Theoretical Economics Letters, 2022, 12, 712-741
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Timing of Revenues and Expenses: Evidence from Finland

Erkki K. Laitinen, Teija Laitinen*

School of Accounting and Finance, University of Vaasa, Vaasa, Finland
Email: *tla@uva.fi

How to cite this paper: Laitinen, E. K., & Laitinen, T. (2022). Timing of Revenues and Expenses: Evidence from Finland. *Theoretical Economics Letters, 12,* 712-741.
https://doi.org/10.4236/tel.2022.123040

Received: April 11, 2022
Accepted: June 12, 2022
Published: June 15, 2022

Copyright © 2022 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/
Open Access

Abstract

In accounting, a critical issue is the timing of revenues and expenses in the annual closing of accounts. If expenditures are expired as expenses faster than revenues are generated, the expense method is accelerated. In the opposite case, the expense method is decelerated. Thus, the timing (time lags) of expenses and revenues is an important determinant of earnings and profitability. The objective of this study is to analyze critically alternative methods to estimate the time lags of revenues and expenses. Six different methods based on correlation, return on investment, logarithmic differences, unrestricted OLS, restricted OLS, and Solver optimization are analyzed. Empirically, these methods are applied to nine-year time series of financial statements from a sample of 697 Finnish firms. Empirical analyses show that these methods still have deficiencies that potentially lead to unreliable results of timing. In future, more research on this important topic is called for.

Keywords

Timing of Expenses, Time Lag Parameters, Time-Series Data, Distributed Lag Model, Profitability, Growth, Finnish Firms

1. Introduction

In accounting, earnings are regarded as an important summary measure of the financial performance of a firm used in valuation and contracting (Bauwhede & Willekens, 2003). Thus, managers can affect valuation and contracting using judgment in financial reporting. They can involve in earnings management (EM) and alter financial reports to mislead stakeholders (Healy & Wahlen, 1999). It leads to losses of investor wealth and misdirects resources from their most productive use (Beneish, Lee, & Nichols, 2013). Since earnings are measured by the difference between revenues and expenses, managers can in EM consider revenues, expenses, or both. In EM, managers can speed up the accumulation of revenues or slow down the accumulation of expenses to improve earnings. Therefore, the time lag between expenditures and generated revenues (that is, revenue lag) and the time lag between expenditures and expired expenses (that is, expense lag) are important determinants of earnings. If the revenue lag is less than the expense lag, expenses are decelerated. In the opposite case, expenses are accelerated. If these time lags are equal, expenses are not decelerated nor accelerated. In this case, the timing of expenses coincides with the timing of expenses so that expenses are properly matched with revenues. In accounting research, the methods to assess empirically these time lags are only little investigated. Therefore, in this study, these methods are critically evaluated.

In accounting, the quality of matching is an important issue, since it has many consequences. Dichev and Tang (2008) define perfect matching as the case where all expenses can be traced directly and specifically to specific revenues. This kind of tracing makes the timing of expenses coincide with the timing of revenues. Based on a specific framework, Dichev and Tang argue that the assumption of perfect matching provides a series of implications. Firstly, in a competitive equilibrium earnings tend to gravitate towards the cost of equity capital. Secondly, deviations in earnings from the long-run mean will gradually diminish over time. Thirdly, according to their framework, there is an economic shock in every period, which is the noise in the matching relation and has a mean of zero. The variance of this economic shock represents the economic volatility of the business environment. Fourthly, in a perfect matching situation, the volatility is driven entirely by economic factors.

Dichev and Tang (2008) use their framework to argue that poor matching has also several implications. Firstly, poor matching decreases the time-series contemporaneous correlation between revenues and expenses. In poor matching, some of the perfectly matched expenses get scattered across different periods, which results in a lower synchronal correlation than the underlying economic correlation of advancing expenses to produce revenues. Secondly, poor matching increases the volatility of earnings. The volatility in earnings that are poorly matched is higher, because the mismatched expenses act as a noise that is not related to the economic process of creating earnings. Thirdly, the persistence of earnings will decrease with poor matching. Thus, poor matching can seriously distort the quality of earnings.

In current matching studies, the contemporaneous correlation between revenues and expenses plays an important role. This correlation or some of its variants has been widely used to reflect the quality of matching, and thus, the quality of earnings (Sivakumar & Waymire, 2003; Dichev & Tang, 2008; Donelson, Jennings, & McInnis, 2011; Srivastava, 2011; He and Shan, 2016; Bushman, Lerman, & Zhang, 2016; Laitinen, 2020). This field of study is essential to accounting since earnings are the most important output of the accounting system having a strong influence on the behavior of stakeholders (Graham, Harvey and Rajgopal, 2005). Furthermore, this field is directly related to the traditional purpose of accounting that is properly to match the expenses against the resulting revenues (Dichev & Tang, 2008; Paton & Littleton, 1940). Thus, matching of expenses with revenues is one of the most important research areas of accounting emphasizing the importance of contemporaneous correlation.

However, it is argued in this study that the contemporaneous correlation between revenues and expenses is a limited measure of matching quality. It is argued that this correlation is associated with the volatility of earnings but that it in steady circumstances cannot recognize a potential bias in earnings. The main limitations will arise when the firm is approaching a steady state. In steady circumstances, the time correlation between revenues and expenses will approach unity irrespective of the time lags. In this case, the volatility of earnings may be limited as indicated by the high correlation. However, depending on the time lags earnings may be strongly biased which, due to the bias, destroys the validity of earnings.

Thus, it is argued in the present study that in order to assess the validity of earnings properly, the time lags of revenues and expenses should be separately estimated to detect the potential bias. The objective of the study is critically to introduce novel approaches to estimate the time lags. Many of these approaches are in this study considered for the first time in accounting research. The idea of the study is firstly to develop a simple analytic framework for accounting concepts and describe the estimation methods in this framework. Secondly, in the empirical part of the study, the usefulness of these approaches is critically analyzed. The empirical data of the study is drawn from the nine-year time series of the financial statements from a sample of Finnish firms.

