementi, 2021; Sun & Yu, 2020; Xia et al., 2022). This versatility of machine learning models in traditional finance tasks is provided by non-linear functional forms, which helps models implicitly learn the relation between input and target data without taking into account explicit formulations or assumptions about underlying processes. The ability to extract complex patterns from raw data through high-dimensional data abstractions also offers this versatility, revealing the outperformance of these machine learning models (Hornik et al., 1989). Many empirical studies have shown that machine learning methodologies are suitable for capturing complex patterns in the correlation between cross-sectional asset return and external data and are also suitable for forecasting financial time series (Gu et al., 2020; Heaton et al., 2017; Hutchinson et al., 1994). The limitations mentioned above in existing portfolio strategies are challenging to address due to the complexity of financial data. For example, the view construction in the Black–Litterman framework requires processing information and abstracting meaningful signals from external financial data. Furthermore, the volatility estimation in synthetic put portfolio insurance strategy also requires excellent forecasting accuracy based on the lagged asset volatility series. Hence, to address these problematic issues, we propose employing machine learning models to perform the aforementioned financial tasks adequately. We expect that the proper application of machine learning methodology successfully mitigates issues of the above limitations.

Ragarding the second procedure of selecting new asset markets, many researchers have conducted numerous studies on various asset markets. These include portfolio research on the stock (Chunhachinda et al., 1997; Fernholz & Shay, 1982; Konno & Yamazaki, 1991), the bond (Caldeira et al., 2016), commodity (Chang et al., 2011; Geman & Kharoubi, 2008; Lean et al., 2015), currency (Jorion, 1994; Walker, 2008), and derivatives (Bookstaber & Clarke, 1984; Holowczak et al., 2006). In the meantime, the cryptocurrency market has overgrown as an emerging asset class. Cryptocurrency is a type of cryptographic-based digital asset using blockchain technology, a decentralized distributed ledger system. Since the development of Bitcoin by Nakamoto (2008), tens of thousands of cryptocurrencies have been developed in the past decade. Many cryptocurrencies, including Bitcoin and Ethereum, are gaining enormous interest among investors, achieving market capitalization significantly and establishing themselves as a financial investment asset. Researchers also showed great interest in these cryptocurrencies and examined various studies, including portfolio research on analyzes of diversified multi-asset investments based on a mean–variance framework. However, studies on dynamic asset allocation for a given portfolio or a single asset aimed at risk management for the cryptocurrency market are limited. In particular, to the best of our knowledge, portfolio insurance strategy has not been studied in the cryptocurrency market. Meanwhile, the NFT market, a secondary market derived from cryptocurrency, has also recently received tremendous interest from investors. NFT refers to a blockchain-based digital asset that tokenizes non-fungible ownership using smart contracts. Although it has received much attention from investors and achieved a significant market capitalization, even basic but important portfolio research on NFTs, such as the mean–variance analysis, has not been done.

The differences between digital assets such as cryptocurrencies and NFTs and traditional assets are four-fold. First, the traditional asset market is centralized in that financial transactions are processed under the supervision of centralized institutions such as firms, banks, or governments. In contrast, the digital asset market is decentralized in that financial transactions take place through a decentralized system called blockchain in the digital asset market (Lee, 2019). Second, traditional assets have an intrinsic value that has been empirically researched and confirmed, while digital assets have no economic consensus on the intrinsic value of the asset yet. In this regard, the types of characteristics used in the valuation are different. For example, for the valuation of a traditional asset, such as a stock, non-technical financial information about a firm is used, whereas, for the valuation of digital assets, additional blockchain-related technical information is used (Liu & Tsyvinski, 2021). Third, the propensities of main market participants are different (e.g., misunderstanding of risk-return trade-off and risk-seeking investors), and accordingly, the behavior of the asset is different (e.g., bubble behavior) (Hasso et al., 2019; Pelster et al., 2019). Finally, the digital asset market is not yet mature compared to the traditional asset market. Various regulations related to investor safety are insufficient in the digital asset market (e.g., Fraud, laundry, Fonzi scheme, cross trading), reinforcing an additional and unnecessary risk (Nabilou, 2019). From a portfolio management point of view, differences in the fundamental characteristics of assets, methods of pricing the intrinsic value of assets, and behaviors as an asset for investment imply the possibility of a lower level of interdependence between the asset classes. That is, there is a high probability that the correlation between digital assets and traditional assets is low. Therefore, the potential for diversification is high when new assets are incorporated into the existing asset portfolio. However, despite the possibility of such a diversification effect, research on new asset markets is limited. Additionally, the fact that investors participating in each asset market have different propensities implies that market participants' degree of risk-averse is different. Hence, even if the same portfolio methodology is applied, it suggests that the portfolio's optimal weights may differ in a new asset market. In other words, the optimal specification of existing portfolio models providing maximum utility to primary investors in the new asset market may be completely different from that in the existing asset market. However, to our knowledge, related studies are limited. Finally, as it is not a mature market, investors who invest in digital assets may suffer a considerable loss due to insufficient regulations or irrational investment decisions based on misinformation, such as fraud, the Fonzi scheme, and cross-trading. Despite these enormous potential risks, research on portfolio management limits or protects against these risks is lacking. Considering the above arguments, empirical research on diverse portfolio management strategies is insufficient in the new digital asset markets. Therefore, motivated by this, this dissertation focuses on the research on portfolio management in the new digital asset markets.

1.2 Aims of the Dissertation

The aims of this dissertation are two folds. First, we attempt to improve the existing portfolio management strategy in model construction and input parameter estimation for the appropriate model specification. In order to mitigate the difficulty of addressing the complex financial data, we propose to use machine learning-based models to extract the meaningful pattern of external financial data and estimate the input parameters required for asset allocation strategy. Second, we investigate the empirical results using portfolio analysis in the emerging digital asset markets. To achieve this aim, we examine the economic value of a new asset in terms of diversification effect on traditional asset-based portfolios and investigate empirical results of portfolio insurance strategy implementation results in terms of various risk measures and investor's utility. Through this, we uncover the risk and return trade-off and optimal asset allocation decisions for portfolio management in the digital asset markets. The specific aims of each chapter are as follows.

In Chapter 2, we first empirically investigate the effect of firm characteristics on the Black–Litterman framework by proposing a novel dynamic Black–Litterman model that incorporates firm characteristics into view distribution. To incorporate such characteristics into the view, we propose the backward-looking view that reflects naively historical information, and the forward-looking view that reflects the predicted information via machine learning.

In Chapter 3, we conduct an econometric analysis to test the hypothesis that NFTs have a hedge and safe haven effects on major traditional asset markets in the global financial system. Moreover, we investigate whether the inclusion of NFTs in portfolio investing in traditional assets provides a significant diversification benefit for constructing a well-diversified portfolio. For the test of a hedge and safe haven effects, we investigate the daily and weekly estimates of these effects in times of extreme market conditions and the COVID-19 crisis. For the test of a diversification effect, we examine Pearson's correlation, the Gerber Statistic for co-movement, the spillover index for volatility transmission, and finally, mean–variance portfolio analysis results.

In Chapter 4, we investigate the impact of volatility misestimation in synthetic put strategy and the comprehensive empirical results on portfolio insurance strategy using various volatility forecasting models, including naive, GARCH-type, HAR-RV-type, and machine learning-type models, to address the protection error problem caused by this issue. To achieve this aim, we examine the performance evaluation using Monte Carlo simulation based on the standard GBM and the GBM with jump models and using S&P 500 index as real-world data.

In Chapter 5, we investigate the comprehensive empirical results of portfolio insurance strategies in the cryptocurrency market. To achieve this aim, we examine the performance evaluation based on the various downside risks by comparing portfolio insurance strategies and benchmarks under several pre-specified economic conditions. Additionally, we explore the impact of the investor's utility and corresponding parameters of their nature based on expected utility and prospect theory.

Therefore, in the dissertation, by investigating the above research questions about portfolio management in terms of asset diversification and risk management, we can propose a novel machine learning-based asset allocation strategy and uncover the effect of digital asset investment. As a result, we expect to achieve an improved portfolio strategy and provide the asset diversification and risk management benefit of a more efficient and well-diversified portfolio for improved portfolio management objectives, as shown in Figure 1.1.

1.3 Organization of the Dissertation

The remainder of this dissertation is organized as shown in Table 1.1.

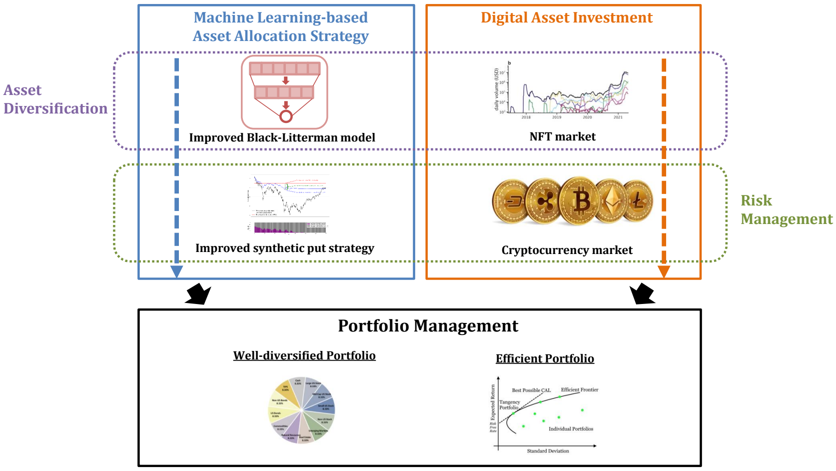


Figure 1.1: The roadmap of the dissertation.

Table 1.1: The outlines of the dissertation

Perspective	Research scope	Market	Content	Chapter
Asset Diversification	ML-based Asset Allocation Strategy	US Stock	Black–Litterman model considering firm characteristic variables	2
	Digital Asset Investment	NFTs	Portfolio analysis for Non-Fungible Token market	3
Risk Management	ML-based Asset Allocation Strategy	US Stock	Volatility forecasting for portfolio insurance strategy	4
	Digital Asset Investment	Cryptocurrency	Portfolio insurance strategy in the cryptocurrency market	5

Chapter 2

Black–Litterman model considering firm characteristic variables

2.1 Chapter overview

In the context of modern finance, a number of firm characteristics have been addressed to explain the cross-section of stock returns (Harvey et al., 2016; Hou et al., 2015; Kogan & Papanikolaou, 2013; McLean & Pontiff, 2016). Banz (1981) demonstrated a strong negative correlation between the average return and the size of a firm. Rosenberg et al. (1985) reported the average return's positive relationship with the book-to-market ratio in the US market. Fama & French (1992, 1993) introduced the three-factor model using size and value in addition to the market beta. Jegadeesh (1990) presented empirical evidence of the predictability of stock returns using momentum. Ang et al. (2006); Baker & Haugen (2012) revealed a low-volatility anomaly in financial markets.

In an attempt to overcome the limitation of the traditional Markowitz (1952) portfolio model, Black & Litterman (1991) proposed the Black–Litterman model (hereafter BL model). This model combines market equilibrium with an expert view based on the Bayesian approach to mitigate the estimation error of covariance and expected return. The construction of view distribution, as demonstrated by He &

Litterman (2002), is at the heart of this framework. Accordingly, many studies have attempted to propose an extension of the BL model in terms of constructing view distribution (Beach & Orlov, 2007; Chiarawongse et al., 2012; Fang et al., 2018; Kara et al., 2019).

One way to generate the view distribution is through the application of firm characteristics. Several studies have improved the BL model using a single characteristic. Fernandes et al. (2018) implemented an autoregressive model to the BL approach using the Price-to-Earnings ratio to estimate the conditional probability distribution of asset returns. Pyo & Lee (2018) exploited the low-volatility anomaly of stocks and applied this anomaly to the BL framework. These studies reveal strong evidence that utilizing stock-level characteristics in constructing view distribution can cause an improvement in the portfolio of the BL framework. However, to the best of our knowledge, limited studies have applied a multitude of firm characteristics simultaneously for the generation of the view distribution of the BL model. Therefore, in this study, we attempt to investigate utilizing a multitude of firm characteristics in the construction of view distribution of the BL framework.

As demonstrated by Fama & French (2008); Green et al. (2017); Lewellen (2015), the expected return of a stock is a function of the multitude of stock-level characteristics. That is, the multitude of firm characteristics provides information explaining the average stock returns. Based on these findings, the aim of our study is to examine the effect of utilizing a multitude of firm characteristics for the construction of the view distribution of the BL framework and to scrutinize the empirical results obtained for the US stock market. To this end, we propose two sets of view strategies, comprising both backward-looking and forward-looking views.

First, for the construction of the backward-looking view, we use the value obtained from the 12-month average of the firm characteristics and expected returns for estimation. Second, for the forward-looking view, we propose a novel dynamic BL model with machine learning to predict a multitude of characteristics and expected returns. Gu et al. (2020) suggested that variables based on firm characteristics can help improve the understanding of asset prices in the empirical context of return prediction using various machine learning models. The authors empirically reveal that Artificial Neural Networks (ANN) displays the best performance compared to other methods. Following Gu et al. (2020), we propose to use ANN as a methodology to predict firm characteristics and expected returns in the BL model to exploit informative estimation[1].

Chapter 2 is organized as follows: Section 2.2 describes the data and the methodology used. Section 2.3 discusses the main empirical results.

2.2 Data and Methodology

2.2.1 Data

Our data sample covers the period from January 1965 to December 2021. We conduct an empirical analysis of large-scale samples with an observed total time period of 57 years and obtain individual asset returns on a monthly basis from the CRSP for all firms that are listed on the AMEX, NASDAQ, and NYSE. The average number of stocks per year is 2,072, and the total number of firms is 29,863. The aforementioned

[1] Our main research question is how the prediction of a multitude of characteristics affects a portfolio in the BL framework. According to Gu et al. (2020), ANN shows the highest explanatory power among various machine learning models in measuring asset risk premiums using characteristics. In other words, the ANN method is considered to be the most suitable machine learning methodology to be used as a predictive model for estimating stock return distribution using characteristics in this study. Therefore, in order to examine the research question clearly, this study proposes to use the ANN model.

stock-level characteristics were taken from data suggested by Gu et al. (2020). We obtained the three-month treasury-bill (t-bill) rate as the risk-free rate proxy used in Welch & Goyal (2008). We calculate individual excess returns using the log return data taken from the WRDS database and the t-bill rate. For every month, we replace the missing value with the cross-sectional median for each stock.

2.2.2 Methodology

Dynamic Black–Litterman Model

The basic assumption of the BL model is that the expected return and covariance are constant over time (Harris et al., 2017). However, as the financial market literature shows, the expected return and covariance of stock return are time-dependent (Devpura et al., 2018; Guidolin et al., 2013; Harvey, 1989; Ng, 1991; Rapach & Zhou, 2013). To reflect this time-varying property, we first obtain the sequential implied equilibrium expected return vectors of the CAPM market portfolio as follows:

$$\pi_{\mathbf{t}} = \lambda \mathbf{\Sigma_t} \mathbf{w_{mkt,t-1}}, \tag{2.1}$$

where $\pi_{\mathbf{t}} \in \Re^N$ is the conditional implied equilibrium expected return vector, λ is the coefficient of risk-aversion, $\mathbf{\Sigma_t} \in \Re^{N \times N}$ is the conditional historical covariance matrix of excess return vector at time t, and $\mathbf{w_{mkt,t-1}} \in \Re^N$ is the weight of the market capitalization of the assets at time $t-1$.

Assume that the prior distribution of the expected excess return is given by $\mu_{\mathbf{t}} \sim \mathcal{N}(\pi_{\mathbf{t}}, \tau\mathbf{\Sigma_t})$, such that the conditional distribution of $\mathbf{q_t}$ is $\mathbf{q_t}|\mu_{\mathbf{t}} \sim \mathcal{N}(\mathbf{P_t}\mu_{\mathbf{t}}, \mathbf{\Omega_t})$, where $\mathbf{P_t} \in \Re^{K \times N}$ is the conditional matrix of experts' view, $\mathbf{q_t} \in \Re^K$ is the conditional expected excess return vector for view, and $\mathbf{\Omega_t} \in \Re^{K \times K}$ is the diagonal conditional covariance matrix of view. Then, following the Bayesian approach, the time-conditional marginal distribution for $\mathbf{q_t}$ and the new combined expected excess returns, $\mu_{\mathbf{posterior,t}}$, can be derived as in Pyo & Lee (2018):

$$\begin{aligned} \mathbf{q_t} &\sim \mathcal{N}(\mathbf{P_t}\pi_\mathbf{t}, \mathbf{\Omega_t} + \mathbf{P_t}(\tau\mathbf{\Sigma_t})\mathbf{P_t}^\top) \\ \mu_{\mathbf{posterior,t}} &\sim \mathcal{N}(\mu_{\mathbf{BL,t}}, \mathbf{\Sigma_{BL,t}}), \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} \mathbf{\Sigma_{BL,t}} &= [(\tau\mathbf{\Sigma_t})^{-1} + \mathbf{P_t}^\top\mathbf{\Omega_t}^{-1}\mathbf{P_t}]^{-1} \\ \mu_{\mathbf{BL,t}} &= \mathbf{\Sigma_{BL,t}}^{-1}[(\tau\mathbf{\Sigma_t})^{-1}\pi_\mathbf{t} + \mathbf{P_t}^\top\mathbf{\Omega_t}^{-1}\mathbf{q_t}]. \end{aligned} \tag{2.3}$$

Artificial Neural Networks

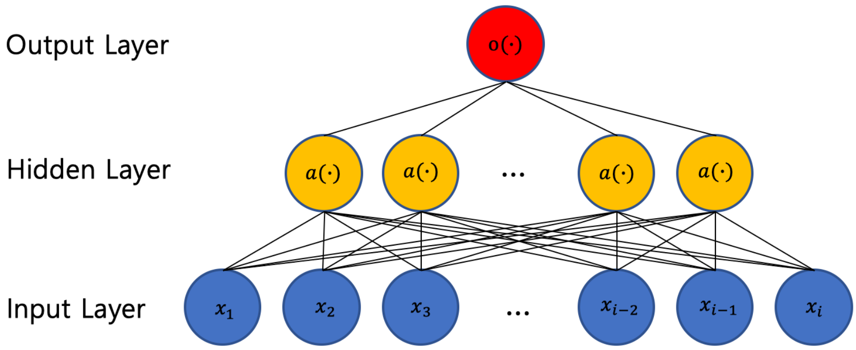


Figure 2.1: Architecture of ANN in the example of one hidden layer structure.

ANN is a widely used machine learning model in various fields such as processing of natural language, computer vision, and finance. It consists of an input layer, hidden layers, and an output layer. The input layer is denoted by $\mathbf{x} = [1, x_1, ..., x_k]^\top \in \Re^{k+1}$ where the number of neurons in this layer represents the dimension of predictors. The hidden layers denoted by $\mathbf{h} = [h_1, ..., h_j]^\top \in \Re^j$ is the nonlinear transformation of predictors. The output layer denoted by $o(x; w) \in \Re^1$ is the aggregated value of the hidden layers for the ultimate output, as illustrated in Figure 2.1. The input data are fed into the neurons in the input layer, and the aggregated value of each neuron with parameter $\mathbf{w_j^{(0)}} \in \Re^{k+1}$ is passed through the nonlinear activation function $a(\cdot)$ to obtain a j-th neuron in the hidden layer h_j, as follows:

$$h_j = a(\Sigma_k w_{j,k}^{(0)} x_k). \tag{2.4}$$

To obtain the final output value in the output layer, the value of each neuron in the hidden layer is linearly aggregated as follows:

$$o(x; w) = \Sigma_j w_j^{(1)} h_j. \tag{2.5}$$

Proposed Procedure

In this study, we propose methods to generate the view matrix reflecting a multitude of characteristics. We use four established firm characteristics: size, book-to-market ratio, momentum, and stock volatility. To incorporate each characteristic into the BL framework, by sorting the i-th stock's estimated values of the k-th characteristic $\hat{c}_{i,t}^{(k)}$ at time t in the ascending order, we divide the stocks into three groups: first decile, last decile, and the remainder. Depending on the economic implication of each characteristic, $w_{i,t}^{(k)}$, $-w_{i,t}^{(k)}$, or 0 are assigned to the stocks in each group, where $w_{i,t}^{(k)}$ represents the weight of the relative view of the i-th stock's k-th characteristic at time t. We use market weights for the weight values, verifying that the sum is equal to zero.

Table 2.1: The economic implication for the firm characteristics reflected on the view matrix

Characteristic	Abbreviation	Econ. Implication	View matrix (P_t)
size	size	Small-cap stock tends to outperform large-cap stock	$[w_{1,t}^{(1)}, w_{2,t}^{(1)}, ..., 0, 0..., -w_{i-1,t}^{(1)}, -w_{i,t}^{(1)}]$
book-to-market	bm	Stock that seems cheaper tends to outperform stock that seems expensive	$[-w_{1,t}^{(2)}, -w_{2,t}^{(2)}, ..., 0, 0..., w_{i-1,t}^{(2)}, w_{i,t}^{(2)}]$
momentum	mom	Stock that outperformed in the past tends to outperform going forward	$[-w_{1,t}^{(3)}, -w_{2,t}^{(3)}, ..., 0, 0..., w_{i-1,t}^{(3)}, w_{i,t}^{(3)}]$
low-volatility	lowvol	Stock with low volatility tends to outperform highly volatile stock	$[w_{1,t}^{(4)}, w_{2,t}^{(4)}, ..., 0, 0..., -w_{i-1,t}^{(4)}, -w_{i,t}^{(4)}]$

The economic implication of each characteristic reflected on the view and the corresponding view matrix are presented in Table 2.1. If a single characteristic is reflected, the view matrix P_t is constructed by using the matrix presented in Table 2.1. When a multitude of characteristics is reflected in the view matrix, P_t is constructed by combining each row of the matrix in Table 2.1. For example, we generate the view matrix that reflects four characteristics simultaneously, as follows:

$$P_t = \begin{bmatrix} w_{1,t}^{(1)} & w_{2,t}^{(1)} & ... & 0 & 0 & ... & -w_{i-1,t}^{(1)} & -w_{i,t}^{(1)} \\ -w_{1,t}^{(2)} & -w_{2,t}^{(2)} & ... & 0 & 0 & ... & w_{i-1,t}^{(2)} & w_{i,t}^{(2)} \\ -w_{1,t}^{(3)} & -w_{2,t}^{(3)} & ... & 0 & 0 & ... & w_{i-1,t}^{(3)} & w_{i,t}^{(3)} \\ w_{1,t}^{(4)} & w_{2,t}^{(4)} & ... & 0 & 0 & ... & -w_{i-1,t}^{(4)} & -w_{i,t}^{(4)} \end{bmatrix}. \tag{2.6}$$

We use the rolling-window architecture for an out-of-sample test to maintain the ordering of the temporal data, rebalancing portfolio weight annually. For each time step t, the proposed model for the forward-looking view consists of two phases. The first phase is to train ANN models to predict the characteristic values and expected return for the estimation of $\hat{c}_{i,t}^{(k)}$ and $\hat{r}_{i,t}$, respectively, for $\forall k, i$ by using the training set. The second phase is to construct the BL portfolio by generating the view distributions $\mathbf{P_t}$, $\mathbf{q_t}$, and $\mathbf{\Omega_t}$, based on the predicted established characteristics and expected return, and by solving optimization to obtain portfolio weight $\mathbf{w_t}$. The overall procedure is shown in Figure 2.2.

In the first phase, for the prediction of $\hat{c}_{i,t}^{(k)}$, each 10-year characteristic of each firm is considered for training. For a subsequence of the 10-year characteristic at each firm, the ANN is trained in batches, where the 12-month data point is used as input data and the characteristic observed in the first month of the next year is set as the target. If all the batches in a 10-year training set are consumed, one fitted model is obtained for each firm. For the prediction of $\hat{r}_{i,t}$, a similar procedure is conducted for the i-th stock return. For consistency, we also use the ANN trained by the training set for prediction, inspired by various studies that show that ANN yields more accurate forecasts compared to the traditional econometric model in the forecasting return of stocks (Aras & Kocakoç, 2016; Ghiassi et al., 2005; Nayak & Misra, 2018; Zhang, 2003; Zhong & Enke, 2019).

In the second phase, this fitted model is used for the inference of a one-year test sample to predict the value of the characteristics of each firm. We use these predicted values $\hat{c}_{i,t}^{(k)}$ to generate $\mathbf{P_t}$, as shown in Eq. 2.6. Similarly, in order to build a return view $\mathbf{q_t}$, we use the predicted value for each firm's expected return, having a forward-looking return view. We get $\mathbf{\hat{r}_t} = [\hat{r}_{1,t}, ..., \hat{r}_{i,t}]$ from the fitted ANN model's inference of the predicted value, where $\hat{r}_{i,t}$ is the predicted expected return of i-th stock at time t. Thus, $\mathbf{q_t}$ is calculated from $\mathbf{q_t} = \mathbf{P_t}\mathbf{\hat{r}_t}$, and $\mathbf{\Omega_t}$ is calculated from $diag(\tau p_{\cdot,t}^{(1)} \Sigma_t p_{\cdot,t}^{(1)'}, \ldots, \tau p_{\cdot,t}^{(k)} \Sigma_t p_{\cdot,t}^{(k)'})$, where $p_{\cdot,t}^{(k)}$ is the k-th row of the view matrix $\mathbf{P_t}$. After obtaining $\pi_\mathbf{t}$, $\mathbf{P_t}$, $\mathbf{q_t}$, and $\mathbf{\Omega_t}$, we get $\mu_{\mathbf{BL,t}}$ and $\mathbf{\Sigma_{BL,t}}$; thus, $\mathbf{w_t}$ is obtained by solving the mean–variance optimization given by

$$\max_{\mathbf{w}_t} \quad \mathbf{w}_t^\top \hat{\mu}_t - \frac{\lambda}{2}\mathbf{w}_t^\top \hat{\mathbf{\Sigma}}_t \mathbf{w}_t \tag{2.7a}$$

$$\text{subject to} \quad \Sigma_{i=1}^{N} w_{i,t} = 1, \tag{2.7b}$$

where $\hat{\mu}_\mathbf{t} = \mu_{\mathbf{BL},\mathbf{t}}$ and $\hat{\mathbf{\Sigma}}_\mathbf{t} = \mathbf{\Sigma}_{\mathbf{BL},\mathbf{t}}$. After obtaining optimal portfolio weight $\mathbf{w}_\mathbf{t}^*$, we apply this weight to the out-of-sample test.

In order to get the proposed model for the backward-looking view, $\hat{c}_{i,t}^{(k)}$ and $\hat{r}_{i,t}$ in the second phase are naively obtained from the 12-month average of the past characteristics and the expected return of the training set, respectively.

Considering the ANN models for $\hat{c}_{i,t}^{(k)}$, we have 13 neurons in the input layer in our model, which implies that one of them is for the intercept, and the 12-month historical characteristic data are used as predictors. We use rectified linear unit (ReLU) as the activation function $a(\cdot)$ of the hidden layer, which is known to have displayed the best performance in previous studies (Glorot et al., 2011; Nair & Hinton, 2010). The model has a single hidden layer of 100 neurons, and all the neurons are fully connected. We use the identity function as the activation function of the output layer for the regression model and aggregated the hidden outputs linearly into an ultimate output. Considering the ANN models for $\hat{r}_{i,t}$, the same structure is used.

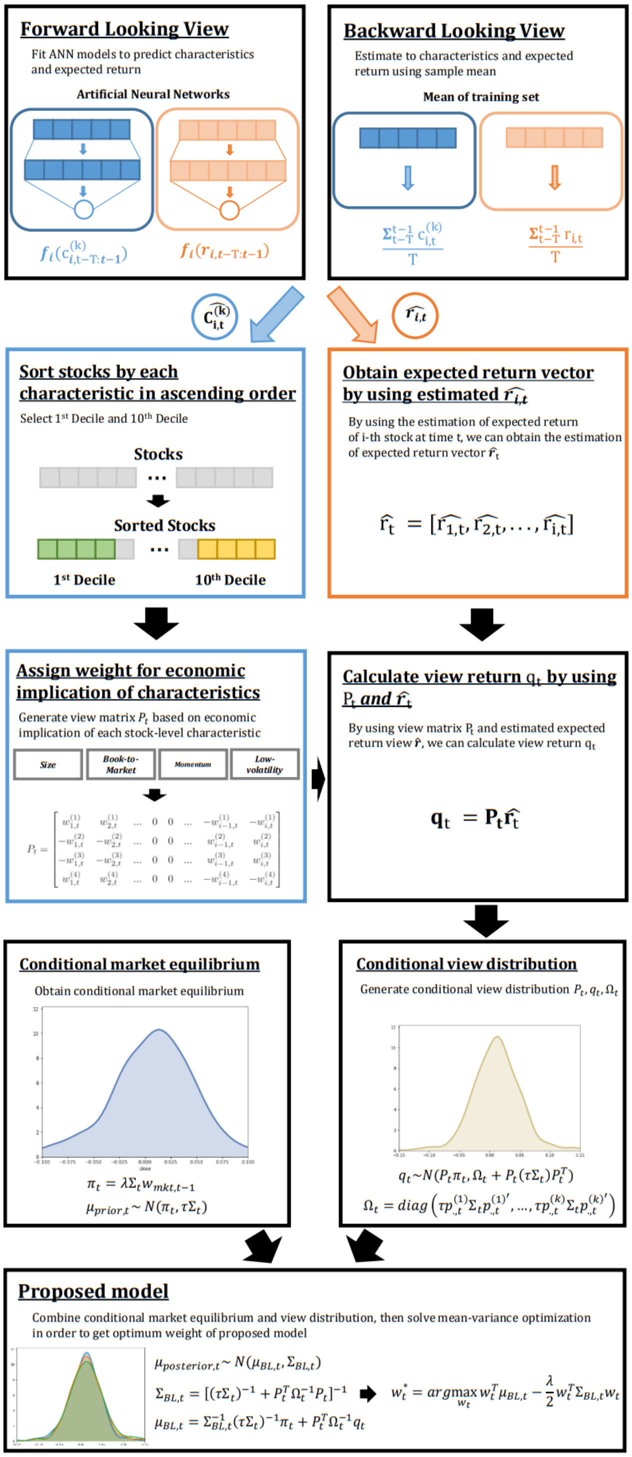


Figure 2.2: Overall procedure of proposed model.

2.3 Empirical results

We use alpha and the annualized out-of-sample Sharpe ratio as the performance measures for the proposed model. Sharpe ratio is calculated as $E(R_p)/\sigma_p$, where $E(R_p)$ is the expectation of the annualized excess return of the portfolio and σ_p is the standard deviation of the annualized excess return of the portfolio. Table 2.2 reports the results of each strategy and benchmark.

Table 2.2: Results on the performance of each benchmark and the proposed model

	Mean Ret.	Std.	Skew.	Kurto.	α	t-stats	SR
Panel A: Long-term (42-year) performance							
S&P 500	0.083	0.15	-0.803	3.036	-	-	0.239
Mkt equilibrium	0.103	0.152	-0.467	2.41	0.021	(3.52)**	0.364
Proposed Backward view	<u>0.138</u>	0.126	-0.232	3.296	<u>0.079</u>	(7.34)***	<u>0.722</u>
Proposed Forward view	**0.2**	0.18	1.996	20.941	**0.138**	(6.27)***	**0.85**
Panel B: Short-term (5-year) performance							
S&P 500	0.156	0.128	-3.16	16.19	-	-	1.135
Mkt equilibrium	0.153	0.159	-0.592	2.508	0.018	(0.32)***	0.898
Proposed Backward view	<u>0.194</u>	0.147	-0.057	2.739	<u>0.083</u>	(1.48)***	<u>1.242</u>
Proposed Forward view	**0.457**	0.32	2.609	9.467	**0.339**	(2.3)***	**1.394**

Notes. This table reports the results of mean, standard deviation, skewness, kurtosis, alpha, t-statistics, and annualized Sharpe ratios of the annualized returns of each strategy and benchmark in the past 42-year (Panel A) and 5-year (Panel B) out-of-sample period. The abbreviations Mean Ret, Std, Skew, and Kurto report the average of the annualized returns, the standard deviation of the annualized returns, skewness of the annualized returns, and kurtosis of the annualized returns, respectively. α and t-stats represent alpha and t-statistics, respectively. SR presents the annualized Sharpe ratios. ** and *** indicate significance at the 5% and 1% level, respectively.

Panel A in Table 2.2 shows the long-term performance result using past 42-year data. The S&P 500 and the market equilibrium portfolio have out-of-sample Sharpe ratios of 0.239 and 0.364, respectively, which are lower than that of 0.722 and 0.85 of the proposed model with backward and forward-looking views, respectively. Additionally, the alpha of the proposed model with backward and forward-looking views is 0.079 and 0.138, respectively, which is larger than that of the market equilibrium portfolio; it is statistically significant. Panel B in Table 2.2 presents the short-term performance result using past 5-year data. The short-term result is essentially similar to the long-term result. The proposed models dominate the market and market equilibrium portfolio, as presented in Figure 2.3, indicating that reflecting a multitude of characteristics improves the performance of the BL portfolio. Especially, the forward-looking view model outperforms the backward-looking view model. This result suggests that the prediction via machine learning provides a significant improvement over naive estimation.

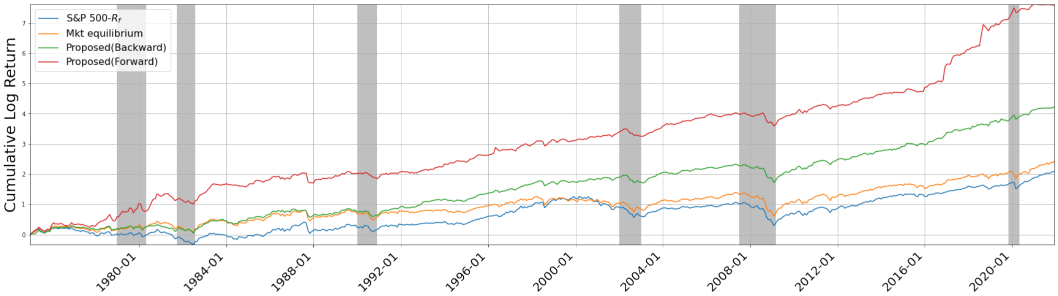


Figure 2.3: Cumulative performance of the proposed model.

Notes. This figure shows the cumulative performance of the proposed model with the backward-looking view and the proposed model with the forward-looking view; the cumulative market excess return and cumulative market equilibrium portfolio return are benchmarks for all test periods. The shaded areas denote global recessions. By a huge margin, the proposed models dominate the other benchmarks.

To sum up the empirical results in Table 2.2, our findings and implication are twofold. First, our proposed procedure to construct the view distribution of the BL model significantly improves the existing naive BL model by reflecting firm characteristics to offer a better opportunity to adjust the subtle weight of allocation. It is clearly demonstrated by the enhancement of the out-of-sample performance in the proposed model compared to the benchmarks, such as S&P 500 and market equilibrium portfolio. This finding implies that the market equilibrium portfolio that is BL model without an investor's view is not a fully efficient portfolio; thus, reflecting four firm characteristics, especially size, book-to-market, momentum, and low-volatility, has a meaningful value of generating more efficient portfolio.

Second, the effect of prediction is clearly revealed by comparing the proposed backward-looking view and forward-looking view model in that the out-of-sample Sharpe ratio of the forward looking-view is larger than that of the backward-looking view. Interestingly, the standard deviation of returns in portfolios built by the forward-looking view tends to be higher on average, implying more aggressive investments. Despite this tendency, the average of the out-of-sample Sharpe ratio of the forward-looking view model is consistently higher than that of the backward-looking view model in all Panel A and Panel B. α shows a similar outcome, suggesting that firm characteristic prediction via ANN provides an improvement in performance, resulting in diversification benefit to portfolio. These results align with the fact that the ANN as a universal approximator is inherently nonlinear and has a rich functional form that adequately extracts the complex pattern of data (Hornik et al., 1989). Empirical studies report that the ANN methodology is suitable for the prediction and estimation of complex financial time-series and cross-sectional asset return data (Gu et al., 2020; Heaton et al., 2017; Hutchinson et al., 1994). Therefore, our results consistently reveal a strong hint that prediction through ANN played a key role in allowing the proposed model to achieve to exploit the pattern of behavior of characteristics in a future return. As a result, it indicates that our proposed model can help construct a more efficient and well-diversified portfolio.

Chapter 3

Portfolio analysis for Non-Fungible Token market

3.1 Chapter overview

Recently, the NFTs market, a new emerging market different from the existing cryptocurrency market, was formed (Bao & Roubaud, 2022; Urquhart, 2021). NFTs are non-fungible ownership recorded on the blockchain by smart contracts (Dowling, 2021a). These tokens function like tradable rights to any assets. These include digital assets such as files and game items, and physical assets like artwork and real estate (Kugler, 2021). NFTs have been attracting public attention, including investors and practitioners, reaching a market capitalization of over $16 billion in early 2022 (Karim et al., 2022a). This NFT market has also begun to receive the attention of academia.

Accordingly, several studies related to NFTs have been conducted. Among these studies, some focused on the behavior of NFT prices. Dowling (2021a) explored the behavior of NFTs in terms of price efficiency on Decentraland, an NFT platform for virtual real estate trading, while Maouchi et al. (2021) investigated the behavior of NFTs in terms of bubbles. Besides research on the behavior of NFT price, studies on the connectedness between NFTs and existing assets have been conducted. Dowling (2021b) investigated the volatility spillover and the wavelet coherence between NFTs and cryptocurrencies, demonstrating that the asset pricing of NFTs is different from the asset pricing of cryptocurrencies. Karim et al. (2022a) looked into the extreme risk spillover among NFTs, decentralized financial (DeFi) tokens, and cryptocurrencies, implying the weak interaction between these asset classes with a strong disconnection of NFTs. Aharon & Demir (2021) NFTs and traditional assets' spillover effect of return, while Umar et al. (2022) investigated the wavelet coherence between NFTs and traditional assets. Ante (2021) confirmed that cryptocurrency markets affect the growth of the NFT markets, but there is no opposite effect. These studies indicate the existence of potential hedge and diversification benefits of NFTs with respect to traditional assets.

Originally, NFTs were designed to be indivisible. However, the concept of divisible NFTs has recently emerged to reflect the demand for a more realistic characteristic of traditional assets, such as shared ownership. The concept of divisible NFTs or fractional NFTs (the so-called F-NFT) is that each unique NFT can be separated into equal and fungible parts (Mazur, 2021). In other words, divisible NFTs are designed for shared ownership of any assets, such as real estate, arts, and digital images (Frye, 2021). Divisible NFTs divide the ownership of any asset into equal parts, allowing the asset to function similarly to stock or shares in the company. In other words, the rights to an asset can be converted into a type of partially investable asset by using divisible NFTs.

If NFTs are divisible, investors can use divisible NFTs to manage their portfolios by hedging or diversifying the assets invested. Therefore, we attempt to show the possibility of portfolio hedge or diversification by including NFTs with traditional assets. Although there are several studies on NFTs, the majority of them study only

NFT price behavior (Dowling, 2021a; Maouchi et al., 2021) or the connectedness between NFTs and existing assets (Dowling, 2021b; Aharon & Demir, 2021; Karim et al., 2022a; Umar et al., 2022). To the best of our knowledge, however, there is no study on the hedge or safe haven property of NFTs against traditional asset classes and no research on portfolios investigating the diversification effect of NFTs as an alternative asset class.

The aforementioned studies (Dowling, 2021b; Karim et al., 2022a; Umar et al., 2022) demonstrate the distinctiveness of the NFT market over traditional markets, implying NFTs have potential as a hedging, safe haven, and diversifying tool against traditional assets. Furthermore, Aharon & Demir (2021) revealed that the connectedness dynamic of NFTs is similar to that of gold[1] in terms of risk absorption during the COVID-19 crisis. Therefore, motivated by these prior studies which suggest the possibility of the hedge or safe haven property of NFTs, in this paper, we raise the question of whether NFTs are a hedge or safe haven against traditional assets. After that, we examine the diversification benefit of NFTs over traditional assets in terms of correlation, co-movement, and volatility transmission. Then, using the Markowitz mean–variance approach, we provide portfolio analysis to confirm better investment opportunities.

Chapter 3 is organized as follows. Section 3.2 describes the data. Section 3.3 presents the methodology. Section 3.4 discusses the empirical results.

[1]According to Bouri et al. (2017), gold has been generally demonstrated as a a hedge or safe haven asset against traditional asset classes. See Agyei-Ampomah et al. (2014); Akhtaruzzaman et al. (2021); Baur & Lucey (2010); Baur & McDermott (2010); Bredin et al. (2015); Capie et al. (2005); Gürgün & Ünalmış (2014); Peng (2020); Reboredo (2013a,b); Śmiech & Papież (2017).

3.2 Data

Table 3.1: Summary statistics of the daily and weekly returns on each asset

	Mean (%)	Median (%)	Std. (%)	Min	Max	Skew.	Kurto.
Panel A: Daily							
NFT	0.847	0.295	24.3	−197.9	172.9	0.32	18.95
US	0.083	0.143	1.43	−10.06	8.670	−0.66	8.650
Canada	0.039	0.104	1.27	−13.18	11.29	−1.78	31.78
Australia	0.025	0.091	1.21	−7.640	5.660	−0.67	7.250
Japan	0.023	0.050	1.55	−8.770	7.730	−0.13	5.060
UK	−0.001	0.072	1.34	−11.51	8.670	−1.19	13.42
Germany	0.023	0.083	1.57	−13.05	10.41	−0.88	13.59
Switzerland	0.037	0.103	1.59	−11.50	12.13	−0.47	19.06
Italy	0.026	0.133	1.73	−18.54	8.550	−2.88	28.72
Finland	0.041	0.110	2.64	−30.25	27.88	−0.53	52.32
Netherlands	0.046	0.113	1.38	−11.38	8.590	−1.19	12.55
Austria	0.019	0.080	3.01	−39.60	37.15	−0.83	79.59
Belgium	0.002	0.039	1.55	−15.33	7.370	−1.76	17.80
Spain	−0.022	0.004	1.62	−15.15	8.230	−1.53	16.71
China	0.017	0.002	1.58	−8.210	11.47	0.20	7.050
Russia	0.071	0.172	1.45	−8.650	7.430	−0.85	8.000
India	0.044	0.123	1.94	−13.12	9.520	−0.89	9.470
South Korea	0.018	0.114	1.49	−7.980	8.750	−0.32	5.670
World Index	0.064	0.135	1.37	−11.02	8.710	−0.86	11.22
Emerging Market Index	0.015	0.134	1.66	−10.55	7.580	−0.65	6.280
Commodity Index	0.048	0.120	1.32	−6.950	4.740	−0.78	3.040
Gold	0.044	0.061	1.13	−7.470	5.810	−0.49	6.230
Oil	0.042	0.207	3.59	−37.34	31.77	−0.64	33.10
Bitcoin	0.147	0.177	5.78	−46.47	26.72	−0.67	8.800
Ethereum	0.169	0.122	7.40	−55.07	32.50	−0.78	6.750
PIMCO Index	0.021	0.030	0.62	−5.080	8.150	1.08	62.16
USD Index	0.014	0.000	0.45	−1.900	2.340	0.24	2.470
	Mean (%)	Median (%)	Std. (%)	Min	Max	Skew.	Kurto.
Panel B: Weekly							
NFT	4.838	3.000	52.1	−202.2	187.9	0.23	2.060
US	0.394	0.787	3.22	−23.19	14.50	−1.82	12.09
Canada	0.182	0.406	3.15	−29.38	17.52	−3.74	34.06
Australia	0.105	0.261	2.89	−24.58	11.04	−2.45	16.61
Japan	0.092	0.340	3.41	−22.67	14.23	−0.97	6.700
UK	−0.016	0.214	3.09	−26.37	13.51	−2.31	20.16
Germany	0.103	0.417	3.78	−31.21	16.95	−2.13	17.43
Switzerland	0.174	0.461	2.87	−21.47	13.80	−1.58	12.81
Italy	0.106	0.444	4.26	−38.76	15.42	−2.80	22.65
Finland	0.193	0.476	4.14	−31.56	26.89	−1.64	16.80
Netherlands	0.214	0.566	3.40	−28.20	17.83	−2.18	17.66
Austria	0.067	0.566	5.36	−41.80	36.67	−2.18	22.32
Belgium	−0.009	0.267	3.92	−33.51	15.38	−2.28	19.46
Spain	−0.129	0.217	3.91	−35.19	18.46	−2.25	20.93
China	0.071	0.202	3.55	−12.09	16.37	0.02	2.500
Russia	0.326	0.607	3.41	−21.72	16.43	−1.31	8.480
India	0.203	0.406	3.99	−25.00	16.22	−1.59	9.010
South Korea	0.068	0.337	3.58	−25.44	15.97	−1.26	9.170
World Index	0.299	0.701	3.23	−26.22	14.53	−2.33	17.21
Emerging Market Index	0.052	0.345	3.65	−25.87	13.51	−1.49	8.950
Commodity Index	0.231	0.678	3.16	−19.35	8.910	−1.38	5.370
Gold	0.214	0.332	2.35	−10.90	10.24	−0.09	3.210
Oil	0.199	1.046	8.69	−59.86	46.28	−1.35	14.81
Bitcoin	0.712	0.352	13.6	−59.36	46.02	−0.16	1.750
Ethereum	0.662	0.966	17.5	−75.95	72.66	−0.26	1.670
PIMCO Index	0.110	0.163	1.45	−14.44	13.08	−2.76	44.17
USD Index	0.072	0.041	1.02	−4.510	7.120	0.70	5.960

Notes. Panels A and B show summary statistics of the daily and weekly log returns on each asset. Regardless of the time horizon (daily or weekly), NFTs are the most volatile and show the highest average return compared to other asset classes. Among other assets, cryptocurrency shows the highest risk, while the bond index and US dollar index show the lowest levels of risk. Stock market and commodity indices show mid-level volatility.

We use different data sets for each test of a hedge or safe haven effect and diversification effect. This is because we attempt to investigate the specific results of the hedge and safe haven effect of NFTs on the traditional asset addressing overall country coverage and regional coverage comprehensively in the test of a hedge and safe haven. On the other hand, in diversification effect of NFTs on the traditional asset, we have to reduce the number of assets since the portfolio with the higher number of assets suffers from the estimation error problem.

3.2.1 Data for a hedge and safe haven effect

For the test of a hedge and safe haven effect, our asset classes consist of stock (country coverage and regional coverage indices), commodity (commodity index, gold, and oil), cryptocurrency (Bitcoin and Ethereum), bond, US dollar index, and NFTs. We obtain prices of the assets from Yahoo Finance[2] except for the NFT price. The country coverage stock market indices consist of each country's stock market. These are clustered as the North American (the US and Canada), Pacific (Australia and Japan), European (the UK, Germany, Switzerland, Italy, Finland, Netherlands, Austria, Belgium, and Spain), and Emerging Markets (China, Russia, India, and South Korea)[3]. The regional coverage stock indices consist of the MSCI World index and MSCI Emerging Market index. We use, as commodity indices, the Invesco DB Commodity index, SPDR Gold Shares, and Brent Crude oil. Cryptocurrencies are represented by Bitcoin and Ethereum prices[4]. Bitcoin is the largest capitalization

[2] www.finance.yahoo.com

[3] Specifically, the US, Canada, Australia, Japan, the UK, Germany, Switzerland, Italy, Finland, Netherlands, Austria, Belgium, Spain, China, Russia, India, and South Korea are represented by S&P 500, S&P/TSX, S&P/ASX 200, Nikkei 225, FTSE 100, DAX 30, SMI, IT 40, OMXH 25, AEX, ATX, BE 20, IBEX, CSI 300, MOEX, INDA, and KOSPI 200, respectively.

[4] We include cryptocurrency into our asset classes under study because of its relevance in that NFTs are emerging assets developed from cryptocurrency.

cryptocurrency. Ethereum is the representative smart contract-based core engine of NFT. We select the Pimco Investment Grade Corporate Bond Exchange-Traded Fund index as the bonds index. The US dollar index represents the US currency.

NFT index is represented by the average value of transaction prices following Aharon & Demir (2021). We obtain average NFT price from NonFungible[5]. At the time of writing, we used 250 different NFT markets and 21,087,054 total NFT trades to calculate the average NFT price. A large number of observations of market trades and the use of average NFT price mitigates the extreme oscillation issue of NFT returns (Dowling, 2021b).

Our data sample covers January 1, 2018, to February 9, 2022. Each price of assets and indices is in US dollars. We obtain daily and weekly log returns of all assets. The total of 869 daily and 173 weekly observations for each asset are used. Table 3.1 summarizes the statistics of the daily and weekly returns on each asset.

3.2.2 Data for a diversification effect

For the test of a diversification effect, we construct our portfolio using classes of the existing asset (stock, bonds, US dollar, commodity index, and cryptocurrencies) and NFTs as a new alternative asset class. We obtain the stock market index, bond index, US dollar index, commodity index, and cryptocurrency price from Yahoo Finance[6]. The WRDS database is used to calculate the market capitalization of each asset in the traditional asset class, while CoinMarketCap[7] is used to obtain the market capitalization of cryptocurrency. We select S&P 500 index, MSCI World index, and MSCI Emerging Market index as stock market indices. Pimco Investment

[5] www.nonfungible.com
[6] www.finance.yahoo.com
[7] www.coinmarketcap.com

Grade Corporate Bond Exchange-Traded Fund index represents Bond. The proxy of the US currency is represented by the US dollar index. We select the Invesco DB Commodity index and SPDR Gold Shares as commodity indices. We select Bitcoin and Ethereum as our cryptocurrency assets.

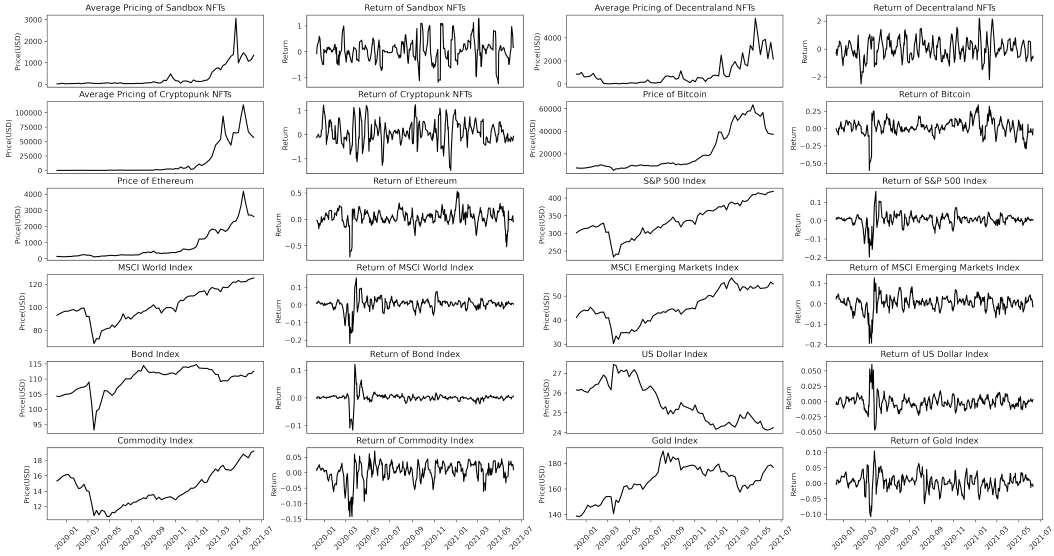


Figure 3.1: Time series of price and return of each asset.

Notes. This figure shows the time series of price and returns of each asset on a weekly basis for the period December 4, 2019, to June 9, 2021.

Based on the concept of the aforementioned F-NFT, we assume that the market index of each NFT can be replicated by aggregating partial shared ownership for tokens in any NFT market. With this assumption, it is acceptable to use the average price of each NFT as a proxy of the corresponding NFT market.

However, there are some critical issues with collecting and preprocessing the relevant data for portfolio analysis. First, there are a large number of missing values because of the low trading volume, particularly in the early period. The early period with excessive missing values is excluded from our empirical results for reliable results. Second, the daily return is subject to extreme fluctuations (Dowling, 2021b). To address this issue, we used the weekly average price of each individual NFT.

We select three of the most liquid and prominent NFTs for our analysis: Sandbox, Decentraland, and Cryptopunks[8]. We have 92,371 trades of Sandbox, 68,500 trades of Decentraland, and 10,704 trades of Cryptopunks. We obtain log returns of all assets on a weekly basis[9].

Table 3.2: Summary statistics of the weekly price and return of each asset

	Mean	Median	Std.	Min	Max	Skew.	Kurto.	ADF
Panel A: Price								
Sandbox	346.94	76.34	542.54	26.38	3054	2.45	7.54	-0.69
Decentraland	948.08	531.9	1179.6	12.24	4745	1.75	2.17	-1.22
Cryptopunks	16338	980.87	28797	56.73	113932	1.8	2.12	-0.49
Bitcoin	21589	11246	17827	5225	59893	1.08	-0.4	-0.6
Ethereum	821.47	366.23	921.8	113.94	3952	1.58	1.71	-0.02
S&P 500	338.25	329.59	46.48	237.77	419.56	0.07	-0.74	-0.9
MSCI World Index	101.46	98.64	13.88	71.69	125.8	-0.0	-0.75	-0.88
MSCI Emerging Mkt. Index	45.62	44.06	6.89	32.24	57.5	-0.02	-1.03	-0.55
Bond Index	109.76	111.08	3.89	96.12	114.76	-1.24	1.67	-2.1
US dollar Index	25.5	25.28	0.99	24.09	27.65	0.32	-1.22	-1.1
Commodity Index	14.33	13.95	2.24	10.84	19.3	0.41	-0.75	-0.29
Gold	166.32	166.84	12.5	138.92	190.15	-0.35	-0.66	-2.24
	Mean	Median	Std.	Min	Max	Skew.	Kurto.	ADF
Panel B: Return								
Sandbox	0.0553	0.0569	0.435	-1.24	1.2944	0.0377	0.9601	-4.32***
Decentraland	0.0106	0.0046	0.7065	-2.4843	2.199	-0.0012	0.6281	-4.78***
Cryptopunks	0.0912	0.0688	0.4706	-1.4802	1.2416	-0.1418	0.7673	-4.34***
Bitcoin	0.0213	0.0229	0.1134	-0.6024	0.3413	-1.025	4.9628	-3.23**
Ethereum	0.0388	0.039	0.149	-0.7133	0.5311	-0.9618	4.9059	-3.91***
S&P 500	0.0044	0.0091	0.0352	-0.1981	0.1601	-1.4135	7.8681	-4.47***
MSCI World Index	0.004	0.0084	0.0359	-0.2194	0.1543	-1.731	9.5309	-4.46***
MSCI Emerging Mkt. Index	0.004	0.0087	0.0371	-0.195	0.1274	-1.4187	6.4634	-3.97***
Bond Index	0.001	0.0019	0.0189	-0.1164	0.1216	-1.0774	19.9531	-5.95***
US dollar Index	-0.001	-0.0016	0.011	-0.0468	0.0605	0.9229	7.1007	-4.46***
Commodity Index	0.0029	0.0084	0.0317	-0.1422	0.07	-1.5816	3.8585	-3.65***
Gold	0.0033	0.0052	0.0259	-0.1081	0.1045	-0.4492	2.4013	-4.5***

Notes. Panels A and B show summary statistics of price and log return of each asset, respectively. We apply the Augmented Dickey-Fuller (ADF) test (Cheung & Lai, 1995) to log price and log return. ADF statistics show that the null hypothesis of a unit root can be rejected for realized variances. ** and *** mean significance at the 5% and 1% levels, respectively.

[8] NFT data is sourced from `www.nonfungible.com`. These three NFTs account for 56.2% of the total trading volume at the time of writing. To demonstrate the reliability of our results, we also performed additional experiments using the top 5 and 7 NFTs by liquidity. The top 5 and 7 NFTs account for 60.1% and 62.6% of the total trading volume at the time of writing. The overall results are available in Table A1 in Appendix. Note that the main results are essentially similar.

[9] A total of 76 weekly observations for each asset are included.

We use a 3-month treasury-bill rate as the risk-free rate proxy. Our data sample covers from December 4, 2019, to June 9, 2021, as NFT trades in Sandbox started in December of 2019. All indices and prices are denominated in US dollars. Figure 3.1 shows the weekly price and returns on each asset. When compared to other traditional assets, NFTs exhibit higher volatility and a disproportionately large price increase. Table 3.2 summarizes the statistics of the weekly price and return for each asset.

3.3 Methodology

3.3.1 Methods for a hedge and safe haven effect

For the test of a hedge and safe haven effect, we define a hedge and a safe haven following the seminal study of Baur & McDermott (2010):

- *A hedge: An asset is a strong (weak) hedge if it is negatively correlated (uncorrelated) with another asset on average.*
- *A safe haven: An asset is a strong (weak) safe haven if it is negatively correlated (uncorrelated) with another asset in a certain period, such as a financial crisis.*

Based on these definitions, we use econometric models introduced by Baur & McDermott (2010) for analyzing the hedge and safe haven properties of NFTs.

Firstly, the equations below present the regression model to statistically analyze the hedge and safe haven effects of NFTs.

$$r_{NFT,t} = a + b_t r_{other,t} + \epsilon_t, \tag{3.1}$$

$$b_t = c_0 + c_1 D(r_{other} q_{10}) + c_2 D(r_{other} q_5) + c_3 D(r_{other} q_1), \tag{3.2}$$

$$h_t = \pi + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}, \tag{3.3}$$

In Eq. 3.1, a and b_t are parameters of the pairwise regression model for the relation between NFTs and another asset, and ϵ_t is the error term. After obtaining b_t from Eq. 3.1, it is regressed on the dummy variables as presented in Eq. 3.2. Eq. 3.2 models the b_t as a dynamic process, where c_0, c_1, c_2, and c_3 are estimated. $D(\cdot)$ denotes the dummy variables capturing extreme asset return movement. If the asset return is below a certain threshold (e.g., 10%, 5%, and 1% quantile of the return distribution), $D(\cdot) = 1$, otherwise $D(\cdot) = 0$. Hence, c_0 represents the coefficient for the hedge property, while c_1, c_2, and c_3 represent the coefficient for the safe haven properties of NFTs against any counterpart asset. Eq. 3.3 represents a GARCH(1,1) model, accounting for heteroscedasticity, where h_t is the conditional variance, π is the constant, α is the short-run persistence parameter, and β is the long-run persistence parameter. With Maximum Likelihood, all the parameters in Eqs. 3.1, 3.2, and 3.3 are jointly estimated.

If the c_0 is zero (significantly negative), NFTs are a weak (strong) hedge, given that the sum of the parameters ($\Sigma_{i=1}^3 c_i$) is not jointly positive, exceeding c_0 (Manuj, 2021). Statistical significance that at least one of the parameters c_1, c_2, or c_3 is non-zero implies evidence of a non-linear relationship between NFTs and other assets. NFTs are a weak (strong) safe haven if the parameters in Eq. 3.2 are non-positive (significantly negative).

Secondly, to identify the effect of a financial crisis, the economic approach is presented as follows:

$$b_t = c_0 + c_1 D(\text{COVID-19 crisis}). \tag{3.4}$$

While Eq. 3.2 specifies the model implicitly, Eq. 3.4 specifies the model explicitly, analyzing the crisis period. This specification is less statistical but more economical in that the selection of the crisis period is more arbitrary. In this model, the dummy variable, $D(\cdot)$, is equal to one if the return overlaps with the COVID-19 crisis period starting on February 20, 2020, to March 23, 2020[10][11], and zero otherwise.

In the COVID-19 crisis period, if the c_0 is zero, NFTs are a weak hedge, while NFTs are a strong hedge if the c_0 is significantly negative. If the parameters in Eq. 3.4 are significantly (insignificantly) negative, NFTs are a strong (weak) safe haven in the COVID-19 crisis period.

3.3.2 Methods for a diversification effect

For the test of a diversification effect, firstly, we examine the diversification effect of NFTs on portfolio investing in traditional assets as a preliminary analysis by analyzing correlation, co-movement, and volatility transmission between the NFT asset class and the traditional asset class.

Pearson's product-moment pairwise correlation coefficients between returns on different assets are used. The correlation coefficients are obtained for all covered

[10] We use a period covering 20 trading days following Baur & Lucey (2010). The authors assume most of the crisis effects occur in the first 20 trading days (one month) after the start date.

[11] The economic approach is more arbitrary in selecting the specific periods of crisis compared to the statistical approach which uses a quantile of the return for a dummy variable threshold. Therefore, to show the reliability of our original results, we also conduct an analysis involving longer periods by incrementally advancing the start date until January 12, 2020, following Aharon & Demir (2021). The results are essentially similar to the original results. The full results are available upon request.

periods.

Next, we examine the co-movement in each asset by using the Gerber Statistic and the corresponding Gerber correlation matrix. The Gerber Statistic, proposed by Gerber et al. (2019, 2021), is a robust co-movement measure for portfolio construction that ignores extreme movements below a certain threshold while also limiting the effects of excessive variation. This is intended to deal with outliers and volatility peaks that can cause traditional correlation coefficients to be distorted. Several studies used the Gerber Statistic for measuring the co-movement of certain assets (Algieri et al., 2021; Zaremba et al., 2021b). Since NFT returns show excessive fluctuations, we calculate the Gerber Statistic for robust co-movement measurements.

Lastly, to quantify volatility transmission between NFTs and traditional assets, we employ the volatility spillover index based on the generalized variance decomposition methods from a vector autoregressive regression (VAR), proposed by Diebold & Yilmaz (2009, 2012, 2014) (hereafter, DY). The volatility spillover index is a general method for measuring the directional connectedness between asset return volatility. In addition to studies on the interconnectedness of traditional finance markets (Grobys, 2015; Symitsi & Chalvatzis, 2018), studies on the spillover between NFTs and cryptocurrencies (Dowling, 2021b) or among cryptocurrencies have been reported (Corbet et al., 2018; Koutmos, 2018).

We also use the enhanced framework of spillover index based on time-varying parameter vector autoregression (TVP-VAR) proposed by Antonakakis & Gabauer (2017) using a time-varying covariance structure of Primiceri (2005). The spillover index based on TVP-VAR methods mitigates the DY framework's shortcomings by providing robust results against outliers and avoiding the loss of valuable observations. As a result, this method is appropriate for low-frequency data and limited time-series data (Antonakakis et al., 2020). Moreover, many studies analyzed the connectedness between the traditional assets by using this TVP-VAR approach (Antonakakis et al., 2019a,b; Gabauer & Gupta, 2018), and Aharon & Demir (2021) examined the return spillover between returns for NFTs and traditional assets. In this paper, we calculate the volatility spillover index based on TVP-VAR for NFTs with small sample size. We briefly investigate two spillover indices between NFTs and traditional assets.

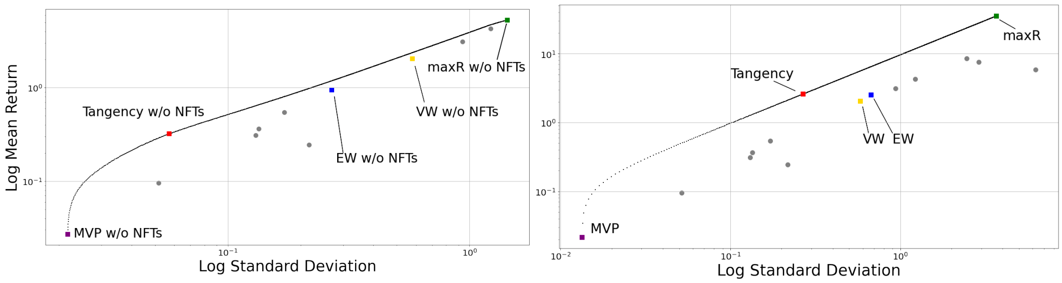
Secondly, the risk-return effect of each portfolio strategy was scrutinized using the portfolio selection via the traditional mean–variance framework proposed by Markowitz (1952). Expected return vector $\mu_{p,t}$ of portfolio p at time t and the covariance matrix $\Sigma_{p,t}$ of portfolio p at time t using h historical days are estimated. The portfolio is rebalanced monthly, and only long positions are allowed. Each portfolio is held for one month (for an out-of-sample test) using $\mu_{p,t}$ and $\Sigma_{p,t}$ estimated by 3-month historical data (for in-sample estimation). The out-of-sample test and in-sample estimation are conducted in a rolling window method for the entire period.

Table 3.3 shows an overview of each portfolio construction strategy. We construct five different portfolios, each for two different investment sets (whether NFTs are included in that portfolio or not). The five strategies consist of an equally weighted portfolio which is a $1/N$ naively diversified strategy (EW), a value-weighted portfolio which is a market capitalization-weighted strategy (VW), a tangency portfolio (tangency), a maximum return portfolio (maxR), and a minimum variance portfolio (MVP).

Table 3.3: Overview of each portfolio construction strategy

Strategy	Abbreviation	Description
Equal weighted	EW	Equally weighted portfolio of all asset
Value weighted	VW	Value-weighted portfolio of all asset
Maximum Sharpe ratio	Tangency	Portfolio that has the highest Sharpe ratio
Maximum return	maxR	Single asset portfolio that has the highest return
Minimum variance	MVP	Portfolio that has the lowest risk

Figure 3.2 exemplarily shows an in-sample risk-return profile of each strategy for the two possible sets of investments and efficient frontier for March 11, 2020. Note that the efficient frontier expands in a more efficient direction when NFTs are included in the portfolio when applied to in-sample.



(a) Portfolios without NFTs

(b) Portfolios with NFTs

Figure 3.2: In-sample risk-return profiles.

Notes. In-sample risk-return profiles are presented by log scale on March 11, 2020. Panel (a) shows an efficient frontier derived from traditional assets and five portfolio strategies investing in the corresponding investment set (traditional assets). Panel (b) shows an efficient frontier derived from traditional assets and NFTs and five portfolio strategies investing in corresponding investment sets (traditional assets and NFTs). Gray circles represent the risk-return profiles of individual assets. Blue, gold, red, purple, and green squares represent equal-weighted, value-weighted, tangency portfolio strategy, minimum variance, and max return, respectively.

3.4 Empirical results

3.4.1 Results of a hedge and safe haven effect

We use two econometric models to test a hedge and safe haven properties of NFTs for extreme market turmoils and the COVID-19 crisis, as introduced in Section 3.3.1. We present empirical results from these models for both time horizons (daily and weekly).

Table 3.4: Daily estimation results on the hedge and safe haven properties of NFTs

	Hedge	10% quantile	5% quantile	1% quantile
Stock indices (country coverage)				
US	−0.464***	−0.214	−0.425	−0.307
Canada	−0.386***	0.135	−0.404	−0.200
Australia	0.288***	0.369***	0.471***	1.009***
Japan	1.049***	1.137***	1.360***	2.230***
UK	−0.279***	−0.036	−0.169	−0.772
Germany	−0.224***	−0.282	−0.019	0.183
Switzerland	−0.363***	−0.256	−0.519***	−0.591
Italy	−0.489***	−0.392	−0.408	−0.433
Finland	−0.051***	0.016	−0.042	0.010
Netherlands	−0.455***	0.099*	−0.196	−1.001
Austria	−0.279***	−0.123	−0.320	−0.127
Belgium	−0.145***	0.028	0.020	−0.172
Spain	−0.343***	−0.096	−0.243	−0.177
China	−0.685***	−0.470***	−0.361***	−0.426
Russia	0.146	0.521	0.632	−0.283
India	0.393***	0.590***	0.680***	0.545
South Korea	0.996***	1.027***	1.165***	2.203***
Stock indices (regional coverage)				
World	−0.322***	−0.080	−0.231	−0.103
Emerging Market	0.394***	0.510***	0.491	1.454***
Commodity				
Commodity index	1.052***	1.145***	1.233***	1.382***
Gold	0.205***	0.406***	0.331***	0.197
Oil	−0.132***	−0.058	−0.219	−0.173
Cryptocurrency				
Bitcoin	0.220***	0.244***	0.255***	0.292***
Ethereum	0.077***	0.080	0.190***	0.048
Bond				
PIMCO index	−4.217***	−4.038***	−3.153	−2.688
Currency index				
USD index	−3.777***	−5.266***	−1.435*	−3.830

Notes. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.
Model: $b_t = c_0 + c_1 D(r_{other} q_{10}) + c_2 D(r_{other} q_5) + c_3 D(r_{other} q_1)$

Tables 3.4 and 3.5 show the daily and weekly estimation results of the regression model, respectively, presented in Eqs. 3.1, 3.2, and 3.3. The hedging effect is evaluated by estimates of c_0, and the total effects of the extreme market movement are estimated. Specifically, $\Sigma_{i=0}^{1} c_i$, $\Sigma_{i=0}^{2} c_i$, and $\Sigma_{i=0}^{3} c_i$ are calculated for the 10%, 5%, and 1% quantile. Hence, each table contains these estimates to investigate the relation between NFTs and the stock, commodity, cryptocurrency, bond, and US currency in normal and extreme conditions of the market.

In Table 3.4, for the country coverage stock markets, looking at the estimation results for the hedge coefficient, NFTs have a strong hedge property for the North American, European, and Chinese markets, while NFTs show no hedge effect for Pacific and Emerging Markets (Russia, India, and South Korea) on average. This trend is similar to the results of regional coverage stock indices, demonstrating that NFTs have a significant negative correlation with the World index and a significant positive correlation with the Emerging Market index. For the commodity markets, it can be seen that commodity index and gold are positively correlated with NFTs, implying that NFTs strongly co-move with these markets on average. Interestingly, however, NFTs have a strong hedge effect against the oil market, showing a significant negative hedging coefficient. Cryptocurrencies (Bitcoin and Ethereum) are positively correlated with NFTs on average. This result is hardly surprising, considering that NFTs are a secondary market derived from the cryptocurrency market (Dowling, 2021b). For the bond and currency markets, NFTs have a strong hedge benefit for the PIMCO and USD indices. We can see that the PIMCO index (-4.217) and the USD index (-3.777) offer a more negative hedging value compared to the stock markets, where the range is between -1 and 0, indicating that the hedging benefits of NFTs vary by asset classes [12].

Furthermore, NFTs are also a safe haven against the same markets. Specifically, for the North American, the European market, with the exception of Switzerland (5%

[12] We can see relatively large coefficients in the results of Tables 3.4 and 3.5. Several coefficients exceed one. This implies that a 1% change in the corresponding asset price leads to more than a 1% change in NFT price. This is hardly surprising, since the average return and risk of NFTs are extremely high compared to those of any other asset classes. Thus, beta coefficients are expected to be large.

quantile), World index, and oil, NFTs show a weak safe haven effect, while a strong haven effect is seen for Switzerland (5% quantile), China (10% and 5% quantiles), bond (10% quantile), and USD index (10% and 5% quantiles). These results suggest that the relationship between NFTs and asset returns is non-linear in the above cases. On the other hand, for the remaining markets, NFTs show positive coefficient estimates, implying the nonexistence of a safe haven effect.

Considering the above estimation results of hedge and safe haven together, we find that the reaction of investors to shocks varies across the asset classes. When huge shocks occur in the market, some investors in the North American, European, and Chinese markets and some investors who invest in World, oil, bond, and USD indices sell some of their assets and buy NFTs. On the other hand, under similar extreme market disturbance, this trend does not persist in the case of other markets, considering that these markets are positively correlated with NFTs. In other words, they may not consider NFTs as protection even though they may sell their assets during times of market turmoil.

Regarding the hedging role of NFTs, the pattern on a weekly basis in Table 3.5 is essentially similar to that on a daily basis, with the exception of the European markets. The result of weekly return shows strong hedge benefits between NFTs and the North American, and Chinese markets, bond, and USD index and weak hedge benefits between NFTs and World index and oil on average (only the hedging effect of NFTs for the European market vanishes compared to daily results). Contrastingly, the Pacific, European, and Emerging Markets, commodity index, gold, and cryptocurrency co-move strongly with the NFT market on average, showing a highly significant and positive coefficient.

Similar patterns are also found in the analysis for the weekly safe haven property of NFTs. NFTs act as a safe haven against the same markets. For Canada (5% quantile), bond (10%, 5%, and 1% quantiles), and USD index (5% quantile), NFTs show a strong safe haven property, while a weak safe haven effect is seen for the US, China, World, and oil. On the other hand, the Pacific and Emerging Markets, commodity index, gold, and cryptocurrency show positive coefficient estimates, which is also consistent with the result of daily returns.

Table 3.5: Weekly estimation results on the hedge and safe haven properties of NFTs

	Hedge	10% quantile	5% quantile	1% quantile
Stock indices (country coverage)				
US	−0.234**	0.093	−0.428	−0.488
Canada	−0.398**	0.759	−1.669**	−0.554
Australia	0.877***	0.718	0.592	0.964
Japan	0.985***	0.980***	−0.112**	1.206***
UK	0.243**	0.223	−1.578***	−0.149
Germany	0.807***	0.943***	0.494	1.226***
Switzerland	1.009***	0.254*	−0.330	−1.304
Italy	0.608***	0.811***	0.069**	0.709
Finland	0.774***	−0.266**	0.444	0.766
Netherlands	0.384***	0.566	−0.438	0.336
Austria	0.295***	0.253	−0.233*	0.411*
Belgium	0.882***	1.165***	0.617***	0.721
Spain	0.995***	1.483***	0.869***	1.336***
China	−0.435***	−0.396	−0.381	0.324
Russia	1.278***	1.342***	0.986***	0.410
India	1.896***	2.090***	2.544***	3.312***
South Korea	1.062***	0.866***	0.986***	1.223***
Stock indices (regional coverage)				
World	0.126	0.059	−0.019	−0.307
Emerging Market	0.862***	0.543	0.884***	1.443***
Commodity				
Commodity index	1.380***	1.822***	1.009***	0.035
Gold	0.115*	−0.012	−0.140	0.536
Oil	−0.005	−0.188	−0.305	−0.148
Cryptocurrency				
Bitcoin	0.633***	0.766***	0.920***	0.667***
Ethereum	0.487***	0.543***	0.603***	0.427***
Bond				
PIMCO index	−2.741***	−4.560***	−9.563***	2.705**
Currency index				
USD index	−5.996***	−4.737	−8.349***	−7.488

Notes. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.
Model: $b_t = c_0 + c_1 D(r_{other} q_{10}) + c_2 D(r_{other} q_5) + c_3 D(r_{other} q_1)$

Similar to the results of the hedge property, the phase of the safe haven property on a weekly basis of the European market differs from that on a daily basis. In the European markets, overall safe haven effects of NFTs in daily data fade when weekly data is used. Hence, only a few of the markets have negative estimates. Furthermore, the strength of the safe haven effect decreases in the case of other stock markets compared to the daily results, implying that the hedging and safe haven properties of NFTs differ across time horizons.

From these results, we find that the return frequency matters to the investors in the stock markets, especially the European markets. This finding implies that the view of investors in the stock markets, particularly the European markets, about the strength of NFTs' protection role against losses from market stress varies across two data frequencies. Two main plausible explanations for this tendency are as follows. Firstly, it is demonstrated that extreme fluctuations of NFT returns exist in daily data, as reported in the study of Dowling (2021b), revealing that this issue is mitigated by using weekly returns to some extent. As a result, the daily fluctuation of NFTs may affect the estimation of the hedge and safe haven properties of NFTs. Secondly, as revealed in the research of Umar et al. (2022), who conducted an empirical analysis to investigate the pairwise coherence between NFTs and traditional assets, risk absorption capacities of NFTs vary between short-run and long-run time horizons. The authors confirm that NFTs offer mainly short-run risk absorption capacity against other asset classes. This result is consistent with our findings in that the strength of a hedging and safe haven effect of NFTs against stock markets with daily data becomes weak on a weekly basis.

We summarize the above results as follows. NFTs are a hedge for the North

American, European (only on a daily basis), Chinese, World, oil, bond, and US currency markets. NFTs are also a safe haven for these markets. The strength of these effects varies across asset classes. NFTs do not act as a hedge or safe haven for the Pacific and Emerging markets, commodity index, gold, and cryptocurrency. The time horizon matters to investors in stock markets, especially the European markets. The hedge (safe haven) role of NFTs for the Europe market, as shown in the results on a daily basis data, vanishes (weakens) in the results on a weekly basis data. In line with this finding, the strength of hedge and safe haven roles of NFTs varies across the data frequency, revealing that the long-term strength decreases compared to the short-term strength.

Table 3.6 shows the daily and weekly estimation results from the model specified in Eq. 3.4. The results are similar to that in Tables 3.4 and 3.5. The daily hedge effects of NFTs are shown in the North American, European, Chinese, World, Oil, Bond, and the USD index. The total effect estimates for the crash during the COVID-19 crisis indicate a negative correlation between NFTs and these markets, except for Germany. Similar to the result in Tables 3.4 and 3.5, these hedge and safe haven properties weaken when we consider the weekly results in the stock markets. The overall hedge effects vanish while the overall safe haven effects decrease in the European markets, suggesting that the time horizon also matters to the investors during a crisis period.

The pronounced difference of these results compared to that in Tables 3.4 and 3.5 is that the strength of the safe haven effects of NFTs during the COVID-19 crisis is larger for bond (daily) and USD Index (daily and weekly), although that of NFTs is similar for any other assets, implying that NFTs can be a more effective refuge against the loss from bond and USD index than from other markets during the COVID-19 crisis compared to times of market turmoil.

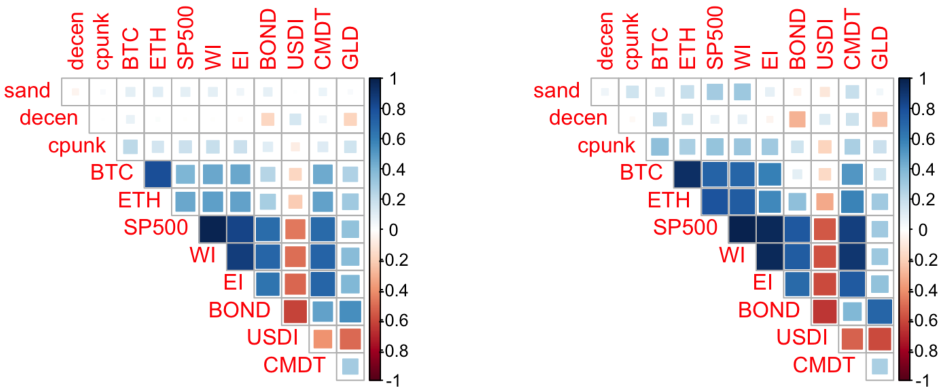
Table 3.6: Estimation results on the hedge and safe haven properties of NFTs during the COVID-19 crisis

	Daily		Weekly	
	Hedge	COVID-19 crisis	Hedge	COVID-19 crisis
Stock indices (country coverage)				
US	−0.455***	−0.305	−0.174	−1.570***
Canada	−0.324***	−1.230***	−0.257	−3.190***
Australia	0.284***	0.982***	0.902***	−0.097**
Japan	1.017***	2.448***	0.978***	0.436**
UK	−0.243***	−0.844***	0.234**	−1.278**
Germany	−0.231***	0.226**	0.805***	0.830***
Switzerland	−0.338***	−1.028***	1.016***	−1.785***
Italy	−0.476***	−0.616***	0.617***	0.226*
Finland	−0.045**	−0.087	0.739***	0.237
Netherlands	−0.401***	−0.920***	0.392***	−0.460**
Austria	−0.253***	−0.661***	0.403***	−1.126***
Belgium	−0.124***	−0.293	0.904***	0.365**
Spain	−0.314***	−0.557***	1.020***	0.989***
China	−0.669***	−0.499***	−0.439***	−0.034*
Russia	0.273***	−1.915***	1.321***	−0.354**
India	0.373***	1.358***	1.878***	3.572***
South Korea	0.942***	2.876***	1.064***	0.822***
Stock indices (regional coverage)				
World index	−0.312***	−0.166	0.164*	−1.117***
Emerging Market Index	0.362***	1.723***	0.857***	0.831***
Commodity				
Commodity index	1.065***	1.170***	1.434***	−0.634**
Gold	0.213***	0.418***	0.105*	−0.034
Oil	−0.116***	−0.521***	0.010	−0.952***
Cryptocurrency				
Bitcoin	0.223***	0.210***	0.649***	0.591***
Ethereum	0.091***	−0.192***	0.497***	0.350***
Bond				
PIMCO index	−3.904***	−9.132***	−3.188***	−1.230
Currency index				
USD index	−3.478***	−10.74***	−5.869***	−10.43***

Notes. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.
Model: $b_t = c_0 + c_1 D(\text{COVID-19 crisis})$

Our main findings in Section 3.4.1 are in line with the study of Aharon & Demir (2021). Aharon & Demir (2021) demonstrated in their study that the spillover effect of NFTs against traditional assets is affected largely by NFTs themselves, compared to other assets, which indicates the possibility of diversification, hedge, or safe haven effect of NFTs against these traditional asset classes. Consistent with this finding, we confirm that NFTs act as a protection against the loss of traditional asset markets. This protection capability of NFTs against losses in these markets on average or under market stress may be attributed to the demand for NFTs from investors who participate in corresponding markets during times of market turmoil and the COVID-19 crisis.

3.4.2 Results of a diversification effect



(a) Pearson's correlation matrix (b) The Gerber Statistic matrix

Figure 3.3: Pearson's correlation matrix and the Gerber Statistic matrix.

Figure 3.3 shows a Pearson's correlation matrix (Panel (a)) and the Gerber Statistic matrix (Panel (b)) between the return of each asset involved over the entire period. While correlations between traditional asset classes are high, correlations between NFTs and traditional assets are low, ranging between -0.2 and 0.2. This tendency is also confirmed by the Gerber Statistic matrix results. The small values of the Gerber

Statistic matrix between NFTs and traditional assets imply low co-movement levels between the two asset classes.

Panels A and B in Table 3.7 report the volatility spillover effects based on DY and TVP-VAR between NFTs and traditional assets in the entire period. The DY spillover index from and to NFT markets is lower than that from and to traditional markets, implying that NFTs are distinct from traditional asset classes in terms of volatility transmission. Furthermore, the spillover index based on TVP-VAR produces consistent results, although the spillover index of NFTs to others rises to the level of the US Dollar index or gold when compared to the DY spillover index.

Considering the results of the preliminary analysis, one may conclude that NFTs have the potential to provide diversification benefits as a new asset class. Hence, it suggests the possibility of achieving a well-diversified portfolio.

Table 3.7: Volatility spillover effects between NFTs and traditional assets

Panel A: DY Spillover index													
	sand	**decen**	**cpunk**	**BTC**	**ETH**	**SP500**	**WI**	**EI**	**BOND**	**USDI**	**CMDT**	**GLD**	**FROM**
sand	**75.3**	0.44	1.86	2.0	0.48	4.03	4.44	2.68	1.11	1.55	2.72	3.38	24.7
decen	2.07	**83.03**	2.29	0.67	4.72	1.09	0.89	1.58	0.67	0.67	1.37	0.96	16.97
cpunk	1.26	1.8	**75.4**	2.76	4.09	2.06	2.52	2.48	0.76	4.91	0.46	1.51	24.6
BTC	1.21	1.7	1.89	**39.86**	26.9	4.35	5.27	5.28	1.74	2.3	8.6	0.9	60.14
ETH	0.83	1.41	2.27	27.17	**47.83**	3.96	4.41	3.6	0.77	1.56	5.36	0.83	52.17
SP500	0.28	0.07	0.47	5.67	5.07	**28.43**	26.05	10.51	5.59	2.66	12.75	2.45	71.57
WI	0.32	0.11	0.56	6.28	5.17	26.94	**25.79**	11.1	5.5	3.01	12.83	2.38	74.21
EI	0.18	0.1	0.79	7.48	6.61	20.12	20.66	**19.72**	5.36	2.96	14.75	1.24	80.28
BOND	0.18	0.08	1.46	10.37	6.97	18.16	19.37	13.91	**11.51**	7.03	8.89	2.09	88.49
USDI	0.18	0.07	2.39	12.15	9.8	15.57	16.69	11.09	6.1	**13.62**	10.21	2.13	86.38
CMDT	1.3	0.5	0.32	2.55	2.71	16.73	14.0	7.11	1.8	4.89	**46.68**	1.41	53.32
GLD	0.65	1.32	1.67	1.73	1.69	6.0	6.21	7.01	4.73	5.34	2.91	**60.74**	39.26
TO	8.47	7.6	15.98	78.83	74.21	119.01	120.51	76.35	34.13	36.87	80.85	19.27	**Total**
In. own	83.78	90.63	91.37	118.69	122.04	147.44	146.3	96.07	45.64	50.49	127.53	80.02	**56**
Panel B: Spillover index based on TVP-VAR													
	sand	**decen**	**cpunk**	**BTC**	**ETH**	**SP500**	**WI**	**EI**	**BOND**	**USDI**	**CMDT**	**GLD**	**FROM**
sand	**81.52**	3.84	4.24	1.72	2.05	0.91	1.14	1.71	0.92	0.58	0.96	0.41	18.48
decen	4.73	**83.48**	2.95	1.25	3.03	1.26	0.89	0.49	0.28	0.39	0.83	0.43	16.52
cpunk	5.33	3.73	**65.46**	3.80	3.91	4.52	4.44	2.32	2.46	1.42	1.32	1.29	34.54
BTC	3.92	2.51	3.49	**43.24**	29.16	3.35	4.35	3.35	1.07	3.18	1.02	1.38	56.76
ETH	2.43	4.06	3.58	30.46	**46.92**	2.13	2.74	2.17	0.81	2.24	0.86	1.60	53.08
SP500	1.48	3.70	2.08	2.47	2.41	**24.78**	24.20	13.81	6.45	2.37	13.18	3.08	75.22
WI	1.73	3.29	2.06	2.81	2.53	23.35	**24.30**	14.46	6.86	2.43	13.05	3.13	75.70
EI	4.34	2.60	1.66	5.72	4.21	15.77	17.91	**24.08**	6.76	3.02	10.46	3.47	75.92
BOND	1.94	1.60	2.37	5.32	4.65	11.78	13.39	10.78	**25.88**	4.95	10.07	7.28	74.12
USDI	2.18	2.34	2.60	6.42	5.32	10.00	11.96	9.73	16.06	**20.82**	5.69	6.86	79.18
CMDT	2.13	3.51	1.94	4.99	4.39	15.76	16.42	10.37	5.34	1.59	**30.08**	3.49	69.92
GLD	2.21	2.97	1.67	4.30	4.19	7.55	8.55	5.64	9.75	4.20	8.52	**40.44**	59.56
TO	32.42	34.15	28.65	69.26	65.85	96.38	105.99	74.82	56.74	26.37	65.95	32.44	**Total**
In. own	113.94	117.62	94.11	112.50	112.77	121.16	130.28	98.90	82.62	47.19	96.03	72.88	**57.42**

We provide empirical results by constructing portfolios investing in individual NFTs and traditional assets using five different strategies. We attempt to demonstrate the impact of the diversification effect of NFTs as distinct individual assets on the portfolio in this experimental setting.