The contents of the study are as follows. First, in this section the idea of the study was shortly discussed. In the second section, the analytic framework of the study is presented in several stages. First, relevant matching studies are shortly reviewed. Secondly, analytical approaches used to describe relevant accounting concepts are briefly discussed. These approaches are based on a narrow analytical research field on the validity of the return on investment ratio (accounting rate of return, ARR) to reflect earnings. Thirdly, an analytic approach is developed and used to extract several methods of estimation for the time lags. In the third section, the empirical data and statistical methods are described. The sample of the study consists of nine-year time series of financial statements finally from 697 Finnish firms. The fourth section includes a discussion of empirical findings. Empirical results indicated that the task to estimate the time lags is very challenging. Thus, all estimation approaches (considered here) potentially have deficiencies in empirical analysis. They should be further improved to get reliable estimates for the time lags. Thus, a call for future research on the topic is presented. Finally, the last section shortly concludes the findings of the study.

2. Framework of the Study

2.1. Matching Studies

Dichev and Tang (2008) introduced a model of matching, where a random variable represented mismatched expense that is unrelated to the well-matched expense and revenue. If expenses are not properly matched against the revenues, it is regarded as poor matching and as a noise in the economic relation of advancing expenses to obtain revenues. Dichev and Tang recommended the contemporaneous correlation of revenues and expenses as a measure of the quality of matching, since poor matching decreases this correlation. In poor matching, some of the perfectly matched expenses get scattered across different periods, which results in a lower synchronal correlation than the underlying economic correlation of advancing expenses to produce revenues. Sivakumar and Waymire (2003) had earlier used this time correlation between revenues and expenses to assess the effects of new rules for depreciation accruals hypothesizing, that new rules will lead to lower correlation due to restrictions on matching.

Dichev and Tang (2008) themselves used the contemporaneous revenue-expense correlation to reflect the quality of matching in large firms showing a significant decline in quality over 40 years. Their results suggested that accounting factors such as the quality of accruals are a substantial determinant of the observed temporal patterns and changes in the real economy play only a secondary role. Donelson, Jennings and McInnis (2011) used a similar sample and concluded that none of the accounting standards has significantly affected special items, which they found to have an impact on the correlation. Srivastava (2011) found that the shift in the U.S. economy towards industries with higher period costs and more research and development activities contributed to the decline in matching quality. He and Shan (2016) investigated the time-series trend and determinants of matching between revenues and expenses in a sample of 42 countries. They found that the decline in matching quality documented by Dichev and Tang (2008) is not unique to the United States, but is a worldwide phenomenon.

In prior research, also different variants of the correlation have been used to reflect matching quality. Bushman, Lerman and Zhang (2016) measured matching using a version of the correlation coefficient. They employed the adjusted coefficient of determination from the cross-sectional regression of revenues on lead, lag, and contemporaneous expense as a more direct measure of the random error component of expense recognition to measure matching. Basu, Cready and Paek (2016) used two different measures for matching. First, they focus how much expense is being matched to revenues. They measure the percentage of expense recognized as a linear function of contemporaneous revenue. The authors multiply the estimate for the level of expenses that are being matched to revenues as a percentage of revenue by the ratio of average revenue to average expense over a ten-year period.

Basu, Cready and Paek also measure mismatching focusing on the total amount of expense that is explained by the introduction of the adjacent period revenue terms. That is, they examine the reduction in residual error obtained by supplementing the contemporaneous revenue with the lag revenue variable in regression using the difference of the mean squared errors. The authors found a more sizable decline in matching quality than Dichev and Tang (2008) since the turn of the century. Laitinen (2020) used logarithmic correlation between revenues and a weighted function of different types of expenses to measure the quality of matching. He also introduced a concept of matching elasticity to describe the functional relationship between revenues and matched expenses. In the present study, the properties of the contemporaneous correlation are analytically investigated. It is argued that the correlation is not a proper measure of timing quality especially in steady circumstances.

In addition to specific matching research, the timing of revenues and expenses is an important issue in earnings management (EM) research in general. EM makes a negative influence on the quality of earnings and weakens the credibility of financial reporting. The traditional way to detect EM in time series analysis is to estimate nondiscretionary accruals from the past levels of the firm during periods when systematic EM is not assumed. Healy (1985), DeAngelo (1986), Jones (1991), and modified Jones (Dechow, Sloan, & Sweeney, 1995) models are examples of these kinds of time series models. The major problem in these approaches is that accruals vary with changes in business circumstances and growth. Therefore, there is missing a reliable benchmark. The recent models try to control for these changes with parameters adjusting accruals to these changes (Spohr, 2005). However, practically all these techniques assume that the residual from a linear regression represents EM (Gerakos, 2012). In this study, time series analysis is used to estimate time lags of revenues and expenses to assess the quality of timing. This kind of approach to estimate time lags from time series is novel and differs notably from the traditional EM time series models.

2.2. Studies on Accounting Rate of Return

The timing of revenues and expenses is in this study analyzed using a similar approach as employed already in early research concentrating on the relationship between the reported accounting rate of return (ARR) (the return on investment ratio, ROI) and the internal rate of return (IRR). Examples of these early studies are Harcourt (1965), Solomon (1966), Kay (1976), Fisher & McGowan (1983), Whittington (1988), and Peasnell (1982, 1996). IRR is a central concept of profitability used by economists in discounting cash flows and assessing profitability of capital investments. Therefore, it is often called the economic rate of return (ERR) (Luckett, 1984; Feenstra & Wang, 2000; Brief, 2013). This body of research is also motivated by the importance of EM. Managers have discretionary power in expense allocation that can yield errors in timing of expenses and thus measurement errors in ARR. In this field of study, researchers usually address the issue whether a periodic ARR serves as a reliable measure of constant IRR. This research however concludes that the reported ARR is generally a poor proxy for IRR reflecting poor quality of earnings (Feenstra & Wang, 2000).

The models used in this research field are useful especially in depicting the relation between revenues and expenses. A typical way to construct a model in ARR research is to use a steady-state approach that leads to tractable mathematical solutions. However, a steady model is not enough to describe revenue-expense correlation properly. Laitinen (2017) introduced a non-steady model depicting the early time-series of startups. For startups, the time-series are usually in the first phases of development non-steady and they will approach a steady state only in the passage of time. Laitinen concluded that in startups, the early ARRs are very poor approximations of IRR and they can easily mislead investors and financers. In this study, a similar non-steady approach is used to investigate the timing of revenues and expenses. It is assumed that the periodic expenditures of the firm grow at a constant rate and generate revenues according to a lag structure following a fixed geometric distribution. The parameter of the distribution describes the form of the lag structure and the length of the revenue lag. It thus reflects timing of revenues. Furthermore, at the end of each accounting period, the firm is assumed to expire a constant proportion of unexpired expenditures as expenses. Thus, this constant proportion reflects the timing of expenses and can compared with the lag parameter of revenues. In the perfect timing, the lag parameter of revenues equals the lag parameter of expenses.

2.3. Non-Steady State Analysis

2.3.1. Expenditures, Expenses, and Revenues

The framework of this study assumes that the expenditures of the firm grow at a constant rate as follows:

$$M_t = M_0 (1+g)^t \tag{1}$$

where M_0 is the initial expenditure in period 0, M_t is expenditure in period t, and g is the constant rate of growth. The constant growth rate is an assumption that remarkably simplifies the framework and makes the analysis mathematically tractable. Although the growth of expenditures is assumed steady, the resulted time series of expenses and revenues are in the early stage non-steady, but converge towards steady growth in the passage of time.

If a firm (for instance, a limited company) runs business, it is obliged to prepare periodically an income statement and a balance sheet according to the accounting law, conventions and doctrine. The going concern convention is based on continuity of activity assuming that the business will be operating indefinitely. The doctrine of consistency requires that financial statements for different accounting periods are based on the same accounting principles making financial results comparable across periods. These general rules justify us to assume that certain accounting parameters are fixed over time although managers are able to change policies on certain reasons (Keating & Zimmerman, 1999). Therefore, it is assumed that the firm periodically expenses a fixed proportion C of periodic expenditure and unexpired expenditure in the balance sheet. This systematic accounting procedure leads to the following time series of expenses

D_t when t goes from 0 to n:

$$D_n = CM_0 \sum_{t=0}^{n} (1+g)^t (1-C)^{n-t} = C(1-C)^n M_0 \sum_{t=0}^{n} (1+g)^t (1-C)^{-t} = CM_n \left[\frac{(1+g)^{n+1} - (1-C)^{n+1}}{(1+g)^n (g+C)} \right] \quad (2)$$

Equation (2) shows that the growth of expenses is a function of the growth rate of expenditure g and the fixed expense proportion C. In this kind of expenditure-based expense, expenses, and thus C, are independent of the revenues generated by expenditures. Since a constant proportion of expenses leads to a geometric series of expenses over expiration time of expenditure, C reflects the time lag between an expenditure and the series of expenses. Thus, the average expense lag can be expressed as $(1 - C)/C$. The higher C, the lower is the average lag and the faster expenditure is expired as expenses.

The framework further assumes that each periodic expenditure generates an infinite series of revenues with an identical lag structure and *IRR*. It is assumed that the series of revenues follow an infinite geometric distribution with a constant lag parameter q. Thus, the growth of revenues from period 0 to n can be expressed as follows:

$$R_n = KM_0 \sum_{t=0}^{n} (1+g)^t q^{n-t} = KM_n q^n \sum_{t=0}^{n} (1+g)^t q^{-t} = KM_n \left[\frac{(1+g)^{n+1} - q^{n+1}}{(1+g)^n (1+g-q)} \right] \quad (3)$$

where K is the level parameter of the lagged revenue distribution. Since the series of revenues follows an infinite geometric distribution, the average lag between revenue and expenditure is $q/(1 - q)$. Thus, in timing of revenues and expenses the lag parameters q and $1 - C$ play the key roles. In the perfect timing, $1 - C = q$. The perfect timing in this framework means that the expiration of expenditure as expenses perfectly follows the realization of revenues leading to a simultaneous recognition of revenue and expense. If $C > 1 - q$, expenditure is expired faster as expenses than revenues generated by this expenditure, are recognized leading to an accelerated pattern of expenses. Similarly, if $C < 1 - q$, a decelerated pattern of expenses is followed.

In Equation (3) K refers to the level parameter of the lagged revenue distribution. It gives the revenue contribution recognized in the same period as the expenditure is invested, as a proportion of this expenditure. This constant level makes it possible to incorporate *IRR* or r into the framework. This level K can be solved in the following way assuming that n approach infinity:

$$M_n = M_n K \sum_{t=0}^{n} q^t (1+r)^{-t} \rightarrow K = \frac{1+r-q}{1+r} \quad (n \rightarrow \infty) \quad (4)$$

Equation (4) shows that the level K is the higher, the higher is r and the lower is q (the faster revenues are generated by expenditures).

The comparison of Equation (2) and Equation (3) shows that if $1 - C = q$, then the synchronal developments of expenses D_t and revenues R_t over time are parallel. If $q = 1 - C$, revenues grow at the same rate as expenses and, thus, the expense-revenue ratio stays constant. However, if $q > 1 - C$, expenses grow faster than revenues and the expense-revenue ratio is increasing over time. If $q < 1 - C$, revenues are growing faster than expenses leading to a decreasing expense-revenue ratio.

2.3.2. Correlation Coefficient

The interpretation of the time series (2) and (3) can be facilitated by moving to the analysis of deflated time series. If expenses D_t in (2) and revenues R_t in (3) are deflated (discounted) by the growth rate of expenditure g, the following results are obtained:

$$D_n^d = CM_0 \left[\frac{1+g-\left(\frac{1-C}{1+g}\right)^{n+1}(1+g)}{g+C} \right] \quad (5a)$$

$$R_n^d = KM_0 \left[\frac{1+g-\left(\frac{q}{1+g}\right)^{n+1}(1+g)}{1+g-q} \right] \quad (5b)$$

where D_n^d and R_n^d refer to the deflated time series of expenses and revenues, respectively.

The time-dependent parts of the discounted time series (5a) and (5b) depend geometrically on the terms $(1 - C)/(1 + g)$ and $q/(1 + g)$, which makes it possible to approximate the correlation coefficient between the time series. The correlation coefficient between two infinite geometric time series with parameters G_1 and G_2 can be presented as follows:

$$\frac{\left[\left(1-G_1^2\right)\left(1-G_2^2\right)\right]^{1/2}}{1-G_1G_2} \quad (6)$$

where $G_1 < 1$ and $G_2 < 1$ to make the series converge. In this analysis, $G_1 = q/(1 + g)$ and $G_2 = (1 - C)/(1 + g)$. Equation (6) shows that if $1 - C = q$, the correlation coefficient equals unity which correctly refers to the perfect timing. The larger the difference between $1 - C$ and q, the lower is the correlation coefficient. Thus, in this case, low contemporaneous correlation refers to a poor timing quality between revenues and expenses as expected by prior research.