We evaluate each strategy's out-of-sample performance using the following criteria: (i) out-of-sample portfolio mean return and standard deviation, (ii) out-of-sample Sharpe ratio (SR), and (iii) certainty equivalent (CEQ) return.

CEQ return is the rate at which an investor with quadratic utility is willing to accept rather than invest in a particular risky portfolio. CEQ return is calculated as follows:

$$CEQ_p = E(R_p) - \frac{\gamma}{2} \cdot \sigma_p^2, \tag{3.5}$$

where γ is the risk aversion of investors. We use $\gamma = 1$ following the prior research of DeMiguel et al. (2009).

Table 3.8: Out-of-sample empirical results of each portfolio strategy using individual NFTs

	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel A: without NFTs						
EW	0.516	0.295	-2.246	10.83	1.749	0.472
VW	0.738	0.507	-1.297	4.555	1.454	0.609
Tangency	0.143	0.179	-0.881	6.839	0.798	0.127
maxR	1.657	1.06	-1.495	5.302	1.564	1.096
MVP	-0.012	0.084	-1.207	20.056	-0.144	-0.016
	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel B: with NFTs						
EW	1.369	0.644	-0.098	0.48	2.126***	1.162
VW	0.7384	0.508	-1.298	4.554	1.455**	0.61
Tangency	0.24	0.248	-0.96	5.043	0.969***	0.21
maxR	0.243	4.678	0.131	0.281	0.052*	-10.698
MVP	-0.0123	0.089	-1.19	19.746	-0.138***	-0.016

Notes. We apply Ledoit & Wolf (2008)'s test where the null hypothesis is no differences in performance in SR of each strategy with NFTs from that in strategy without NFTs. In the test, the robust statistical inference method is performed to test the null hypothesis. ***, **, and * mean significance at the 1%, 5%, and 10% level, respectively. The covered period is from December 4, 2019, to June 9, 2021, on a weekly basis.

In Table 3.8, Panels A and B report out-of-sample results of each portfolio strategy without and with NFTs as a new asset class on a weekly basis. The mean return of the maxR portfolio, 1.657, has the highest value in Panel A, while the MVP portfolio has the lowest risk, 0.084. The EW portfolio has the highest SR of all strategies, 1.749. The CEQ of the maxR portfolio without NFTs outperforms the CEQ of other strategies. In Panel B, the EW strategy outperforms the others in terms of mean return (1.369), and the MVP strategy outperforms the others in terms of risk (0.089). EW portfolio shows the highest SR, 2.126, compared to other strategies. CEQ of EW, 1.162, also exhibits the highest value.

Considering Panels A and B in Table 3.8 together, including NFTs in the investment basket dramatically reduces the maxR portfolio's SR and CEQ. The maxR strategy involves using a single asset that has the highest return compared to other assets in investment decisions without taking the level of risk into account, so choosing a NFT asset as a single investment asset[13]. Hence, this causes deterioration in risk-adjusted measures, SR and CEQ. This tendency is revealed in the result of the standard deviation of maxR strategy with NFTs, 4.678, which shows an extremely high value of risk.

MVP portfolio exhibits only a small performance improvement in SR and minor change in CEQ. MVP pursues the lowest risk investment strategy, resulting in the exclusion of NFTs, a highly volatile asset, from its portfolio. Similarly, the VW portfolio shows little improvement in SR and CEQ because the inclusion of NFTs in its portfolio is very small due to NFTs' low market capitalization relative to other traditional assets.

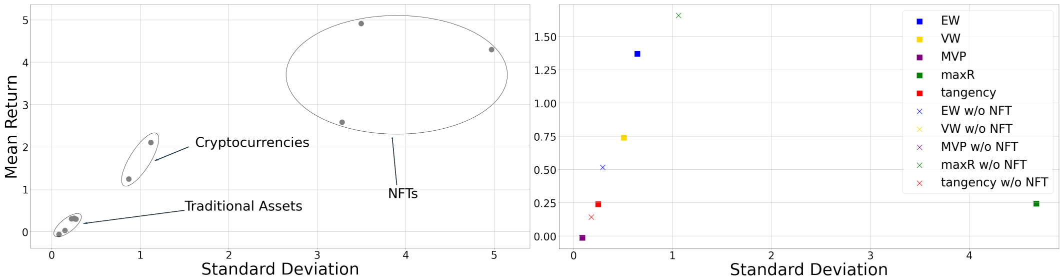
SR and CEQ of EW and tangency portfolio strategies with NFTs largely improve

[13]We checked that the maxR strategy selects one of the NFTs as a single investment asset at every rebalancing point.

compared to strategies without NFTs. The results show that the SR and CEQ of the EW portfolio and the tangency portfolio are improved by 0.377 and 0.171, respectively, and by 0.69 and 0.083, respectively, when NFTs are included. In terms of SR, we use the Ledoit & Wolf (2008) method to test whether the difference in performance between strategies is statistically significant or not, depending on the inclusion of NFTs. As shown in Panel B, the SR of all strategies significantly differs, implying that the inclusion of NFTs is highly beneficial to a given portfolio in terms of diversification.

Interestingly, irrespective of the inclusion of NFTs, the EW portfolio outperforms the rest of the strategies on a weekly basis. This is consistent with DeMiguel et al. (2009)'s research, which discusses a superior risk-return pattern of an EW strategy compared to optimized strategies, revealing that an inaccurate estimation of correlation increases the out-of-sample error of optimized strategy with limited observations. It is worth noting that including individual NFTs in the investment basket enhances the performance of the EW strategy to be greater than that of the tangency portfolio. We take this as a strong hint that excessive fluctuation of return on individual NFTs makes the correlation estimation for NFT-included portfolio unstable.

In Figure 3.4, Panels (a) and (b) display a risk-return perspective of every single asset and portfolio. As illustrated in Panel (a), NFTs, cryptocurrencies, and traditional assets form three clusters: high-risk, mid-risk, and low-risk. In Panel (a), NFTs gather in the northeast of the risk-return plot, while traditional assets gather in the southwest. Notice how the position of the squares in Panel (b) changes from X-shaped to square-shaped for each strategy. The addition of NFTs to a portfolio shifts it in the direction of higher risk and higher return. Except for the maxR portfolio, the slopes of other strategies increase when NFTs are included, which implies that the strategies are improved in a more efficient direction.



(a) Single assets (b) Portfolios

Figure 3.4: A risk-return perspective of every single asset.

Notes. These figures show a risk-return perspective of every single asset (Panel (a)) and portfolios (Panel (b)). In Panel (a), gray circles correspond to single individual assets. In Panel (b), squares and X-shaped represent each portfolio strategy with and without NFTs.

However, one can question whether the portfolio performance of our results is a consequence of the properties of selected NFTs. Therefore, to present the reliability of our results in Table 3.8, we use the aggregate NFT index as a representative of the entire NFT markets by averaging each NFT prices[14], similar to Aharon & Demir (2021). In this experimental setting, we can examine the impact of the entire NFT market's diversification benefit on the portfolio as a single index rather than individual NFTs. To avoid unreliable results due to excessive missing values, we drop the data during the early period, so the data sample is from March 5, 2018, to June 9, 2021. We have 5,239,211 total NFT trades.

[14] We used 165 different NFTs for calculating the aggregate NFT index at the time of writing.

Table 3.9: Out-of-sample empirical results of each portfolio strategy using aggregate NFT index

	Mean Ret.	Std.	Skew.	Kurto.	SR	CEQ Ret.
Panel A: without NFT						
EW	0.19	0.244	-1.609	9.051	0.781	0.161
VW	0.256	0.383	-1.096	6.331	0.668	0.183
Tangency	0.077	0.119	-1.053	18.163	0.647	0.07
maxR	0.628	0.813	-1.088	6.708	0.773	0.298
MVP	0.031	0.069	-2.032	43.696	0.454	0.029
	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel B: with NFT						
EW	0.454	0.462	-0.056	1.962	0.981***	0.347
VW	0.256	0.383	-1.096	6.33	0.669***	0.183
Tangency	0.115	0.141	0.54	12.194	0.815***	0.105
maxR	2.287	3.545	0.355	4.224	0.645*	-3.997
MVP	0.03	0.07	-2.024	44.031	0.428***	0.028

Notes. We apply Ledoit & Wolf (2008)'s test. The covered period is from March 5, 2018, to June 9, 2021, on a weekly basis.

In Table 3.9, Panels A and B report out-of-sample results of each portfolio strategy without and with an aggregate NFT index on a weekly basis[15][16][17]. When an aggregate NFT index is included in the investment basket of the portfolio, the SR of 0.781 and CEQ of 0.161 by EW strategy in Panel A increase to 0.981 and 0.347 in Panel B. A similar trend can be seen in the SR and CEQ of the tangency portfolio strategy. When an aggregate NFT index is included in the VW and MVP strategies,

[15] A preliminary analysis using an aggregate NFT index was also conducted. The results are essentially similar. The full results are available upon request.

[16] The issue of extreme fluctuation of daily returns on NFTs can be addressed to some extent by using the data of all trades in the entire NFT markets. As a result, portfolio analysis on a daily basis can be performed. The results are shown in Appendix Table A2. We discover that analysis yields similar results. For weekly and daily covered periods, the total number of observations for each asset is 164 and 818, respectively.

[17] Portfolio analysis only covers December 4, 2019, to June 9, 2021, since earlier data is not available. However, portfolio analysis using the aggregate NFT index covers March 5, 2018, to June 9, 2021, which includes the crisis caused by the COVID-19 pandemic. As a result, to test the consistency of our main argument, we presented the results of portfolio analysis using pre-COVID-19 periods and during COVID-19 periods in Appendix Tables A3 and A4. The results are essentially similar to those from full periods in Table A2, although the market conditions between the two periods are fundamentally different.

there is little change in SR and CEQ. The SR and CEQ of the maxR strategy in Panel B are lower than that in Panel A. Irrespective of the inclusion of NFT, the EW portfolio outperforms the rest of the strategies on a weekly basis, similar to Table 3.8. All the above results are consistent with the results of individual NFTs, confirming that our main results are robust and reliable.

Taking the results of Section 3.4.2 together, our empirical results suggest that the inclusion of NFTs as a new asset class in portfolios has a diversification effect, thus improving the diversified portfolio with improved risk-adjusted performance and efficiently augmenting investment opportunities for investors. We consider these findings to be evidence that the inclusion of NFTs improves a target portfolio and that the EW portfolio is the best choice for building an NFT-included portfolio on a weekly basis.

Chapter 4

Volatility forecasting for portfolio insurance strategy

4.1 Chapter overview

The portfolio insurance strategy is one of the most widely used dynamic asset allocation frameworks. It helps limit the downside risk by specifying a risk tolerance level for determining the portion of a risky asset (Bertrand & Prigent, 2001). In other words, it is a popular strategetic allocation model that aims to protect the risk and hedge the loss of underlying assets in market stress conditions. Due to its flexibility in implementation and hedging ability, this portfolio insurance strategy has globally attracted enormous attention among institutional and retail investors for a long time in the financial system (Dichtl & Drobetz, 2011). Many researchers have also obtained great interest; thus, various studies on portfolio insurance strategies have been reported. First of all, diverse forms of portfolio insurance strategies are proposed, including synthetic put portfolio insurance (Leland & Rubinstein, 1988; Rubinstein & Leland, 1981), stop-loss portfolio insurance (Bird et al., 1988; Rubinstein, 1985), constant proportion portfolio insurance (Black & Jones, 1987, 1988; Black & Perold, 1992; Perold & Sharpe, 1988), and time-invariant portfolio protection (Estep & Kritzman, 1988) strategies.

In another strand of literature, researchers investigated the performance evaluation that is adequate to the structure of portfolio insurance strategy to address the evaluation problem caused by its skewness, non-normality, and asymmetry (Annaert et al., 2009; Bertrand & Prigent, 2011; Leland, 1999; Zieling et al., 2014). Furthermore, other researchers have reported uncovering the rationale of preference of this strategy, taking account of investor's utility on portfolio choice in portfolio insurance strategy (Benninga & Blume, 1985; Dichtl & Drobetz, 2011; Gaspar & Silva, 2021). Based on these various portfolio insurance strategies and performance measures, empirical evidence of outperformance of portfolio insurance strategies in the global financial system has been reported (Agić-Šabeta, 2016; Agic-Sabeta, 2017; Annaert et al., 2009; Bertrand & Prigent, 2011; Dehghanpour & Esfahanipour, 2018; Dichtl & Drobetz, 2011; Dichtl et al., 2017; Garcia & Gould, 1987; Jiang et al., 2009; Lee et al., 2011; Zieling et al., 2014). These studies support its popularity among practitioners, corroborating the arguments of Bird et al. (1990). The authors mentioned that portfolio insurance could be considered a suitable alternative strategy for investors who want their portfolio value to be directly hedged by limiting the downward risk while benefiting from upward participation.

Despite its large popularity provided by numerous advantages, the portfolio insurance strategy, in particular, the synthetic put strategy, is widely known to have a critical problem in its implementation. As an original version of the option-based portfolio insurance (OBPI) strategy, a portfolio strategy whose position is protected by the put option is called *protective put* (Figlewski et al., 1993; Pozen, 1978). However, obtaining a protective put requires sufficient put options with liquidity, strikes, and maturity for the underlying asset in the market. To mitigate the limitation of the insufficiency of the put option, a novel portfolio insurance strategy that replicates the position of a protective put, the so-called synthetic put strategy, is introduced by Leland & Rubinstein (1988); Rubinstein & Leland (1981). This strategy creates the position of a protective put portfolio synthetically by dynamically adjusting the weight of risky and risk-free assets based on the Black-Scholes option pricing model. Even though this strategy offers flexibility to an investor, the dependency of the Black-Scholes model, which requires the estimation of input parameters, particularly volatility estimation, makes the insured portfolio unstable and, thus, implementation difficult in real-world market conditions. That is, the synthetic put strategy fails in creating a synthetically precise put option due to its estimation error of volatility because its estimation is a complicated problem in the actual market (Chu & Freund, 1996).

This estimation error issue in the synthetic put strategy has been addressed by a few researchers. Hill et al. (1988) examined the volatility estimation error problem and demonstrated that this causes a serious problem in terms of mispricing. Rendleman Jr & O'Brien (1990) investigated the effect of volatility misestimation on synthetic put strategy in terms of the total cost of misallocation effect as well as mispricing effect. They found that the misestimation of volatility can provide a substantial impact on the terminal payoffs of a synthetically insured portfolio. Zhu & Kavee (1988) studied the performance of synthetic put strategy in terms of protection level error provided by volatility estimation error, using the standard GBM simulation. They confirmed that sustaining the floor value to achieve the target protection level requires considerable cost in terms of volatility misestimation. In this study, we focus on the synthetic put strategy in terms of the impact of volatility estimation[1]. A large amount of studies on portfolio insurance strategy after the studies of Hill et al. (1988); Rendleman Jr & O'Brien (1990); Zhu & Kavee (1988) have been done and, most of these studies all refer to the adverse effects of this misestimation. However, no one reported more specific and advanced studies related to the effect of this volatility misestimation using more closer to real-world data rather than only using simple simulated data.

In the meantime, countless studies proposed novel volatility forecasting models which are widely used in the global financial market, such as GARCH-type (Bollerslev, 1986; Glosten et al., 1993; Nelson, 1991), HAR-RV-type (Andersen et al., 2007; Corsi, 2009), and machine learning-type (Bucci, 2020; D'Ecclesia & Clementi, 2021; Dunis & Huang, 2002; Khashanah & Shao, 2022; Kristjanpoller & Minutolo, 2015; Liu, 2019b, 2022; Roh, 2007; Santamaría-Bonfil et al., 2015; Sun & Yu, 2020; Xia et al., 2022) models until now. In the context of volatility forecasting literature, many researchers have justified conducting their research on volatility forecasting by explaining the importance of volatility estimation in option pricing and accurate hedge strategy implementation. To the best of our knowledge, however, there is no study investigating the impact of volatility forecasting as estimators for the input volatility parameter in synthetic put strategy using various existing forecasting models[2].

Especially among volatility forecasting models, machine learning models have elicited substantial attention from academia for time-series forecasting due to their

[1]Therefore, in this chapter, synthetic put strategy and portfolio insurance strategy will be used alternately with the same meaning.

[2]Even if there are studies on synthetic put strategy, most of the studies (Annaert et al., 2009; Dichtl & Drobetz, 2011) report only the results of synthetic put strategy with the naive volatility estimation (e.g., the standard deviation of asset return).

excellent predictability. Furthermore, without considering explicit formulation or assumption for underlying processes, these machine learning models are successfully applied to traditional financial tasks by implicitly training non-linear function form and extracting the complex pattern from data via learning procedure. Therefore, many researchers have conducted empirical studies comparing the traditional econometric models (GARCH-type and HAR-RV-type) and machine learning models in a variety of time-series tasks in a financial context, revealing the superiority of machine learning models. However, no one addresses these types of forecasting methods jointly in portfolio insurance literature.

Hill et al. (1988); Rendleman Jr & O'Brien (1990) did not address the protection level error directly. Rendleman Jr & O'Brien (1990) and Zhu & Kavee (1988) assumed constant volatility rather than variable volatility from the forecasting model. They also used only simulation data instead of real-world data. Zhu & Kavee (1988) only conducted simulation using the standard GBM rather than GBM with a jump model closer to realistic market assumption. Simply, the literature on the impact of volatility misestimation is limited. Motivated by these limitations, we want to address and uncover the scope of the shade. For this, we investigate three research questions as follows. First, is there any clear degradation in portfolio insurance strategy performance due to volatility misestimation even under more realistic conditions closer to the real world? Second, can a significant performance improvement be achieved in a synthetic put portfolio insurance strategy if accurate estimation is performed through various volatility forecasting models when using real-world data? Finally, how do the traditional time series forecasting models compare to machine learning methods, and what significant differences are there? Therefore, in this study, we investigate the comprehensive and extensive empirical study of synthetic put portfolio insurance strategies using various volatility forecasting models to fill our research gap.

Our findings in Monte Carlo simulation results using the standard GBM with and without jump phenomena indicate the existence and the effect of volatility misestimation on the protection accuracy of the synthetic put strategy. The impact of volatility estimation on the performance of portfolio insurance becomes more pronounced as market volatility increases and can be further strengthened by the jump phenomenon. These findings imply the importance of a more accurate volatility forecasting model. This issue caused by volatility estimation errors is also confirmed from the statistical point of view in real-world data. Interestingly, we find that the protection error of portfolio insurance positively correlates with the accuracy of the volatility forecasting model in terms of ranking correlation, implying a statistically direct link between them. We also find that other sophisticated volatility forecasting models are always superior to naive methodologies used as general conventions in portfolio insurance literature. In particular, machine learning model, especially XGB, consistently outperforms other traditional forecasting models regardless of market conditions. Overall, these results suggest that investors can employ a tailored investment strategy in implementing a synthetic put strategy based on our findings on the performance of forecasting models. From our findings, we attempt to shed light on the existence, impact, and improvement of volatility estimation error on the synthetic put strategy in terms of empirical results of various volatility forecasting models, giving investors valuable alternatives to a naive way.

Chapter 4 is organized as follows: Section 4.2 describes the data. Section 4.3

introduces the details of the synthetic put portfolio insurance strategy and related performance measures. Section 4.4 explains the concept of various types of volatility forecasting models. Section 4.5 presents the experimental design and procedure. Section 4.6 discusses the empirical results of portfolio insurance strategies in terms of volatility estimation error and forecasting based on GBM and real data.

4.2 Data

4.2.1 The Monte Carlo simulation data

In this study, we analyze by using Monte Carlo simulations. For this, we utilize two different simulations under two assumptions about the stock price process; standard geometric Brownian motion (GBM) and GBM with jump process.

In the standard GBM setup, the log return of risky asset follows a Brownian motion with linear drift, and constant parameters (μ and σ) are assumed. This setup postulates unrealistic stock markets, neglecting the effect of autocorrelation, skewness, and heavy tails, which are observed in actual financial data. Despite this, we can confirm the impact of precise volatility estimation on the performance of synthetic put strategy by using *true* volatility that we can explicitly pre-specify and thus be known in advance. We assume that the price of the risky asset (S_t) follows a diffusion process following a stochastic differential equation is:

$$dS_t = S_t(\mu d_t + \sigma dW_t) \tag{4.1}$$

where μ is the expected return, σ is the volatility, W_t is a standard Brownian motion, and dW_t is a Wiener process. The period is [0, T]. The solution of Eq. 4.1 can be obtained as:

$$S_t = S_0 e^{(\mu - \frac{1}{2}\sigma^2)(T-t)+\sigma W_t}. \tag{4.2}$$

Dimson et al. (2006) estimated the excess return to be approximately 4.5% per year in their study on the developed stock markets covering periods of more than one hundred years. Furthermore, the authors argued that the long-run return volatility in the stock market is roughly 20% per year[3]. Following these findings, by adding a 4.5% risk-free rate to the equity risk premium, in their portfolio insurance simulation study, Dichtl & Drobetz (2011) obtained the expected annual return on the stock market as 9%. Then, they use 9% expected annual return and 20% annual volatility as values of their pre-specified parameters of GBM simulations. Following Dichtl & Drobetz (2011), we use this 9% as the input parameter for our simulation as the true market expected returns[4]. In terms of volatility, we analyze the influence of volatility estimation versus true volatility in a systemic way by using four different economic scenarios; 10%, 20%, 30%, and 40%. We set these values as our *true* volatility. Hence, the corresponding parameters in our numerical example we use are $T = 1$, $r = 4.5\%$, $S_0 = 1$ $\mu = 9\%$, and $\sigma = 10\%, 20\%, 30\%$, and 40%.

On the contrary, in GBM with a jump process setup, the log return of risky asset follows a Levy process with double-exponential jumps. This model, introduced by Kou & Wang (2003, 2004), incorporates two independent components; a diffusion component and a jump component. A diffusion component corresponds to that of standard GBM. In contrast, a jump component involves the jump times and sizes.

[3]Benninga (1990); Figlewski et al. (1993) also use 20% as a value of volatility in their portfolio insurance studies (Dichtl & Drobetz, 2011).

[4]We also conducted a simulation using 11.5% as expected return in the high-risk premium state as used in Dichtl & Drobetz (2011). The results are essentially similar to the results in the 9% scenario. To maintain our main argument clearly, we omit to report these results. The full results are available upon request.

The jump times follow exponential distribution (Poisson process), and the jump sizes follow double-exponential distribution (Bertrand & Prigent, 2011). The price of the risky asset (S_t) following the stochastic differential equation is assumed:

$$dS_t = S_t(\mu d_t + \sigma dW_t + d(\Sigma_{i=1}^{N_t} \frac{\Delta S_{T_n}}{S_{T_n}})) \tag{4.3}$$

where N_t is a Poisson process with intensity $\lambda > 0$ and $\frac{\Delta S_{T_n}}{S_{T_n}} \in (-1, \infty)$ denotes relative jumps which are i.i.d. The double-exponentially distributed random variable $Z_n = \log(1 + \frac{\Delta S_{T_n}}{S_{T_n}})$ has probability density function as follows:

$$f_Z(z) = p_z \cdot \eta_1 e^{-\eta_1 z \mathbb{1}_{\{z \geq 0\}}} + q_z \cdot \eta_2 e^{\eta_2 z \mathbb{1}_{\{z<0\}}}, \ \eta_1 > 1, \ \eta_2 > 0, \tag{4.4}$$

where p_z and q_z ($p_z + q_z = 1$ and $p_z, q_z \geq 0$) represent the probabilities of moving upside and downside, respectively. In other words, $z_n = \xi^+$ with probability p_z and $z_n = \xi^-$ with probability q_z, where ξ^+, ξ^- are exponentially distributed with means $1/\eta_1, 1/\eta_2$, respectively. The solution of Eq. 4.3 can be obtained as:

$$S_t = S_0 e^{(\mu - \frac{1}{2}\sigma^2)(T-t) + \sigma W_t + \Sigma_{i=1}^{N_t} \log(1 + \frac{\Delta S_{T_n}}{S_{T_n}})}. \tag{4.5}$$

where W_t, N_t, and Z_n are independent. Similar to the standard GBM setup, we four different volatility scenarios. The numerical example parameters we use are $T = 1$, $r = 4.5\%$, $S_0 = 1$ $\mu = 9\%$, $\sigma = 10\%, 20\%, 30\%$, and 40%, $\eta_1 = 182.08$, $\eta_2 = 172.86$, $\lambda = 1.4615$, $p_z = 0.496$, and $q_z = 0.504$[5].

[5]We use the parameters $\eta_1 = 182.08$, $\eta_2 = 172.86$, $\lambda = 1.4615$, $p_z = 0.496$, and $q_z = 0.504$, following the research of Ramezani & Zeng (2007) on the estimations of the S&P 500 Composite index during past 40 years.

4.2.2 The real-world data

As for our real-world data simulation, we use the S&P 500 index as our base data covering the period of 4 January 2000 to 19 May 2022. We obtain our data from the Oxford-Man Institute's Quantitative Finance Realized Library[6]. A total of 5,610 daily observations are included. We use the 3-month Treasury Bill (T-bill) Rate as a proxy for the risk-free rates.

Unlike the GBM simulation, where we know about the true return volatility, in actual financial data, the volatility measures are unobservable; thus, true volatility is unknown. Therefore, it is difficult to evaluate the performance of volatility estimation or forecasting tasks. A conventional solution to this issue is to set an appropriate proxy for the true volatility and conduct the performance evaluation by comparing estimated or forecasted volatility to this risk proxy. As a widely used alternative to dealing with this issue, many related studies (Bentes, 2015; Byun & Cho, 2013; Liu et al., 2019, 2020a; Mittnik et al., 2015; Wei, 2012) use realized variance as a risk proxy of actual volatility from a statistical point of view (Andersen & Bollerslev, 1998a).

Realized variance, proposed by Andersen & Bollerslev (1998b), is a widely used effective proxy for a variance of an asset return. Realized variance at a specific day t ($RV_t^{'}$) is defined as the sum of squared intra-day high-frequency return data shown as follows:

$$RV_t^{'} = \sum_{j=1}^{M} r_{t,j}^2, \ (t = 1, ..., T), \tag{4.6}$$

where M is the number of high-frequency observations and $r_{i,j}$ denotes the j-th intraday return of t-th day. As reported by various related studies, this realized

[6]https://realized.oxford-man.ox.ac.uk/data

variance is widely known to be a better measure of the ex-post variance than the squared returns, which is a poor approximation of actual volatility (Andersen & Bollerslev, 1997; Andersen et al., 2001, 2003). We use the closing price of real data at 5-minute intervals[7] to obtain log returns[8]. Consequently, we calculate the daily realized variance by summing obtained squared log return.

The limitation involved in $RV_t^{'}$ in Eq. 4.6 is that this measure fails to consider the effect of overnight returns. To address this limitation, Hansen & Lunde (2005) proposed to use the scale parameter to capture this overnight effect as follows:

$$RV_t = c \cdot RV_t^{'}, \tag{4.7}$$

where the scale parameter c is given by:

$$c = \frac{\Sigma_t^N r_t^2/N}{\Sigma_t^N RV_t/N}. \tag{4.8}$$

Hansen & Lunde (2005) mention that this scaled RV is an approximately unbiased estimator, and many studies also apply a similar scale parameter to obtain a measure of variance accounting for the whole day (Martens, 2002; Fleming et al., 2003).

Figure 4.1 shows the return and realized volatility of the S&P 500 index daily for the covered period. As illustrated in Figure 4.1, the most volatile moments of the Dot-com bubble, the Global Financial Crisis, and the COVID-19 crisis can be found

[7] Numerous studies have reported that the selection of 5-minutes as sampling frequency is a rule of thumb (Corsi et al., 2010; Gong & Lin, 2017; Liu et al., 2015, 2017; Sévi, 2014). That is, this is the best parameter value to consider the trade-off between evaluation accuracy and noise of microstructure. Consequently, we use this sampling frequency of realized variance in our study.

[8] In the financial context, log return is widely used in volatility forecasting studies (Bentes, 2015; Byun & Cho, 2013; Liu et al., 2020a; Wei, 2012). In the vein of portfolio insurance strategy, although several studies use the simple return (Annaert et al., 2009; Dichtl & Drobetz, 2011; Zieling et al., 2014), most other studies (Bertrand & Prigent, 2011; Buccioli & Kokholm, 2018; Dichtl et al., 2017; Jiang et al., 2009; Tawil, 2018) use log return for their empirical analysis. Hence, we use log return in our empirical study.

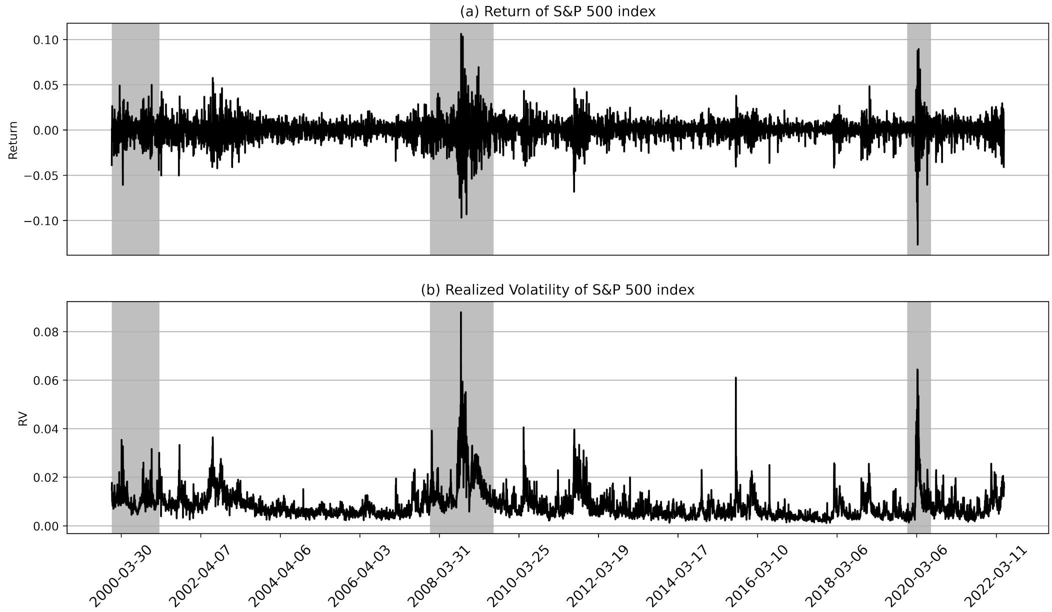


Figure 4.1: Time series of return and realized volatility of S&P 500 index.

Notes. This figure shows the time series of return and realized volatility of the S&P 500 index on a daily basis for the period 4 January 2000 to 19 May 2022.

Table 4.1: Summary statistics of the S&P 500 index

	Avg	Std.	Skew	Kurto	ADF	Q(1)	Q(5)	Q(10)	ARCH(1)	ARCH(3)	J-B
Return	0.044	0.197	-0.39	10.4	-13.8^{***}	0.0	5.2	27.3^{***}	593.4^{***}	1375.2^{***}	25221.0^{***}
Realized volatility	0.159	0.115	3.33	19.5	-8.4^{***}	2.5	43.4^{***}	83.5^{***}	2696.4^{***}	3163.8^{***}	99290.0^{***}

Notes. We apply the Augmented Dickey-Fuller(ADF) test (Cheung & Lai, 1995). ADF statistics show that the null hypothesis of a unit root can be rejected for return and realized volatility. Ljung-Box tests up to leg ten are conducted to detect serial correlation. $Q(\cdot)$ denotes the test statistics of the Ljung-Box test. Engle (1982)'s Lagrange multiplier (LM) test detects heteroskedasticity up to lag 3. Values in columns $ARCH(1)$ and $ARCH(3)$ denote the values of LM statistics. The J-B columns denote the test statistic of the Jarque-Bera test, where the null hypothesis is that return distribution is normally distributed. *, **, and *** mean significance at the 10%, 5%, and 1% level, respectively.

during these periods as marked by the grey shaded area. Table 4.1 summarizes the statistics of the annualized return and realized volatility for S&P 500 index over the covered period. In Table 4.1, we report the values of average return, standard deviation, skewness, and kurtosis. While the average return shows 4.4% per year, the average realized volatility is 15.9% per year. As widely reported by many studies on the stock market, the return of the S&P 500 index shows negative skewness, whereas realized volatility shows positive skewness. Furthermore, it is shown that there might be a fat-tail in return distributions of the S&P 500 index, taking into account that the kurtosis shows greater value[9].

We applied the ADF test (Cheung & Lai, 1995) to the return and realized volatility, either. The results show that the null hypothesis of a unit root can be rejected at a 1% significance level. Hence, we confirm that returns and realized volatility of the S&P 500 index are stationary. Furthermore, a Ljung-Box test up to leg ten is performed to detect the existence of autocorrelations. The results of $Q(1)$, $Q(5)$, and $Q(10)$ in Table 4.1 demonstrate that there are autocorrelations in both returns and realized volatility. The return has the autocorrelation only at a higher lag (10). On the other hand, realized volatility shows the autocorrelation at a higher and relatively lower lag (5 and 10). Moreover, we conducted Engle (1982)'s Lagrange multiplier test to detect heteroskedasticity. The result implies statistically significant heteroskedasticity effects in returns and realized volatility up to lag 3. Finally, the Jarque-Bera (J-B) test statistics are presented. These J-B results confirm that the null hypothesis of a normal distribution must be rejected in returns and realized volatility of the S&P 500 index.

4.3 Portfolio insurance strategy

4.3.1 Synthetic put strategy

A synthetic put strategy has known to be prominent among all the portfolio insurance strategies. Leland & Rubinstein (1988); Rubinstein & Leland (1981) proposed this strategy aiming to replicate the protective put strategy, which is an option-based portfolio insurance strategy (Figlewski et al., 1993). One of the most significant

[9] If a return is normally distributed, the corresponding kurtosis should be three.

weaknesses of the original protective put strategy is that it requires a put option that is obtainable in the market and has adequate liquidity, strike price, and maturity. Without the presence of an appropriate put option, the protective put strategy becomes unavailable. However, the synthetic put strategy synthetically creates an effect of a put option with a risky and risk-free asset as insurance against the risky portion of the portfolio. It makes the insurance scheme more flexible and highly executable in various conditions.

In order to replicate the effect of a put option, the synthetic put strategy simultaneously purchases both risky and risk-free assets to create a long position in the underlying asset S_t plus European put option P_t with the maturity of T and strike price K. With the Black-Scholes option pricing formula proposed by Black & Scholes (1973), the value of a put option P_t can be derived as follows:

$$P_t = Ke^{-r(T-t)}N(-d_2) - S_tN(-d_1), \tag{4.9}$$

where $N(x)$ is the standard normal cumulative distribution function. d_1 and d_2 are defined as follows:

$$d_1 = \frac{ln(S_t/K) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \tag{4.10}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{4.11}$$

In Eq. 4.10, the true volatility of the underlying asset denoted by σ is used to calculate d_1 and d_2 values. The problem is that the exact value of σ is unknown. In order to utilize the Black-Scholes formula, we should correctly estimate the true volatility σ otherwise, the whole calculation would be biased. As long as we use the Black-Scholes formula to calculate a put option value in a synthetic put strategy, the strategy's performance would be inevitably highly dependent on the performance of volatility estimation (Rendleman Jr & O'Brien, 1990). Our main focus in this study is to show the importance of precise volatility estimation with empirical results on synthetic put strategy using various forecasting methods.

By using Eq. 4.9, the portfolio value with a synthetic put strategy can be expressed as follows:

$$S_t + P_t = S_t N(d_1) + Ke^{-r(T-t)} N(-d_2). \tag{4.12}$$

In order to implement a synthetic put strategy and make the portfolio value equal to Eq. 4.12, the concept of delta must be introduced. The delta is defined as the price change of a derivative given a unit change in the price of an underlying asset. This definition implies that the delta value can be calculated as a partial derivative of the option value with respect to the underlying asset value (Hull & White, 2017). Accordingly, from Eq. 4.12, the delta of the portfolio in a synthetic put strategy can be calculated as follows:

$$\frac{\partial(S_t + P_t)}{\partial S_t} = N(d_1). \tag{4.13}$$

In Eq. 4.13, the delta implies the amount of the underlying asset to be purchased to replicate the protective put strategy. In order to make a position's delta in a synthetic put strategy identical to the one in a protective put strategy, a certain amount of position in the underlying asset should be maintained. It means that the investor must hold a proportion $N(d_1)$ of the underlying asset at any given time. It could be done by rebalancing the portfolio by selling or buying the underlying asset or the risk-free asset to make a proportion of a risky asset to $N(d_1)$. In other words, with the strike price and maturity of a put option, we can calculate $N(d_1)$

value at any time and decide the portion of the risky asset in the portfolio value in a synthetic put strategy. It emphasizes the importance of precise estimation of σ, which is directly used in calculating the value of $N(d_1)$.

In our study, the strike price K of a put option with the maturity of T is decided as follows:

$$K = a \cdot S_0, \tag{4.14}$$

where a is the predetermined percentage floor, and S_0 is the initial price of the underlying risky asset.

In the original Black & Scholes (1973) framework, transaction costs are not considered. However, transaction costs are not negligible in a practical manner since the more frequently an investor rebalances his asset, the more cost occurs by his decision, which leads to an increase in total cost. In this study, we decide to include transaction costs for implementing a synthetic put strategy to make our study more realistic. Following the modified volatility estimation in the synthetic put strategy proposed by Leland (1985), the estimated volatility in our setting can be calculated as follows:

$$\sigma_{Leland} = \sigma\sqrt{1 + \sqrt{\frac{2}{\pi}} \cdot \frac{k}{\sigma\sqrt{\Delta t}}} \tag{4.15}$$

where k denotes the round-trip transaction costs, and Δt is the time length of the rebalancing interval[10].

[10]Boyle & Vorst (1992)'s model is also known to be an alternative to adjusted volatility considering transaction cost. This model takes $\sigma_{Boyle/Vorst} = \sigma\sqrt{1 + 2 \cdot \frac{k}{\sigma\sqrt{\Delta t}}}$ as adjusted volatility. Dichtl & Drobetz (2011) used this estimation model in their study on portfolio insurance strategy, testing performance in Monte Carlo, and real-world data simulation. We also applied this model in this study, revealing there are no different results qualitatively.

4.3.2 Protection level error

The synthetic put portfolio insurance strategy aims to directly limit the downward market movement while retaining the potential of upward gains rather than guarantee a higher level of return or risk-adjusted return. Even if volatility estimation in implementing portfolio insurance strategy is performed well, no one guarantees their portfolio has higher performance measures traditionally used, such as the Sharpe ratio. It is trivial, considering that this kind of performance measure does not directly and logically link to the accuracy of volatility estimation in portfolio insurance strategy. In this line, in measuring the impact of volatility estimation accuracy on the performance of portfolio insurance strategy, seeking explicit metrics to measure the degree of capital damage protection beyond a given tolerance level is reasonable. Hence, to address this aim, we use protection level error, which is appropriate to evaluate the protection capability of a portfolio insurance strategy.

Suppose investors have no error in estimating parameters of market condition (e.g., *true* volatility of stock market). In that case, they can replicate and obtain the synthetic put with the protection value that is equal to that of a *true* protective put (risky asset hedged by a *true* put option). Consequently, a portfolio insurance strategy investor's portfolio value cannot fall below that put protection value. However, the *true* volatility is unknown in the real world; thus, investors cannot but resort to estimating the proxy of market volatility (Bird et al., 1990; Zhu & Kavee, 1988). As a result, the synthetic put portfolio insurance strategy fails in maintaining the target floor level due to unavoidable errors in estimating market parameters. That is, the minimum value of a synthetic put portfolio goes below the level of the *true* protective put's target protection value (TPV) which is defined as follows:

$$TPV = \frac{S_0}{S_0 + P_0} \tag{4.16}$$

where S_0 is the initial price of a risky asset, P_0 is the initial price of the put option, which is synthetically replicated. That is, $S_0 + P_0$ indicates the initial price of the portfolio, which synthetically replicates (*true*) protective put strategy. By plugging Eq. 4.12 into Eq. 4.16, we obtain another form reflecting the Black-Scholes model assumption as follows:

$$TPV = \frac{S_0}{S_0 N(d_1) + K e^{-rT} N(-d_2)}. \tag{4.17}$$

To address the estimation error problem measurement, Zhu & Kavee (1988) introduced protection level error (PLE). The authors defined it as the ratio of the difference between TPV and the sample minimum value (SMV) to the TPV as follows:

$$PLE = \frac{\max\{TPV - SMV, 0\}}{TPV}, \tag{4.18}$$

where $SMV = \min_{t \leq T}\{V_t : t \in \{1, ..., T\}\}$ is the sample minimum value and V_t is the portfolio value at time t.

The upper part in Figure 4.2 exemplarily illustrates the price of the underlying risky asset, the value of the synthetic put portfolio insurance strategy, and their TPV, SMV, and PLE. The lower part in Figure 4.2 presents the portfolio weights between the risky and risk-free assets over time. For simplicity, the underlying asset's price is divided by the initial value. Hence, our portfolio value is scaled and starts as a 1$ value. The underlying risky asset's price drops sharply at inception. Then, it is bound to the upside after reaching the local minimum value at the mid-time point, showing extreme market turbulence in all interim time horizons. The value of a synthetic put portfolio, on the other hand, limits the additional capital loss at some point and maintains this protected position until maturity, even though it also slightly drops down like the value of the risky asset at the inception period. Obviously, the synthetic put portfolio insurance strategy makes the weight of a risky asset in the portfolio decrease as the price of the underlying risky asset goes down, shifting the budget to the risk-free asset, thereby resulting in the protection of our portfolio value to some extent as illustrated in the lower part in Figure 4.2.

However, the insured portfolio suffers from deteriorated performance caused by estimation error of market volatility. The example in Figure 4.2 is under assumption that $S_0 = 1$, $K = S_0$, $r = 4.5\%$, $T = 1$, and $\sigma = 19.6(\%)$. In other words, we create a synthetic put portfolio with a 100% protection level[11] and 1-year duration under the market condition that volatility is 19.6% and the initial price of the underlying risky asset is 1$. Under this condition, we can obtain the price of the put option in Eq. 4.9 as 0.056. Thus, we can calculate TPV as $S_0/(S_0+P_0) = 1.0/(1.0+0.056) = 0.947$. If an investor can know true volatility exactly under frictionless market and continuous trading, they can obtain a synthetic put portfolio where their portfolio value is not below this TPV. However, our portfolio insurance strategy is implemented using incorrectly estimated volatility (23.9%); thus, the SMV (0.876) of our portfolio is below the TPV (0.947). As a result, our portfolio insurance seems to fail to achieve the objective of sustaining the target floor value, showing the protection level error $((0.947 - 0.876)/0.947 = 7.49(\%))$.

[11] In this case, the strike price (K) equals a 100% of the initial price (S_0).

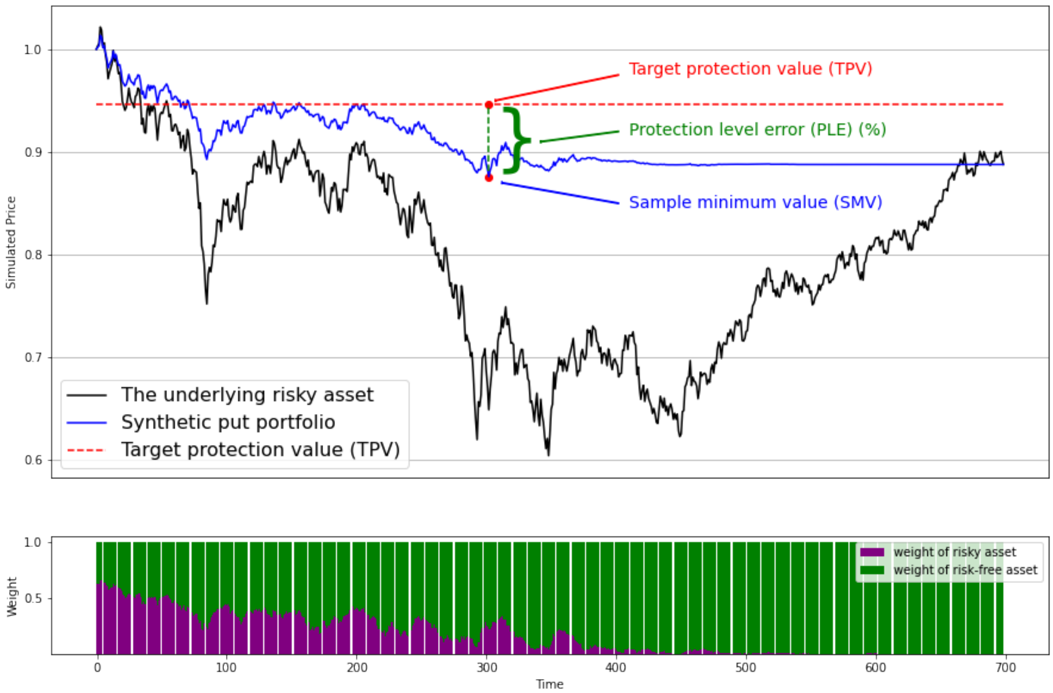


Figure 4.2: Example of protection level error.

Notes. This figure shows an example of the price of an underlying risky asset and the value of a synthetic put portfolio. The red dotted line, solid blue line, and the solid black line represent target protection value, the value of the portfolio obtained by synthetic put strategy, and the price of the underlying risky asset, respectively. The red point on the blue dotted line denotes the sample minimum value, and the green dotted line refers to protection level error. Note that we refer to protection level error (green dotted line) as the ratio of the difference between the target protection value and the sample minimum value to the target protection value, rather than the difference only.

4.4 Volatility forecasting models

4.4.1 Naive model

In order to implement a synthetic put strategy, an investor must prespecify the volatility of an underlying risky asset. One of the most widely used textbook approaches is to use volatility estimated by calculating standard deviations of historical returns, as shown in Eq. 4.19.

$$\hat{\sigma_t}^2 = \frac{\Sigma_{i=t-N}^{t-1}(r_i - \bar{r})^2}{N} \tag{4.19}$$

This method is the convention in the portfolio insurance context, as shown in a significant body of portfolio insurance studies (Annaert et al., 2009; Dichtl & Drobetz, 2011; Dichtl et al., 2017). Therefore, we use the rolling window-based standard deviation of 252 daily returns for the volatility estimate as our benchmark naive forecasting model for our comparative study.

4.4.2 GARCH-type models

In order to address the drawback of the ARCH model proposed by Engle (1982) about lack of flexibility, the standard GARCH model was introduced by Bollerslev (1986) by incorporating a moving average component together with the autoregressive component. The standard GARCH(p,q) jointly estimates conditional mean and conditional variance. This model can be described as follows:

$$r_t = \mu + \epsilon_t, \tag{4.20}$$

$$\epsilon_t|\psi_{t-1} \sim N(0, \sigma_t), \tag{4.21}$$

$$\hat{\sigma_t}^2 = \omega + \Sigma_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \Sigma_{i=1}^{p}\beta_i\sigma_{t-i}^2, \tag{4.22}$$

where ψ_t is the set of information available at time t, p is the order of moving average (ARCH) term, q is the order of autoregressive (GARCH) term, μ, ω, α_i, and β_i are constant parameters, $\omega > 0$, $\alpha_1, \beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$ for restrictions to avoid negative variance. The conditional variance at current time t ($\hat{\sigma_t}^2$) is estimated by the function of constant (ω) and weighted previous squared shocks (ϵ_{t-i}^2) and variances (σ_{t-i}^2) term. In the financial context, the GARCH has been the most widely used model to describe stock return volatility due to the main advantage of the GARCH model; fewer parameters required and better performance compared to the ARCH model.

However, despite this parsimonious, a standard GARCH model fails to capture several features of financial time series. One of the most representative features is the asymmetry of return series (the so-called leverage effect), as reported in many studies on the stock market (Chou & Kroner, 1992; Engle & Ng, 1993; Pagan & Schwert, 1990). These researchers have observed that negative shock appears to make volatility to be more increased compared to positive shock in the financial time series. The problem is that the traditional GARCH model fails to explain this leverage effect. To address this issue, the Exponential GARCH (EGARCH) model, the modified version of the GARCH model, is proposed (Nelson, 1991). By making positive and negative shocks have a different impact on volatility, the author mitigates the limitation of the GARCH model. This EGARCH(p,q) model can be described as follows:

$$\log \hat{\sigma_t}^2 = \omega + \Sigma_{i=1}^{q} \alpha_i g(\epsilon_{t-i}) + \Sigma_{i=1}^{p} \beta_i \log \sigma_{t-i}^2, \tag{4.23}$$

$$g(z_t) = \theta z_t + \gamma(|z_t| - E(|z_t|)),\ z_t = \epsilon_t / \sqrt{\sigma_t^2}, \tag{4.24}$$

where θ and γ are the constant parameters, γ shows the asymmetric effect, and there are no sign restrictions on the parameters, unlike the GARCH model.

Another extension addressing the asymmetric effect is the so-called GJR-GARCH model, which is introduced by Glosten et al. (1993). GJR-GARCH model explicitly reflects asymmetry in obtaining a conditional variance, using a dummy variable whose value differs depending on the sign of previous shocks. GJR-GARCH model can be written as follows:

$$\hat{\sigma_t}^2 = \omega + \Sigma_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \Sigma_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \Sigma_{i=1}^{q} \gamma_i I_{t-i} \epsilon_{t-i}^2, \tag{4.25}$$

where dummy variable I_{t-i} is one if $\epsilon_{t-i} < 0$, and zero otherwise.

4.4.3 HAR-RV-type models

For forecasting stock market volatility, various time-series auto-regressive models are introduced. Among them, the heterogeneous autoregressive model of realized volatility (HAR-RV) proposed by Corsi (2009), based on the Heterogeneous Market Hypothesis of Müller et al. (1997), is known to show the best performance for forecasting realized volatility. The author incorporates different dynamics in volatility structure within different time horizons into the model to capture market participants' short-term, mid-term, and long-term behavior. Accordingly, it uses volatility information from the previous day, week, and month as heterogeneous volatility components for forecasting. The standard formation of the HAR-RV model can be expressed as follows:

$$\hat{RV_t} = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + \epsilon_t \tag{4.26}$$

where RV_{t-1} is realized variance at time $t-1$ and $RV_{t-k:t-1}$ is average realized variance from time $t-k$ to $t-1$. $RV_{t-5:t-1}$ and $RV_{t-22:t-1}$ is calculated as follows:

$$RV_{t-5:t-1} = \frac{1}{5} \sum_{T=t-5}^{t-1} RV_T, \tag{4.27}$$

$$RV_{t-22:t-1} = \frac{1}{22} \sum_{T=t-22}^{t-1} RV_T. \tag{4.28}$$

This simple model is known to capture well the features of financial asset return series, such as long memory behavior of volatility. Moreover, various studies have demonstrated its simplicity, parsimony, and superiority (Chen et al., 2020; Fernandes et al., 2014; Ma et al., 2014; Čech & Baruník, 2017; Zhang et al., 2019).

According to the theory introduced by Barndorff-Nielsen & Shephard (2004), as $M \to \infty$, the $RV_t^{'}$ in Eq. 4.6 converges as follows:

$$RV_t^{'} \to \int_0^t \sigma^2(s)ds + \Sigma_{0<s\leq t}\kappa^2(s), \tag{4.29}$$

where the first term denotes the continuous component (the so-called integrated variance) and the second term represents the discontinuous component of the quadratic variation process as the jump component. As $M \to \infty$, the continuous component is equal to the realized bipower variation (BV_t), approximately as follows:

$$BV_t = u^{-2}\Sigma_{j=2}^{M}|r_{t,j}||r_{t,j-1}|, \tag{4.30}$$

where $u = \sqrt{\frac{2}{\pi}}$ is the mean of the absolute value of the random variable, which is the standard normally distributed.

Accounting for the jump component, Andersen et al. (2007) proposed the HAR-RV-J model where the jump component is added as an explanatory variable as follows:

$$\hat{RV_t} = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + \beta_j J_{t-1} + \epsilon_t \tag{4.31}$$

where the jump component (J_t) is defined as follows:

$$J_t = max(RV_t^{'} - BV_t, 0). \tag{4.32}$$

Since more realistic model specifications are appropriate to actual realized volatility series in the financial market, this HAR-RV-J model has been widely used and obtained popularity in forecasting volatility (Chen et al., 2020; Liu & Zhang, 2015; Ma et al., 2018).

4.4.4 Machine learning-type models

In the financial context, there have been a large number of studies on volatility forecasting via machine learning-type models. This subsection reviews several related kinds of research and briefly reviews our comparative machine learning methodologies.

Firstly, Support Vector Regression (SVR) has been used as a forecasting model for asset volatility. Santamaría-Bonfil et al. (2015) proposed a volatility forecasting model using SVR with a hybrid genetic algorithm. They demonstrated that their SVR volatility forecasting model outperforms GARCH(1,1) in terms of the mean absolute percentage error and directional accuracy functions, claiming that their proposed model overcomes the drawbacks of a traditional method. Furthermore, Sun & Yu (2020) proposed the hybrid forecasting model for financial returns volatility by mixing the SVR with GARCH. The author shows the empirical results of one-step-ahead forecasts suggesting that their hybrid models improve the volatility forecasting capability. Khashanah & Shao (2022) also introduced a forecasting model with kernel SVR for short-term volatility, revealing the outperformance of the proposed model.

Various studies using ANN have been reported. For example, Roh (2007) proposed hybrid models with neural networks and traditional time series models in volatility forecasting tasks for stock price index, demonstrating the utility of the hybrid model for volatility forecasting. Furthermore, Kristjanpoller & Minutolo (2015) investigated the empirical results of a hybrid volatility forecasting model with ANN and GARCH on the volatility of gold. Additionally, D'Ecclesia & Clementi (2021) examined the volatility forecasting performance using parametric, and ANN approaches, showing that the ANN approach is more accurate than parametric models.

Recurrent Neural Network (RNN) and Long Short-Term Memory (LSTM) are representative machine learning models specialized for time series. They have also been studied in the finance literature because of their superiority in the time series domain. Dunis & Huang (2002) investigated currency volatility forecasting using the RNN model. The authors showed that RNN models seem to be the best single modeling approach compared to benchmarks in forecasting currency volatility. Bucci (2020) examined the predictive performance of ANN, RNN, and LSTM compared with traditional econometric approaches. The authors demonstrated that RNN outperforms all the traditional econometric models, and LSTM also appears to enhance the forecasting accuracy in a highly volatile period capturing long-range dependence. Liu (2019b) proposed a novel volatility forecasting model using deep learning–long short-term memory recurrent neural networks. The results revealed that with big data, their proposed model improves the volatility prediction compared to SVR-based models and the GARCH model when applied to the forecasting task of stocks volatility.

Attention mechanism Bahdanau et al. (2015); Vaswani et al. (2017) is a famous deep learning model to capture complex patterns in time-series data. It also extracts the importance of the input feature (the so-called attention) on an element-by-element basis based on the internal attention layer component incorporated between layers in a deep neural network. There are a few of study in the volatility forecasting vein. For example, Lin & Sun (2021) proposed a novel volatility prediction model based on sparse multi-head attention. The author claimed that their model shows effectiveness in empirical financial data by addressing the gradient issue due to long-range propagation, revealing that this model is more suitable than traditional methods for forecasting tasks on long time series.

The eXtreme Gradient Boosting (XGB) is another strand of machine learning models specialized time series. Xia et al. (2022) investigated the volatility forecasting results of a green bond by proposing tree-based ensemble models, including XGB, which utilizes exogenous predictors, showing significant outperformance relative to the benchmark models. Liu (2022) examines the empirical results of applying XGB to forecasting stock volatility. They concluded that XGB-based models show the best performance in terms of root mean square percentage error compared to other machine learning-based models such as logistic regression and SVM.

Therefore, taking into account the aforementioned broad strands of financial literature, we select these machine learning-type models as our comparative volatility forecasting method for portfolio insurance strategy. By this, we investigate the empirical results of comparing these models with traditional volatility time-series models such as GARCH-type and HAR-RV-type models.

Support vector regression

After the rise of the Support Vector Machine (SVM), which is the cornerstone machine learning model for classification proposed by Cortes & Vapnik (1995), SVR was proposed as a variation of SVM for regression problems (Drucker et al., 1997). In SVR, the maximized distance from the hyperplane $\mathcal{H} \in \Re^{n-1}$ to support vectors is obtained by solving a quadratic optimization problem as follows:

$$\begin{aligned}
&\text{min} && \frac{1}{2}||\mathbf{w}||^2 + C(\Sigma_{i=1}^n \zeta^i + \zeta_*^i) && \text{(4.33a)}\\
&\text{subject to} && y^i - g(\mathbf{w}^T\mathbf{x}^i) \leq \epsilon + \zeta^i && \text{(4.33b)}\\
& && g(\mathbf{w}^T\mathbf{x}^i) - y^i \leq \epsilon + \zeta_*^i && \text{(4.33c)}\\
& && \zeta^i, \zeta_*^i \geq 0, && \text{(4.33d)}
\end{aligned}$$

where ϵ is a tube depending on a nonlinear function, $g(\cdot)$, ζ^i and ζ_*^i are 0 if the i-th point is in the tube, and x_i is a slack variable for i-th point.

From the Karush–Kuhn–Tucker conditions, the predicted value, y_* of x_*, which is new data, is obtained as $y_* = \Sigma_{i=1}^n(\alpha_i - \beta_i)k(x^i, x_*) + b$, where $k(x, x_*)$ is a kernel gives SVR a nonlinear nature by mapping the data points to feature space, and α and β are Lagrangian multipliers.

Recurrent neural network

A RNN is designed to keep and use the historical information as a sequence. In dealing with sequential data, computation with historical data is as important as the computation with the current input. However, there is no way to include previous outputs in the training phase in the standard ANN structure inside the neural network. In RNN, the model uses previous outputs with the current input data in hidden layers by adding a recurrent layer. Owing to this structural modification, RNN shares weights across time and overcomes this major shortcoming of the standard ANN model. The basic formula of hidden state at time t in the RNN model is as follows:

$$h_t = a(h_{t-1}, x_t; \theta). \tag{4.34}$$

where function a is the activation function, x_t is the current input, and θ is the parameters of function a. Implementing hidden state values could be done by adding computation results from h_{t-1} to the input of the activation function of ANN models described in Eq 2.4.

Long short-term memory

LSTM model was proposed to overcome a significant weakness of the standard RNN model. The standard RNN model suffers from vanishing gradient problems; the gradient could easily die during the backpropagation of sequential data in the training phase. Furthermore, gradient values could also burst during the training, so the weight update could not be done appropriately. This inherent problem during the training phase makes the model's weight updates difficult so that the whole optimization process might get stuck. This LSTM model suggests a solution by introducing a concept of the cell state and gates to handle both the long-term and the short-term dependencies within the data. The cell state runs through the entire network, and gates control updates in this state. An LSTM cell consists of three different gates (a forget gate, an input gate, and an output gate) (Gers et al., 2000). A forget gate decides the information be thrown away from the cell state, while an input gate decides the information to be stored in the cell state. An output gate decides the final output of an LSTM block at time t. The equation for each gate at time t is as follows:

$$i_t = \sigma(w_i[h_{t-1}, x_t] + b_i) \tag{4.35}$$

$$f_t = \sigma(w_f[h_{t-1}, x_t] + b_f) \tag{4.36}$$

$$o_t = \sigma(w_o[h_{t-1}, x_t] + b_o) \tag{4.37}$$

where i_t is the value of an input gate, f_t for a forget gate, and o_t for an output gate. σ is the sigmoid function. w_x is the weight of neurons at gate x, and b_x is the bias term at gate x. h_{t-1} is the output from time $t-1$.

The value of cell state and the final output of an LSTM block at time t are as follows:

$$\tilde{c_t} = tanh(w_c[h_{t-1}, x_t] + b_c) \tag{4.38}$$

$$c_t = f_t * c_{t-1} + i_t * \tilde{c_t} \tag{4.39}$$

$$h_t = o_t * tanh(c_t) \tag{4.40}$$

where c_t is the value of cell state and $\tilde{c_t}$ is the candidate value for a cell state. The final output of block h_t is calculated with o_t from Eq. 4.37.

Attention

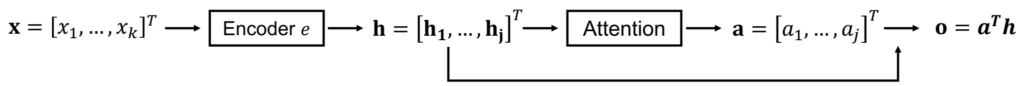


Figure 4.3: Illustration for attention.

Attention mechanisms have been widely used in various fields, such as machine translation and question-answering (Bahdanau et al., 2015; Vaswani et al., 2017). The weights of attention often offer insights into the decision process of deep learning models by interpreting the output softmax probabilities as the important of information (Koh & Liang, 2017; Vaswani et al., 2017). The overall mechanism is expressed in Figure 4.3. Given an input sequence $\mathbf{x} = [x_1, ..., x_k]^\top \in \Re^k$, the output sequence of encoder is $\mathbf{h} = [\mathbf{h_1}, ..., \mathbf{h_j}]^\top \in \Re^j$, which is formulated as

$$\mathbf{h} = e(\mathbf{x}), \tag{4.41}$$

where e is an encoder function. Then, we compute the attention score for $\mathbf{h_t}$

$$\hat{a}_t = \mathbf{c}^T \tanh(\mathbf{W}\mathbf{h_t} + \mathbf{b}), \tag{4.42}$$

where $\mathbf{W} \in \Re^{d' \times j}$ and $\mathbf{b}, \mathbf{c} \in \Re^{d'}$ are the parameters of attention mechanism. Attention weights can be obtained from using the softmax function to the attention scores as follows:

$$\boldsymbol{a} = \text{softmax}(\hat{\boldsymbol{a}}), \tag{4.43}$$

where the t-th element of $\boldsymbol{a}$ is $\frac{\exp(\hat{a}_t)}{\sum \exp(\hat{a}_t)}$.The final output vector $\boldsymbol{o}$ of the attention model becomes

$$\boldsymbol{o} = \boldsymbol{a}^T \mathbf{h}. \tag{4.44}$$

Extreme gradient boosting

XGB is a scalable tree ensemble boosting algorithm. This model is proposed by Chen & Guestrin (2016) and is one of the boosted tree models that incorporate the baseline decision tree methods into the gradient boosting algorithms. The concept of the boosting algorithm indicates combining simple and weak machines incrementally to obtain a more accurate and powerful ultimate model. Let the complete model is f, and k is the number of complement steps. Then, k-th complemented model f_k is obtained by:

$$f_k = f_{k-1} + h_k(x), \tag{4.45}$$

where $f_k(x) = f_k - f_{k-1}$ is a basic model to be trained model that aims to capture the residual of $y - f_k$ for the forecasting model at step k. A gradient of the objective function is used to train this basic model.

Although the performance of a created model is inferior, another model complements the weaknesses of the previous model, and this improvement procedure is repeated. Several weak models are created and combined step by step in terms of the gradient of the loss function based on the ensemble method, thereby creating a strong classifier (Friedman, 2001).

Based on this gradient boosting algorithm, XGB combines multiple weak classifiers into a strong classifier as a linear-additive model. The XGB model uses a tree ensemble model, which uses K additive functions as follows:

$$\hat{y_i} = \sum_{k=1}^{K} f_k(x_i) \tag{4.46}$$

where $f_k \in \mathcal{F}$ is the space of regression trees (e.g. CART).

In order to train the set of functions, the following regularized objective function is minimized based on an additive manner where f_t, which most improves the model, is added:

$$\mathcal{L}^{(t)} = \Sigma_{i=1}^{n} l(y_i, \hat{y}_i^{(t-1)} + f_t(x_i)) + \Omega(f_t), \tag{4.47}$$

where $\Omega(f) = \gamma T + \frac{1}{2}\lambda \Sigma_{j=1}^{T} w_j^2$, l is a differentiable convex loss function, T is the number of leaves, and w_j is the score on a j-th leaf. This regularized loss function is the difference between other tree boosting algorithms and XGB, improving learning speed and avoiding over-fitting problems.

The XGB has an advantage as an engine for the volatility forecasting task. The excellent forecasting power has been proven through numerous pieces of literature in the field of time-series analysis. The method shows a dominant forecastability provided by ensembles of decision trees as a state-of-art prediction model.

4.4.5 Forecasting performance measure and statistical test

The volatility forecasts ($\hat{\sigma_t}^2$) obtained by various models are compared to the realized volatility (RV_t) based on the following volatility forecasting criteria in evaluating the accuracy of models:

$$MAPE = n^{-1}\Sigma_{t=1}^{n}|(RV_t - \hat{\sigma_t}^2)/RV_t| \cdot 100, \tag{4.48}$$

$$MAE = n^{-1}\Sigma_{t=1}^{n}|RV_t - \hat{\sigma_t}^2|, \tag{4.49}$$

where n is the number of forecasts, MAPE is the mean absolute percentage error, and MAE is the mean absolute error.

In order to test the statistical significance of the above performance criteria, we conducted the Diebold-Mariano (DM) test introduced by Diebold & Mariano (2002) for comparison of the forecasting model and benchmark model. The DM statistic of two h-step ahead forecasts is defined as follows:

$$DM = [Var(\bar{d})]^{-\frac{1}{2}}\bar{d},\ \bar{d} = \frac{1}{T}\sum_{t=1}^{T} d_t, \tag{4.50}$$

where $d_t = g(e_{1,t}) - g(e_{2,t})$, $g(e)$ is certain specified function, and $e_{i,t}$ denotes forecast errors from method i at time t. We use the function $g(e) = |e|$ as for $d_t = |e_{1,t}| - |e_{2,t}|$. Specifically, we use $e_{i,t} = (RV_t - \hat{\sigma}_{i,t}^2)/RV_t$ for test of MAPE and $e_{i,t} = RV_t - \hat{\sigma}_{i,t}^2$ for test of MAE. The null hypothesis of $d = 0$ based on the student's t-distribution is used to obtain a p-value.

Moreover, for additional statistical evidence of forecasting model performance, we employ the model confidence set (MCS) test introduced by Hansen et al. (2011).

The concept of MCS is related to testing for superior predictive ability (Hansen, 2005) by comparing forecasts. Hence, the MCS test offers a set of models identifying the best performance model using a specific loss with confidence level α. The procedure of MCS starts at the full universe of comparative models, rather than only depending on a benchmark model. It drops the model based on an elimination rule by alternately testing a null hypothesis of equal predictive ability at significance level α. The procedure provides a set of significantly inferior models as eliminated models and a set of superior models.

4.5 Experimental design and procedure

We use the synthetic put strategy for all empirical studies we investigate with a 100% protection level. 100% is a commonly used protection level in the context of portfolio insurance research, as revealed in Dichtl & Drobetz (2011). This study focuses on the investigation of the impact of volatility estimation error in synthetic put strategy rather than the impact of other parameters such as protection level (strike price K) and maturity time (T). As a result, to maintain our main argument clear, we use only a fixed level of these parameters as in the values mentioned above ($K = S_0$ and $T = 1$). Next, our benchmark is the buy-and-hold strategy, where 100% of the portfolio is invested in the risky asset at the initial date and held until maturity.

To consider the impact of transaction costs due to the frequent rebalancing, we use the transaction cost scheme as in Eq. 4.15 for the synthetic put portfolio insurance strategy. Ten basis points are used as a value of k, which is round-trip transaction costs, following the studies of Dichtl & Drobetz (2011); Herold et al.

(2007). To reflect the transaction cost of the buy-and-hold strategy, we also use ten basis points using an explicit scheme in our implementation of backtesting.

4.5.1 The Monte Carlo simulation

To investigate the results of synthetic put portfolio insurance strategy based on the standard GBM and GBM with a jump, we generate numerous simulation paths, assuming that GBM simulation contains 252 trading days as a 1-year duration[12]. In creating random path of simulated price, we use $T = 1$, $r = 4.5\%$, $S_0 = 1$ $\mu = 9\%$, and $\sigma = 10\%, 20\%, 30\%$, and 40% as input parameters in our numerical example of the standard GBM. For simulation of GBM with jump, we use $T = 1$, $r = 4.5\%$, $S_0 = 1$ $\mu = 9\%$, $\sigma = 10\%, 20\%, 30\%$, and 40%, $\eta_1 = 182.08$, $\eta_2 = 172.86$, $\lambda = 1.4615$, $p_z = 0.496$, and $q_z = 0.504$ as fixed parameters.

The procedure of our Monte Carlo simulation is as follows. First, the level of true volatility is selected out of four possible values (10%, 20%, 30%, and 40%). Second, to obtain reliable results taking into account various market conditions, the diffusion process model is selected following the stochastic differential equation in Eq. 4.1 (the standard GBM) or Eq. 4.3 (the GBM with a jump). Third, 1,000 paths are created based on selected true volatility and selected diffusion process model using the aforementioned pre-specific input parameters. Finally, a total of 1,000 repeated synthetic put strategy simulation is implemented based on four estimated volatility (10%, 20%, 30%, and 40%) as an input strategy parameter using simulated prices, and the performance of the strategy are evaluated based on various performance measures.

[12] A 1-year is a popular investment horizon for a large amount of institutional or retail investors in their investment strategy (Benartzi & Thaler, 1995).

4.5.2 The real-world data simulation

For portfolio insurance simulation in real-world data, we use S&P 500 index as our empirical results on real-world data. In order to obtain more reliable results, we apply our portfolio insurance simulation-based rolling window scheme (moving the window forward by one day by deleting the first observation and adding one observation) following the research of Dichtl & Drobetz (2011)[13]. Moreover, this rolling windows simulation ensures that each window consisting of the corresponding period is exactly considered once. We use $K = S_0$ and $T = 1$ as input parameters of synthetic put strategy with 100% protection level, and transaction cost scheme is same as in Monte Carlo simulation. For the risk-free rate at time t, the T-bill rate corresponds to that time is used.

For volatility forecasting, the total data whose observations are 5,610 on a daily basis are divided into two subsamples; in-sample and out-of-sample. In-sample consists of 2,000 observations covering 4 January 2000 to 31 December 2007 and is used to estimate or train the models. Out-of-sample consists of 3610 from 1 January 2008 to 19 May 2022 and is used to test[14]. Volatility forecasting also uses the rolling window scheme to generate a one-step-ahead forecast. Thus, it delivers a $3,610 - 252 = 3,358$ overlapping series of data in implementing our strategies by using 3,610 one-step ahead forecasts.

[13]Dichtl & Drobetz (2011) claim that the rolling window basis method uses the available data most efficiently since it preserves the effect of dependency in the financial time series, such as autocorrelation and heteroskedasticity.

[14]We also examined the result using a case of 70% in-sample and 30% out-of-sample since it is widely known the convention in the field of machine learning is that the percentage of out-of-sample does not exceed 50% of the total sample. However, we report only the main result, which is not in accordance with this convention, since we want to investigate the empirical out-of-sample result of the period of the 2009 financial crisis event. Nevertheless, we confirmed that the result following the convention is essentially similar to our main result. The full results are available upon request.

For our volatility forecasting models, we first use naive models with 252 daily rolling windows standard deviation as the benchmark model. We select the best performance model among various possible models (e.g., the window size equals 5, 22, 132, and 252.). As for GARCH-type models, we use GARCH(1,1), EGARCH(1,1), and GJR-GARCH(1,1) models. We also use HAR-RV and HAR-RV-J as our HAR-RV-type models. For machine learning-type models, we select the best models by searching the hyperparameter space, thereby resulting in the best set of hyperparameters. As a result, for SVR, we use Radial Basis Function (RBF) kernels, L2 regularization parameter $C = 1$, and 0.1 of epsilon. In the case of deep learning models (ANN, RNN, LSTM, and Attention), we use ReLU as the activation function and mean squared error as the loss function. We train all models 100 epochs with the Adam optimizer. Unless specified otherwise, the learning rate is fixed to 1e-3. For ANN, we use three hidden layers (128, 64, and 32 neurons). For RNN and LSTM, we use one hidden layer followed by one fully connected layer. For Attention, we combine the aforementioned LSTM model and an attention module with two convolution layers. We fix a learning rate to 1e-4 for Attention. For XGB, we use 1,000 gradient boosted trees, six maximum tree depths for base learners, 0.3 learning rate, gamma of zero, one L2 regularization term on weights, and root mean square error as loss function.

The detailed procedure to obtain the result for the impact of volatility forecasting in synthetic put strategy is as follows. First, we obtain forecasting models using in-sample and conduct out-of-sample volatility forecasting using those models based on the rolling window method. Second, using these forecasts as an input parameter in the Black-Scholes formula of synthetic put strategy (estimators for volatility), synthetic put strategy is implemented based on t-th 1-year S&P 500 index data obtained from out-of-sample. Third, a total of $3,358$ repeated synthetic put strategy simulation is implemented based on the rolling window method. Finally, the strategy's performance is evaluated in terms of PLE, and forecasting performance evaluation is conducted using two performance measures (MAPE and MAE).

4.6 Empirical results

4.6.1 The Monte Carlo simulation results

The standard GBM result

Table 4.2 shows the performance evaluation results of the synthetic put strategy using the standard GBM simulation. Panels A, B, C, and D correspond our simulation setup of *true* volatility as 10%, 20%, 30%, and 40%, respectively. Each column implies that the estimated volatilities as an input parameter of synthetic put strategy are also 10%, 20%, 30%, and 40%, respectively. Hence, this table demonstrates the overall impact of the estimation error of volatility in the synthetic put portfolio insurance strategy. The last column includes the result of the buy-and-hold strategy as a benchmark.

In the first column of Panel A, where true volatility is 10%, if there is no estimation error, that is, if estimated volatility is equal to true volatility, the PLE is 4.627%, which is the lowest value. However, in the second column of Panel A, if volatility is incorrectly estimated, it can be seen that the PLE increases to 5.448%. Moreover, as the estimation error is significant, the PLE becomes larger, reaching 6.713% PLE when the estimated volatility is 40%.

In Panel B, the tendency of estimation error problem is consistent with the results of Panel A. In other words, the more volatility is overestimated, the greater

Table 4.2: The Monte Carlo simulation results using GBM data

	$\hat{\sigma} = 0.1$	$\hat{\sigma} = 0.2$	$\hat{\sigma} = 0.3$	$\hat{\sigma} = 0.4$	B&H
Panel A: $\sigma = 0.1$					
Average return	0.063	0.0586	0.0568	0.056	0.0838
Standard deviation	0.0783	0.0714	0.0681	0.0666	0.0998
Skewness	0.585	0.637	0.617	0.574	0.037
Sharpe ratio	0.23	0.191	0.173	0.165	0.389
Sample min value	0.9342	0.9262	0.9191	0.9138	0.7847
Protection level error (%)	**4.627**	5.448	6.175	6.713	-
Panel B: $\sigma = 0.2$					
Average return	0.047	0.047	0.0474	0.0479	0.0723
Standard deviation	0.1463	0.1373	0.1329	0.1308	0.2006
Skewness	0.958	0.975	0.929	0.861	0.016
Sharpe ratio	0.013	0.015	0.018	0.022	0.136
Sample min value	0.8682	0.8829	0.8576	0.8337	0.5313
Protection level error (%)	8.153	**6.6**	9.278	11.809	-
Panel C: $\sigma = 0.3$					
Average return	0.0337	0.0346	0.0352	0.0357	0.0448
Standard deviation	0.2116	0.2009	0.1954	0.1928	0.3001
Skewness	1.319	1.376	1.344	1.275	0.078
Sharpe ratio	-0.053	-0.052	-0.05	-0.048	-0.001
Sample min value	0.7473	0.8125	0.8232	0.7964	0.4037
Protection level error (%)	18.106	10.957	**9.785**	12.72	-
Panel D: $\sigma = 0.4$					
Average return	0.0128	0.0168	0.0198	0.0223	0.0281
Standard deviation	0.2799	0.2679	0.2612	0.258	0.4002
Skewness	1.296	1.387	1.389	1.343	-0.069
Sharpe ratio	-0.115	-0.105	-0.096	-0.088	-0.042
Sample min value	0.6783	0.7268	0.7636	0.7802	0.2482
Protection level error (%)	23.086	17.583	13.408	**11.523**	-

Notes. This table shows the Monte Carlo simulation performance evaluation results of the synthetic put strategy with a 100% protection level using the standard GBM according to the different true and estimated volatility conditions. The input parameters of GBM are fixed as $\mu = 9\%$, $r_f = 4.5\%$, and $T = 1$.

the PLE. Specifically, when true volatility is 20%, if there is no estimation error, it is shown that PLE has 6.6%, whereas when estimated volatility is 30% or 40%, PLE is 9.278% or 11.809%. Similarly, when volatility is underestimated as 10%, PLE shows 8.153%, which is also more extensive than the no-estimation error case.

In Panels C and D, we also find results that are essentially consistent with the results of Panels A and B, indicating that the problem of volatility estimation errors exists regardless of whether volatility is overestimated or underestimated. The interesting point is that considering the Panels A, B, C, and D together, the problem of volatility estimation is pronounced as true market volatility is larger, implying that extreme market turbulence makes it more difficult for investors to protect their insured portfolios[15].

Note that PLE is lowest when estimation error is lowest in all Panels. We implement a synthetic put strategy by modifying estimated volatility to adjusted volatility in Eq. 4.15, rather than just using estimated volatility. This modified version of volatility makes input volatility slightly larger, leading to slightly overestimated. Hence, we demonstrate that slightly overestimation provides more accurate synthetic put protection, consistent with the findings of Leland (1985); Zhu & Kavee (1988). Leland (1985) argues that larger volatility should be considered for taking into account transaction costs and replicating a put option more accurately. Zhu & Kavee (1988) demonstrate performance evaluation of synthetic put strategy, revealing that the protection level error becomes larger when using correctly estimated volatility than the slightly greater estimated volatility when they do not adjust their input volatility. They also argue that in implementing the synthetic put strategy, PLE can be explained more accurately by Leland (1985)'s model. Our study empirically demonstrates that these arguments are correct by confirming the improved PLE in simulation when using Leland (1985)'s alternative option pricing model.

Furthermore, we find that as more volatility is overestimated, the synthetic put strategy invests in being less risky irrespective of true volatility value. We can see

[15]It can be seen that PLE is more than zero even though there is no estimation error in all Panels. These are unsurprising results considering realistic trading conditions since our portfolio insurance strategy applies weight rebalancing on a daily basis rather than a continuous basis, thereby suffering from the sudden downward movement before adjustment. As a result, portfolio value can fall below the TPV, and this tendency can be potentially reinforced when market volatility becomes larger.

that the standard deviation decrease as the estimated volatility value increases in all Panels. These results indicate that portfolio insurance strategy limits the risk exposure based on their estimation of volatility level given a market condition and seeks to reduce a risky position as the market uncertainty enlarges gradually. On the contrary, the other performance measures widely used in traditional studies on portfolio analysis, such as average return, skewness, and Sharpe ratio, do not show any similar pattern to the results mentioned above according to the change of volatility estimation error.

Compared to the synthetic put strategy, the buy-and-hold strategy shows the largest value of average return and standard deviation in all Panels. This result is not surprising, considering the concept of portfolio insurance that investor pays an upside participant as a cost of downward protection for their portfolio. Contrary to the buy-and-hold strategy, which shows a small skewness value, the portfolio insurance strategy has substantially higher positive skewness. Generally, higher positive skewness implies that the strategy is more desirable and provides frequent small losses and a few significant gains rather than vice versa in investment (Harvey & Siddique, 2000; Post et al., 2008). On the other hand, the buy-and-hold strategy outperforms portfolio insurance strategies in terms of the Sharpe ratio regardless of volatility estimation error[16]. Despite the inferiority in terms of Sharpe ratio, synthetic put strategy considerably better level of sample minimum value compared to a buy-and-hold strategy, implying that portfolio insurance strategy successfully protects their portfolio value to the downside risk of the underlying risky asset.

[16]It is a trivial result since many portfolio insurance strategy studies have reported that the Sharpe ratio might not be a proper performance measure for portfolio insurance due to its asymmetry and non-normality in their return distribution (Annaert et al., 2009; Bertrand & Prigent, 2011; Dichtl & Drobetz, 2011; Dichtl et al., 2017; Gaspar & Silva, 2021; Zieling et al., 2014).

The GBM with jump result

Table 4.3: The Monte Carlo simulation results using the jump-diffusion model

	$\hat{\sigma} = 0.1$	$\hat{\sigma} = 0.2$	$\hat{\sigma} = 0.3$	$\hat{\sigma} = 0.4$	B&H
Panel A: $\sigma = 0.1$					
Average return	0.0558	0.0517	0.05	0.0494	0.0754
Standard deviation	0.0728	0.0666	0.0639	0.0628	0.0974
Skewness	0.334	0.326	0.257	0.183	-0.384
Sharpe ratio	0.148	0.101	0.079	0.069	0.312
Sample min value	0.926	0.9095	0.8915	0.8783	0.7438
Protection level error (%)	**5.471**	7.151	8.992	10.335	-
Panel B: $\sigma = 0.2$					
Average return	0.0363	0.0368	0.0373	0.0379	0.0556
Standard deviation	0.1329	0.1246	0.1212	0.1202	0.1988
Skewness	0.892	0.886	0.803	0.7	-0.602
Sharpe ratio	-0.066	-0.066	-0.063	-0.059	0.053
Sample min value	0.8528	0.8751	0.8482	0.8284	0.3649
Protection level error (%)	9.787	**7.43**	10.273	12.366	-
Panel C: $\sigma = 0.3$					
Average return	0.004	0.0067	0.0087	0.0101	0.005
Standard deviation	0.1885	0.1782	0.1742	0.1735	0.3168
Skewness	0.987	1.029	0.952	0.833	-0.847
Sharpe ratio	-0.217	-0.215	-0.208	-0.201	-0.126
Sample min value	0.752	0.8	0.8169	0.7909	0.2143
Protection level error (%)	17.588	12.324	**10.474**	13.319	-
Panel D: $\sigma = 0.4$					
Average return	-0.0186	-0.0178	-0.0178	-0.0181	-0.0766
Standard deviation	0.2349	0.2242	0.2202	0.2201	0.4867
Skewness	1.049	1.12	1.077	0.98	-1.347
Sharpe ratio	-0.271	-0.28	-0.285	-0.286	-0.25
Sample min value	0.6227	0.7096	0.7469	0.7472	0.0083
Protection level error (%)	29.382	19.526	15.306	**15.265**	-

Notes. The Monte Carlo simulation performance evaluation results of the synthetic put strategy with a 100% protection level using the Kou jump-diffusion model according to the different true and estimated volatility conditions. The input parameters of GBM are fixed as $\mu = 9\%$, $r_f = 4.5\%$, $T = 1$, $\eta_1 = 182.08$, $\eta_2 = 172.86$, $\lambda = 1.4615$, $p_z = 0.496$, and $q_z = 0.504$.