Thus, in theory, the correlation coefficient may be a reliable measure of timing. However, in its standard form, it only gives a rough estimate how much the lag parameters of expenses and revenues in absolute terms potentially differ from each other. It can however also be used to approximate the lag parameter q when $1 - C$ is known. The lag parameter of expenses $1 - C$ is quite simple to estimate from time series of expenditures and expenses. However, the lag parameter of revenue q is a latent variable and very difficult to estimate properly (Hall, 2007; Laitinen, 2017). Therefore, correlation coefficient may be useful in drawing a rough estimate of q from estimated $G_2 = (1 - C)/(1 + g)$ in the following way:

$$G_1 = \frac{G_2 P^2 - \sqrt{G_2^4\left(-P^2\right) + G_2^4 + 2G_2^2 P^2 - 2G_2^2 - P^2 + 1}}{G_2^2 P^2 - G_2^2 + 1} \tag{7a}$$

or

$$G_1 = \frac{G_2 P^2 + \sqrt{G_2^4\left(-P^2\right) + G_2^4 + 2G_2^2 P^2 - 2G_2^2 - P^2 + 1}}{G_2^2 P^2 - G_2^2 + 1} \tag{7b}$$

where $G_1 = q/(1 + g)$ and P is the Pearson time correlation between the discounted time series of revenues and expenses. The equation has two roots because q can either exceed $1 - C$ or be below $1 - C$ so that both roots give the same P. If the expense-revenue ratio is increasing, q is higher than $1 - C$ and (7a) holds. In the opposite case when the ratio is decreasing, $1 - C$ exceeds q and (7b) is valid. The choice of the root is crucial, because it determines whether the expiration of expenses is decelerated or accelerated. This method to estimate q can give useful results if $1 - C$ can first be estimated properly. However, the selection of the root as either (7a) or (7b) should be made carefully. Furthermore, in the passage of time the differences in the growth rates of expenses and revenues diminish, when the firm is approaching a steady state. This kind of development will increase correlation, but this increase is not a consequence of improving timing but of approaching a steady state.

2.3.3. Logarithmic Differences of Discounted Time Series

The correlation method of estimation makes it possible to approximate the time lag parameter of revenues q, when the lag parameter of expenses $1 - C$ and the correlation coefficient P between the time series are known. However, it is also possible to estimate the lag parameter of revenues q directly from the time series of discounted revenues (5b) in the following way. The first-order difference of the time series in period n can be presented as follows:

$$R_n^d - R_{n-1}^d = KM_0 \cdot \left[\frac{1+g}{1+g-q}\right] \cdot \left[\frac{q}{1+g}\right]^n \cdot \left[\frac{q}{1+g} - 1\right] \tag{8}$$

which gives the following solution as presented in the (natural) logarithmic terms:

$$\log\left[R_n^d - R_{n-1}^d\right] = n \cdot \log\left[\frac{q}{1+g}\right] + \log\left\{KM_0 \cdot \left[\frac{1+g}{1+g-q}\right] \cdot \left[\frac{q}{1+g} - 1\right]\right\} \tag{9}$$

Following Equation (9) the coefficient $q/(1 + g)$ can now be estimated by an ordinary linear model as the exponent of the coefficient of the time index n. However, this estimation procedure for q in a linear form can be carried out only using the logarithmic discounted time series of revenues.

The estimation of the lag parameter 1 − *C* can be carried out following the similar procedure. Taking the difference of the discounted time series of expenses in Equation (5a) and, then, transforming it to the logarithmic form, the following equation is obtained:

$$\log\left[D_n^d - D_{n-1}^d\right] = n\cdot\log\left[\frac{1-C}{1+g}\right] + \log\left\{CM_0\cdot\left[\frac{1+g}{g+C}\right]\cdot\left[\frac{1-C}{1+g}-1\right]\right\} \tag{10}$$

which makes it possible to estimate the coefficient (1 − *C*)/(1 + *g*) by a linear model as the exponent of the coefficient of the time index *n* similarly as *q*/(1 + *g*) in (9).

The estimation methods that are based on logarithmic difference time series are not problem-free. The estimation models (9) and (10) are properly applicable only for increasing monotonic discounted time series. The first-order differences of the discounted time series should be positive to make it possible to use logarithmic transformation. Otherwise, the negative differences should be excluded from the estimation data or they must be replaced by appropriate positive values to enable taking logarithms. This limitation can be restrictive in estimation if the firms in the sample report very low or even negative growth rates.

2.4. Steady state Analysis

2.4.1. Approaching the Steady State

Equation (1) assumes that the expenditures of the firm grow at a steady rate g. Moreover, Equations (2) and (3) show that in this framework the non-steady time series of expenses and revenues are converging towards a steady state with the passage of time. In the steady state, these three variables grow at the same rate *g* and the ratios of the variables are constant over time so that the time (contemporaneous) correlations between them equal unity. In this steady case, correlation is thus supposed to indicate that the timing of expenses and revenues is perfect. However, this indication does not necessarily hold.

The steady state will be reached regardless of the values of 1 − *C* (<1) and *q* (<1) since the growth rates of expenses and revenues are continuously converging towards *g*. Thus, the correlation coefficient between the (discounted) time series of expenses and revenues continuously increases towards unity, when the growth rates of revenues and expenses are approaching each other. Factually, this convergence means that on steady state circumstances the correlation equals unity although *q* and 1 − *C* may significantly differ. The correlation coefficient can therefore be a biased measure of the quality of timing for a firm approaching a steady state. In an extreme case, this characteristic of steady state can lead to the situation where the contemporaneous correlation is unity and the volatility of earnings is limited but, at the same time, earnings are strongly biased due to low quality of timing when 1 − *C* and *q* notably differ from each other.

2.4.2. Steady Proportion of Expenses 1 − *C*

If the firm has reached a steady state so that expenditures, expenses, and revenues grow at the same rate g, expenses D_t in Equation (2) converge towards the following steady value:

$$D_n = CM_n \left[\frac{1+g}{g+C} \right] \quad (n \to \infty) \tag{11}$$

Equation (11) shows that the steady ratio of periodic expenses to periodic expenditures depends on g and C. This relation (11) in the steady state is decreasing in g whereas the marginal effect of C is positive for $g > 0$ and negative for $g < 0$. If $g = 0$, $D_n = M_n$ and Equation (11) holds for any value of C.

Equation (11) makes it possible to estimate C using the following solution:

$$C = \frac{D_n g}{M_n (1+g) - D_n} = \frac{\frac{D}{M} g}{1 + g - \frac{D}{M}} \tag{12}$$

which shows that C is closely associated with the steady rate of growth g. In Equation (12), D/M refers to the constant ratio of expenses to expenditures. The estimate of C in (12) is obviously sensitive to the growth rate. Moreover, according to the assumptions of the method, the firm should grow approximately at steady rates to get an appropriate estimate of C. When C is estimated through (12), the time lag of expenses is obtained as $1 - C$. Similarly to (9) and (10), the use of (12) can be questioned when the firms in the sample data grow at very low or even negative rates. When the growth rate g is close to zero, the estimate of C can be very slippery due to the sensitiveness for g.

2.4.3. ROI as an Approximation of *r*

Equation (3) shows that the steady value of revenues R_n can be presented in the following form:

$$R_n = M_n \left[\frac{(1+r-q)(1+g)}{(1+r)(1+g-q)} \right] \to \frac{R}{M} = \frac{(1+r-q)(1+g)}{(1+r)(1+g-q)} \quad (n \to \infty) \tag{13}$$

where R/M is the constant steady ratio of revenues to expenditures. Equation (13) implicates that the ratio R/M equals unity for $r = g$. It exceeds unity if $r > g$ and is below unity if $g > r$. Thus, in summary, R/M is symmetric with respect to r and g.

Equation (13) shows the steady ratio of revenues to expenditures that can be useful in approximating the lag parameter q. However, since the ratio is a function of parameters r and q, it only allows calculating the relation between these parameters as follows:

$$q = \frac{\left(-\frac{R}{M} + 1 \right)(1+g)}{-\frac{R}{M} + \frac{1+g}{1+r}} \tag{14}$$

Because r is a latent variable that cannot be directly observed, it must be approximated in some way to get an estimate for q. In Equation (14), q is sensitive to the estimate of r (=: r^*) making this estimation challenging, since even small errors in r^* may distort the estimate of q. Equation (13) implies that r^* should exceed g for $R/M > 1$ and be below g for $R/M < 1$. If $R/M = 1$, r^* simply equals g.

Let us assume that there is the perfect timing of revenues and expenses so that $1 - C = q$. In this special case of perfect timing, the steady state assets of the firm A_t can be can be determined by summing up the unexpired proportions of periodic expenditures to infinity as follows:

$$\begin{aligned} A_n &= (1-C) M_0 \sum_{i=0}^{\infty} (1+g)^i (1-C)^{n-i} \\ &= (1-C) M_n \left[\frac{1+g}{g+C}\right] = q M_n \left[\frac{1+g}{1+g-q}\right] \quad (n \to \infty) \end{aligned} \tag{15}$$

The steady return on assets ratio ROI can now be solved deducting D_t in (11) from R_t in (13) and finally dividing the result (that is, profit) by the assets in the beginning of the period t, that is by A_{t-1}.

When simplifying the results, the following ROI_t and r^* are resulted from this procedure:

$$ROI_t = \frac{R_t - D_t}{A_{t-1}} \to ROI = r \cdot \frac{1+g}{1+r} \to r^* = \frac{ROI}{1+g-ROI} \quad (q = 1-C) \tag{16}$$

If there is perfect timing so that $q = 1 - C$, r can be approximated by ROI following Equation (16). However, if perfect timing is assumed, q can be approximated simply by $1 - C$.

If $1 - C = q$ is assumed, the firm is factually using a neutral (proportional) declining-balance depreciation method leading to perfect timing of expenses and revenues. In a more general case where perfect timing is not assumed, another depreciation method must be assumed. For example, it can be assumed that the rate-of-return or realization theory of expense is considered. In this accelerated depreciation method, expenses are determined by the presents value of realized revenue, discounted by IRR to the moment of acquisition. Laitinen (2018) has shown that for this depreciation method, IRR can be approximated using the following relation:

$$ROI = r \cdot \frac{(1+r-q)(1+g)}{(1+r)(1+g-q)} = r \cdot \frac{R}{M} \to r^* = ROI \cdot \frac{M}{R} \tag{17}$$

Thus, in this case ROI is a biased measure of IRR. The relation between ROI and r^* is proportional to the expenditure-revenue ratio M/R showing that $ROI = r^*$ only when $r = g$ (that is, $R/M = 1$). A decelerated depreciation can be described for example by the compound interest or annuity method (of depreciation). In this method, the assets of the firm are determined as the present value of future revenues generated by past and present expenditures. In this present value, IRR is used as the rate of discount. Laitinen (2018) among others has showed that for this depreciation method $ROI = r$ leading to $r^* = ROI$. If the firm uses this kind of decelerated depreciation, ROI may give a reasonable estimate of *r*.

2.4.4. Koyck Transformation

Equation (13) makes it possible to present the lag parameter q as a coefficient of the lagged revenue R_{t-1} following traditional Koyck (1954) transformation. This transformation is a procedure used generally in time series analysis to transform an infinite geometric lag model into a model with lagged dependent variable (Franses & van Oest, 2004). In this case, the following equation is obtained:

$$R_t = M_t \cdot K \cdot \frac{1+g}{1+g-q} \rightarrow R_t \cdot \frac{1+g-q}{1+g} = K \cdot M_t \rightarrow R_t = K \cdot M_t + q \cdot R_{t-1} \quad (18)$$

Equation (18) shows that it is possible to estimate K (and thus r) and q simultaneously from the time series of R_t and M_t. Although Equation (18) seems to be intuitively appealing for estimation, estimation of q may suffer from serious statistical problems. Firstly, the transformed model is likely to have serial correlation in errors. Secondly, in steady circumstances M_t and R_{t-1} are strongly correlated leading to obvious multicollinearity. The distributed lag model as (18) generally gives statistically highly significant estimates that, however, can be remarkably biased.

Koyck transformation can also be applied to estimate the lag parameter $1 - C$ in the same way as q in Equation (18). In this application, the estimation is however simpler, since expenditures generate expenses exactly 100% of expenditures, provided that all expenditures are expired as expenses. This simplification means that, using concepts of revenue-expenditure flows in Equation (18), $r = 0$ and so $K = C$. Therefore, Koyck transformation in this case only shows that:

$$D_t = C \cdot M_t + (1-C) \cdot D_{t-1} \rightarrow C = \frac{D_t - D_{t-1}}{M_t - D_{t-1}} \quad (19)$$

If steady-state growth holds so that $D_{t-1} = D_t/(1 + g)$, C in Equation (19) equals to (12) that represents the steady value of C. Thus, r is a latent variable that makes it challenging to estimate the lag parameter q according to Equation (18) following Koyck transformation. However, the lag parameter of expenses $1 - C$ is observable and can be calculated directly from observations using Equation (19) or (12). If the firm does not grow at steady rates, the estimate of C may not be reliable, since the proportion of expenses can strongly vary over time.