Table 4.3 shows the performance evaluation results of the synthetic put strategy using the GBM with jump-diffusion model simulation. Panels A, B, C, and D correspond our simulation setup of input volatility (σ term) as 10%, 20%, 30%, and 40%, respectively. Overall, it can be seen that the results considering jump phenomena are qualitatively similar to those based on only the standard GBM, which does not consider jump. In other words, the more volatility is incorrectly estimated, the greater the PLE, no matter what volatility is overestimated or underestimated. Furthermore, this tendency is more highlighted as the market goes extremely fluctuated.

Although the tendency is similar between these two results in Table 4.2 and Table 4.3, the specific intensity in the two results greatly differs. This result indicates that the account for jump phenomena matters in evaluating the impact of volatility estimation error in synthetic put portfolio insurance strategy. The overall PLE in GBM simulation with jump is greater than without jump, looking at the results in Table 4.2 and Table 4.3 together. These results imply that it is more difficult to implement synthetic put strategy in real-world market conditions than in idealistic conditions.

The crucial point suggested in the results in Table 4.2 and Table 4.3 can be summarized as follows. Obviously, there exists the protection error in the synthetic put strategy caused by volatility misestimation. Furthermore, this protection error is enlarged as the estimation error becomes greater. This impact can be more pronounced in the market condition which is more similar to the real world considering jump phenomena.

4.6.2 The real-world data simulation results

Volatility forecasting models results

Table 4.4 shows the overall performance evaluation results of portfolio insurance and volatility forecasting using the S&P 500 index. We report all results from various forecasting models. Panel A, B, C, and D represent the result of naive, GARCH-type, HAR-RV-type, and machine learning-type forecasting models, respectively. In Table 4.4, each column presents the performance of portfolio insurance in terms of

PLE with t-value of paired t-test, the performance of volatility forecasting in terms of MAPE and MAE, DM test results of all forecasting models based on MAPE and MAE measures, and rank for PLE, MAPE, and MAE, respectively.

More precisely, the PLE of the naive model shows 21.79%, while the PLE of GARCH, EGARCH, and GJR-GARCH show 19.06%, 21.52%, and 19.38%, respectively, revealing the outperformance of GARCH-type models compared to the naive model. Furthermore, HAR-RV-type models outperform the naive model showing a PLE of 18.41%. Compared to the GARCH-type model, HAR-RV-type models show a better level of PLE, demonstrating that HAR-RV-type seems more suitable for portfolio insurance strategy. However, this result should be interpreted with caution since the PLE improvement of HAR-RV-type is not statistically significant. In contrast, GARCH and GJR-GARCH show statistical significance based on paired t-tests. This result implies that even though the degree of improvement of GARCH and GJR-GARCH is not greater than that of HAR-RV-type models, the PLE of GARCH and GJR-GARCH is significantly improved compared to the PLE of naive from the statistical point of view, unlike HAR-RV-type.

Interestingly, looking at Panel D, machine learning-type forecasting models, with the exception of SVR, show absolute dominance compared to all traditional approaches. Among them, XGB (12.88%) shows overwhelming improvement in terms of PLE, followed by Attention (15.08%) which is a substantial enhancement. Compared with naive, in terms of PLE, XGB improved by 8.91%, and Attention improved by 6.71%. Additionally, except for SVR, machine learning-type forecasting models show statistical significance, supporting the improvement of these models compared to the naive way.

Table 4.4: The performance evaluation results of the S&P 500 index

	Portfolio insurance		Volatility forecasting		DM test		Rank		
	PLE (%)	t-value	MAPE	MAE	MAPE	MAE	PLE	MAPE	MAE
Panel A: Naive									
Standard deviation	21.79	-	2.14347	0.00016376	-	-	11	12	11
Panel B: GARCH-type									
GARCH	19.06	-2.45^{**}	1.13206	0.00011182	-23.28^{***}	-13.51^{***}	8	8	7
EGARCH	21.52	0.37	1.21811	0.00012740	-16.67^{***}	-12.76^{***}	10	10	10
GJR-GARCH	19.38	-2.15^{**}	1.15530	0.00011096	-9.86^{***}	-12.48^{***}	9	9	6
Panel C: HAR-RV-type									
HAR-RV	18.41	-1.30	0.88339	0.00011401	-22.38^{***}	-10.54^{***}	6	5	9
HAR-RV-J	18.41	-1.30	0.88331	0.00011399	-22.38^{***}	-10.54^{***}	7	4	8
Panel D: ML-type									
SVR	22.54	0.68	1.51166	0.00017482	-10.01^{***}	1.69^{*}	12	11	12
ANN	15.83	-5.97^{***}	0.93554	0.00010788	-24.84^{***}	-15.93^{***}	4	6	5
RNN	16.42	-2.12^{**}	0.94762	0.00010355	-27.38^{***}	-21.8^{***}	5	7	4
LSTM	15.11	-4.94^{***}	0.87346	0.00009961	-26.41^{***}	-17.23^{***}	3	3	3
Attention	15.08	-3.52^{***}	0.84996	**0.00008408**	-22.11^{***}	-18.68^{***}	2	2	**1**
XGB	**12.88**	-12.21^{***}	**0.78788**	0.00009613	-25.63^{***}	-23.64^{***}	**1**	**1**	2
							Rank corr.	0.930^{***}	0.916^{***}

Notes. This table shows the performance evaluation results of portfolio insurance and volatility forecasting based on the S&P 500 index using various forecasting models.

Taking into account the overall result of PLE, the PLE shows great difference depending on the selection of forecasting models. This result implies a strong hint that the influence of volatility estimation error exists even in real-world data, even though, at this point, we cannot identify the exact tendency yet.

Next, we report the value of MAPE and MAE as the performance measure of volatility forecasting, which measures the accuracy of predicted volatility value relative to realized volatility as a target. MAPE and MAE of naive show the largest value among all models, whereas those of GARCH-type and HAR-RV-type modes show a moderate level of values[17]. In Panel D, with the exception of SVR, most machine learning-based models outperform traditional forecasting models in terms of MAE, and XGB, Attention, and LSTM show lower MAPE levels than other

[17] Although there is no substantial difference in MAE, HAR type shows better performance than GARCH type in MAPE. It is presumed that this is because, unlike the GARCH-type model that generates latent volatility, the HAR-RV-type model is a methodology that directly targets realized volatility.

models. Specifically, XGB shows the lowest MAPE, and Attention is the second, while Attention is the first and XGB is the second in terms of MAE. In both MAPE and MAE, the LSTM model is the third most excellent forecasting model. All results of these MAPE and MAE are confirmed to be statistically significant by Diebold & Mariano (2002) test, indicating that all forecasting models improve the realized volatility forecasting compared to the naive method in terms of MAPE and MAE. All models, including traditional and machine learning models, outperform the naive model. Furthermore, three machine learning models (XGB, Attention, and LSTM), widely known to be specialized in the task related to time-series data, outperform other traditional models in terms of MAPE and MAE.

The most shocking point is that the tendency of the degree of improvement in PLE of each volatility forecasting model is very similar to that in MAPE and MAE. In other words, if a particular forecasting model has better forecasting ability and shows a better value of MAPE and MAE, this model appears to show a better level of PLE. More precisely, machine learning models (especially XGB and Attention) outperform the traditional and naive approach in terms of PLE, demonstrating that similar patterns are shown in terms of MAPE and MAE. This tendency is confirmed by the result of the rank of PLE, MAPE, and MAE in Table 4.4[18]. Specifically, it can be seen that the lowest value of PLE of XGB can be corroborated by the lowest MAPE and second lowest MAE. In addition, the second PLE rank of Attention is explained by the second lowest MAPE and lowest MAE of Attention. The rank

[18] We also applied similar experiments based on HMAE and HMSE as volatility forecasting performance measures. These results are presented in Table A5 in Appendix and are essentially consistent with the main result. The rank correlation of PLE and HMAE (0.944) and PLE and HMSE (0.937) also support the statistically significant positive relationship between PLE and realized volatility forecasting accuracy.

of PLE of LSTM is also linked to the rank of MAPE and MAE. Most inferior methodologies, such as SVR and naive, also show the highest PLE and second highest PLE, while the highest level of MAPE and MAE is shown in both of those models.

To show our main argument clearly, we attempt to report the ranking correlation between PLE and MAPE or PLE and MAE by simultaneously applying the statistical test. As a result, the ranking correlation of PLE and MAPE is 0.93, and that of PLE and MAE is 0.916 showing statistical significance. We consider these results as strong evidence that improved realized volatility forecasting is directly correlated with the improvement in the performance of portfolio insurance strategy in real-world conditions. Therefore, an enhanced volatility forecasting model means one can benefit from the better performance of portfolio insurance.

Table 4.5: The evaluation results of MCS p-values with $\alpha = 0.1$

	MCS p-value		PLE rank	MCS rank	
	MAPE	MAE	PLE	MAPE	MAE
Panel A: Naive					
Standard deviation	0.0000	0.0000	11	12	11
Panel B: GARCH-type					
GARCH	0.0000	0.2986	8	8	7
EGARCH	0.0000	0.0000	10	10	10
GJR-GARCH	0.0000	0.4830	9	9	6
Panel C: HAR-RV-type					
HAR-RV	1.0000	0.1124	6	4	8
HAR-RV-J	1.0000	0.1114	7	5	9
Panel D: ML-type					
SVR	0.0000	0.0000	12	11	12
ANN	0.5288	0.7608	4	6	5
RNN	0.2364	1.0000	5	7	4
LSTM	1.0000	1.0000	3	2	3
Attention	1.0000	1.0000	2	3	2
XGB	**1.0000**	**1.0000**	**1**	**1**	**1**
			Rank corr.	0.930***	0.930***

In order to show the reliability of our main results, we also apply the MCS test as a supportive statistical test for the rank of our volatility forecasting models. Table 4.5 presents the evaluation results of the MCS test, demonstrating the MCS p-value of MAPE and MAE, PLE rank, and MCS rank of MAPE and MAE with the confidence level $\alpha = 0.1$. More precisely, XGB, Attention, LSTM, and HAR-RV-type models show the p-value of 1 in terms of MAPE, and XGB, Attention, LSTM, and RNN reveal the p-value of 1 in terms of MAE. XGB always has the best performance in terms of MAPE and MAE. Looking at the PLE rank and MCS rank of MAPE and MAE together, we confirm the consistent tendency with the main results. This finding is supported by the rank correlation reported in Table 4.5, showing 0.93 based on each rank pair (PLE and MAPE, and PLE and MAE).

The impact of market condition

Table 4.6: The performance under the different market conditions

	Total		Low volatility		High volatility	
	PLE (%)	t-value	PLE (%)	t-value	PLE (%)	t-value
Panel A: Naive						
Standard deviation	21.79	-	16.42	-	25.98	-
Panel B: GARCH-type						
GARCH	19.06	-2.45^{**}	15.36	-2.19^{**}	23.40	-1.66
EGARCH	21.52	0.37	15.31	-2.39^{**}	25.73	1.29
GJR-GARCH	19.38	-2.15^{**}	15.47	-2.03^{**}	23.70	-1.65
Panel C: HAR-RV-type						
HAR-RV	18.41	-1.30	<u>12.27</u>	-3.80^{***}	22.78	-3.48^{**}
HAR-RV-J	18.41	-1.30	12.27	-3.80^{***}	22.78	-3.48^{**}
Panel D: ML-type						
SVR	22.54	0.68	13.28	-7.33^{***}	26.68	0.11
ANN	15.83	-5.97^{***}	12.49	-9.07^{***}	20.34	-7.85^{***}
RNN	16.42	-2.12^{**}	15.68	0.74	20.90	-2.61^{***}
LSTM	15.11	-4.94^{***}	12.29	-5.11^{***}	19.65	-8.18^{**}
Attention	<u>15.08</u>	-3.52^{***}	13.28	-2.76^{***}	<u>19.63</u>	-4.04^{***}
XGB	**12.88**	-12.21^{***}	**10.60**	-13.45^{***}	**17.54**	-3.30^{***}

In order to analyze our result from the economic point of view, we investigate the empirical results of PLE according to different market conditions. On the contrary to the main result where the market condition is not ex-ante known to the investor at the inception, this analysis is an ex-post exercise obviously, in that we already know about the realized volatility. Even if we analyze the ex-post backtest, it can add value to our study in that it makes us know about the impact of the market condition on the volatility forecasting and, thus, the performance of portfolio insurance strategy. Table 4.6 shows the results of the total, low-volatility, and high-volatility samples based on the ex-post realized volatility. Each Panel represents the model type, and we report the PLE and t-value of these three groups. Low and high volatility groups consist of the top lowest 33.3% and top highest 33.3% of out-of-sample in terms of realized volatility, respectively. We present these two groups because the portfolio insurance strategy aims to maintain the value of the insured portfolio not to be below the floor value in highly volatile conditions (high potential to downside risk) as well as normal conditions. Therefore, a desirable portfolio insurance strategy should show no higher relative performance degradation under high volatility conditions than under low volatility conditions.

The low-volatility condition results show that the PLE of naive (16.42%) shows the highest value. In contrast, machine learning-based models (except for RNN) generally show decent performance among forecasting models, which is similar to the main result. On the other hand, interestingly, although the performance of GARCH-type models is still not so good, HAR-RV-type models show relatively enhanced performance compared to that of the total sample in terms of PLE ranking. These results demonstrate that the HAR-RV-type model outperforms all the models, including machine learning-type models, except for XGB. Furthermore, unlike the result in the total sample, the HAR-RV-type model becomes statistically significant in terms of PLE. It can be seen that the traditional HAR-RV-type models can be a good choice in the low-volatility condition as well as machine learning models.

On the other hand, in the high volatility condition, we can find shocking results which are not similar to the low volatility results. Although overall PLE values increase due to increased market risk, the rank of PLE in traditional models shows different results. HAR-RV-type models show substantial deterioration compared to low volatility conditions, whose PLE rank is lower than most machine learning models in high volatility conditions. Additionally, it is shown that all GARCH-type models become insignificant in terms of t-value, still maintaining the inferior level of PLE rank. Considering the results of HAR-RV-type and GARCH-type models together, we conclude that the traditional models suffer from the crucial deterioration in sustaining the target protection value of portfolio insurance strategy when the market condition changes from low to high volatility condition. On the contrary, all machine learning models except for SVR show pronounced outperformance, revealing the superiority in terms of the level of PLE. Considering the above results, we conclude that traditional and machine learning models can capture the low volatile market condition in forecasting volatility. However, if market conditions become different, the results are different, implying the hint that machine learning models might abstract the complex pattern of realized volatility series such as peak or turbulence in highly volatile fluctuation better than traditional forecasting models.

The results in Table 4.6 are summarized as follows. First, all models outperform the naive model in terms of PLE, regardless of market condition. This result is consistent with the main findings supporting that volatility forecasting improves portfolio insurance performance. Second, machine learning models (except for SVR in high volatility conditions) are always a consistently good choice in volatility forecasting for portfolio insurance strategy. Among them, XGB always shows the best PLE performance irrespective of the degree of market fluctuation. Third, although traditional models appear to be a good choice in low-volatility condition, these models' performance deteriorates rapidly when market turbulence become reinforced. Again, the instability of traditional models supports the superiority of machine learning models in implementing portfolio insurance in both lowly and highly volatile market conditions.

Chapter 5

Portfolio insurance strategy in the cryptocurrency market

5.1 Chapter overview

In 2009, Bitcoin, a novel peer-to-peer distributed ledger system, was proposed by Nakamoto (2008). Since then, hundreds of alternative cryptocurrencies have emerged, topping a market capitalization of 1.9 trillion USD by early 2022. Accordingly, shaping huge market and media coverage, cryptocurrencies have not only captured the attention of investors but also fascinated the world of academia, prompting numerous studies on the cryptocurrency market. These studies include the study of cryptocurrencies' behavior, market analysis, and asset pricing. Although some authors have focused on the research topic, such as price (Bouri et al., 2019c; Cai et al., 2021; Dimpfl & Peter, 2021; Köchling et al., 2019; Stosic et al., 2019), return (Akyildirim et al., 2021; Balcilar et al., 2017; Caporale et al., 2018; Long et al., 2020; Nguyen et al., 2019; Punzo & Bagnato, 2021), and volatility (Bouri et al., 2019b; Chaim & Laurini, 2018; Cross et al., 2021; Liu & Serletis, 2019; Omane-Adjepong et al., 2019; Qiao et al., 2020), others have investigated connectedness across the cryptocurrencies (Bouri et al., 2021; Fousekis & Tzaferi, 2021; Ji et al., 2019; Koutmos, 2018; Nguyen et al., 2020; Omane-Adjepong & Alagidede, 2019; Sensoy et al., 2021; Shahzad et al., 2021; Xu et al., 2021; Yi et al., 2018). In line with the findings from these studies, other researchers confirmed the inefficiency of the cryptocurrency market (Bariviera, 2017; Urquhart, 2016; Vidal-Tomás et al., 2019b; Wei, 2018) or explored the common risk factors to explain excess returns of cryptocurrencies through the lens of empirical asset pricing (Jia et al., 2022; Kosc et al., 2019; Kozlowski et al., 2021; Li et al., 2020; Liebi, 2022; Liu et al., 2020b, 2022; Liu & Tsyvinski, 2021; Zaremba et al., 2021a; Zhang & Li, 2020, 2021).

In addition to the studies on the behavior of cryptocurrencies, market analysis, and asset pricing, various studies from the portfolio analysis viewpoint have been conducted. For instance, Brauneis & Mestel (2019) empirically investigated cryptocurrency portfolios using a mean–variance framework. The authors demonstrated that the Markowitz optimal portfolio strategy shows a higher Sharpe ratio than the buy-and-hold strategy and the naively diversified portfolio outperforms other optimized portfolio strategies. Similarly, Liu (2019a) scrutinized portfolio diversification across cryptocurrencies in terms of the Sharpe ratio and the investor's utility. The author showed significant enhancement in the out-of-sample Sharpe ratio and utility of various cryptocurrency portfolio strategies compared to the benchmark strategy. Meanwhile, Čuljak et al. (2022) examined the effect of the inclusion of sectoral cryptocurrency in the cryptocurrency-based investment basket under the Markowitz framework, revealing the benefit of the inclusion of sectoral cryptocurrency.

Unlike the aforementioned studies on portfolio strategies for cryptocurrency under the Markowitz framework, some authors focused on other strategies under the different philosophy of portfolio construction. For instance, Platanakis & Urquhart (2019) explored the empirical result of the cryptocurrency portfolio under the Black–Litterman framework and compared this to the strategy under the

Markowitz framework and equal-weighted strategy. They demonstrated that Black–Litterman approach outperforms other benchmarks by imposing variance-based constraints of Levy & Levy (2014) to control estimation errors of the input parameters in managing cryptocurrency portfolios. Burggraf (2021) provided the empirical result of cryptocurrency portfolio using the Hierarchical Risk Parity strategy proposed by De Prado (2016), one of the risk-based portfolio construction strategies like the risk parity approach, to overcome the estimation error of the Markowitz approach and risk parity's ignorance of useful covariance structure. The study confirms the Hierarchical Risk Parity strategy outperforms the benchmarks (minimum risk portfolio and inverse volatility portfolio) in terms of tail risk-adjusted return, thus providing cryptocurrency investors better opportunity to manage the risk of cryptocurrency portfolio. The attempt behind the studies of Burggraf (2021); Platanakis & Urquhart (2019) mitigates the estimation error problem of traditional Markowitz frameworks in estimating the covariance of cryptocurrency portfolio under the limited number of samples versus the number of cryptocurrencies included.

As another strand of literature, several studies on the strategies under technical trading rules in the cryptocurrency market have been investigated. For instance, Corbet et al. (2019) first reported the results of various technical trading rules. By using the form of the break-out strategies of trading range and the oscillation of the moving average, their results reveal significant support for the value of moving average strategies in the cryptocurrency market. Supporting the findings of Corbet et al. (2019), Grobys et al. (2020) demonstrated that investors can obtain the opportunities for significant excess returns via simple technical trading rule strategies in the cryptocurrency market. These results from Corbet et al. (2019); Grobys et al.

(2020) imply possible inefficiency in the cryptocurrency market, which is consistent with the findings of the inefficiency in the cryptocurrency market (Bariviera, 2017; Urquhart, 2016; Vidal-Tomás et al., 2019b; Wei, 2018).

Despite these various portfolio strategies, many investors in the cryptocurrency market still suffer from extreme fluctuations that may result in a huge potential loss. The hypothesis that this huge potential loss is related to some extent to the high downside risk or tail risk of cryptocurrencies has started to receive attention. In this vein, several studies on the downside and tail risks in the cryptocurrency market have been studied. For example, Tan et al. (2021) investigated the relationship between downside risk (value at risk) and return in the cryptocurrency market using the fractionally cointegrated vector autoregression model. The authors showed that cryptocurrencies exhibit risk-return trade-off after the crisis, but the effect does not exist before the crisis, suggesting inconsistency in the existence of risk-return trade-off before and after the crisis. The authors concluded that their findings imply it is not wise to adopt the buy-and-hold strategy. Meanwhile, Borri (2019) studied the vulnerability of cryptocurrencies concerning the tail risk. The important finding of the study is that cryptocurrencies are highly exposed to tail risk within the cryptocurrency market. The author also suggested that idiosyncratic risk in the cryptocurrency market can be reduced, and better risk-adjusted conditional returns can be obtained by a proper portfolio strategy.

Related to the heavy tail of cryptocurrency, bubble behavior is one of the reported challenges for the cryptocurrency investors. In this regard, some researchers investigated the bubble behavior of cryptocurrency as a speculative asset. Fry & Cheah (2016) used econophysics models to examine shocks and crashes in the cryptocurrency markets, confirming that cryptocurrency markets have a significant speculative component and are extremely volatile. By combining long-tails with realistic measures of risk and return, Fry (2018) identified evidence of bubbles in the cryptocurrency market, arguing that liquidity risks generate heavy tails in the cryptocurrency markets. The authors suggest caution to cryptocurrency investors, noting that these markets are inherently risky. Additional study related to tail risk in terms of interconnectedness, which is highly correlated with bubbles, has been conducted by Ahelegbey et al. (2021). Specifically, Ahelegbey et al. (2021) examined bubbles in crypto assets, presenting a general method to incorporate systemic connectedness and tail risk. They showed a positive and significant relationship between the cryptocurrencies' tail risk and the weighted average market index. Additionally, they clustered cryptocurrency into two groups and identified the asymmetric nature of these groups. In particular, the first group's role is being the main agents of tail contagion, whereas the second group is vulnerable to tail contagion. The implications of the aforementioned research on the nature of cryptocurrencies, such as high downside risk, heavy tail risk, and bubbles, are clear. The buy-and-hold strategy is not always wise for investors in the cryptocurrency market.

In this vein, the first consideration is portfolio diversification strategies aimed at diversifying crypto assets in the investment basket of investors (Brauneis & Mestel, 2019; Burggraf, 2021; Čuljak et al., 2022; Liu, 2019a; Platanakis & Urquhart, 2019). Generally, these diversified portfolio strategies can offer the investor the benefit of the diversification effect. However, the diversified portfolio strategies in the cryptocurrency market may not be sufficient for an investor's cryptocurrency portfolio to be well-diversified, thereby not having a sufficiently lower risk. The required assumption that all assets included in the investment basket are uncorrelated (low-correlated) is not adequately applied to crypto assets,[1] as shown in many studies on the evidence of significant herding behavior in the cryptocurrency market (Amirat & Alwafi, 2020; Ballis & Drakos, 2020; Bouri et al., 2019a; Raimundo Júnior et al., 2022; Ren & Lucey, 2022; Vidal-Tomás et al., 2019a; Yarovaya et al., 2021; Youssef, 2020). Bouri et al. (2019a) studied herding behavior in cryptocurrencies by conducting a rolling window analysis. They showed that herding behavior exists and varies over time. Their logistic regression results reveal that herding is stronger as uncertainty increases. This is evidence of the high degree of co-movement in the cross-sectional returns across cryptocurrency markets, implying the tendency of crypto investors to mimic the investment decisions of others. Vidal-Tomás et al. (2019a) analyzed the presence of herding behavior among cryptocurrencies based on the cross-sectional absolute deviation of returns. The authors show that the cryptocurrency market is characterized by herding during bearish markets, arguing that inefficiency is evidenced by the presence of herding behavior. Another finding is that the smallest cryptocurrencies are herding with the largest ones; thus, investors base their investment decisions on the behavior of the main cryptocurrencies. Ballis & Drakos (2020) investigated the presence of herding behavior in the cryptocurrency market using a GARCH model and found that herding exists in both up and down markets. In other words, cryptocurrencies show a move in tandem and do not necessarily reflect their fundamentals. Moreover, Raimundo Júnior et al. (2022)

[1] From the mathematical point of view, a high correlation among asset returns exacerbates the estimation error problem. This is consistent with the finding of Brauneis & Mestel (2019), who specified that an equal-weighted portfolio strategy outperforms optimized portfolio strategies such as the lowest-risk portfolio strategy. We conjecture that this outperformance of naive equal-weighted strategy relative to the optimized portfolio is caused by estimation error which is substantially sourced from the high correlation among crypto assets.

also confirmed the existence of herding in the cryptocurrency market and identified a positive relationship between herding and market stress. All of these studies on cryptocurrency herding behavior have important implications for the portfolio management perspectives. Above all, evidence of herding implies the possibility of insufficient portfolio diversification and the exposure of investors to additional risk. Furthermore, as herding is stronger in a bearish market disturbance, this risk tends to be extreme[2] (Raimundo Júnior et al., 2022; Vidal-Tomás et al., 2019a). If all assets in our investment basket behave in the same direction, particularly if they have a stronger tendency in a negative market condition, exploiting sufficient diversification benefits from diversifying these assets is difficult.

The second alternative is to include an asset that has a hedge effect on the cryptocurrency portfolio. However, there is a scarcity of literature on assets with a hedge effect on crypto assets[3] (Hassan et al., 2021; Karim et al., 2022b). Hassan et al. (2021) investigated the hedge effect of precious metals on cryptocurrency risk. The authors find that only gold shows a reliable and consistent safe-haven effect on the uncertainty of crypto assets among other precious metals. Similarly, Karim et al. (2022b) studied quantifying the hedge and safe haven features of bonds against the cryptocurrency uncertainty indices, revealing that some bond indices show the hedge and safe haven effect on the cryptocurrency market, whereas none of the others offer these effects. A challenging point for this consideration is that although one finds

[2]These results of Raimundo Júnior et al. (2022); Vidal-Tomás et al. (2019a) are in line with the findings of studies on the asymmetric volatility structure (Baur & Dimpfl, 2018; Cheikh et al., 2020) of cryptocurrency that behavior of the cryptocurrency volatility varies across the regime of market conditions (e.g., positive or negative).

[3]At the time of writing, related studies are limited, although there are many studies on the hedge effect in the opposite direction (Bouri et al., 2017; Hasan et al., 2022; Selmi et al., 2018; Stensås et al., 2019; Urquhart & Zhang, 2019; Wang et al., 2019).

some assets having this hedging effect on a crypto asset, like gold or some bond index, we cannot perfectly hedge our position of the portfolio using this asset in a practical perspective. Simply, it is difficult to find an asset that moves in reverse completely unless someone artificially makes the asset move like that on purpose. In this regard, the put option may also be one of the desirable alternatives since the purpose of this asset is to hedge against the loss of the underlying asset as originally designed. However, put options with sufficient liquidity, strikes, and maturity with cryptocurrency as underlying assets are not always available in the market[4]. Hence, investors have no choice but to resort to other alternatives to protect the naked position of their cryptocurrency portfolio.

To sum up the literature, cryptocurrency investment has several critical challenging issues: (i) cryptocurrency has extreme volatility, particularly, high downside and heavy tail risk, and these are related to the bubble, which makes investment perilous; (ii) it shows herding behavior, and this trend is outstanding in negative market condition, thereby disturbing the construction of well-diversified portfolios with sufficient diversification effect; and (iii) it lacks counterpart assets aimed at hedging against a crypto asset. Considering the whole argument, we must consider a portfolio strategy that lowers its downside risk and aims at directly hedging against the potential loss of the cryptocurrency portfolio. Therefore, we consider that the cryptocurrency investor should select a strategy that directly addresses the aforementioned issues, such as a portfolio insurance strategy.

Portfolio insurance strategies are the framework for asset allocation that helps limit the downside risk by setting a risk tolerance level for determining the portion

[4]Hou et al. (2020) noted the lack of existence of the officially traded options in their study on option pricing of cryptocurrency.

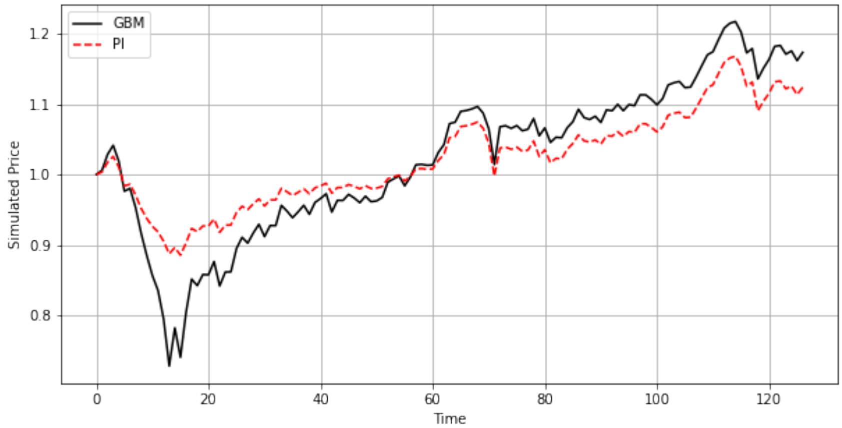


Figure 5.1: Simulated price and insured portfolio value.

Notes. This figure shows simulated price based on Geometric Brownian Motion and a portfolio value of portfolio insurance strategy using simulated price. When applied to the simulated stock price, the portfolio insurance strategy effectively limits the downside risk.

of a risky assets (Bertrand & Prigent, 2001), as illustrated in Figure 5.1. It is the strategic asset allocation model aimed at directly protecting the downside risk and hedging the loss of underlying assets under market stress conditions. Because of these advantages of an explicit approach for protection and flexibility for implementation, this portfolio insurance strategy has obtained enormous popularity among practitioners and investors for a long time in the global financial system (Dichtl & Drobetz, 2011). Many academic researchers have also expressed great interest, leading to the development of various portfolio insurance strategies. Several researchers have studied portfolio positions protected by put options, also known as protective put, as an original version of the option-based portfolio insurance (OBPI) strategy (Figlewski et al., 1993; Pozen, 1978). This protective put strategy, however, lacks flexibility because it requires a sufficient number of put options with liquidity, strikes, and maturity for the underlying asset in the market. To overcome the limitation of a lack of sufficient put options, Leland & Rubinstein (1988); Rubinstein & Leland

(1981) proposed a novel portfolio insurance strategy called the synthetic put (SP) strategy. Given the level of protection and volatility, the SP strategy synthesizes the portfolio's position in the protective put strategy by dynamically adjusting the weight of risky and risk-free assets. A stop-loss (SL) portfolio insurance strategy is another type of naive portfolio insurance strategy (Bird et al., 1988; Rubinstein, 1985). This SL strategy is a very simple method in that investors are only required a fixed level of SL to guarantee that the portfolio value at maturity time has given a level of protection. In addition to SP and SL portfolio strategies, as a successful strategy with popularity, a constant proportion portfolio insurance (CPPI) strategy was proposed by Black & Jones (1987, 1988); Black & Perold (1992); Perold & Sharpe (1988), where the strategy allocates more weight to risky assets when the portfolio value is high and vice versa. The popularity of this strategy is easily explained by its flexibility in that it only requires the specification of two parameters (protection level and risk exposure), and it mitigates the volatility estimation issues in the SP strategy. Estep & Kritzman (1988) proposed the time-invariant portfolio protection (TIPP) strategy as an extension of the CPPI strategy. When compared to the CPPI strategy, this TIPP strategy provides investors with elastic time-varying protection bound by dynamically adjusting the floor value. Jiang et al. (2009) proposed a value at risk-based portfolio insurance (VBPI) strategy as another novel portfolio insurance model. This strategy incorporates the concept of value at risk (VaR) into the portfolio insurance framework, thereby aligning the value of VaR of the investor's portfolio with the level of protection. This model's contribution is that it analytically incorporates the concept of downside risk into the portfolio insurance strategy as an explicit protection level of strategy.

As another strand of study, some researchers have focused on the performance evaluation that is more adequate to the structure of portfolio insurance strategy (Annaert et al., 2009; Bertrand & Prigent, 2011; Zieling et al., 2014). These studies are motivated by the issue that the Sharpe ratio widely used for performance measure of portfolio strategy is actually not suitable for measuring the performance of the portfolio insurance strategy because the return distribution generated by portfolio insurance strategy is a non-normal distribution rather than normal[5]. Under the assumption that returns are normally distributed, investors know all about the return distribution from expectations and standard deviation. In other words, the Sharpe ratio is an adequate performance measure to capture the entire distribution of normal returns due to the symmetry of the normal distribution. However, return distribution generated by portfolio insurance strategy is highly (positively) skewed than the normal by nature because it is a strategy aimed at directly cutting the loss from the negative value of the investor's portfolio. In this respect, some researchers proposed using the novel performance measure for portfolio insurance strategy. For instance, Annaert et al. (2009) proposed a block-bootstrap simulation and evaluated portfolio strategies using the measures that capture the non-normality of the strategy's return distribution. VaR, expected shortfall (ES), Omega ratio, and stochastic dominance criteria are among these measures. The authors found that although the buy-and-hold strategy has a higher average return and Sharpe ratio, other performance measures that capture the non-normality show opposite results, indicating that the average return and Sharpe ratio are not always proper measures of performance for the portfolio insurance framework. In line with this study, Bertrand &

[5] Leland (1999) argued that portfolio insurance strategies will be under-evaluated based on the traditional CAPM performance measure.

Prigent (2011) analyzed the Omega ratio, which takes account of the entire return distribution as a performance measure for portfolio insurance strategy. The authors investigated the empirical results from the simulation using geometric Brownian motion and the US stock market data, revealing that CPPI outperforms OBPI strategy in terms of Omega ratio.

Although the aforementioned studies demonstrated that these novel performance measures can provide the proper criteria to evaluate the results of the implementation of portfolio insurance strategies, the rationale for portfolio choice and preference of each investor over portfolio insurance strategies is not uncovered. In this regard, studies taking into account investor's utility on justification of choice for portfolio insurance strategies have been reported (Benninga & Blume, 1985; Dichtl & Drobetz, 2011; Gaspar & Silva, 2021). First, Benninga & Blume (1985) studied the portfolio choice of portfolio insurance strategies based on the investor's utility. The authors theoretically revealed that the optimality of portfolio insurance strategies depends on the investor's utility. After the research of Benninga & Blume (1985), additional research on the impact of investor utility in the portfolio insurance strategy context has been investigated. The basic argument behind these studies is that depending on the type of utility function of investors and corresponding parameters (e.g., risk aversion and loss-aversion coefficient), the perceived performance of outcomes might differ greatly. Dichtl & Drobetz (2011) evaluated the outcomes of portfolio insurance and benchmark strategies using the concept of prospect theory introduced by Kahneman & Tversky (1979); Tversky & Kahneman (1992). The study shows that a portfolio insurance strategy is preferred to a buy-and-hold strategy for the type of prospect theory investor. Similarly, Gaspar & Silva (2021) studied the results of portfolio insurance strategy using the expected utility theory and prospect theory, explaining the popularity of portfolio insurance.

Finally, based on these various portfolio insurance strategies and performance measures, many studies have shown the empirical results of better investment opportunities caused by the outperformance of portfolio insurance strategies. Numerous pieces of evidence have been shown in the research on the global financial markets. The empirical results in the US (Annaert et al., 2009; Bertrand & Prigent, 2011; Dehghanpour & Esfahanipour, 2018; Garcia & Gould, 1987; Lee et al., 2011; Zieling et al., 2014), European (Agić-Šabeta, 2016; Dichtl & Drobetz, 2011; Dichtl et al., 2017), and other emerging markets (Agic-Sabeta, 2017; Jiang et al., 2009; Lee et al., 2011), and under simulation (Buccioli & Kokholm, 2018; Gaspar & Silva, 2021; Lee et al., 2011; Tawil, 2018) support the popularity of portfolio insurance strategies among practitioners who participate in the financial market. Bird et al. (1990) mentioned that a portfolio of highly volatile assets under negative market conditions sees minimum degradation in performance via a portfolio insurance strategy. As they noted, portfolio insurance can be considered a suitable alternative strategy for investing in cryptocurrency that has the aforementioned challenges to crypto assets investors who want their portfolio value to be directly hedged by limiting the downside risk while benefiting from the upside market.

Considering the literature on cryptocurrency and portfolio insurance together, many studies have dealt with various strategies for the cryptocurrency market and portfolio insurance in various markets. To the best of our knowledge, no research has been conducted on portfolio insurance in the cryptocurrency market, although investors in this market have suffered from extreme fluctuations in market conditions regularly. The cryptocurrency market has become more mature than before, and many retail and institutional investors have decided that this is a viable form of investment vehicle for managing their portfolios to seek significantly positive alphas (Bianchi & Babiak, 2022). Along this line, as cryptocurrency is increasingly recognized as an investable asset by many investors in the financial market, the demand for extensive research on investment strategies to protect downside risk, such as portfolio insurance in the cryptocurrency market, is increasing. Therefore, this study investigates the comprehensive empirical results from portfolio insurance strategies in the cryptocurrency market to fill the related research gap. In particular, the research questions we will investigate are two folds. First, what meaningful empirical results (i.e., supportive evidence of a better level of performance compared to benchmarks) are shown in the cryptocurrency market through portfolio insurance analysis in terms of downside risk and traditional performance measures of portfolio strategy? Second, how will the impact of an investor's utility form on portfolio choices or preferences be demonstrated in the cryptocurrency market compared to the traditional stock market? If our research successfully clarifies the empirical findings from a portfolio insurance strategy in the cryptocurrency market in terms of these research questions, it can assist investors in reaping the benefits of the bull market while limiting the downside risk in the bearish market.

Our findings in downside risk results indicate a dominant performance of portfolio insurance strategies compared to buy-and-hold in the cryptocurrency market in terms of downside risks, showing that portfolio insurance strategies reduce the simple buy-and-hold risk of cryptocurrency. For the test of the impact of parametric conditions, we check protection level, money market condition, and frequency. Risk exposure is empirically demonstrated to be closely related to protection level and money market conditions. If the frequency is changed while remaining fundamentally similar to the overall result, the outstanding evidence for the degradation of SP and VBPI due to volatility estimation error is confirmed. Another finding in the utility results of investors suggests that expected utility theory investors prefer portfolio insurance strategies to buy-and-hold as risk aversion increases, whereas an interesting finding confirming prospect theory investors show the opposite trend in their portfolio choice. In other words, investors are more sensitive to risk-seeking in the loss domain than risk aversion in the gain domain. Furthermore, the greater the investor's loss-aversion, the more they prefer portfolio insurance. In a comparison between the cryptocurrency and traditional stock market results, we confirm the significant difference between the two market results, implying the pronounced benefit of portfolio insurance for the crypto asset investment. Overall, our research provides investors with the criteria for designing portfolio insurance as a customized strategy in terms of downside risks and utility.

Chapter 5 is organized as follows: Section 5.2 describes various portfolio insurance strategies. Section 5.3 introduces the details of the downside risk measures. Section 5.4 explains the concept of expected utility and prospect theory. Section 5.5 presents the data and experimental design. Section 5.6 discusses the empirical results of portfolio insurance strategies in the cryptocurrency market in terms of downside risk and utility, respectively.

5.2 Portfolio insurance strategies

As aforementioned, various portfolio insurance strategies have been suggested in the literature. The objective of the portfolio insurance strategy is to limit the potential downward loss of the underlying assets to a specified floor value while exploiting the benefit of participation in the upside market movement. We use the following portfolio insurance strategies: SL, SP (in Section 4.3.1), CPPI, TIPP, and VBPI strategies. In this section, we provide reviews of these strategies.

5.2.1 SL strategy

One of the simplest methods of implementing a portfolio insurance strategy is the SL portfolio insurance strategy (Bird et al., 1988; Rubinstein, 1985). The investor only needs the pre-specified level of protection (floor value, F) and takes a 100% risky asset position at the initial time in this strategy. The main idea of this strategy is that to guarantee a given level of protection of SL to be a portfolio value at maturity time, investors liquidate their position of risky assets and move to risk-free assets when their portfolio value goes under the current discounted value of the protection level[6]. This condition is described as follows:

$$V_t \leq Fe^{-r(T-t)}, \tag{5.1}$$

where V_t is the portfolio value at time t, T is maturity, and r is the risk-free rate, respectively. As long as the condition of $V_t > Fe^{-r(T-t)}$ holds, the investor's risky asset position maintains and helps to obtain the positive upside part of market movement.

[6]After liquidation, this risk-free position is held until the time point at maturity T.

This strategy is very popular due to its flexibility and easy implementation, in that it does not require any assumptions or estimation of parameters. Meanwhile, the limitation of this simple method is that by cutting the loss based on naive conditions, investors no longer obtain the potential gain from the upside market after the liquidation of their position. Another disadvantage might be transaction costs since all risky assets are sold in the investor's portfolio only once but are substantial.

5.2.2 CPPI strategy

As a successful alternative strategy to portfolio insurance, a constant proportion portfolio insurance strategy is proposed by Black & Jones (1987, 1988); Black & Perold (1992); Perold & Sharpe (1988). CPPI strategy aims at mimicking the movement of allocation strategy where investor's capital moves to risky assets when the portfolio value goes up, whereas capital shifts to risk-free assets in opposite market condition, thereby guaranteeing the insured value F at maturity time T. Hence, floor value F, the first parameter for CPPI strategy, has to be decided as the insured value that is the final protection level of an investor's portfolio as follows:

$$F = a \cdot V_0, \tag{5.2}$$

where a is the percentage floor and V_0 is the initial value of the portfolio. After determining this floor value, the investor can calculate the so-called "cushion" (C_t) which is the investor's risk capital at time t. The cushion (C_t) is the difference between the portfolio value at time t (V_t) and the current discounted value of the floor F as follows:

$$C_t = V_t - Fe^{-r(T-t)} \tag{5.3}$$

The multiplier m is the second pre-specified input parameter. The multiplier m, a constant value, is assumed to be greater than one and serves to provide an option-like payoff structure for the insured portfolio (Jiang et al., 2009). In economic terms, m denotes the degree of sensitivity to market movement. Following the determination of m, the risky exposure (the amount invested in the risky asset) at time t (E_t) can be calculated as:

$$E_t = m \cdot C_t \tag{5.4}$$

However, this unconstrained specification of Eq. 5.4 can lead to short positions in the risk or risk-free asset (Dichtl & Drobetz, 2011). To rule out short-sale, implementing strategy as simple and practical as possible in terms of commercial applications to the practitioner, we used the modified version of the specification of the cushion (Annaert et al., 2009; Benninga, 1990; Dichtl & Drobetz, 2011; Huu Do, 2002):

$$E_t = max(min(m \cdot C_t, V_t), 0) \tag{5.5}$$

At every rebalancing date, the amount E_t is dynamically invested in a risky asset, and the remainder is invested in the risk-free asset. As shown in Eq. 5.5, the cushion will increase when the market goes up. As a result, the risk exposure increase, leading to further capital shift to the risky asset. In contrast, the cushion will decrease when market conditions are negative, thereby causing a fund shift to the risk-free asset. If the market goes down continuously so that the value of the cushion reaches zero, all capital in the portfolio is fully invested in the risk-free asset, thereby limiting the investor's wealth to below the floor value. Additionally, a higher value of multiplier m and a lower value of floor F imply a more risky investment since these lead to larger risk exposure. Hence, if investors want a higher degree of protection level, they must specify a higher floor value and a lower multiplier that trade-off participation in a potential upside market.

5.2.3 TIPP strategy

Estep & Kritzman (1988) proposed a TIPP strategy, a variant of the CPPI strategy. In this TIPP strategy, floor value is dynamically ratched-up when the portfolio value goes up. From this ratched-up floor value, the portfolio value is guaranteed the original protection level as pre-specified and an increased protection level by combining interim upside. Generally, the TIPP strategy with this dynamic ratched-up floor value is expected to lead to a higher percentage proportion of risk-free assets in an investor's portfolio compared to the CPPI strategy. Likewise to the CPPI strategy, in the TIPP strategy, investors must decide on the initial floor value F_0 and the multiplier m. Contrary to the CPPI strategy, the floor value is dynamically adjusted by comparing this to the new floor value calculated using the current portfolio value. Specifically, when the multiplication of the current portfolio value by the floor percentage is larger than the previous floor value, this value is set to the new floor value; otherwise, the previous value is kept, as described in the following:

$$F_t = \begin{cases} a \cdot V_t & \text{if } a \cdot V_t > F_{t-1} \\ F_{t-1} & \text{otherwise} \end{cases}, \tag{5.6}$$

where a is the percentage floor, and V_t is the current value of the portfolio. As a result, the cushion (C_t) of TIPP can be calculated as follows:

$$C_t = V_t - F_t e^{-r(T-t)} \tag{5.7}$$

Risk exposure E_t is also obtained similar to the CPPI strategy as follows:

$$E_t = max(min(m \cdot C_t, V_t), 0) \tag{5.8}$$

TIPP strategy is proposed to address the disadvantages of SP and CIPP strategies. In particular, Estep & Kritzman (1988) argued that in the TIPP strategy, protection is continuous, computation is simple, and the cost of an equivalent protection level is lower. Additionally, investors do not have to consider the ending date, the effect of time, or volatility estimation. In contrast to the attractiveness of the TIPP strategy, Choie & Seff (1989) criticized its limitations of the TIPP strategy. They argued that the TIPP strategy decreases the likelihood of participation in any upside market due to excessive capital shift from risky assets to risk-free assets via the ratched-up floor value.

5.2.4 VBPI strategy

Due to discrete portfolio adjustments or insufficient liquidity, the final insured portfolio value is practically not guaranteed at the pre-specified protection level in standard portfolio insurance strategies. In contrast, the VBPI strategy proposed by Jiang et al. (2009) seeks to achieve the insurance level through a probabilistic approach based on the concept of confidence level rather than a strictly fixed percentage value of protection level, to provide flexibility to a strategy. As a result, the VBPI strategy dynamically adjusts the proportion of investor's capital allocated to the risky and risk-free assets in order to achieve an insured portfolio with a VaR at a confidence level based on the return distribution as its worst case of loss.

The VBPI strategy is under the assumption that the price of risky asset S_t

follows a geometric Brownian motion as follows:

$$dS_t = S_t(\mu dt + \sigma dW_t),\ S_0 > 0, \tag{5.9}$$

where μ is the expected return, σ is the volatility, and W denotes a standard Brownian motion. The value of the risk-free asset (B_t) is also described as follows:

$$dB_t = rB_t dt,\ B_0 > 0, \tag{5.10}$$

where r is a constant risk-free rate, and $\mu > r$.

If w denotes the proportion weight of risk-free assets in the insured portfolio and $1-w$ that of the risky asset, the final value (V_T) at maturity time T over the initial value (V_0) of the insured portfolio can be described as follows:

$$\begin{aligned}\frac{V_T}{V_0} &= w\frac{B_T}{B_0} + (1-w)\frac{S_T}{S_0} \\ &= we^{rT} + (1-w)e^{(\mu-0.5\sigma^2)T+\sigma W_T}.\end{aligned} \tag{5.11}$$

Then, based on the VaR approach, the VaR for the portfolio at pre-specified confidence level α ($VaR_{1-\alpha}$) over V_0 is obtained as:

$$\frac{VaR_{1-\alpha}}{V_0} = E(\frac{V_T}{V_0}) - \frac{F}{V_0}, \tag{5.12}$$

where $E(\frac{V_T}{V_0})$ is the expectation of $\frac{V_T}{V_0}$ and F is the insured amount. According to Jiang et al. (2009), since the horizon of VaR coincides with the period of investment, $\frac{VaR_{1-\alpha}}{V_0}$ can be obtained by plugging Eq.5.11 into Eq.5.12 as follows:

$$\frac{VaR_{1-\alpha}}{V_0} = we^{rT} + (1-w)e^{\mu T} - \frac{F}{V_0}. \tag{5.13}$$

From the other side, the VaR for the insured portfolio is equal to the VaR for the risky asset since the insured portfolio includes only risky assets and risk-free assets.

Hence, $\frac{VaR_{1-\alpha}}{V_0}$ can be described as follows:

$$\frac{VaR_{1-\alpha}}{V_0} = (1-w)(e^{\mu T} + e^{(\mu-0.5\sigma^2)T + z_{1-\alpha}\sigma\sqrt{T}}), \tag{5.14}$$

where $z_{1-\alpha}$ denotes the $(1-\alpha)$-quantile of the standard normal distribution. By plugging Eq. 5.13 into Eq. 5.14, we have

$$w = \frac{\frac{F}{V_0} - e^{(\mu-0.5\sigma^2)T + z_{1-\alpha}\sigma\sqrt{T}}}{e^{rT} - e^{(\mu-0.5\sigma^2)T + z_{1-\alpha}\sigma\sqrt{T}}}. \tag{5.15}$$

Eq. 5.15 denotes the weight of the risk-free asset of the static VBPI strategy, if the proportion of the portfolio is not rebalanced until the terminal end of the investment horizon. However, the weight of risk-free assets of the dynamic version of the VBPI strategy, which dynamically adjusts the investment proportion between risky assets and risk-free assets, can be described as follows:

$$w = \frac{\frac{F}{V_t} - e^{(\mu-0.5\sigma^2)(T-t) + z_{1-\alpha}\sigma\sqrt{T-t}}}{e^{r(T-t)} - e^{(\mu-0.5\sigma^2)(T-t) + z_{1-\alpha}\sigma\sqrt{T-t}}}. \tag{5.16}$$

At every rebalancing point, investors adjust the proportion[7] of their portfolio between the risk-free asset (w) and risky asset ($1-w$) based on Eq. 5.16. For practical reasons, w is set to zero (one) when w is less than zero (larger than one) to avoid leverage and short sales. As seen in Eq. 5.16, two parameters, final insured value (F) and the confidence level (α), must be decided in advanced[8].

The major advantage of the VBPI strategy is that, in incorporating the concept of the VaR to the protection level of the insured portfolio, this strategy can control

[7]Note that there is a V_t term in Eq. 5.16 rather than V_0. V_t denotes the value of the insured portfolio, reflecting the market condition. A higher value of V_t implies a higher value of w, suggesting that if the market goes up, the investment in the risk-free asset decreases and vice versa (Jiang et al., 2009).

[8]As the economic implication, these two parameters represent the degree of the investor's risk tolerance (Jiang et al., 2009).

the capability to meet the protection level. In contrast, the drawback of the VBPI strategy is that the investor must estimate the expected return and volatility of the risky asset. As a result, the performance of the strategy depends on the precision of the estimation of these parameters.

5.3 Downside risks

Portfolio insurance strategies seek to directly limit downward market movement while maintaining the potential for upward gains. In addition, the payoffs of portfolio insurance strategies are generally non-linear in relation to the underlying risky asset (Bertrand & Prigent, 2011). As a result, the insured portfolio's return distribution becomes skewed and asymmetric, with a left short tail and a right heavy tail. Because of their skewness and asymmetry, general performance measures, such as the Sharpe ratio, are argued to be inappropriate performance measures in the context of portfolio insurance (Annaert et al., 2009; Bertrand & Prigent, 2011; Dichtl & Drobetz, 2011). To address this issue of inadequacy of traditional performance measures, several researchers have proposed and studied various downside risks as performance measures for portfolio insurance strategies in the literature (Acerbi, 2004; Artzner et al., 1999; Pedersen & Satchell, 1998; Szegö, 2002); This consists of maximum drawdown (MDD), average drawdown (AvDD), VaR, ES, semideviation, and Omega ratio. Therefore, this section briefly reviews these performance measures.

5.3.1 MDD and AvDD

A drawdown is a percentage of how much the value of an investor's portfolio given a portfolio strategy is down from the peak until it recovers back to the peak during a specific investment horizon. After being introduced by Grossman & Zhou (1993) for the first time, many studies have used it as one of the measures of the downside risk of portfolio strategy (Chekhlov et al., 2004, 2005; Hamelink & Hoesli, 2004; Johansen & Sornette, 2000; Pospisil & Vecer, 2010). The drawdown process of the portfolio strategy at time t is defined as follows:

$$d_t = \{\frac{V_t}{\max_{\tau \leq t} V_\tau} - 1 : t \in \{1, ..., T\}\}, \tag{5.17}$$

where V_t is the value of the portfolio at time t and $\max_{\tau \leq t} V_\tau$ is the maximum of all values up to time t.

A MDD is the maximum value of the series of drawdown. This MDD refers to the loss suffered when an investor buys an asset at a local maximum and sells it at the next local minimum. The MDD at time T is defined as follows:

$$MDD_T = \max_{t \leq T} d_t. \tag{5.18}$$

Next, an AvDD is the average value of the series of drawdown. Hence, AvDD at time T is defined as follows:

$$AvDD_T = \frac{1}{T} \Sigma_{t=1}^{T} d_t. \tag{5.19}$$

In the context of portfolio insurance, investors can use MDD and AvDD as the performance measures of downside risk. To some extent, these can serve as an evaluation for protection against the downside risk of comparative portfolio insurance strategies because the main demand of investors for the desirable property of an insured portfolio is ultimately to increase the likelihood of limiting the downward losses when the market condition begins to be negative.

5.3.2 VaR

In finance, VaR is widely used as the standard downside risk measure to quantify the market or portfolio risk (Hotta et al., 2008). The definition of VaR is a measure of the risk of possible financial losses of an asset or portfolio over a specific period. Initiated by Dowd (1998); Jorion (1997), a theoretical study on the VaR as a risk measure has been conducted. Since then, a considerable number of studies have applied risk management methods based on VaR for portfolio management (Alexander & Baptista, 2003; Gaivoronski & Pflug, 2005; Huang et al., 2009; Quaranta & Zaffaroni, 2008).

Let X be a random variable of an asset and $F(x)$ be the cumulative distribution function of the corresponding random variable, such that $F(x) = P(X \leq x)$. Then, VaR_α at a fixed level α[9] is defined as the α-quantile of X as follows:

$$VaR_\alpha(X) = F^{-1}(\alpha) \tag{5.20}$$

Its popularity is easily explained by its simplicity. It is intuitively understood by a single number for a value of potential loss over a given period. Another reason is that VaR, as an asymmetric risk measure, mitigates the problem of the symmetric volatility measure by focusing on the left tail of the return distribution (Ibragimov & Walden, 2007; Jorion, 1997). Due to this advantage, VaR can be considered to be an appropriate performance measure for a portfolio insurance strategy under which return distribution is non-normally and asymmetrically distributed.

[9] A larger confidence level implies a higher degree of risk aversion since larger confidence levels will involve more negative VaR.

5.3.3 ES

A crucial drawback of VaR is the absence of an indication of excessive losses beyond VaR. This can lead to critical real-world problems since risk information via VaR may misguide investors (Yamai & Yoshiba, 2005). To address this limitation, the ES that measures the average loss below the VaR is introduced (Acerbi & Tasche, 2002). Further research on justifying the advantages of an ES over VaR has also been conducted in terms of tail risk management on allocating economic capital (Acerbi & Tasche, 2002; Yamai & Yoshiba, 2005).

The ES refers to the conditional expectation of loss given the loss beyond the level of VaR. Let X be a random variable of the loss of an asset. The ES is defined as follows:

$$ES_\alpha(X) = E[X|X \geq VaR_\alpha(X)]. \tag{5.21}$$

That is, the ES represents the average loss under conditions where the loss exceeds VaR. ES, like VaR, aims to capture tail risk and is thus regarded as an appropriate performance measure for the framework of portfolio insurance strategies.

5.3.4 Semideviation

The semideviation refers to the square root of the expected squared deviation from the mean under the condition that the random variable (X) does not exceed the mean ($E(X)$). This semideviation (σ_-) is defined as follows:

$$\sigma_- = E[(E[X] - X)^2 \mathbb{1}_{X \leq E[X]}]^{\frac{1}{2}}, \tag{5.22}$$

where $\mathbb{1}_{X \leq E[X]}$ denotes the indicator function of $X \leq E[X]$.

The semideviation of returns is a plausible measure of risk, considering many advantages. Due to these abundant advantages, various studies have used semideviation as a downside risk measure to use as a criterion for the optimal portfolio choice problem (Chiodi et al., 2003; Ogryczak & Ruszczyński, 1999, 2001; Pınar & Paç, 2014; Vercher & Bermúdez, 2015). The superiority of semideviation as a downside risk measure is three-fold. First, investors obviously focus mainly on the downside volatility rather than the upside volatility. Additionally, the semideviation is more useful than the simple standard deviation in the case of the asymmetric underlying return distribution as well as the symmetric case (Jafarizadeh & Khorshid-Doust, 2008). Furthermore, the semideviation incorporates the information on standard deviation and skewness into one measure (Estrada, 2004, 2007). Accordingly, in that semideviation well addresses the downsides of return, asymmetry, and skewness, it is considered an appropriate performance measure for the evaluation of portfolio insurance strategies under this study.

5.3.5 Omega ratio

Keating & Shadwick (2002) proposed the Omega performance measure which is one of the powerful measures that consider the entire distribution of portfolio return. The Omega ratio evaluates the gains and losses with respect to the pre-specified threshold of return. By doing so, the Omega ratio can consider the whole distribution rather than a particular moment of the return (e.g., volatility or skewness) while requiring no assumption of parameters on the return distribution. Additionally, the authors successfully incorporated the concept of loss-aversion studied by Hwang & Satchell (2010); Tversky & Kahneman (1992) into this measure, by using the downside lower partial moment.

In particular, the Omega ratio is defined as the probability-weighted ratio of the expectation of gains to the expectation of losses relative to a return threshold (Keating & Shadwick, 2002). At this point, the gains refer to a return that is above the threshold, whereas the losses refer return that is below the threshold[10]. In this way, the Omega ratio splits the return into two sub-parts by using a threshold. The exact mathematical form of the Omega ratio is represented by the following:

$$\Omega_x(L) = \frac{\int_L^b (1 - F(x))dx}{\int_a^L F(x)dx}, \tag{5.23}$$

where $F(\cdot)$ denotes the cumulative distribution function of the return $X \in (a, b)$ and L denotes the pre-specified threshold of return.

Interestingly, Kazemi et al. (2004) showed that the Omega ratio can be written as:

$$\Omega_x(L) = \frac{\mathbb{E}_{\mathbb{P}}[(X - L)^+]}{\mathbb{E}_{\mathbb{P}}[(L - X)^+]}, \tag{5.24}$$

where $\mathbb{P}$ denotes the probability distribution. In other words, the Omega ratio is the ratio of the expected gains above the threshold to the expected losses below the threshold. More specifically, from the mathematical point of view, Kazemi et al. (2004) observed that the Omega is equivalent to the ratio of the expected value of a call option payoff over the expected value of payoff of a put option for the underlying risky asset X with a strike price L corresponding to the threshold evaluated under the historical probability $\mathbb{P}$ rather than risk neutral one (Bertrand & Prigent, 2011), as follows:

[10] A threshold implies a minimum acceptable return. According to Bertrand & Prigent (2011), thresholds between zero and the risk-free rate are proper in evaluating an investment using the Omega ratio. The authors mentioned that the threshold corresponds to the concept of protection of funds in the economic term.

$$\Omega_x(L) = \frac{e^{-rT}\mathbb{E}_{\mathbb{P}}[(X-L)^+]}{e^{-rT}\mathbb{E}_{\mathbb{P}}[(L-X)^+]} = \frac{Call(L)}{Put(L)}. \tag{5.25}$$

Generally, a strategy with a higher Omega ratio is considered more appealing since the investors prefer the strategies with higher gains above the threshold return and lower losses below the threshold return.

The major advantage of the Omega ratio is that it is an appropriate performance measure to compare non-normally distributed portfolio returns since it considers all the moments of the return distribution, such as volatility, skewness, and kurtosis, compared to traditional measures (e.g., Sharpe ratio). Due to this advantage, the Omega has been applied across an amount of literature in financial studies, including portfolio insurance strategies with non-linear payoff features and asymmetric returns (Bertrand & Prigent, 2011).

5.4 Investor's utility

5.4.1 Expected utility theory

Expected utility theory, which was initially introduced by Bernoulli in 1738, was developed by Von Neumann & Morgenstern (1947). Von Neumann & Morgenstern (1947), who modeled the decisions of economic agents based on their preference under uncertainty. This theory assumes that agents are rational investors, and thus, their decisions are rational. Each agent bases his or her decision on the utility provided by wealth (w), rather than the monetary value of the wealth. As a result, the optimal choice problem for investors is reduced to a maximization problem of the final value of investors' expected utility, which is described as follows:

$$E[u(w)] = \Sigma_{n=1}^{N} p_i \cdot u(w_i), \tag{5.26}$$

where $u(w)$ is the utility function provided by wealth w and p_i is the probability that i-th outcome w_i will occur.

There are three principles behind this expected utility theory. First, the expectation of the utility of the investor's choice is equal to the summation of the utilities weighted by the probability of all possible outcomes (Dichtl & Drobetz, 2011). Second, if the alternative portfolio choice generates excessive value compared with the existing asset portfolio, it is considered more acceptable. Third, every investor is assumed to be strictly risk-averse rather than risk neutral or risk-seeking.

Utility is a function that maps the monetary value of wealth to the magnitude of an investor's perceived preference. Among various utility functions, hyperbolic absolute risk aversion (HARA) utility is widely used in the general form of Von Neumann & Morgenstern (1947)'s utility functions. This HARA utility function is described as follows:

$$u(w) = \frac{1-\gamma}{\gamma}(\frac{aw}{1-\gamma} + \beta)^{\gamma}, \tag{5.27}$$

with the parameters a and β such that $a > 0$ and $\frac{aw}{1-\gamma} + \beta > 0$. Various special forms of this HARA utility are commonly used in prior studies related to an investor's utility (e.g., quadratic utility and exponential utility function). Quadratic utility can be obtained by HARA utility if $\gamma = 2$, as follows:

$$u(w) = w - \frac{b}{2}w^2, \tag{5.28}$$

where $b > 0$ implies risk aversion. Exponential utility can also be obtained by HARA utility if γ goes negative infinity and $\beta = 1$ as follows:

$$u(w) = -e^{-bw}, \tag{5.29}$$

where $b > 0$ also implies risk aversion. The expected utility theory takes into account the investor's risk aversion, which is the economic concept that the investor would prefer certain outcomes over lottery risk by avoiding a fair gamble. Expected utility theory investor's utility functions such as quadratic and exponential utility show concavity and diminishing marginal wealth utility. As a result, the utility function's curvature b measures the level of risk aversion.

5.4.2 Prospect theory

Several behavioral phenomena that contradict the expected utility theory have been reported; thus, related literature has discussed the rational framework to describe how investors assess the possible outcome of gains and losses. As a seminal study of behavioral finance, which addresses this issue, Kahneman & Tversky (1979) proposed a prospect theory[11] where the investor's choice is determined by preference under framing of gains and losses. More specifically, prospect theory investors evaluate their choices based on the potential gains and losses by comparing to a pre-specific reference point, contrary to the expected utility theory of investors who evaluate their choices based on the overall value of expected wealth. This phenomenon refers

[11] After research on this prospect theory, Tversky & Kahneman (1992) proposed cumulative prospect theory, which is the modified version of the original prospect theory. The crucial difference is that cumulative prospect theory weights the prospect value based on the cumulative weights obtained by the probability weighting function. From the economic perspective, this indicates the accounts for the investor's sensitivity to extreme events. To obtain reliable results, we also conducted the simulation under a cumulative prospect framework. However, we confirm essentially similar result to the result from the original prospect theory. To maintain our main argument clearly, we omit the cumulative prospect results in our study. Full results are available upon request.

to the *framing* concept.

The key point is that, unlike expected utility theory investors, who are always risk-averse, prospect theory investors are assumed to be risk-averse in the domain of gains while seeking risk in the domain of losses. Hence, value function[12] of the gain domain is concave, whereas the value function of the loss domain is convex, implying an S-shaped value function. Another important aspect of prospect theory is that it assumes investors have a loss-aversion personality. Loss-aversion implies that the impact of losses is greater than the corresponding gains with the same deviation from the reference point. Prospect theory investors, in other words, pay more attention to potential losses than potential gains. As a result, the value function should be steeper in the domain of losses relative to the domain of gains.

Accordingly, to capture the aforementioned-assumed properties of investors, the authors suggest the value function considering different features of the domain of gains and losses, considering deviations from a reference point (Δx) as outcomes as follows:

$$v(\Delta x) = \begin{cases} (\Delta x)^{\alpha} & \text{if } \Delta x \geq 0 \\ -\lambda \cdot (-\Delta x)^{\beta} & \text{otherwise} \end{cases}, \tag{5.30}$$

where α denotes the coefficient of risk aversion in the gain domain, β denotes the coefficient of risk-seeking in the loss domain, and λ denotes the coefficient of loss-aversion[13]. By averaging the value function in Eq. 5.30, the mean prospect value of

[12]In prospect theory, the value function indicates the standard utility function in expected utility theory.

[13]Kahneman & Tversky (1979) suggested using $\alpha = \beta = 0.88$ and $\lambda = 2.25$. These α and β represent the concavity in gains and convexity in losses, respectively. λ denotes the loss-aversion of the investor. $\lambda = 2.25$ implies that investors respond to the impact of losses more than twice as much as the impact of gains.

a portfolio strategy (p) is obtained as follows:

$$MPV_p = \frac{\Sigma_{i=1}^{N} v(\Delta x_i)}{N} \tag{5.31}$$

where Δx_i is the i-th outcome sorted in ascending order.

5.5 Data and experimental design

5.5.1 Data

To obtain reliable and robust results, we use the following criteria for the filtering process to select cryptocurrencies for our empirical study. First, cryptocurrency price data must cover at least a 5-year period to include a sufficiently long time series that captures most of the dynamics in the cryptocurrency market during periods. Second, since portfolio insurance strategies require risky assets as investment assets, stable coins (e.g., Tether) are excluded. Finally, the selected cryptocurrencies in our data must be included in the list of the 50 largest cryptocurrencies in terms of market capitalization at the time of the writing.

As a result of the filtering process, we obtain the price of seven major cryptocurrencies from May 2017 to April 2022. Bitcoin, Ethereum, Ethereum Classic, Litecoin, Ripple, Stella, and Monero are all part of it. Figure 5.2 depicts the daily price and return of these seven cryptocurrencies over the covered period. These seven cryptocurrencies account for 89.6% of market capitalization in May 2017 and 64.5% in April 2022, respectively. The daily prices of these cryptocurrencies are obtained from CoinMarketCap[14]. A total of 1,827 daily observations for each cryptocurrency are included. Prices are denominated in USD. Based on price data, we calculate simple returns of all cryptocurrencies on a daily basis, similar to Annaert et al. (2009);

[14]www.coinmarketcap.com

Zieling et al. (2014). As a proxy for risk-free rates, the 3-month Treasury Bill (T-bill) Rate is used. For the covered period, the average annual return and annual volatility are 1.05% and 0.89%, respectively. Table 5.1 summarizes the return statistics for each asset based on annualized returns over the covered period.

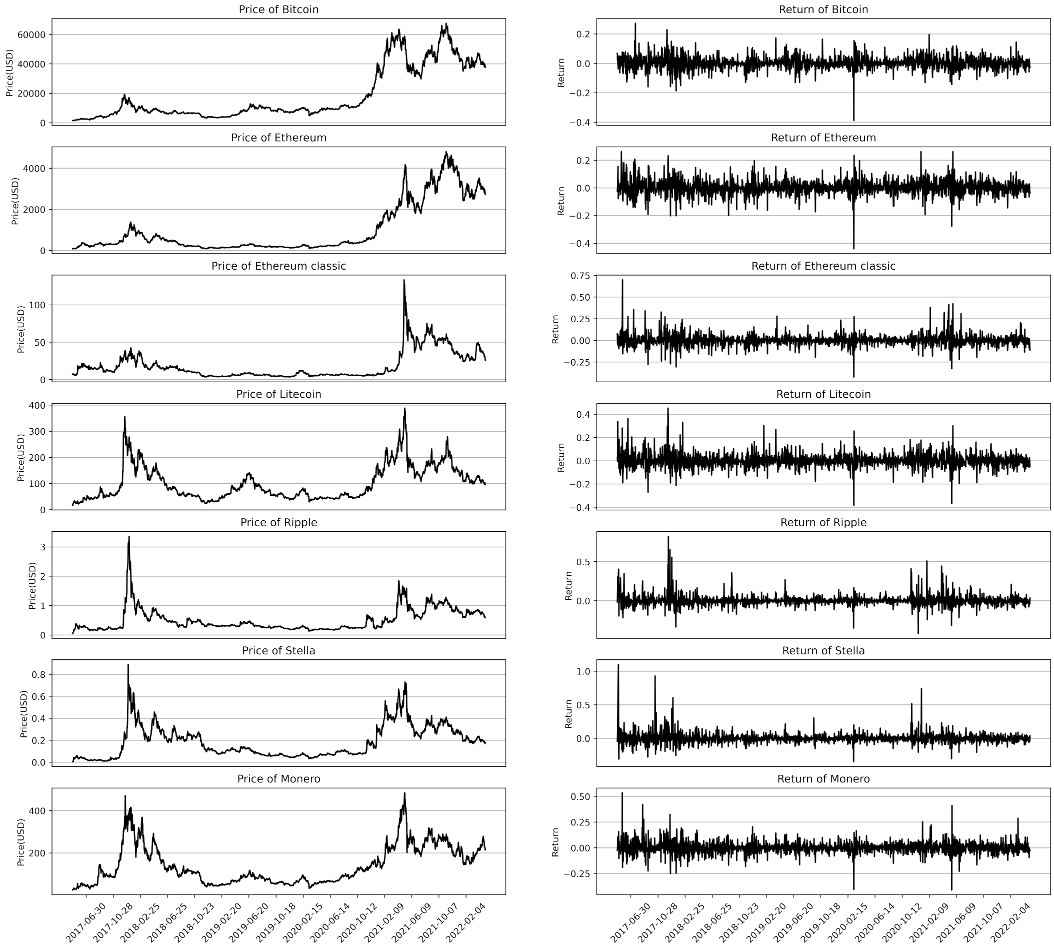


Figure 5.2: Time series of price and return of each cryptocurrency.

Notes. This figure shows time series of price and return of each cryptocurrency on a daily basis for the period May 2017 to April 2022.

In Table 5.1, we report the values of the average return, standard deviation, skewness, and kurtosis for each cryptocurrency. The overall average return and standard

Table 5.1: Summary statistics of return of each cryptocurrency

	Avg	Std.	Skew	Kurto	ADF	Q(10)	Q(40)	Q(70)	ARCH(1)	ARCH(3)	J-B
BTC	0.682	0.677	-0.08	6.7	-30.0^{***}	9.0	47.0	85.9*	24.1***	28.8***	3388.0***
ETH	0.863	0.869	-0.03	4.8	-12.4^{***}	22.3**	53.3*	73.1	58.2***	64.2***	1748.0***
ETC	0.74	1.071	1.3	12.9	-8.4^{***}	20.8**	85.5***	127.0***	97.3***	101.8***	13027.0***
LTC	0.698	0.964	0.75	7.4	-16.2^{***}	12.3	51.9*	88.9*	92.9***	121.5***	4337.0***
XRP	0.938	1.146	2.5	21.9	-9.6^{***}	8.0	63.3**	93.5**	39.9***	92.7***	38090.0***
XLM	1.25	1.324	3.95	40.9	-19.0^{***}	16.5*	49.0	83.5	307.0***	311.7***	131349.0***
XMR	0.751	0.957	0.59	9.6	-15.7^{***}	16.2*	64.1***	86.5*	66.1***	71.1***	7056.0***

Notes. We apply the Augmented Dickey-Fuller (ADF) test (Cheung & Lai, 1995). ADF statistics show that the null hypothesis of a unit root can be rejected for returns. Ljung-Box tests up to leg 70 are conducted to detect serial correlation. $Q(\cdot)$ denotes the test statistics of the Ljung-Box test. Engle (1982)'s Lagrange multiplier (LM) test detects heteroskedasticity up to lag 3. Values in columns $ARCH(1)$ and $ARCH(3)$ denotes the values of LM statistics. The J-B columns denote the test statistic of the Jarque-Bera test, where the null hypothesis is that return distribution is normally distributed. *, **, and *** mean significance at the 10%, 5%, and 1% level, respectively.

deviation show relatively large values. While Bitcoin shows the lowest average return and risk (68.2% and 67.7%), Stella shows the highest average return and risk (125% and 132.4%) among cryptocurrencies. These high returns and risks are shown in Figure 5.3, representing the overall growth of each cryptocurrency. Furthermore, although Bitcoin and Ethereum have small negative skewness values, the remainders have positive skewness values, implying that, except for these two cryptocurrencies, whose return distributions are slightly left-skewed, the remainder's return distributions are right-skewed. Next, fat-tail seems to exist in the return distributions of all cryptocurrencies, considering that the value of all cryptocurrencies' kurtosis is greater than 3[15]. Considering the aforementioned descriptive statistic results of each cryptocurrency together, we found that the cryptocurrency market is characterized by high average return, risk, skewness, and kurtosis[16].

We apply the ADF test (Cheung & Lai, 1995), and the result shows that the null hypothesis of a unit root can be rejected at 1% significance level. Hence, we confirms

[15] If a return distribution is normal, the corresponding kurtosis should be 3.

[16] In fact, many articles have reported these characteristics of the cryptocurrency market (Liu & Tsyvinski, 2021; Liu et al., 2022). They demonstrate that the cryptocurrency market shows a higher value of average return, standard deviation, skewness, and kurtosis compared to the stock market, which is consistent with our results.

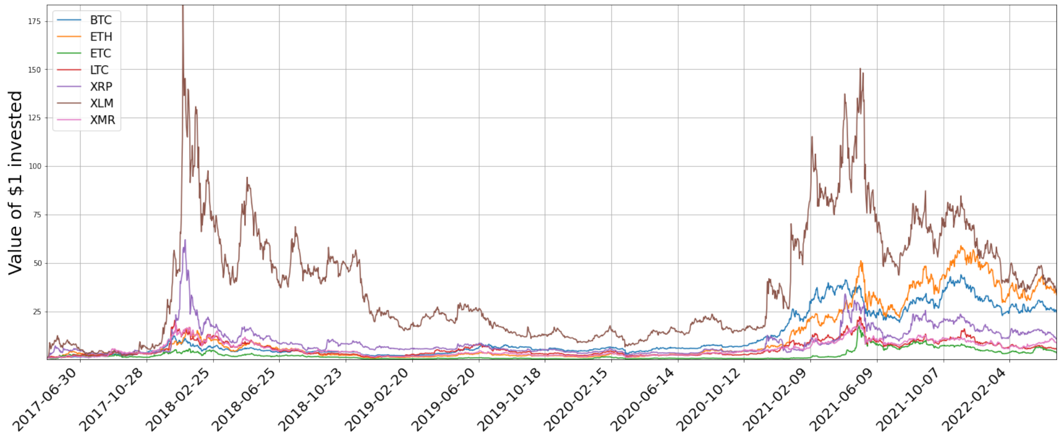


Figure 5.3: The value of $1 invested in the initial date of the covered periods.

that returns of cryptocurrencies are stationary. Furthermore, a Ljung-Box test up to leg 70 is conducted to detect the existence of autocorrelations. The results of $Q(10)$, $Q(40)$, and $Q(70)$ in Table 5.1 reveal autocorrelations in all returns. Bitcoin, Litecoin, and Ripple have autocorrelation only at a higher lag (40 or 70). Meanwhile, Ethereum, Ethereum classic, Stella, and Monero show the autocorrelation at lower lag (10), although Ethereum, Ethereum classic, and Monero show the autocorrelation at higher lag (40), either. Next, we performed Engle (1982)'s Lagrange multiplier test to check heteroskedasticity, revealing that there exist statistically significant heteroskedasticity effects in all cryptocurrency returns at up to lag 3. Finally, the Jarque-Bera (J-B) test statistics are reported. These J-B results indicate that a normal distribution null hypothesis can be rejected in all cryptocurrency returns.

5.5.2 Experimental design

Simulation setup for downside risks results

The first research question we want to address in our study is how the impact of risk and return structure of our portfolio in portfolio insurance strategies compared with benchmark strategies are shown in the cryptocurrency market. In order to investigate this, the block-bootstrap simulation[17] introduced by Annaert et al. (2009) is used in our study on selected cryptocurrencies for the past five years. We assume that the bootstrap simulation contains 252 trading days[18] as 1-year duration since various institutional or retail investors prefer using a 1-year horizon of investment (Benartzi & Thaler, 1995). The procedure of the block-bootstrap approach is as follows. First, among our selected cryptocurrencies, one cryptocurrency is randomly drawn with the replacement. Second, a continuous block of 252 daily cryptocurrency returns is obtained by randomly drawing a random starting date with replacement. A total of 10,000 repeated simulations using this procedure are performed to reliably evaluate the performance of all strategies.

To implement the SP strategy, investors must specify the volatility of an underlying risky asset. However, since true volatility is unknown to investors in advance, they should estimate this volatility. In this study, we attempt to investigate the empirical results of various portfolio insurance strategies in the cryptocurrency market rather than attempt to propose an enhanced volatility estimation model in SP. Thus, we use the rolling window-based standard deviation of 252 daily returns before the random initial date of simulation for the estimate of volatility, following the various portfolio insurance studies (Annaert et al., 2009; Dichtl & Drobetz, 2011;

[17] The authors stated that they can obtain return series without making any distribution assumptions using block-bootstrap simulation. Furthermore, bootstrapping preserves skewness and kurtosis structure, avoiding the dependency effects associated with autocorrelation and heteroscedasticity. The selection of the simulated periods has a significant impact on the performance evaluation of portfolio insurance strategies. As a result, we believe that using this block-bootstrap simulation effectively mitigates this issue.

[18] Some cryptocurrency portfolio studies (Grobys et al., 2020; Liu, 2019a; Silahli et al., 2021) used the assumption that a 1-year is about 365 trading days. We conduct the simulation under this assumption, demonstrating essentially similar result to the main ones. Full results are available upon request.

Dichtl et al., 2017). This is consistently applied to the VBPI strategy. Moreover, the VBPI strategy requires estimation of expected return as well as volatility; we also use rolling a window-based average of 252 daily returns before the random initial date of simulation as an estimated expected return. In CPPI and TIPP, we use the value that is equal to the SP's initial risk exposure for selecting the initial value of that risk exposure.

Our benchmarks consist of a buy-and-hold strategy, a buy-and-hold strategy with 50/50 proportion (B&H 50/50), 70/30 portfolio strategy (70/30), and 50/50 portfolio strategy (50/50). In buy-and-hold strategy, 100% of portfolio is invested in cryptocurrency at the starting date and held until maturity. In B&H 50/50, 50% of the portfolio is invested in cryptocurrency and 50% of the portfolio is invested in risk-free assets at the starting date and held until maturity. In contrast, in the 70/30 portfolio strategy, 70% of the portfolio is invested in cryptocurrency and 30% in risk-free assets at the start date, and portfolio rebalancing occurs at each rebalancing point (daily) until maturity in order to maintain its initial proportion. Similarly, in the 50/50 portfolio strategy, 50% of the portfolio is invested in cryptocurrency and 50% in risk-free assets at the start date, with rebalancing occurring at each rebalancing point until maturity.

To consider the effect of transaction costs from the frequent rebalancing, we use a transaction cost of 0.1% for our overall portfolio insurance strategies and benchmark analysis[19].

[19] For the SP strategies, we apply the transaction cost scheme as shown in Eq. 4.15. Following Dichtl & Drobetz (2011); Herold et al. (2007), we use ten basis points as the value of k, which is the round-trip transaction cost.

Investor's utility simulation setup

The second research question of our study is how the choice of portfolio strategies is affected by the investors' utility and level of parameters in the cryptocurrency market. To investigate this question, we consider two types of investors: (i) expected utility investors and (ii) prospect theory investors. In this study on investor's utility over portfolio insurance strategies, we also performed 10,000 repeated block-bootstrap simulations. Likewise to downside risk simulation, we calculated the utility value and mean prospect value by using the obtained 10,000 return series. As a result, the contribution of different features across utility types, such as risk aversion in expected utility theory, a curvature of S-shaped value functions, and loss-aversion in prospect theory is investigated. We use the buy-and-hold strategy as a unique benchmark because we want to investigate the impact of these investor characteristics in the cryptocurrency market by comparing a group of portfolio insurance strategies and a 100% risky asset invested portfolio.

In expected utility theory, we use two widely used utility functions (e.g., quadratic and exponential), revealing the impact of cryptocurrency investor's risk aversion on the choice of investors among portfolio strategies. In prospect theory, the value function in Eq. 5.30 is used to investigate the impact of curvature (α and β) of the S-shaped value function on the investors' preferences with a fixed level of loss-aversion. The curvature in the S-shaped value function indicates the degree of risk aversion (α) in the gain domain and the degree of risk-seeking (β) in the loss domain. Since regression results in Tversky & Kahneman (1992) demonstrated that parameters α and β estimated as the same values (0.88) are in accord with the empirical data, following the research, we explore the space in the condition that $\alpha = \beta$ which implies that the degree of investor's risk aversion in the gain domain is assumed to be the same as the degree of the investor's risk-seeking in the loss domain. For this result, we use the two fixed levels of loss-aversion ($\lambda = 1.0$ and $\lambda = 2.25$). $\lambda = 1.0$ means that it does not account for loss-aversion while $\lambda = 2.25$ means vice versa. By comparing the expected utility theory and prospect theory results, we uncover the impact of S-shaped value functions, which implies the additional consideration of risk-seeking features in the loss domain. Additionally, we also explored the impact of loss-aversion λ at a fixed level of curvature ($\alpha = \beta = 0.88$). To obtain the total effect of the S-shaped value function, we use the average value function as the mean prospect value.

5.6 Empirical results

5.6.1 Downside risk results

Comparison of portfolio insurance strategies and benchmarks

Table 5.2: Performance evaluation results based on downside risks

	SL	SP	CPPI	TIPP	VBPI-S	VBPI-D	B&H	B&H 50/50	70/30	50/50
Average return	0.109	0.537	0.107	0.026	0.012	0.027	0.766	0.419	0.531	0.38
Volatility	0.19	0.615	0.16	0.019	0.021	0.046	0.846	0.482	0.592	0.423
Skewness	3.032	1.402	3.369	6.949	22.16	3.761	0.801	1.354	0.8	0.8
Sharpe ratio	0.516	0.855	0.601	0.824	0.086	0.372	0.892	0.847	0.878	0.873
MDD	-0.145	-0.441	-0.12	-0.009	-0.016	-0.038	-0.56	-0.367	-0.426	-0.321
AvDD	-0.094	-0.209	-0.067	-0.005	-0.007	-0.016	-0.255	-0.159	-0.18	-0.128
VaR 5%	-0.176	-0.313	-0.077	-0.002	-0.017	-0.031	-0.509	-0.249	-0.352	-0.245
ES 5%	-0.244	-0.35	-0.154	-0.006	-0.06	-0.072	-0.581	-0.285	-0.418	-0.298
Semideviation	0.081	0.279	0.045	0.007	0.019	0.022	0.42	0.21	0.289	0.202
Omega ratio	1.192	1.186	1.205	1.499	1.209	1.219	1.185	1.18	1.183	1.184

Notes. This table shows performance evaluation results using performance measures, including downside risks in the cryptocurrency market.

Table 5.2 shows the overall results from comparative portfolio insurance strategies and benchmarks in the cryptocurrency market. For the main results, we studied

SL, SP, CPPI with multiplier 7, TIPP with multiplier 7[20], static VBPI (VBPI-S) with 99% confidence level, and dynamic VBPI (VBPI-D) with 99% confidence level[21], as our portfolio insurance strategies. We implemented all portfolio insurance strategies under 100% protection level[22]. As shown in Table 5.2, the buy-and-hold strategy offers the largest value of average return and volatility. Despite having lower average returns, portfolio insurance strategies show substantially lower volatility than the buy-and-hold strategy. This result is not surprising given the portfolio insurance philosophy that investors pay an upward capture as a cost of downward protection for the insured portfolio. Specifically, SP has the highest average return and volatility (53.7% and 61.5%, respectively), while TIPP, VBPI-S, and VBPI-D strategies have the lowest average return and risk among portfolio insurance strategies. The average returns and volatility of the SL (10.9% and 19%) and CPPI (10.7% and 16%) strategies are both in the middle. These findings imply that, despite the same level of protection, the risk exposures of each strategy are very different[23]. The other benchmarks (B&H 50/50, 70/30, and 50/50) show average returns and volatility roughly intermediate between SP and CPPI.