2.5. Summary of the Framework

The framework of this study is based on two important areas of accounting and financial reporting. Firstly, the matching research is fruitful when concentrating on the timing of accounting flows and introducing the contemporaneous correlation between revenues and expenses as the measure of matching quality. Dichev and Tang (2008) recommended the contemporaneous correlation of revenues and expenses as a measure of the quality of matching, since poor matching decreases this correlation. In poor matching, some of the perfectly matched expenses get scattered across different periods, which results in a lower synchronal correlation than the underlying economic correlation of advancing expenses to produce revenues. In perfect matching all expenses can be traced directly and specifically to specific revenues. This kind of tracing makes the timing of expenses coincide with the timing of revenues and makes the correlation approach unity. Thus, the matching research provides us with a good accounting emphasis to consider the relation between the revenue lag and the expense lag.

In order to analyze the lags an analytic framework is needed. The framework should allow us to analyze the revenue lag and the expense lag separately. The body of research analyzing the relationship between the internal rate of return (IRR) and the return on investment ratio (ROI) provides us with an appropriate framework to investigate the lags. The framework comes from the early studies of Harcourt (1965) (a continuous model) and Solomon (1966) (a discrete model) where the accumulation of revenues and expenses are investigated by a steady state model. The present framework includes a set of results derived on steady circumstances. However, many of the approaches are built beyond steady assumptions. In this study, a similar model as developed by Laitinen (2017) for startups is employed here to analyze methods on less restricted circumstances. These kinds of models provide us with useful tools to analyze potential methods to estimate the lags.

The theoretical framework allows us to analyze four different basic models potentially useful in the estimation of the lags. First, the non-steady framework makes it possible to analyze the contemporaneous correlation between revenues and expenses. This analysis showed that when the firm is approaching the steady state, the correlation approaches unity, irrespective of the lags. However, assuming steady growth for revenues and expenses it is possible to extract estimates for the lags. Secondly, on steady conditions it is possible to get estimates for the revenue lag assuming an expense method and approximating the lag using the analytical relationship between IRR and ROI. Thirdly, using the first differences of the logarithmic series of revenues and expenses it is possible to use a linear model to estimate the lags. Fourthly, the Koyck (1954) transformation of the geometric lag structure makes it possible to get estimates for both the revenue lag and IRR. In this form, these parameters can be estimated by a set of statistical methods, for example OLS, restricted OLS, and a more general optimization model. In the empirical part of the study, the performance of these methods is assessed.

3. Empirical Data and Methods

3.1. Original Sample

The purpose of the empirical part is to assess the usefulness of alternative approaches in reflecting the timing of revenues and expenses according to the theoretical framework. The estimation of the lag parameters requires quite steady and long time series of expenses, revenues, and expenditures. Therefore, empirical data are gathered from firms fulfilling a minimum size limit and having a longer time-series of financial statements available. Very small firms usually exhibit an unstable development that is not consistent with steady assumptions making estimation extremely challenging. The empirical data of the study are extracted from the ORBIS database of Bureau Van Dijk (a Moody's Analytics Company) that includes detailed financial information on 41 million companies across the globe (see https://www.bvdinfo.com/en-gb/?). The extraction of observations was made under restrictions that the selected firm must be Finnish, have successive financial statements available for at least 10 years, and employ at least 50 employees in each year. For the sample firms, financial statements from the last ten years were extracted. There were originally 1 015 firms fulfilling these preliminary three criteria set for the sample firms.

The sample firms were consisted of middle-size and large firms. The average number of employees in the sample was in the last year 893.8 and in the first year (ten years earlier) 886.0 reflecting only a slow average growth. However, the median size was only 202.0 employees in the last year and 177.0 employees in the first year referring to a skewed size distribution. In fact, the size distributions in each year showed a high skewness and a high kurtosis. The last years of the time-series were selected from the period before the Covid-19 pandemic to avoid the effect of the crisis on the results. In spite of that, the average growth of the sample firms during the research period was slow or even negative (due to the earlier financial crisis). In the same way, the average profitability of the sample firms was not high.

The sample firms were from different industries but the majority of firms were concentrated on few industries. In the sample, there were 393 (38.7%) firms from manufacturing industry, 177 (17.4%) from wholesale and retail trade, 71 (7.0%) from professional, scientific and technical activities, 65 (6.4%) from information and communication, and 64 (6.3%) from construction. The sample mainly consisted of 971 (95.7%) industrial firms and only 37 (3.6%) financial companies. The legal forms of the firms were mainly private limited companies (86.8%) and public limited companies (10.3%). The most frequent status of the sample firms was active (93.6%), but the sample also included dissolved (5.2%) and few bankrupt (0.7%) firms.

3.2. Statistical Methods

The empirical part of the study is based on using time series models to extract estimates of the revenue and expense lags. In this estimation, different econometric methods (Johnston, 1972; Fomby, Hill, & Johnson, 1984) are used to estimate distributed lag structures (Koyck, 1954; Hall, 2007) and parameters of steady and non-steady models (Laitinen, 2017; Laitinen, 2018). The present empirical analysis of the lags is for the main part based on three nine-year time series, which are revenues, expenses, and expenditures calculated and analyzed for each sample firm separately. These variables are measured by standard concepts. Firstly, revenues (R_t) are measured by net sales. Net sales are the result of gross revenue minus sales returns, allowances, and discounts. Secondly, expenses (D_t) are calculated as the difference between net sales and earnings before interest and taxes (EBIT). Thirdly, expenditures (M_t) are obtained as the sum of expenses and changes in inventories and fixed assets. Using the time series of nine years, the steady growth rates for revenues and expenditures are estimated by the ordinary least squares (OLS) method explaining the (natural) logarithmic time series by the year index t as follows:

$$R_t = R_0 \cdot (1+g)^t \cdot \mathrm{e}^{\varepsilon} \rightarrow \log R_t = \log R_0 + t \cdot \log(1+g) + \varepsilon \quad (20a)$$

$$M_t = M_0 \cdot (1+g)^t \cdot \mathrm{e}^{\varepsilon} \rightarrow \log M_t = \log M_0 + t \cdot \log(1+g) + \varepsilon \quad (20b)$$

where ε is a random residual. The estimate of firm-level steady growth rate g is calculated as the weighted average of these growth estimates using the sum of time-series over nine observations as weights.