Although the benchmark strategies show a moderate positive value of skewness, all portfolio insurance strategies show substantially higher positive skewness than

[20]We also investigated the impact of multiplier m. Since the aim of this study is to investigate portfolio insurance strategies in the cryptocurrency market, rather than to propose a new portfolio insurance strategy, we present these results in Table A6 in Appendix in order to maintain our main argument in this section.

[21]We also investigated the impact of confidence level. These results are presented in Table A7 in the Appendix.

[22]The commonly used level of protection is 100% in the vein of study on the portfolio insurance strategy, as shown by Dichtl & Drobetz (2011).

[23]In that TIPP strategy is the modified version of CPPI with dynamically adjusted floor level (only adjusting upward side), it is a plausible result that risk exposure of TIPP is less than that of CPPI given the same level of protection since the degree of protection might be tight compared to that of CPPI.

the benchmarks in the cryptocurrency market. Generally, higher positive skewness of strategy indicates that the strategy is more desirable since the positive skewness of a return distribution implies frequent minor downward losses and a few large upward gains rather than frequent minor upward gains and a few large downward losses in their investment (Harvey & Siddique, 2000; Post et al., 2008). However, in terms of the Sharpe ratio, it is shown that most of the benchmarks outperform portfolio insurance strategies. However, it is a non-surprising result, as already reported in many portfolio insurance strategy studies. As aforementioned, the Sharpe ratio might not be an adequate performance measure for a portfolio insurance strategy due to its non-normality[24].

As a result, to adequately address this issue, we also conducted a performance evaluation of all strategies using various downside risk measures (MDD, AvDD, VaR, ES, semideviation, and Omega ratio), which are proper to capture non-normality and asymmetry in return distribution. Specifically, the buy-and-hold strategy (−56% and −25.5%) shows the highest level of MDD and AvDD (in absolute value) compared to the portfolio insurance strategies and other benchmarks in the cryptocurrency market. Among portfolio insurance strategies, SP (−44.1% and −20.9%) show the highest MDD and AvDD, while TIPP (−0.9% and −0.5%) showed the lowest MDD and AvDD. Overall results of MDD and AvDD demonstrate that all portfolio insurance strategies significantly improve compared with buy-and-hold. Similar results are shown in terms of VaR and ES with a confidence level of 5%. The buy-and-hold strategy (−50.9% and −58.1%) has the highest VaR and ES (in absolute value),

[24] Much research related to this issue has been reported. See Annaert et al. (2009); Bertrand & Prigent (2011); Dichtl & Drobetz (2011); Dichtl et al. (2017); Gaspar & Silva (2021); Zieling et al. (2014).

indicating that all portfolio insurance strategies have a considerably better level of VaR and ES than the buy-and-hold strategy. TIPP (−0.2% and −0.6%, respectively) outperforms other portfolio insurance strategies in terms of VaR and ES level.

Furthermore, the buy-and-hold semideviation (42%) is higher than that of all other portfolio insurance strategies and benchmarks. The result of semideviation should be interpreted with caution because, in addition to the absolute value of semideviation, the relative difference in volatility and semideviation of comparative strategies over the baseline has significant implication. More precisely, the volatility of buy-and-hold (84.6%) decreases by 50.4% to the semideviation of buy-and-hold (42%), as expected; thus, the upside deviation of buy-and-hold return distributions is similar to the downside deviation of buy-and-hold since its return distribution is close to the symmetry. Meanwhile, volatility of SL, SP, CPPI, TIPP, VBPI-S, and VBPI-D (19%, 61.5%, 16%, 1.9%, 2.1%, and 4.6%, respectively) decreases to the semideviation of those (8.1%, 27.9%, 4.5%, 0.7%, 1.9%, and 2.2%, respectively) by 57.4%, 54.6%, 71.9%, 63.2%, 9.5%, and 52.2%, respectively. With the exception of VBPI-S, the degree of reduction in portfolio insurance strategies is larger than that of the buy-and-hold strategy. That is, the downside deviations of these portfolio insurance strategies are lower than the upside deviations of those due to asymmetry and positive skewness, and portfolio insurance strategies in the cryptocurrency market effectively improve the degree of risk reduction relative to the standard risk. Considering the results in terms of MDD, AvDD, VaR, ES, and semideviation together, we confirm that portfolio insurance strategies in the cryptocurrency market can deliver a better level of downside risk to the investors compared to buy-and-hold, thereby corroborating lower risk investment.

Next, we investigate the results based on the Omega ratio with a zero return threshold. Portfolio insurance strategies outperform the buy-and-hold strategy and other benchmarks in the cryptocurrency market in terms of the Omega ratio. This result implies that portfolio insurance strategies' expectation of gains to expectation of losses is greater than that of buy-and-hold and other benchmark strategies in the cryptocurrency market. TIPP strategy has the best Omega ratio performance among portfolio insurance strategies, whereas the SP strategy has the worst.

Interestingly, although most results demonstrated substantial outperformance of portfolio insurance strategies in terms of downside risks, there exist several exceptions, such as VBPI-S in terms of semideviation and the insignificant outperformance of SP in terms of Omega ratio. These exceptions show a strong hint that under the requirement of assumption of the Black–Scholes option pricing model, SP, and VBPI strategies suffer from underperformance due to the absence of consideration of sudden jumps in cryptocurrency prices (Bouri et al., 2020) and due to unbiased volatility estimation in the complex real-world market condition. This result is consistent with the results of Annaert et al. (2009). The authors demonstrate that under high volatility market condition, the portfolio insurance strategies with a requirement of Black–Sholes assumptions show underperformance, while the portfolio insurance strategies without the requirement of Black–Sholes assumption substantially outperforms the benchmarks in terms of various performance measures in part.

The main results from Table 5.2 are summarized as follows. First, portfolio insurance strategies make investment less risky than the buy-and-hold strategy in the cryptocurrency market, as shown in all risk measures (MDD, AvDD, VaR, ES, and semideviation as well as volatility), and this lower risk entails a lower average return. Second, portfolio insurance strategies are inferior to the buy-and-hold strategy in terms of the Sharpe ratio in the cryptocurrency market. However, the argument that the Sharpe ratio in a portfolio insurance context is not necessarily an appropriate measure is also corroborated by the higher skewness and asymmetry of the insured portfolio compared to the buy-and-hold portfolio[25]. Third, the results from all downside risks (MDD, AvDD, VaR, ES, semideviation, and Omega ratio) clearly demonstrate the outperformance of portfolio insurance relative to the benchmarks in the cryptocurrency market. Despite this superiority, several portfolio insurance strategies based on Black–Scholes assumptions seem to suffer from an estimation error problem.

The impact of the protection level

A common pre-determined specification in all portfolio insurance is floor value F (i.e., protection level a). As a result, we investigate how performance measures of portfolio insurance strategies change as the level of protection changes in the cryptocurrency market. Because it is commonly used for protection levels, three protection levels (100%, 95%, and 90%) are considered (Dichtl & Drobetz, 2011). As the level of protection is reduced, the average return and volatility of all portfolio insurance strategies rise. This volatility tendency is reflected in the results of other downside risk measures in all portfolio insurance strategies. It is not surprising given that the protection level, or floor value, denotes the willingness to accept potential losses. Simply put, it refers to the level of risk exposure. Consistent with this im-

[25] This result is consistent with the prior research on adequate performance measures of the portfolio insurance strategies in the stock market (Annaert et al., 2009; Bertrand & Prigent, 2011; Zieling et al., 2014).

Table 5.3: Performance evaluation results according to protection levels

	SL	SP	CPPI	TIPP	VBPI-S	VBPI-D	B&H	B&H 50/50	70/30	50/50
Panel A: Protection level of 95%										
Average return	0.232	0.553	0.244	0.079	0.06	0.088	0.766	0.419	0.531	0.38
Volatility	0.337	0.632	0.336	0.098	0.079	0.124	0.846	0.482	0.592	0.423
Skewness	2.021	1.354	2.356	7.738	4.657	4.33	0.801	1.354	0.8	0.8
Sharpe ratio	0.66	0.859	0.697	0.697	0.623	0.625	0.892	0.847	0.878	0.873
MDD	-0.263	-0.451	-0.25	-0.059	-0.063	-0.1	-0.56	-0.367	-0.426	-0.321
AvDD	-0.161	-0.213	-0.151	-0.048	-0.028	-0.044	-0.255	-0.159	-0.18	-0.128
VaR 5%	-0.224	-0.326	-0.152	-0.058	-0.061	-0.061	-0.509	-0.249	-0.352	-0.245
ES 5%	-0.292	-0.363	-0.239	-0.067	-0.12	-0.095	-0.581	-0.285	-0.418	-0.298
Semideviation	0.107	0.291	0.068	0.042	0.042	0.042	0.42	0.21	0.289	0.202
Omega ratio	1.198	1.186	1.198	1.349	1.201	1.209	1.185	1.18	1.183	1.184
Panel B: Protection level of 90%										
Average return	0.322	0.57	0.329	0.129	0.102	0.134	0.766	0.419	0.531	0.38
Volatility	0.44	0.649	0.437	0.177	0.129	0.182	0.846	0.482	0.592	0.423
Skewness	1.71	1.303	1.918	7.608	2.854	3.644	0.801	1.354	0.8	0.8
Sharpe ratio	0.708	0.862	0.73	0.668	0.713	0.677	0.892	0.847	0.878	0.873
MDD	-0.34	-0.461	-0.323	-0.111	-0.102	-0.145	-0.56	-0.367	-0.426	-0.321
AvDD	-0.205	-0.217	-0.198	-0.095	-0.044	-0.064	-0.255	-0.159	-0.18	-0.128
VaR 5%	-0.272	-0.34	-0.224	-0.109	-0.089	-0.097	-0.509	-0.249	-0.352	-0.245
ES 5%	-0.34	-0.376	-0.305	-0.127	-0.149	-0.124	-0.581	-0.285	-0.418	-0.298
Semideviation	0.135	0.306	0.108	0.078	0.061	0.062	0.42	0.21	0.289	0.202
Omega ratio	1.197	1.185	1.195	1.331	1.197	1.197	1.185	1.18	1.183	1.184

plication, overall results are presented as expected. In other words, a lower level of protection leads portfolio insurance strategies to be more volatile.

However, except for several cases (TIPP and VBPI in 100% and 95% protection level), overall skewness increases as the protection level increases, revealing that a higher protection level leads to a return distribution of the strategy to be more positive-skewed. Next, it is unclear which protection level provides a better level of Omega ratio. None of the major changes in the Omega ratio are shown, which indicates the ambiguity of enhancement according to the protection level.

There are a few differences between the results of SL and SP. The most highlighted is that the SP strategy is riskier than the SL strategy. As well as values of volatility, all downside risks (MDD, AvDD, VaR, ES, and semideviation) show higher values in SP than in SL under the same level of protection. Presumably, this is because the SP strategy might have a longer opportunity of risk exposure than the SL strategy since the SL strategy maintain its 100% risk-free asset position after switching its proportion of risky assets when the value of the portfolio reaches the (discounted) floor value. Interestingly, as protection level varies, the Sharpe ratio of SL changes significantly while the Sharpe ratio of SP changes only slightly. This indicates that there is room for improvement in the risk-return trade-off relationship based on the level of protection in SL. Meanwhile, the protection level does not allow for any Sharpe ratio improvement in the SP strategy.

In a comparison of all risk measures of CPPI and TIPP, CPPI seems riskier than TIPP, regardless of the protection level. Although the impact of a dynamically adjusted floor in TIPP is unclear, at least in the cryptocurrency market, it seems that it acts as a stabilizer for the insured portfolio by reducing the exposure to the position in the cryptocurrency asset. Consistent with the results in downside risks, TIPP outperforms the CPPI in terms of Omega ratio, irrespective of the level of protection.

Considering Tables 5.2 and 5.3 together, we confirm the impact of the level of protection in all portfolio insurance strategies in the cryptocurrency market. Its implications are clear. The lower the level of protection, the higher the level of volatility in strategies due to the rise in risk exposure. Furthermore, all metrics of downside risk at lower levels of protection show higher values compensated by higher average returns.

The impact of frequency

We also scrutinized the impact of data frequency on the performance evaluation of portfolio insurance and benchmark strategies in the cryptocurrency market. We

Table 5.4: Performance evaluation results on a weekly basis

	SL	SP	CPPI	TIPP	VBPI-S	VBPI-D	B&H	B&H 50/50	70/30	50/50
Average return	0.199	0.484	0.202	0.026	0.015	0.034	0.825	0.476	0.34	0.406
Volatility	0.269	0.511	0.204	0.018	0.022	0.056	0.886	0.47	0.336	0.397
Skewness	5.172	4.097	6.362	5.994	10.016	1.178	3.245	2.846	2.846	4.194
Sharpe ratio	0.702	0.927	0.941	0.863	0.193	0.42	0.919	0.989	0.981	0.996
MDD	-0.182	-0.335	-0.135	-0.009	-0.013	-0.042	-0.538	-0.299	-0.221	-0.259
AvDD	-0.109	-0.147	-0.068	-0.005	-0.005	-0.013	-0.246	-0.122	-0.087	-0.104
VaR 5%	-0.24	-0.259	-0.125	-0.007	-0.015	-0.057	-0.531	-0.253	-0.179	-0.178
ES 5%	-0.309	-0.287	-0.183	-0.009	-0.054	-0.138	-0.604	-0.303	-0.217	-0.208
Semideviation	0.133	0.25	0.061	0.008	0.019	0.04	0.539	0.25	0.17	0.196
Omega ratio	1.556	1.53	1.637	2.226	1.59	1.477	1.506	1.555	1.555	1.569

construct portfolios based on all strategies and present the results on a weekly basis[26] in Table 5.4 at the same protection level of 100% as in the main results. First, by comparing the results on a weekly and a daily basis, the buy-and-hold strategy shows higher average return, volatility, skewness, Sharpe ratio, and Omega ratio on a weekly basis than on a daily basis. These tendencies are similar to results in SL and CPPI. However, other portfolio insurance strategies show a change differently. In SP, even if average return and volatility show lower value, skewness, Sharpe ratio, and Omega ratio shows higher values on a weekly basis than on a daily basis. In TIPP, volatility and skewness show lower value, although Sharpe ratio and Omega ratios show higher value on a weekly basis than on a daily basis. In VBPI-S and VBPI-D, average return, volatility, Sharpe ratio, and Omega ratio shows higher value. However, skewness shows lower values on a weekly basis than on a daily basis. In terms of downside risks, risks of SL, CPPI, TIPP, and VBPI-D show higher values, while SP and VBPI-S show lower values on a weekly basis than on a daily basis, with the only exception of AvDD in VBPI-D.

[26] A total of 261 weekly observations for each cryptocurrency were used. We conduct the same analysis on a monthly basis, confirming the essentially consistent, but pronounced results with the results on a weekly basis due to the limited number of observations provided by the lower frequency. Full results are available upon request.

In a comparison of buy-and-hold and portfolio insurance strategies on a weekly basis, almost all metrics are qualitatively similar to the daily results. Portfolio insurance strategies on a weekly basis in the cryptocurrency market also lead to less risky investments than a buy-and-hold strategy based on volatility. Furthermore, although the buy-and-hold strategy outperforms portfolio insurance strategies in terms of the Sharpe ratio, the results from downside risks reveal that portfolio insurance strategies outperform buy-and-hold on a weekly basis in the cryptocurrency market, effectively reducing the overall downside risks.

The shocking point, however, is that even though this superiority of downside risk measures, several portfolio insurance strategies are inferior to the benchmarks in terms of Omega ratio, such as SP (1.53 in SP vs. 1.555 in B&H 50/50) and VBPI-D (1.477 in VBPI-D vs. 1.506 in B&H). This result on a weekly basis is inconsistent with the results on a daily basis in Table 5.2, which shows the outperformance of SP and VBPI compared to buy-and-hold. Considering the outperformance of the CPPI and TIPP strategies in both frequencies, it is a strong hint that there might be critical issues in the estimation error problem from the extracted cryptocurrency return data under the SP and VBPI framework[27]. SP and VBPI are intrinsically under the assumption of the Black–Scholes option pricing model. True volatility is unknown to investors. Thus, a precise estimation of volatility is required to obtain an accuracy of strategy implementation (Hentschel, 2003). This tendency to underperformance is more pronounced on a weekly basis than on a daily basis. Precisely, the Omega ratio of SP and VBPI-D at least shows higher values than the benchmarks on a daily basis, while that of SP and VBPI-D shows lower values than those of the benchmarks on

[27]This volatility estimation issue in the Black–Scholes formula in portfolio insurance strategies are well organized in the research of Zhu & Kavee (1988).

a weekly basis. We think of this finding as strong evidence to support our argument that underperformance is caused by estimation error since the estimation of return volatility on a weekly basis is considered more difficult than the estimation of that on a daily basis due to insufficient observations[28].

Table 5.5: Performance evaluation results according to a level of the risk-free rate

	SL	SP	CPPI	TIPP	VBPI-S	VBPI-D	B&H	B&H 50/50	70/30	50/50
Panel A: Risk-free rate 0.55%										
Average return	0.093	0.555	0.075	0.012	0.007	0.019	0.766	0.417	0.531	0.38
Volatility	0.174	0.629	0.121	0.009	0.013	0.033	0.846	0.483	0.592	0.423
Skewness	3.185	1.238	3.864	7.33	22.859	3.815	0.801	1.353	0.8	0.8
Sharpe ratio	0.5	0.874	0.575	0.758	0.088	0.412	0.898	0.853	0.887	0.885
MDD	-0.132	-0.452	-0.092	-0.004	-0.01	-0.028	-0.56	-0.367	-0.426	-0.321
AvDD	-0.087	-0.205	-0.054	-0.003	-0.004	-0.012	-0.255	-0.159	-0.18	-0.128
VaR 5%	-0.17	-0.342	-0.071	-0.001	-0.01	-0.025	-0.509	-0.252	-0.352	-0.245
ES 5%	-0.238	-0.389	-0.142	-0.003	-0.044	-0.068	-0.581	-0.288	-0.418	-0.298
Semideviation	0.078	0.287	0.04	0.003	0.015	0.019	0.42	0.21	0.289	0.202
Omega ratio	1.185	1.184	1.194	1.506	1.203	1.227	1.185	1.179	1.183	1.184
Panel B: Risk-free rate 1.55%										
Average return	0.123	0.558	0.134	0.04	0.017	0.034	0.766	0.531	0.38	0.421
Volatility	0.206	0.632	0.196	0.03	0.028	0.056	0.846	0.592	0.423	0.482
Skewness	2.786	1.232	3.227	6.672	15.038	3.711	0.801	0.8	0.8	1.355
Sharpe ratio	0.521	0.859	0.601	0.833	0.053	0.335	0.886	0.87	0.862	0.841
MDD	-0.158	-0.454	-0.145	-0.014	-0.022	-0.047	-0.56	-0.426	-0.321	-0.366
AvDD	-0.102	-0.206	-0.081	-0.009	-0.01	-0.02	-0.255	-0.18	-0.128	-0.158
VaR 5%	-0.184	-0.344	-0.086	-0.004	-0.026	-0.035	-0.509	-0.352	-0.245	-0.247
ES 5%	-0.253	-0.391	-0.169	-0.009	-0.074	-0.078	-0.581	-0.418	-0.298	-0.282
Semideviation	0.084	0.289	0.049	0.012	0.023	0.024	0.42	0.289	0.202	0.21
Omega ratio	1.196	1.184	1.207	1.484	1.202	1.212	1.185	1.183	1.184	1.181

The impact of money market conditions

Since portfolio insurance strategies adjust their proportion of investment to the risk-free asset and risky asset, we consider that money market conditions might affect the overall performance of strategies. Thus, we investigate the empirical results from the assumed condition of different money market conditions by comparing the results in Tables 5.2 and 5.5. We consider two levels of risk-free rate assumptions;

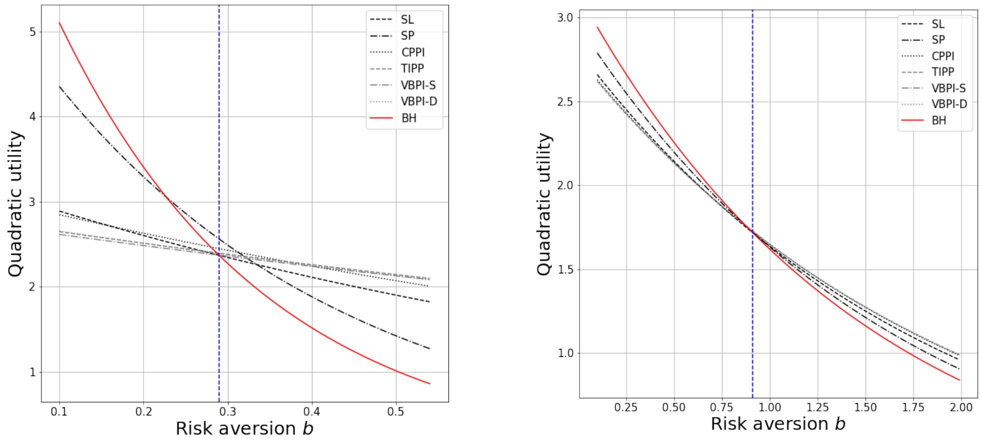
[28]Some researchers have addressed this type of issue (Dokuchaev, 2014). The author mentioned that under the condition where data frequency is limited and only short time series of prices are available, the estimation error can be significantly enlarged.

$r_f = 1.55\%$ and $r_f = 0.55\%$. Based on the baseline of the average 3-month T-bill rate for the covered periods as the risk-free rate (1.05%) in Table 5.2, we obtain these two scenarios by adding + and – of 50 base points to the baseline risk-free rate.

Panel A results show that an undervalued risk-free rate (0.55%) makes portfolio insurance strategies less risky than before, with lower average return and volatility across all portfolio insurance. This lower risk is supported by lower downside risks, as demonstrated by the values of MDD, AvDD, VaR, ES, and semideviation. Although the condition of an undervalued risk-free rate leads to a lower level of risk in portfolio insurance, the skewness of all insured portfolios increases. However, the Sharpe and Omega ratios show no clear tendency, implying a hazy causal relationship between the risk-free rate and the improvement of portfolio insurance strategies in terms of Sharpe and Omega ratios.

Looking at Panel B, we confirm the exact opposite trends compared to the results in Panel A. The overvalued risk-free rate (1.55%) provides a riskier insured portfolio than before, revealing the higher average return and volatility. This higher risk also aligns with higher downside risks shown in the value of MDD, AvDD, VaR, ES, and semideviation. Skewness shows a lower value than before, and we cannot detect any tendency to change in the Sharpe ratio and Omega ratio relative to before.

Taken the results together, we can interpret the results in the economic term as follows. A higher risk-free rate leads to a higher degree of a discount factor for pre-specified floor value at the maturity time T. As a result, strategies' floor value at current time t is specified less than before by this higher discount factor, thereby providing the potential of longer time and a larger level of risk exposure. As such, we



(a) The cryptocurrency market (b) The traditional stock market

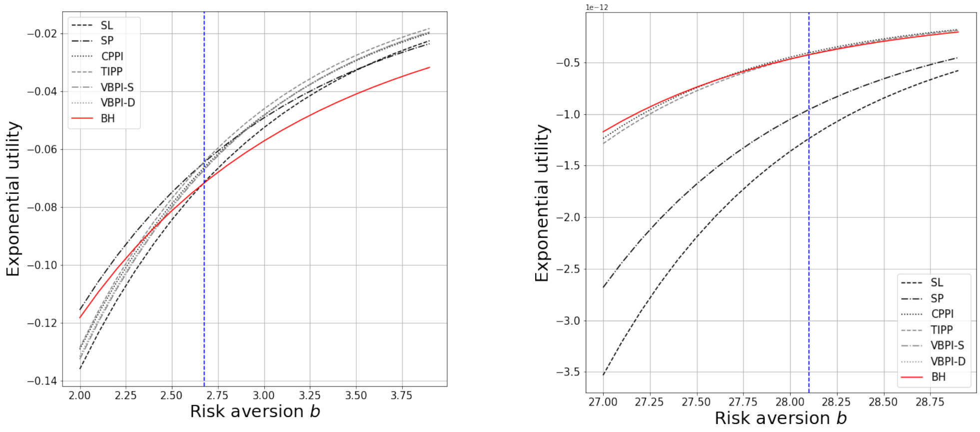
Figure 5.4: Quadratic utility function: the impact of the risk aversion b of expected utility theory investors.

present the preceding interpretation as an implication behind the result from Table 5.5.

5.6.2 Investor's utility results

Expected utility investors

First, we investigate the impact of the risk aversion b of expected utility theory investors. Figures 5.4 and 5.5 show the utility function values according to the changes in risk aversion coefficient b. We want to confirm the difference between results in the cryptocurrency market and the traditional stock market when implementing the portfolio insurance strategies. To achieve this, we present the empirical results using both cryptocurrencies and S&P 500 index, respectively. The quadratic utility in Eq. 5.28 is presented in Figure 5.4 and the exponential utility in Eq. 5.29 is presented in Figure 5.5. Looking at the result in the cryptocurrency market shown in Panel (a) in Figure 5.4, we confirm that the utilities of all strategies decrease as risk aversion



(a) The cryptocurrency market

(b) The traditional stock market

Figure 5.5: Exponential utility function: the impact of the risk aversion b of expected utility theory investors.

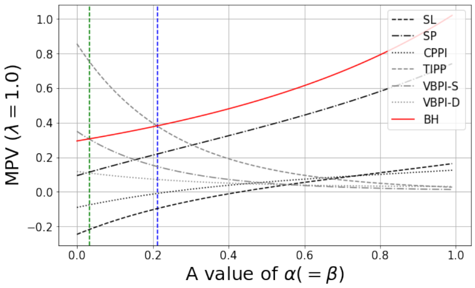
b increases. More precisely, the degree of reduction in the utility of buy-and-hold is greater than that in other strategies, implying that buy-and-hold is more sensitive to the degree of the changes in the risk aversion of investor. Meanwhile, in Panel (a) in Figure 5.5, utilities of all strategies increase as risk aversion b increases. The degree of increase in the utility of buy-and-hold is lower than that in other strategies, implying that buy-and-hold is less sensitive to the degree of the changes in risk aversion of the investor.

Even though there is an opposite trend in utility function concerning risk aversion, a common tendency is clearly shown in these two utility cases. At low risk aversion, the utility of buy-and-hold is greater than the overall utility of other portfolio insurance strategies, but as risk aversion increases, after a certain point[29], the utility of the buy-and-hold strategy is smaller than that of all other portfolio in-

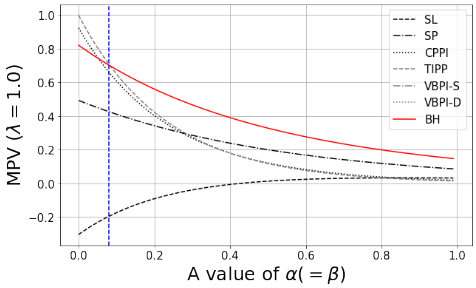
[29]Specifically, $b = 0.29$ is cut-off point in quadratic utility (Panel (a) in Figure 5.4) and $b = 2.675$ is cut-off point in exponential utility (Panel (a) in Figure 5.5). These cut-off points are marked by blue vertical dotted lines.

surance strategies. Taking into account the utility function and its risk aversion coefficient, these findings show that less risk-averse expected utility investors prefer buy-and-hold to other portfolio insurance strategies, whereas more risk-averse expected utility theory investors prefer portfolio insurance strategies to buy-and-hold in the cryptocurrency market. These findings are in line with our expectations. Given the expected utility theory's assumption that investors are risk averse (presented by b), it is unsurprising that risk-averse investors tend to obtain higher utility in portfolio insurance strategies (less risky strategies) than in buy-and-hold strategy (more risky strategy), as our results in Section 5.6.1 demonstrate.

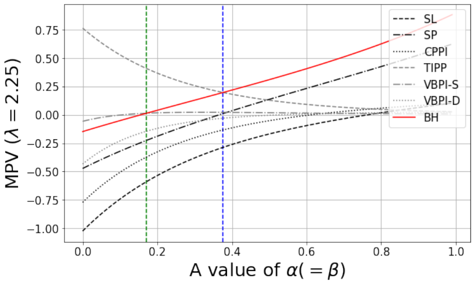
Next, looking at the results in the traditional stock market shown in Panel (b) in Figures 5.4 and 5.5, we can confirm the critical difference between the results in the cryptocurrency market and the traditional stock market. Although there is a difference in the degree of increase or decrease of each utility function according to the change in risk aversion, the aforementioned trend is consistent. In other words, when the risk aversion value is above a certain cut-off value, portfolio insurance strategies' utility is higher than buy-and-hold's utility. Interestingly, however, a significant difference is found between the cryptocurrency and traditional stock markets results. The cut-off value is much larger in the traditional stock market[30]. It implies that a wider range of expected utility theory investors can achieve greater utility in the cryptocurrency market through portfolio insurance strategies than in the traditional stock market when compared to a buy-and-hold strategy.



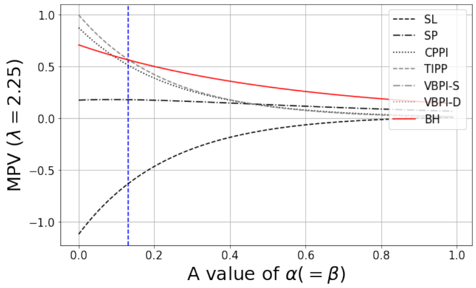
(a) MPV ($\lambda = 1.0$) according to curvature in the cryptocurrency market



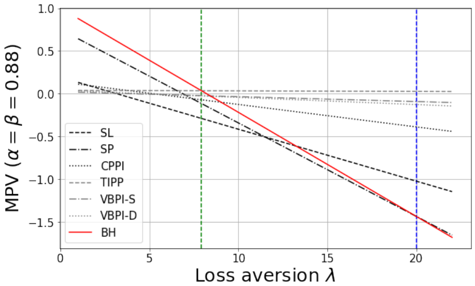
(b) MPV ($\lambda = 1.0$) according to curvature in the traditional stock market



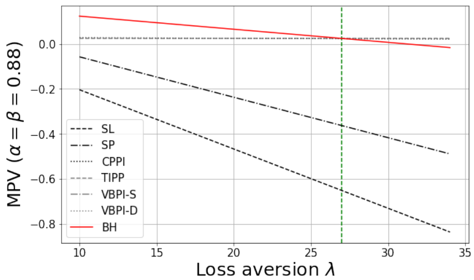
(c) MPV ($\lambda = 2.25$) according to curvature in the cryptocurrency market



(d) MPV ($\lambda = 2.25$) according to curvature in the traditional stock market



(e) MPV ($\alpha = \beta = 0.88$) according λ in the cryptocurrency market



(f) MPV ($\alpha = \beta = 0.88$) according λ in the traditional stock market

Figure 5.6: The impact of curvature or loss-aversion on MPV in prospect theory investors.

Prospect theory investors

In the prospect theory, we explore the impact of both the S-shaped value function's curvature (α and β) and the loss-aversion (λ) of investors. Likewise to expected

[30] In the traditional stock market, $b = 0.91$ is cut-off point in quadratic utility (Panel (b) in Figure 5.4) and $b = 28.1$ is cut-off point in exponential utility (Panel (b) in Figure 5.5).

utility theory results, we present the cryptocurrency market and the traditional stock market results. Firstly, we discuss the cryptocurrency market result shown in Panels (a), (c), and (e) in Figure 5.6. Panel (a) and (c) in Figure 5.6 show the mean prospect value (MPV) of prospect theory investors as in Eq 5.31 according to the changes in curvature (α and β) under the condition that the loss-aversion $\lambda = 1.0$ and $\lambda = 2.25$ is fixed, respectively. As mentioned in Section 5.5, we only explore the space under the condition that $\alpha = \beta$. In Panel (a), MPVs without loss-aversion ($\lambda = 1.0$) of all strategies except for TIPP and VPBI increase as the value of curvature increases. More precisely, in the domain of $\alpha < 0.0325$ (marked by a green vertical dotted line), TIPP shows the highest MPV, VBPI-S shows second, and buy-and-hold shows third, while TIPP shows the highest MPV, buy-and-hold shows the second, and VBPI-S third in the domain of $0.0325 \leq \alpha < 0.212$. Finally, in the domain of $\alpha \geq 0.212$ (marked by a blue vertical dotted line), the buy-and-hold strategy has the highest MPV of all strategies. Even after accounting for loss-aversion ($\lambda = 2.25$), the results in Panel (c) are qualitatively similar to the results in Panel (a), despite the fact that the cut-off values shift to the upper direction. TIPP has the highest MPV in the domain of $\alpha < 0.17$, VBPI-S is second, and buy-and-hold is third. Meanwhile, in the domain of $0.17 \leq \alpha < 0.375$, TIPP has the highest MPV, buy-and-hold is the second, and VBPI-S is the third. The buy-and-hold strategy has the highest MPV among all strategies in the domain of $\alpha \geq 0.375$. These findings imply that, regardless of the presence of loss-aversion, prospect theory investors have higher utility in buy-and-hold strategy than portfolio insurance strategies as curvature increases. We believe that this is a very interesting result because the opposite results are demonstrated in the expected utility theory investors' results in Figures 5.4 and 5.5. Taking into account the difference between expected utility theory and prospect theory, which is a risk-seeking assumption in the loss domain, this result indicates a strong hint that the impact of losses on return distribution is significantly greater than that of gains, and this impact is effectively highlighted by our study.

In the prospect theory, we also explore the loss-aversion (λ) of investors. Panel (e) in Figure 5.6 presents MPV according to the changes in loss-averse coefficient λ given a fixed level of $\alpha = \beta$ as 0.88. As loss-aversion increases, MPVs of all strategies decrease, although the degree of reduction significantly varies across the strategies. More precisely, the buy-and-hold strategy shows the highest MPV when $\lambda < 7.9$ (marked by a green vertical dotted line). Then, as loss-aversion increases, this MPV is dramatically reduced, thereby most portfolio insurance strategies outperform the buy-and-hold strategy. After a certain cut-off point ($\lambda = 20$ marked by a blue vertical dotted line), the buy-and-hold strategy finally demonstrates the lowest MPV values among all strategies. This is another interesting result that indicates the impact of loss-aversion on the prospect theory investor's choice among portfolio strategies.

Next, looking at the cryptocurrency market results (Panels (a), (c), and (e)) and the traditional stock market results (Panel (b), (d), and (f)) together, we can confirm the pronounced differences between markets. As the curvature increases, the buy-and-hold strategy's MPV increases in the cryptocurrency market, whereas that in the traditional stock market decreases. It implies that the collective impact of gains and losses in the cryptocurrency market substantially differs from that in the traditional stock market. Additionally, as the most shocking result, the cut-off value shifts so that the domain that the portfolio insurance strategies outperform the buy-and-hold strategy is wider in the cryptocurrency market than in the traditional stock market[31]. This result is consistent with the result of expected utility theory investors.

In economic terms, the implications of the results from Figures 5.4, 5.5, and 5.6 are clear. First, the more sophisticated criterion of portfolio insurance strategy implementation can be achieved based on the empirical results of investor's utility by considering our findings. Prospect theory investors with a higher value of curvature tend to prefer a more risky strategy (buy-and-hold strategy) to a less risky strategy (portfolio insurance strategies) rather than a less risky strategy to a riskier one, regardless of the consideration of loss-aversion, indicating the pronounced impact of risk-seeking in the loss domain relative to risk aversion in gain domains. However, the result should be interpreted with caution because the collective impact of curvature in the gain and loss domain differs between the cryptocurrency and the traditional stock market. Additionally, prospect theory investors with higher loss-aversion at a fixed level of curvature tend to prefer portfolio insurance strategies to a buy-and-hold strategy, implying that the degree of loss-aversion affects the utility and preference of investors over strategies. Second, portfolio insurance can provide effective cryptocurrency investment risk management opportunities by offering higher utility to the larger number of both expected utility and prospect theory investors. This is because the area where portfolio insurance strategies offer higher utility than buy-and-hold strategies is wider in the cryptocurrency market

[31] The cut-off value shifts to the lower direction in the results of MPV with $\lambda = 1.0$ (Panel (a) and (b)), and MPV with $\lambda = 2.25$ (Panel (c) and (d)). Meanwhile, the cut-off value shifts to the upper direction in the results of MPV with $\alpha = \beta = 0.88$ (Panel (e) and (f)). Although the shift directions are opposite, these are consistent results in that the area of the higher utility of portfolio insurance strategies is wider in the cryptocurrency market than in the traditional stock market.

than in the traditional stock market, as shown in our results.

Chapter 6

Conclusion

6.1 Summary and contributions

This dissertation attempts to address the vital topic and critical issues in modern portfolio management in the field of finance regarding the two core procedures (the model improvement and implementation and asset selection) in terms of two perspectives (asset diversification and risk management). First, we attempt to improve the existing portfolio management strategies using machine learning models in terms of model construction in the Black–Litterman framework (in Chapter 2) and input parameter estimation in the synthetic put strategy for the appropriate model specification (in Chapter 4). Second, we investigate the result of portfolio analysis in the emerging digital asset market, including the NFT (in Chapter 3) and the cryptocurrency (in Chapter 5) market using the mean–variance and portfolio insurance frameworks, respectively. Our main findings and corresponding economic implications can be summarized four-fold.

First, in Chapter 2, we investigate the effect of firm characteristics on the BL framework proposing a novel dynamic BL model that incorporates characteristics into the framework with or without prediction via ANN. We find that when firm characteristics are successfully reflected in the BL framework, our proposed firm characteristic-based view construction procedure leads to improving the model, demonstrating that the proposed models show higher out-of-sample Sharpe ratio, cumulative return, and significant alpha compared to benchmarks. Additionally, among proposed strategies, the forward-looking view model shows outperformance over the backward-looking view model, revealing that the prediction is meaningful for improvement. Our main results imply that the proposed model not only exploits the implicated information on stock returns with a multitude of characteristics but also appropriately identifies the time-varying attributes and interactions of firm characteristics through prediction via ANN. Our study confirms that a more efficient and well-diversified portfolio can be achieved through the proposed procedure in constructing the BL portfolio.

Second, in Chapter 3, we investigate the role of NFTs in the traditional asset markets in the global financial system in terms of hedge, safe haven, and diversification effect. We conduct a hypothesis test regarding the significance of the hedge and safe haven effects of NFTs against traditional assets by using econometric analysis methods to estimate the benefit of these properties. Empirical analyses are applied to investigate the effect in both times of extreme market turmoil and the COVID-19 crisis in terms of short-run and long-run perspectives. Additionally, for a test of a diversification effect, we examine Pearson's product-moment pairwise correlation coefficient, the Gerber Statistic for co-movement, and volatility transmission via spillover index as a preliminary analysis. From the mean–variance point of view, we also provide the first empirical results from portfolio analysis to construct various comparative strategies using NFTs. We evaluate the performance measures for out-of-sample data and apply the SR test of Ledoit & Wolf (2008).

As an empirical result of a hedge or safe haven effect, we find evidence that NFTs are a hedge and safe haven for stock markets (the North American, European, Chinese, and World index), oil, bond, and USD index, while NFTs show positive co-movement with the Pacific, Emerging Market, commodity index, gold, and cryptocurrency. Additionally, these hedging and safe haven benefits of NFTs vary by asset class. Another finding is that the hedge and safe haven properties of NFTs disappear or shrink as data frequency changes from daily to weekly, confirming that time horizons are important to investors in the stock markets, especially in the European market. This result is consistent with the finding of Umar et al. (2022) that there exist differences between the short-run and long-run risk absorption capacities of NFTs over traditional asset markets. In terms of the effect of the COVID-19 crisis, we find results that are consistent with those in the time of extreme market stress, showing that the strength of the safe haven effects of NFTs for bond and USD index is much stronger. That is, NFTs act as a hedge or safe haven for the aforementioned markets during the crisis and show a consistent tendency in that the strength in the long-term is less compared to the strength in the short-term.

As an empirical result of a diversification effect, Pearson's correlation and the Gerber Statistic results show that NFTs are low correlated with the traditional asset class. Furthermore, we confirm that NFT markets are distinct from traditional markets in terms of volatility transmission by investigating volatility spillover indices based on DY and TVP-VAR. These findings are consistent with previous research on NFTs (Dowling, 2021b; Aharon & Demir, 2021), shedding light on the potential of NFTs to construct a well-diversified portfolio. Our main finding is that the inclusion of moderate amounts of NFTs statistically significantly improves the risk-adjusted performance of EW and tangency portfolio strategies. On the other hand, VW and MVP show minor changes in performance because the inclusion of NFTs in the portfolio is very small, whereas maxR, the strategy with a large inclusion of NFTs, shows a significant deterioration in performance. These findings imply that the diversification effect of NFTs can be advantageous, particularly when constructing an EW or tangency portfolio. Another finding is that the EW portfolio outperforms other optimized strategies on a weekly basis, and the tangency strategy is enhanced when the number of observations is increased on a daily basis[1]. When NFTs are included in a portfolio, the performance increase in an EW portfolio is greater than in a tangency portfolio, indicating that estimation error is primarily caused by extreme fluctuations in NFT return. Our finding suggests that when investing in traditional assets with NFTs as a new asset class on a weekly basis, a simple EW portfolio may be the best choice for constructing a well-diversified portfolio.

Third, in Chapter 4, we address research questions on the performance evaluation of portfolio insurance strategies under various virtual and real-world conditions in terms of volatility estimation error. We uncover the adverse effect of misestimation in terms of protection error and the positive relationship of volatility forecasting model's performance with protection error by presenting empirical results of comparison of various models, including naive, GARCH-type, HAR-RV-type, and machine learning-type models. As shocking results, our findings suggest the novel implication in the economic term as summarized as follows.

We examine the overall impact of the estimation error of volatility in synthetic put portfolio insurance strategy under the standard GBM and the GBM with jump

[1]See Table A2 in Appendix

simulations. We find that PLE is lowest when there is no volatility estimation error. When there is an estimation error, on the other hand, the PLE becomes higher, regardless of whether it is overly estimated or under-estimated. These results imply that performance degradation due to volatility estimation error obviously exists in terms of protection error in the synthetic put strategy. Also, this tendency becomes more pronounced as true market volatility increases. The more unstable market conditions, the more likely investors may suffer from deterioration in the protection accuracy of portfolio insurance strategies. Moreover, when the jump phenomenon is added, the intensity of the overall PLE degradation due to the volatility estimation error becomes more considerable. It suggests that the synthetic put strategy is more difficult to adequately achieve its goals with less sophisticated estimation methods in an environment closer to the reality in which the jump phenomenon exists.