In the analysis of the timing of revenues and expenses, six different approaches are used. First, the correlation method of estimation is applied. The time series of R_t and D_t are discounted by the growth rate g, and the time (contemporaneous) correlation coefficient between these two series is calculated. The estimate of C is calculated through Equation (12) using the final firm-level growth rate estimate and determining D/M as the relation of the sum of the time series of D_t and M_t for the nine years. This estimate is then substituted as $(1 - C)/(1 + g)$ in Equations (7a) and (7b) to solve an estimate for the timing of expenses as $q/(1 + g)$. The formula of correlation has two roots and, therefore, three different criteria were used to select the root to estimate q. Firstly, the higher root (7b) is selected if it less than 1. Secondly, the lower root (7a) is selected if it is positive. Thirdly, the higher root (7b) is selected if D_t/R_t is increasing referring to higher q according to Equations (5a) and (5b), otherwise the lower root (7a) is used in estimation.

Secondly, ROI method of estimation is applied. Thus, an estimate for the lag parameter q is solved through Equation (14) when R/M is determined as the relation of the sum of the time series R_t and M_t and r is approximated by ROI assuming alternative expense methods (neutral declining-balance, annuity (compound-interest), and rate-of-return (realization) expense methods). The neutral declining-balance method is based on the assumption that $1 - C = q$ leading to the approximation r^* presented in Equation (16). The annuity method is simply based on the approximation $r^* = ROI$. Finally, the approximation for the rate-of-return method follows Equation (17). The estimate of steady ROI is calculated dividing the sum of the time series of profit (EBIT) by the sum of the time series of assets (inventories and depreciable fixed assets) in the beginning of the period. This method to estimate q is sensitive to the approximation of r and, thus, the approach may give a slippery estimate.

Thirdly, the lag parameters are estimated using the logarithmic difference method. In this method, the lag parameters q and $1 - C$ are estimated from the logarithmic differences of the discounted time series of revenues R_t and expenses D_t by the linear regression analysis (OLS) following the procedures presented in Equations (9) and (10). However, since the method is based on logarithms, its full application requires that the time series are increasing so that the first-order differences are positive. However, due to slow average growth, this condition is not always valid in the sample firms. Therefore, a simplified transformation of $\log(-X) = -\log(X)$ is used when negative differences are considered. This simplification obviously affects the estimates, which should therefore be interpreted cautiously.

Finally, the last three estimation methods are associated with the Koyck transformation to estimate the timing parameter q simultaneously with r. This method is a challenging issue due to the sensitivity of estimates (Laitinen, 2017; Hall, 2007). The parameters q and r are in this analysis estimated using Equation (18) applied to the distributed revenue lag function. This transformation shows that the relation between the time series of R_t and M_t can be presented in the form of the following equation:

$$R_t = a + K \cdot E_t + q \cdot R_{t-1} + \varepsilon \tag{21}$$

where ε is a random variable. In Equation (21) the constant a is expected to be zero and $K = (1 + r - q)/(1 + r)$ according to Equation (4). The lag parameter q can be directly got as the coefficient of R_{t-1} and, thus, r can be solved from the estimate of K as $r = (1 - q - K)/(K - 1)$. The distributed lag equation will be estimated by three different statistical methods. Firstly, this equation is here estimated using the ordinary least squares method (OLS) to give an estimate for the revenue lag parameter q.

Secondly, the parameters q and r are simultaneously estimated using the direct OLS estimation with incorporated equality restrictions. The first restriction is to set $a = 0$. The second (linear) restriction is based on the relation (13) describing the revenue-expenditure ratio R/M. Because in a steady state $R/M = K\cdot(1 + g)/(1 + g - q)$, there is the following relation:

$$(1+g)\cdot K + \frac{R}{M}\cdot q = \frac{R}{M}\cdot(1+g) \tag{22}$$

which together with $a = 0$ lead to the following matrices of restrictions:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+g & \frac{R}{M} \end{bmatrix} \tag{23a}$$

$$\boldsymbol{h} = \begin{bmatrix} 0 \\ \frac{R}{M}\cdot(1+g) \end{bmatrix} \tag{23b}$$

These restrictions can be presented in the matrix form as $\boldsymbol{h} = \boldsymbol{H}\cdot\boldsymbol{B}$ where $\boldsymbol{B}$ is the 3×1 matrix of estimates. In these restrictions, the estimate of g and R/M are used to stabilize the estimates of q and r. Linear restrictions are incorporated in the estimation to make the estimates more stable (Johnston, 1972: pp. 155-159; Fomby, Hill, & Johnson, 1984: pp. 82-85). The variance of the restricted estimators is lower than that of the unrestricted ones. Factually, the linear restriction means that the relation between q and r is fixed as in Equation (13) so that there is only one parameter to be estimated.

Thirdly, an optimization routine is used to solve q and r simultaneously from the Koyck transformation (21). In this method, Equation (13) of R/M is used as a restriction in the same way as in the restricted OLS model to make the solution consistent. In the restricted OLS model, the estimate of q fulfilling the restriction can however be negative. Therefore, in the optimization model, restrictions in an inequality form are set to lead to acceptable results. In this optimization, the squared sum of errors in (21) is minimized using inequality constraints $r > -0.5$, $r < 1$, $q > 0$, and $q < 1$ as restrictions. The optimization procedure is carried out by Excel Solver using the Generalized Reduced Gradient (GRG) method (see Lasdon, Fox, and Ratner, 1973). This method looks at the gradient or slope of the objective function as the input values change and determines that it has reached an optimum solution when the partial derivatives equal zero.

3.3. Final Sample

The original sample of 1 015 Finnish firms included a number of firms that either had missing values in the research variables or did not conform properly to the assumptions of the theoretical framework. Firstly, the sample only included 927 (91.3%) firms that did not have any missing values. Secondly, the consistency with the framework was tested simply by the steady estimate of C (or the lag parameter $1 - C$). This estimate can be approximated by the steady values of time series of D_t and M_t and the estimated rate of growth g according to Equation (12). In the estimation of C, the number of non-missing observations was 968 (95.4%) firms. However, only for 781 (77.0%) firms the estimate was within reasonable limits being positive or less than 1.5 (150%). Consequently, all firms with missing values or an estimate of C outside these broad limits were dropped from the sample.