Next, we investigate the results of portfolio insurance strategy using comprehensive volatility forecasting models using real-world data. The degree of improvement in PLE differs significantly according to the volatility forecasting model. This result shows evidence that the adverse effect due to volatility estimation error also exists in real-world data. By checking the performance of portfolio insurance in terms of PLE, we find that all forecasting models do better than the naive approach. Specifically, as an interesting result, no traditional method can beat machine learning-based models except for SVR, followed by HAR-RV better than GARCH. The statistical significance of the improvement of machine learning is also confirmed. Among them, XGB, Attention, and LSTM, specialized in time-series, show excellent performance in order. The performance of the volatility forecasting model is obtained in terms of MAPE and MAE, confirming that the results are qualitatively similar to those of PLE. The performance improvement of MAPE and MAE is tested through DM and MCS tests, and we find that all volatility forecasting models outperform the benchmark. As the most shocking result, based on the ranks of PLE, MAPE, and MAE, and the result of the MCS test, we find that realized volatility is forecasted better, the better the portfolio insurance's performance is. That is, even in the real market, based on the rank correlation, it indicates that the volatility forecasting accuracy is directly linked to the protection performance of the synthetic put portfolio insurance strategy.

Lastly, we perform additional analyzes to investigate the impact of the various market condition (low volatility and high volatility condition). When the market is calm, traditional methodologies perform pretty well. Interestingly, however, when the market fluctuates, the performance of the traditional method deteriorates significantly in terms of PLE and t-value. In contrast, it is confirmed that the machine learning model always performed robustly and consistently well regardless of market conditions. This result implies that machine learning can adequately capture macro-trends and complex micro-oscillations of realized volatility, irrespective of the actual amplitude of market volatility, compared to traditional methodologies. However, considering this experiment is an ex-post, it is impossible to know the market conditions in advance and select the volatility forecasting model type. Therefore, we suggest that machine learning models (especially XGB or Attention), rather than naive or traditional models, as a forecasting model for portfolio insurance would be a good option.

Finally, in Chapter 5, we address the two research questions on the performance evaluation of portfolio insurance strategies compared to benchmarks under various parametric conditions and the impact of the type of utility and corresponding parameters on the portfolio choice in portfolio insurance relative to a buy-and-hold strategy in the cryptocurrency market. We present empirical results by performing 10,000-year block-bootstrap simulations of various portfolio insurance strategies and benchmarks using seven major cryptocurrencies cover past five years. We obtained several observations that are very surprising because the findings are opposite to our expectations, revealing the novel implication in the economic term. Our findings are three folds as follows.

We examine the empirical results of portfolio insurance strategies compared to benchmark strategies based on the downside risk performance results. We discovered that portfolio insurance strategies make investing in cryptocurrencies less risky by significantly lowering the potential downside risk at the expense of some upside participation. Although portfolio insurance strategies are inevitably undervalued in terms of Sharpe ratio due to their skewness and asymmetry of the return distribution, we confirmed that in the cryptocurrency market, portfolio insurance strategies clearly outperform buy-and-hold strategy in terms of high positive skewness, low downside risks (MDD, AvDD, VaR, ES, and semideviation), and high Omega ratio. Overall, although the SP and VBPI methodologies, which require Black & Scholes (1973) model assumption, appear to be inferior to other portfolio insurance strategies due to performance degradation caused by the volatility estimation error, the TIPP, which has a flexible protection level update structure (time-varying floor value) and is not subject to the Black & Scholes (1973) assumption, demonstrated the best performance among portfolio insurance strategies in the cryptocurrency market.

Next, we perform additional analysis to investigate the impact of various specifications on portfolio insurance strategies. We find that the change in the level of protection implies the level of risk tolerance, revealing that the lower level of protection leads to a higher level of risk due to higher risk exposure of portfolio insurance strategies in terms of all risk metrics in the cryptocurrency market. Next, when comparing portfolio insurance strategies and benchmarks, we find that the results of most downside risks and performance measures on a weekly basis are essentially similar to those on a daily basis, with a few exceptions (e.g., SP and VBPI). What is shocking is that in the cryptocurrency market, regardless of the frequency, CPPI and TIPP always show decent performance compared to benchmarks, whereas SP and VBPI (as the aforementioned exceptions) show noticeable degradation considering the performance difference relative to benchmarks, especially on a weekly basis. Given that the CPPI and TIPP do not require the Black–Scholes assumption, and that volatility estimation on a weekly basis can be more unstable, we conclude that these results provide strong evidence to support the existence of volatility estimation error in the cryptocurrency market. Hence, we suggest that SP and VBPI in the cryptocurrency market should be implemented with caution. Additionally, similar to the results on the impact of protection level, the impact of money market conditions is also revealed to be related to the level of risk exposure in portfolio insurance strategies in the cryptocurrency market. More precisely, the higher risk-free rate implies a lower current (discounted) floor value caused by a higher discount factor, thereby indicating the larger risk exposure.

Lastly, as for investors' utility simulation results, we investigate the empirical result on the impact of an investor's utility in terms of utility type and corresponding parameters that imply the investor's features by comparing the results from portfolio insurance strategies and buy-and-hold. We perform two sets of independent experiments in the cryptocurrency and traditional stock market, respectively. We find that in expected utility theory, at the higher level of risk aversion, Von Neumann & Morgenstern (1947) type utility shows higher values in portfolio insurance strategies than in buy-and-hold strategy, implying unsurprising results that more risk-averse expected utility theory investors prefer portfolio insurance to buy-and-hold strategy in the cryptocurrency market. In prospect theory, however, the shocking opposite results are shown compared to expected utility theory. Prospect utility theory investors show higher utility in buy-and-hold strategy rather than portfolio insurance strategies at a higher level of the corresponding parameter (the curvature of the S-shaped value function), regardless of the level of loss-aversion. This finding implies a strong hint that the impact of risk-seeking on losses is greater than risk aversion in gains on preferences for an investor, and this pronounced effect can be detected by a more realistic assumption of risk-seeking nature in the loss domain for prospect theory investors. Additionally, in terms of loss-aversion in prospect theory, the degree of reduction is larger in buy-and-hold than in portfolio insurance strategies, thereby leading to the lowest MPV of buy-and-hold at the higher level of loss-aversion, among other strategies. These imply a higher chance of implementing subtle portfolio insurance tailored to the broad crypto asset investors based on our empirical findings on utility theory. Another shocking finding is that portfolio insurance strategies in the cryptocurrency market cover a more comprehensive range of investors than in the traditional stock market, in terms of higher utility, irrespective of the investor's utility type. It suggests that the economic benefits of portfolio insurance strategies are more prominent in the cryptocurrency market than in the traditional stock market, confirming the usefulness of an insured portfolio investing in cryptocurrency for effective risk-managed investments.

The main contribution of this dissertation can be summarized as follows. First, in Chapter 2, we show that irrespective of the adoption of the backward or forward-looking view, combining firm characteristics into view distribution enhances the portfolio performance of the BL framework. Additionally, our empirical results indicate that prediction via ANN further increases the degree of improvement in the BL portfolio compared to using naively historical averages. Empirically, we take all the results obtained from large-scale data for all the stocks listed in the US stock market for a period of 57 years; our proposed models outperform all other benchmarks, revealing the evidence that the prediction of a multitude of characteristics through ANN enhances the performance of BL framework in terms of out-of-sample Sharpe ratio and alpha.

Second, in Chapter 3, as a first attempt, we investigate the hedge, safe haven, and diversification property of NFTs. With an established econometric analysis model based on the test of Baur & McDermott (2010), we fill a research gap regarding the NFTs' role in the global financial system. In particular, we investigate whether NFTs act as a hedge or safe haven in times of extreme market conditions and the COVID-19 crisis, presenting our empirical results in terms of different data frequencies, daily and weekly. As a result, we confirm that NFTs can have significant hedging and safe haven benefits against several traditional assets, and these tendencies vary across the data frequency in both times of extreme market conditions and the COVID-19 crisis. Furthermore, with the preliminary analysis of Pearson's correlation, the Gerber statistic, and volatility spillover effect, we construct portfolios by the inclusion of NFTs in traditional assets under the Markowitz mean–variance framework. We confirm that NFTs can provide significant diversification benefits to traditional asset-based portfolios by interpreting our empirical findings.

Third, in Chapter 4, we show the existence and impact of volatility estimation error in synthetic put strategy based on more realistic conditioned Monte Carlo simulation such as GBM with jump model as well as the standard GBM model, and actual data such as S&P 500 index. Since the literature in the field of related studies only focused on the standard GBM model, which is insufficient to capture the real-world phenomena, we contribute to related literature. Next, as a first attempt, we investigate comprehensive and extensive empirical results of the effect of the attempt to correct volatility estimation using various existing volatility forecasting models in the literature. Most researchers on portfolio insurance use, at best naive-way estimation schemes in their studies. Contrarily, we use the most popular forecasting methods, widely used in the related literature, devoting the in-depth comparison. Lastly, we attempt to shed light on the economic value of machine learning-type volatility forecasting in implementing a portfolio insurance strategy. Little attention has been devoted to the ability and the economic implication of machine learning-type volatility forecasting models through the lens of the protection level error in the vein of a portfolio insurance strategy.

Finally, Chapter 5 is the first study of the empirical results on the portfolio insurance strategies in the cryptocurrency market. Since the literature in the vein of portfolio insurance only focuses on traditional assets such as stock, we contribute to cryptocurrency-based portfolio construction literature. Next, we investigate comprehensive and extensive empirical results using various established portfolio insurance strategies and benchmarks, and performance measures including numerous downside risks proposed in the literature. Most research on portfolio insurance consists of at best two or three portfolio insurance methods and several performance measures in part in their studies. We use the most reported portfolio insurance methods (six portfolio insurance strategies and four benchmarks) and performance measures (return, volatility, skewness, Sharpe ratio, and six downside risks) in the related literature, intensively, devoting to in-depth comparison. Lastly, using expected utility theory and the prospect theory framework, we investigate the impact of investor's utility on the portfolio choice and preference in the cryptocurrency market over portfolio insurance and benchmark strategies in terms of the type of utility function and corresponding parameters (risk aversion, risk-seeking, and loss-aversion). Additionally, we compare the results of the cryptocurrency market with those of the traditional stock market, confirming the economic implication of portfolio insurance strategies' higher utility in the cryptocurrency market. Little attention has been devoted to portfolio choice through the lens of investor's utility in the context of cryptocurrency investment. From our findings, we shed light on the tendency of crypto asset investors' preference on the portfolio insurance strategies, giving investors valuable alternatives to a buy-and-hold.

6.2 Future work

The research on portfolio management in the financial field has attracted significant attention. Thus, many studies on improvement of portfolio models and new asset markets have been attempted. However, despite these attempts, research on portfolio management still has the potential for improvement, and several limitations of this dissertation should be addressed in future work.

For the research on portfolio strategy improvement, future studies should include examining the effect of other characteristics or the error mitigation of the Black–Litterman model compared to the traditional portfolio model. Next, we should investigate the impact of aggregating volatility forecasting models that exploit and incorporates the advantages of each forecasting model for portfolio insurance strategy in detail. Additionally, we should study the methods that aim to address the drawback of each traditional and machine learning forecasting model for the synthetic put strategy. Finally, another approach of AI-based models that, based on a novel proposed loss-function, directly maximizes the Sharpe ratio of the mean–variance portfolio and minimizes protection level error in portfolio strategy can be investigated.

Furthermore, for the study on the selection of new asset classes, future research should examine the empirical results of other emerging assets, such as Decentralized Financial (DeFi) tokens and Central Bank Digital Currency (CBDC), in detail. Additionally, future research should investigate the results of the portfolio analysis with NFTs using the extensions of the mean–variance framework, such as the Black-Litterman model and risk parity model, to address the estimation error problem. Finally, we should study the methods that aim to manage the potential risk of that emerging asset market by proposing a novel risk managing strategy that is specialized in that emerging market.

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Appendix

A Appendix to Chapter 3

Table A1: Out-of-sample empirical results of top 5 and 7 individual NFTs

	Mean Ret.	Std.	Skew.	Kurto.	SR	CEQ Ret.
Panel A: Top 5 Liquidity						
EW	1.637	0.663	-0.229	-0.229	2.468***	1.417
VW	0.738	0.507	-1.297	4.554	1.454**	0.609
Tangency	0.293	0.216	-0.479	3.976	1.356***	0.269
maxR	3.796	4.129	0.297	1.439	0.919	-4.729
MVP	-0.001	0.089	-1.233	19.839	-0.012***	-0.005
	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel B: Top 7 Liquidity						
EW	1.813	0.784	-0.251	0.597	2.312***	1.505
VW	0.7384	0.508	-1.297	4.554	1.456**	0.609
Tangency	0.377	0.233	-0.687	3.188	1.621***	0.351
maxR	-1.181	4.731	0.122	0.898	-0.249**	-12.371
MVP	-0.002	0.09	-1.217	19.687	-0.025***	-0.006

Notes. We apply Ledoit & Wolf (2008)'s test. The covered period is from December 4, 2019, to June 9, 2021, on a weekly basis. Top 5 NFTs consist of top 3 NFTs, Axie infinity, and Superrare. Top 7 NFTs consist of top 5 NFTs, Makersplace, and Gods unchained.

Table A2: Out-of-sample empirical results of an aggregate NFT index on a daily basis

	Mean Ret.	Std.	Skew.	Kurto.	SR	CEQ Ret.
Panel A: without NFT						
EW	0.186	0.232	-2.278	22.702	0.801	0.159
VW	0.25	0.375	-1.835	16.19	0.666	0.179
Tangency	0.141	0.112	-0.17	16.199	1.262	0.135
maxR	0.809	0.801	-1.986	22.177	1.01	0.488
MVP	0.038	0.048	1.852	41.213	0.806	0.037
	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel B: with NFT						
EW	0.432	0.408	0.745	12.034	1.058***	0.348
VW	0.25	0.375	-1.835	16.188	0.667**	0.18
Tangency	0.19	0.148	1.005	21.933	1.285***	0.179
maxR	1.925	3.137	2.175	26.215	0.614***	-2.993
MVP	0.039	0.048	1.877	41.769	0.818	0.038

Notes. We apply Ledoit & Wolf (2008)'s test. The covered period is from March 5, 2018, to June 9, 2021. The inclusion of NFTs increases the performance of the EW and tangency portfolio in terms of SR and CEQ. This result is consistent with the main result in Table 3.9. DeMiguel et al. (2009) show that an EW strategy outperforms other optimized strategies on a monthly basis due to inaccurate estimation of correlation from sample error of optimized strategy. The authors suggest that in order to enhance the optimized strategies, the number of observations for the estimation needs to be increased. In this table, the tangency strategy outperforms the EW strategy, which is consistent with the suggestion of DeMiguel et al. (2009) that a higher number of observations can mitigate the error of estimation. However, the increase in SR of the EW strategy is larger than that of the tangency strategy, implying that the estimation error caused by extreme fluctuations of NFT return greatly affects the optimization in the tangency strategy. These results are also consistent with the main results discussed in Section 3.4.2.

Table A3: Out-of-sample empirical results of an aggregate NFT index before COVID-19 period on a daily basis

	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel A: without NFT						
EW	-0.061	0.175	-0.34	1.808	-0.347	-0.076
VW	-0.108	0.23	-0.383	2.145	-0.467	-0.134
tangency	0.075	0.039	-0.682	2.264	1.932	0.074
maxR	0.158	0.521	0.446	8.784	0.303	0.022
MVP	0.064	0.025	0.086	0.729	2.535	0.064
	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel B: with NFT						
EW	0.085	0.41	1.869	18.252	0.207***	0.001
VW	-0.108	0.23	-0.383	2.145	-0.467	-0.134
tangency	0.09	0.042	-0.385	2.007	2.131***	0.089
maxR	0.73	3.415	3.059	33.295	0.214***	-5.103
MVP	0.066	0.026	0.106	0.593	2.591	0.066

Notes. This table shows out-of-sample empirical results of each portfolio strategy without and with an aggregate NFT index on a daily basis before the COVID-19 period. We apply Ledoit & Wolf (2008)'s test. The covered period is from March 5, 2018, to January 12, 2020. We use January 12, 2020, as the cut-off date following the prior research of Aharon & Demir (2021). The inclusion of an aggregate NFT index increases the performance of the EW and tangency portfolio in terms of SR and CEQ. This result is consistent with the main conclusion.

Table A4: Out-of-sample empirical results of an aggregate NFT index during the COVID-19 period on a daily basis

	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel A: without NFT						
EW	0.747	0.202	-0.405	1.639	3.693	0.727
VW	1.221	0.39	-0.259	2.585	3.127	1.145
tangency	0.352	0.132	-0.089	3.307	2.665	0.343
maxR	2.774	0.836	-0.027	2.163	3.318	2.425
MVP	-0.006	0.034	-0.38	1.502	-0.177	-0.007
	Mean Ret.	**Std.**	**Skew.**	**Kurto.**	**SR**	**CEQ Ret.**
Panel B: with NFT						
EW	1.389	0.356	0.536	2.696	3.901***	1.326
VW	1.222	0.391	-0.259	2.585	3.13***	1.146
tangency	0.546	0.173	0.447	3.512	3.145***	0.531
maxR	6.483	2.784	0.592	6.143	2.329*	2.608
MVP	-0.005	0.034	-0.373	1.492	-0.148	-0.006

Notes. This table shows out-of-sample empirical results of each portfolio strategy without and with an aggregate NFT index on a daily basis during the COVID-19 period. We apply Ledoit & Wolf (2008)'s test. The covered period is from January 13, 2020, to June 9, 2021. We use January 12, 2020, as the cut-off date following the prior research of Aharon & Demir (2021). The inclusion of an aggregate NFT index increases the performance of the EW and tangency portfolio in terms of SR and CEQ. This result is consistent with the main conclusion.

B Appendix to Chapter 4

In addition to MAPE and MAE, we report the result using additional volatility forecasting criteria in evaluating the accuracy of models as follows:

$$HMSE = n^{-1}\Sigma_{t=1}^{n}(1 - \hat{\sigma_t}^2/RV_t)^2, \quad (A1)$$

$$HMAE = n^{-1}\Sigma_{t=1}^{n}|1 - \hat{\sigma_t}^2/RV_t|, \quad (A2)$$

where HMSE and HMAE are the heteroskedasticity-adjusted mean squared error and mean absolute error, respectively.

Table A5: The performance evaluation results in terms of MAPE, MAE, HMAE, and HMSE

	Performance measures					Rank				
	PLE (%)	MAPE	MAE	HMAE	HMSE	PLE (%)	MAPE	MAE	HMAE	HMSE
Panel A: Naive										
Standard deviation	21.79	2.14347	0.00016376	2.143	15.29	11	12	11	12	11
Panel B: GARCH-type										
GARCH	19.06	1.13206	0.00011182	1.132	3.890	8	8	7	8	8
EGARCH	21.52	1.21811	0.0001274	1.218	8.210	10	10	10	10	9
GJR-GARCH	19.38	1.1553	0.00011096	1.155	32.22	9	9	6	9	12
Panel C: HAR-RV-type										
HAR-RV	18.41	0.88339	0.00011401	0.883	3.29	6	5	9	4	6
HAR-RV-J	18.41	0.88331	0.00011399	0.883	3.29	7	4	8	5	7
Panel D: ML-type										
SVR	22.54	1.51166	0.00017482	1.512	12.32	12	11	12	11	10
ANN	15.83	0.93554	0.00010788	0.936	2.790	4	6	5	6	4
RNN	16.42	0.94762	0.00010355	0.948	2.890	5	7	4	7	5
LSTM	15.11	0.87346	0.00009961	0.873	2.760	3	3	3	3	3
Attention	15.08	0.84996	**0.00008408**	0.850	**1.900**	2	2	1	2	1
XGB	**12.88**	**0.78788**	0.00009613	**0.788**	2.070	1	1	2	1	2
						Rank corr.	0.930***	0.916***	0.944***	0.937***

Notes. This table shows the performance evaluation results of portfolio insurance and volatility forecasting based on various volatility forecasting models in terms of HMAE and HMSE, as well as MAPE and MAE.

C Appendix to Chapter 5

CPPI and TIPP require special pre-specified parameters, multiplier m, and VBPI-S and VBPI-D also require confidence level $1-\alpha$. Each portfolio insurance strategy can be affected depending on these parameters. Hence, in this Appendix, we investigate the impact of multiplier m of CPPI and TIPP and the impact of the confidence level of VBPI-S and VBPI-D in the cryptocurrency market. We aim to examine the empirical results from portfolio insurance strategies in the cryptocurrency market. Therefore, we present our result in terms of the impact of changes in parameters on the empirical results rather than attempt to find the optimal multiplier or confidence level.

Table A6: The impact of the multiplier on CPPI and TIPP

	CPPI m = 3	CPPI m = 5	CPPI m = 7	CPPI m = 9	TIPP m = 3	TIPP m = 5	TIPP m = 7	TIPP m = 9
Average return	0.086	0.096	0.107	0.094	0.021	0.024	0.026	0.026
Volatility	0.128	0.152	0.16	0.16	0.012	0.016	0.019	0.02
Skewness	4.45	3.771	3.369	3.11	3.224	4.941	6.949	9.0
Sharpe ratio	0.593	0.565	0.601	0.523	0.854	0.87	0.824	0.762
MDD	-0.103	-0.117	-0.12	-0.122	-0.007	-0.008	-0.009	-0.009
AvDD	-0.045	-0.06	-0.067	-0.071	-0.003	-0.005	-0.005	-0.006
VaR 5%	-0.053	-0.068	-0.077	-0.089	0.001	-0.001	-0.002	-0.007
ES 5%	-0.086	-0.119	-0.154	-0.175	-0.003	-0.005	-0.006	-0.009
Semideviation	0.03	0.037	0.045	0.048	0.005	0.007	0.007	0.008
Omega ratio	1.184	1.185	1.205	1.188	1.458	1.471	1.499	1.54

First, we investigated the impact of the multiplier on the performance evaluation of CPPI and TIPP in the cryptocurrency market. As shown in Table A6, we consider four multipliers (3, 5, 7, and 9) which are commonly used in the portfolio insurance context (Annaert et al., 2009; Dichtl & Drobetz, 2011), for both the CPPI and TIPP strategies. All values of each metric are presented according to the value of m for CPPI and TIPP at the protection level of 100%. It can be seen that the average return increases as the multiplier increases up to $m = 7$, then the average return decreases (CPPI) or maintains (TIPP) at $m = 9$, peaking at $m = 7$. By contrast, volatility increases as m increases. The results on average returns and volatility imply that risk-return trade-off is not always positive, indicating excessive risk relative to the return. That is, risk, which is not compensated by the level of return, might exist at some point. However, as expected, volatility shows a consistent tendency with the implication of m as the level of risk exposure.

As m increases, the value of skewness decreases in the CPPI strategy while it increases in the TIPP strategy. When compared to the CPPI strategy, this opposite tendency manifests the impact of TIPP's dynamic floor value. In the case of the Sharpe ratio, there is no clear tendency, revealing once again the Sharpe ratio's inadequacy in evaluating performance for portfolio insurance strategies. When the Sharpe ratios of CPPI and TIPP are compared, it is clear that TIPP outperforms CPPI in terms of Sharpe ratio at the same multiplier level. Similar to volatility, most of the downside risks (MDD, AvDD, VaR, ES, and semideviation) increase as m increases, showing the consistency with the implication of parameter m as risk exposure, and these downside risks are also lower in TIPP than in CPPI. When $m = 7$, CPPI shows the highest value of Omega ratio, while TIPP shows the highest value of Omega ratio at $m = 9$. Similar to the results in downside risks, TIPP outperforms the CPPI in terms of Omega ratio, given the same level of m.

Next, we also investigated the impact of confidence level on the performance evaluation of VBPI-S and VBPI-D in the cryptocurrency market at the protection level of 100%, as shown in Table A7. For selecting the confidence level, we follow the research of Jiang et al. (2009). All the metrics were presented according to the confidence level for VBPI-S and VBPI-D. The average return increases as the confidence level decreases in VBPI-S. Meanwhile, average returns in VBPI-D were

Table A7: The impact of confidence level in VBPI-S and VBPI-D

	VBPI-S confidence level 99%	VBPI-S confidence level 95%	VBPI-S confidence level 90%	VBPI-D confidence level 99%	VBPI-D confidence level 95%	VBPI-D confidence level 90%
Average return	0.012	0.014	0.02	0.027	0.013	0.024
Volatility	0.021	0.042	0.055	0.046	0.076	0.1
Skewness	22.16	9.798	10.156	3.761	5.144	6.977
Sharpe ratio	0.002	0.003	0.006	0.009	0.001	0.005
MDD	-0.016	-0.033	-0.044	-0.038	-0.065	-0.081
AvDD	-0.007	-0.014	-0.016	-0.016	-0.029	-0.029
VaR 5%	-0.017	-0.066	-0.09	-0.031	-0.087	-0.15
ES 5%	-0.06	-0.124	-0.152	-0.072	-0.171	-0.252
Semideviation	0.019	0.037	0.043	0.022	0.047	0.067
Omega ratio	1.209	1.134	1.156	1.219	1.073	1.108

highest at the confidence level of 99% and lowest at 95% in VBPI-D. These results on average return and volatility imply similar results in Table A6. That is, risk-return trade-off is not always captured well in the VBPI-D strategy, demonstrating that risk is not always compensated by the level of return at some point.

The clearest thing is that the smaller the confidence level, the more risky the strategy tends to be in both VBPI-S and VBPI-D. Furthermore, almost downside risk metrics show a consistent tendency with the implication of confidence level for the level of risk exposure. That is, investors expect that a lower level of confidence provides a longer and higher risk exposure opportunity, thereby making the strategy to be riskier. Overall, VBPI-S and VBPI-D seem to be riskier as the confidence level decreases in terms of volatility, MDD, AvDD, VaR, ES, and semideviation, even if there exist minor exceptions. In contrast, we cannot find any clear tendency in terms of skewness and Sharpe ratio. In terms of the Omega ratio, interestingly, the value of the Omega ratio at a confidence level of 99% is always the highest, and that at a confidence level of 95% is always the lowest irrespective of strategies. Comparing the overall results of VBPI-S and VBPI-D, it seems that the VBPI strategy tends to be riskier when the dynamic adjustment is included in the cryptocurrency market. In terms of other performance measures such as skewness, Sharpe ratio, and Omega ratio, we cannot detect any clear conclusion for the ranking of the two strategies.

Prospect theory has been criticized for the fact that the likelihood of violation of first-order stochastic dominance exists (Rieger & Wang, 2008). In other words, despite becoming a worse outcome, one prospect might be more preferred, with probability one. To address this issue, Tversky & Kahneman (1992) proposed cumulative prospect theory as the developed version of prospect theory. Contrary to prospect theory, where single probabilities are weighted, under the cumulative prospect theory framework, the cumulative probabilities are weighted as follows:

$$\pi_i := \begin{cases} \pi_i^- = w^-(p_1 + ... + p_i) - w^-(p_1 + ... + p_{i-1}) \\ \pi_i^+ = w^+(p_i + ... + p_N) - w^+(p_{i+1} + ... + p_N) \end{cases}, \tag{A3}$$

where i is the index of sorted outcomes in ascending order. For w^- and w^+, Tversky & Kahneman (1992) suggested the probability weighting function in the following functional form:

$$w^+ = \frac{p^{\gamma^+}}{(p^{\gamma^+} + (1-p)^{\gamma^+})^{1/\gamma^+}} \quad \Delta x \geq 0 \tag{A4}$$

$$w^- = \frac{p^{\gamma^-}}{(p^{\gamma^-} + (1-p)^{\gamma^-})^{1/\gamma^-}} \quad \Delta x < 0, \tag{A5}$$

where γ^- and γ^+ denote the curvature of function. However, Ingersoll (2008) specified the disadvantage of this form of Tversky & Kahneman (1992) that uses a single parameter. Another alternative is the other form of the probability weighting function, proposed by Lattimore et al. (1992) can be considered as follows:

$$w^+ = \frac{\delta^+ p^{\gamma^+}}{\delta^+ p^{\gamma^+} + (1-p)^{\gamma^+}} \quad \Delta x \geq 0 \tag{A6}$$

$$w^- = \frac{\delta^- p^{\gamma^-}}{\delta^- p^{\gamma^-} + (1-p)^{\gamma^-}} \quad \Delta x < 0, \tag{A7}$$

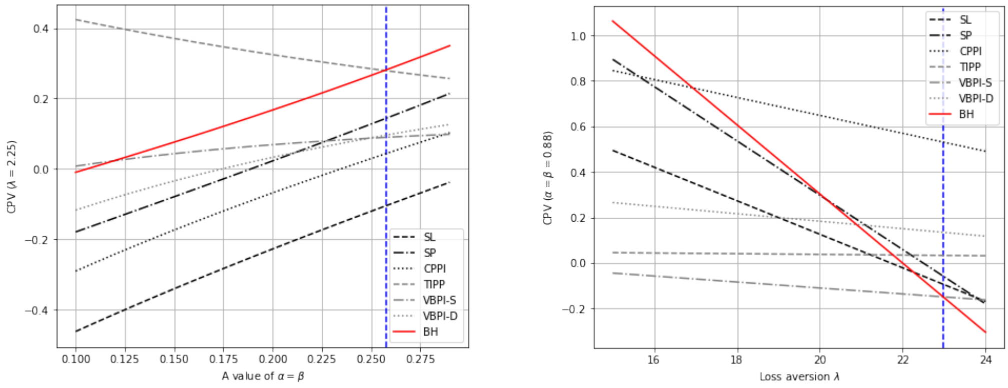
where γ^- and γ^+ denote the curvature and δ^- and δ^+ denote the elevation. Based on these parameters, this probability weight function well captures the tendency for investors to overweight small probability events[2]. Using the value function in Eq. 5.30 and decision weight (π_i) in Eq. A3, the cumulative prospect value of a portfolio strategy (p) is obtained as follows:

$$CPV_p = \Sigma_{i=1}^{N} \pi_i \cdot v(\Delta x_i) \tag{A8}$$

where Δx_i is the i-th outcome sorted in ascending order. Dichtl & Drobetz (2011) use cumulative prospect value (CPV) with Lattimore et al. (1992)'s probability weighting function for their portfolio insurance study. Following Dichtl & Drobetz (2011), we also used Lattimore et al. (1992)'s probability weighting function to obtain CPV for our study.

Panel (a) in Figure A1 shows the CPV of cumulative prospect theory investors as in Eq A8 according to the changes in curvatures at the fixed level of loss-aversion $\lambda = 2.25$. The results for CPV (with $\lambda = 2.25$) are essentially similar to the results in MPV (with $\lambda = 2.25$) in Panel (c) in Figure 5.6. When the value of curvature is higher than a certain cut-off value, the buy-and-hold strategy shows the highest CPV among all strategies. The only difference is that the cut-off value shifts to the

[2] According to the empirical result of Abdellaoui (2000); Gurevich et al. (2009), $\gamma^+ = 0.6$, $\gamma^- = 0.65$, $\delta^+ = 0.65$, and $\delta^- = 0.84$ are suggested.



(a) CPV ($\lambda = 2.25$) according $\alpha(= \beta)$ (b) CPV ($\alpha = \beta = 0.88$) according λ

Figure A1: The impact of curvature or loss-aversion on CPV in cumulative prospect theory investors.

lower direction in the results of CPV ($\alpha = \beta = 0.258$ in CPV vs. $\alpha = \beta = 0.377$ in MPV).

Panel (b) in Figure A1 illustrates the cumulative prospect value (CPV) according to the changes in loss-aversion at the fixed level of curvature $\alpha = \beta = 0.88$. CPV in Panel (b) shows also similar results to the results of MPV in Panel (e) in Figure 5.6 at the fixed degree of curvature in that all strategies' CPV decreases as loss-aversion increases, and the degree of reduction is highest in buy-and-hold among all strategies, thereby CPV of CPPI was highest while CPV of buy-and-hold was lowest among all strategies, after at a certain cut-off point. However, we can perceive two pronounced differences between the results of CPV and MPV. First, the cut-off value shifts to the upper direction in the results of CPV ($\lambda = 23$ in CPV vs. $\lambda = 20$ in MPV). Second, the CPV of TIPP and VBPI strategies was evaluated to be lower among all portfolio insurance strategies, and the utility of CPPI showed relatively higher CPV compared with other portfolio insurance strategies, contrary to the result of MPV[3]. Taking into account the implications of decision weight and probability weighting function, that is, cumulative prospect theory investors are assumed to be more sensitive to extreme events than normal ones, we believe that these postulated specifications affect the parameter cut-off value and the ranking of portfolio strategies in terms of CPV, compared to MPV.

Despite the impact of decision weight and probability weighting function in cumulative prospect theory, our findings support the main argument of the results of MPV. In other words, the cumulative prospect theory investors with higher curvature values prefer more buy-and-hold strategy over portfolio insurance strategies, demonstrating the impact of risk-seeking in the loss domain. Furthermore, cumulative prospect theory investors with higher loss-aversion at a fixed level of curvature also tend to prefer portfolio insurance strategies to a buy-and-hold strategy, demonstrating the impact of loss-aversion on investor preferences.

[3]This second result indicates the more extreme positive return or the less extreme negative return in CPPI compared to TIPP and VBPI.

국문초록

자산 분산화와 위험 관리는 포트폴리오 관리의 핵심 요소이다. 자산 분산화란 자산간 상관관계를 추정하여 자산 배분을 기반으로 다중 자산 포트폴리오에 대한 분산 효과를 극대화하는 것을 의미한다. 위험 관리란 자산의 잠재적 위험과 변동성을 추정하여 자산 배분을 기반으로 주어진 포트폴리오에 대한 하방 위험을 최소화하는 것을 의미한다. 또한 포트폴리오 관리의 두 가지 중요한 절차는 다음과 같다. 첫째, 적절한 자산 배분 전략 시행을 위한 모형 개선 및 시행이다. 모형이 가진 내재적 한계로 인해 자산 배분 전략을 적절하게 수행하지 못하는 경우, 해당 포트폴리오 모형이 추구하는 목표를 달성하지 못하게 되어 바람직하지 않은 포트폴리오가 구축되는 문제가 발생할 수 있다. 이러한 목표는 여러 개의 자산을 포함하는 포트폴리오에 대한 분산 효과와 한가지 자산에 대한 포트폴리오 가치 방어를 통한 위험 관리를 포함한다. 둘째, 투자를 위한 자산군 선택이다. 기존의 자산군과 상관관계가 작은 새로운 자산군에 대한 선택이 효율적인 포트폴리오 구축에 있어 잠재적으로 큰 도움을 줄 수 있다. 본 논문은 포트폴리오 관리에 대한 이러한 두 가지 핵심과 절차에 초점을 맞추어 연구를 수행하였다. 자산 분산화와 위험 관리 각각의 관점에 대하여, 첫째, 기존 포트폴리오 모형의 구축 및 모수 추정에 대한 한계점을 개선하는 연구를 수행하였다. 둘째, 새로운 디지털 자산 시장에 대한 포트폴리오 분석을 수행하였다.

이에 따라, 본 논문의 구체적인 목표는 다음과 같이 두 가지로 정리될 수 있다. 첫째, 모형 구축 및 모수 추정에 대한 한계점을 갖는 기존 포트폴리오 관리 전략의 개선에 관한 연구를 수행하는 것이다. 구체적으로, 블랙-리터만 모형의 전망 구축과 합성 풋 옵션 전략의 모수 추정에 대한 문제를 다루었다. 둘째, 대체불가능 토큰과 암호화폐 시장을 포함하는 디지털 자산 시장에 관한 포트폴리오 분석 및 실증 결과를 살펴보는 것이다.

이때, 대체불가능 토큰에 대해서는 마코위츠의 평균-분산 모형을, 암호화폐에 대해서는 포트폴리오 보험 모형을 사용한다. 첫 번째 연구를 위해, 자산 수익률 이외의 외부적인 금융 데이터로부터 의미 있는 패턴을 추출할 수 있는 기계학습 모형을 사용하여 블랙-리터만 모형의 전망 구축을 수행하는 모형을 제안하였고, 이에 대한 실증 결과를 살펴보았다. 또한, 합성 풋 옵션 전략에서 요구하는 변동성 모수 추정의 문제를 해결하기 위해 기계학습 기반 변동성 예측 모형을 사용하여 개선된 합성 풋 옵션 전략을 제안하고, 이에 대한 실증 연구를 수행하였다. 두 번째 연구를 위해서는, 기존 자산 기반 포트폴리오에 대해 대체불가능 토큰이 새로운 자산군으로써 분산 효과를 제공할 수 있는지를 살펴봄으로써 그 경제학적 가치를 검증해 보았고, 다양한 위험 측정 지표와 투자자 효용 측면에서 포트폴리오 보험 전략에 대한 암호화폐 시장에서의 실증 결과를 살펴보았다.

본 논문의 주요 실증 결과는 다음과 같다. 첫째, 기업 특성 변수를 결합하여 전망에 반영하였을 때, 블랙-리터만 모형에서 산출된 포트폴리오의 수익률 분포가 개선됨을 확인하였다. 또한, 기업 특성 변수를 반영할 때, 과거의 정보를 단순히 반영하는 것보다 기계학습 기법을 활용하여 미래에 대한 예측 방식으로 반영할 때 표본 외 성능 측면에서 훨씬 큰 개선이 나타났다. 해당 연구 결과는, 본 논문에서 제안된 기업 특성 변수 기반 전망 구축 방법론을 바탕으로 한 블랙-리터만 모형을 통해 더 잘 분산되고 더욱 효율적인 포트폴리오를 구축하는 것이 가능하다는 것을 보여준다는 점에서 의의가 있다.

둘째, 계량 경제 모형 및 포트폴리오 실증 분석 결과, 대체불가능 토큰은 기존 자산에 시장에 대해 헤지, 안전 피난처, 분산 효과를 갖는다는 증거를 발견하였다. 구체적으로, 대체불가능 토큰은 여러 국가의 주식 시장, 원유 시장, 채권 시장, 달러 지수에 대해 헤지 및 안전 피난처 효과를 보이며, 이러한 경향성은 자산 수익률 데이터의 해상도가 변함에 따라 그 정도가 달라진다. 특히 COVID-19 위기 동안, 채권 시장 및 달러 지수에 대해 더욱 강한 강도의 안전 피난처 효과를 보였다. 또한, 대체불가능 토큰 시장은 기존

자산 시장과 매우 구별되는 자산 시장으로써, 상관관계, 공행성, 변동성 스필오버 효과 및 마코위츠의 평균-분산 포트폴리오 모형을 통한 분석 결과, 대체불가능 토큰이 기존 자산군에 대한 강한 분산 효과를 가짐을 확인하였다. 이를 통해, 대체불가능 토큰의 편입이 균등 배분 포트폴리오 모형과 접점 포트폴리오 모형을 샤프 비율 측면에서 크게 개선 시킬 수 있음을 확인하였다.

셋째, 포트폴리오 가치 방어 오차 측면에서 합성 풋 옵션 전략에 변동성 모수 추정 오차에 의한 악영향이 존재함을 시뮬레이션 및 실제 금융 시장 데이터를 통해 확인하였다. 흥미롭게도, 포트폴리오 가치 방어 오차는 이러한 변동성 예측의 정확도와 직접적으로 연관되어 있다는 것을 통계적으로 확인하였다. 이는, 더욱 정확한 변동성 예측 모형을 통해 합성 풋 옵션의 모수 추정 오차 문제를 완화할 수 있다는 사실을 실증적으로 확인했다는 점에서 의의가 있다. 또 다른 결과로써, 전통적인 변동성 예측 방법론 및 기계학습 기반 변동성 예측 방법론 모두 단순 모형보다 성능이 좋다는 것을 확인하였다. 또한, 기계학습 모형이 가장 우수한 성능을 보였으며, 그중 익스트림 그라디언트 부스팅 (XGB) 모형이 포트폴리오 가치 방어 오차 및 변동성 예측 오차 측면에서 가장 좋은 성능을 보임을 확인하였다. 이러한 경향성은 기계학습 모형이 기존의 모형 보다 실현 변동성 (realized volatility)의 복잡한 파동 패턴을, 매우 변동성이 큰 시장 상황에서도 더욱 잘 잡아낸다는 사실을 지지하는 결과라 할 수 있다.

마지막으로, 하방 위험 측면에서, 포트폴리오 보험 전략들이 암호화폐 시장에서 벤치마크 방법론보다 더 좋은 성능을 보임을 실증적으로 확인하였다. 이러한 포트폴리오 보험 전략들은 매수 후 보유 전략보다 더 작은 위험을 보이는 것을 알 수 있었다. 또한, 흥미롭게도, 효용함수의 곡률 측면에서, 전망 이론 투자자의 포트폴리오 선택과 기대효용 이론 투자자의 포트폴리오 선택의 경향성이 서로 반대로 나타남을 발견 하였다. 이러한 결과는, 전망 이론 투자자에 대하여 이익 대비 손실의 영향력이 더 클 수 있음을 나타낸다. 이와 더불어, 투자자의 손실 회피 경향이 포트폴리오 보험 전략에 대한 투

자자의 선호를 더욱 강화시킬 수 있음을 확인하였다. 가장 놀라운 결과로써, 투자자가 어떤 효용 함수를 따르는지에 관계없이, 암호화폐 시장에서 포트폴리오 보험 전략이 매수 후 보유 전략 보다 높은 효용을 주는 영역이 기존 자산 시장에서보다 더 넓음을 확인하였다. 이는 포트폴리오 보험 전략이 더 많은 수의 암호화폐 투자자에 대해 위험 관리 측면에서 더 큰 경제학적 가치를 제공해 줄 수 있음을 실증하는 결과라는 점에서 의의가 있다.

본 논문은 블랙-리터만 모형의 다중 자산 포트폴리오와 합성 풋 옵션 전략의 개별 자산 포트폴리오에 대한 포트폴리오 관리 모형을 자산 분산화와 위험 관리 측면에서 개선하는 연구를 수행하였다. 또한, 마코위츠의 평균-분산 모형과 포트폴리오 보험 모형을 사용하여 대체불가능 토큰과 암호화폐 시장을 포함한 새로운 디지털 자산 시장에서의 포트폴리오 분석을 수행하였다. 본 논문의 결과를 바탕으로, 투자자들은 자산 분산화와 위험 관리 관점에서 더욱 개선된 포트폴리오 전략을 달성할 수 있으며, 이를 통해 개선된 포트폴리오 관리를 위한 더욱더 효율적이고 바람직한 투자 포트폴리오를 구축할 수 있게 될 것으로 기대된다.

주요어: 포트폴리오 관리, 기계학습, 자산 배분, 위험 관리, 디지털 자산, 암호 화폐, 대체불가능 토큰

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