The final sample included 697 (68.7% of the original sample) firms. In the final sample, the average of the number of employees in the last research year was 938.6 whereas the median was only 216. Thus, the size distribution is skewed conforming to the distribution of the original sample. Finally, it is also probable that the negative values of growth rate g and IRR (r) have an impact on the estimation results due to the violence of the convergence conditions in the theoretical framework. Therefore, the sample was split into two parts. In the latter part, all firms have positive estimates for both g and r (as approximated by the weighted ROI) whereas in the former part this condition does not hold. The first sub-sample (part 1) includes 265 (36.6%) firms and the second sub-sample (part 2) 442 (63.4%) firms.

4. Empirical Results

4.1. Descriptive Statistics

Table 1 presents descriptive statistics of the main variables of the research. For most firms, the steady estimate of *R/M* exceeds unity so that revenues exceed expenditures while the estimate of *D/R* tends to be less than unity. In part 1 of the sample (*g*, *ROI*, or both are non-positive), the steady estimates of these ratios are typically higher than in part 2 (*g* and *ROI* are positive). The estimate of *D/M* in part 1 exceeds unity for a majority of the firms, which refers to a negative growth rate of expenditure. This implication holds since for most firms in part 1 the estimate of expenditure growth rate $g(M_t)$ is negative. For part 2, the estimate of growth rate is constrained to the positive value but it is generally very low telling that the research period is characterized by slow growth in Finnish firms. The multiple correlation squared R^2 in the growth rate estimation equation tends, especially in part 1, be low implying that the growth of firms does not well conform to the steady rate assumption.

Table 1. Descriptive statistics of the sample (697 firms).

Estimate of	Sample	Percentiles:				
		10	25	50	75	90
R/M	Part 1	0.96885	1.00185	1.03073	1.07179	1.13088
	Part 2	0.98941	1.00623	1.02862	1.06198	1.10914
D/R	Part 1	0.90108	0.95694	0.98161	1.01188	1.03719
	Part 2	0.87077	0.91071	0.94910	0.97005	0.98506
D/M	Part 1	0.97596	1.00016	1.00956	1.02759	1.05658
	Part 2	0.92957	0.96124	0.97924	0.99212	0.99789
$g(M_t)$	Part 1	−0.08932	−0.05630	−0.02422	−0.00193	0.00902
	Part 2	0.00620	0.02116	0.04221	0.07393	0.10579
$g(M_t)$ R^2	Part 1	0.00118	0.03470	0.32833	0.62827	0.76411
	Part 2	0.02542	0.17999	0.52155	0.79622	0.93060
$g(R_t)$	Part 1	−0.09289	−0.05659	−0.02717	−0.00673	0.01319
	Part 2	0.00821	0.02353	0.04206	0.07261	0.10668
$g(R_t)$ R^2	Part 1	0.01547	0.09050	0.40829	0.71903	0.86217
	Part 2	0.05284	0.27986	0.63978	0.87299	0.94683
g (weighted)	Part 1	−0.09024	−0.05566	−0.02609	−0.00423	0.00859
	Part 2	0.00906	0.02059	0.04153	0.07167	0.10660
ROI (weighted)	Part 1	−0.13142	−0.02723	0.04616	0.13658	0.27464
	Part 2	0.04207	0.07339	0.14560	0.29210	0.60789

Legend: Part 2 = Estimates of *g* and *ROI* are both positive, Part 1 = other firms. R^2 = Multiple correlation squared R^2 of the estimation equation.

The statistical distribution of the growth rate of revenue $g(R_t)$ is almost identical with that of $g(M_t)$. However, the steady growth assumption fits with the time series of revenues better than with the time series of expenditures. In part 2, that is characterized by positive values of growth and profitability, the fit in the growth estimation as measured by R^2 is better than in part 1. Since the distributions of $g(M_t)$ and $g(R_t)$ are nearly identical, the distribution of the weighted growth rate is also very similar to these. The percentiles of the estimate of *ROI* show that even in part 2 profitability in general is quite low the median value being less than 15%. Hence, the research period is characterized by both low growth and low profitability, which may have an impact on the estimation results.

4.2. Estimation Methods for Revenue and Expense Lags

Correlation method of estimation. The correlation method provides an estimate of the lag parameter q but this estimation requires that an estimate of $1 - C$ is firstly given. Thus, the accuracy of the estimate of q depends on the estimate of C. In this estimation, the steady estimate as in Equation (12) is used to approximate C. **Table 2** shows the results for this method of estimation. The correlation coefficient between expenses D_t and revenues R_t is generally very high reflecting a combination of a high quality of timing and a steady state potentially approached by the sample firms. Because the time series of M_t and R_t do not properly conform to the steady growth, the quality of timing may play an important role in this correlation. The correlation tends to be high also in part 1 where the steady-state growth assumption is not widely supported.

In parts 1 and 2, the median estimate of $1 - C$ is about 0.34 referring to an average lag of 0.5 years between expenditure and expenses. Panel 1 shows the percentiles for the estimates of q when the higher root is selected if it is less than unity. In this case, the median estimate of q is in part 1 0.62 (referring to 1.7 year time lag) and in part 2 0.58 (referring to 1.4 year time lag) which significantly exceed the median estimates of $1 - C$. This result is as expected since the higher root leads to higher q than $1 - C$. Consequently, the differences $1 - C - q$ are heavily negative.

However, Panel 2 shows that when the lower root is selected (if positive), the median estimates of q are in parts 1 and 2 around 0.38 referring to time lag of about 0.6 year time lag. The median differences between $1 - C$ and q are positive. This is again as expected, since the lower root gives for q a lower value than $1 - C$. Panel 3 shows that if the higher root is selected instead of the lower one when D_t/R_t is growing, the median estimates of q are lower than for the higher root but higher than for the lower root. The median estimate of q is in part 1 0.55 (referring to 1.3 year time lag) but in part 2 only 0.45 (referring to 0.8 year time lag). These estimates may be most useful, since the selection of the root is based on Equations (5a) and (5b). Thus, the correlation method indicates that a majority of Finnish firms seems to use an accelerated expense policy, where

Table 2. Results of the correlation method of estimation (697 firms).

Estimate of	Sample	Percentiles: 10	25	50	75	90
Correlation (D_t, R_t)	Part 1	0.73018	0.89273	0.96174	0.98775	0.99601
	Part 2	0.82554	0.92798	0.97431	0.99098	0.99593
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05850	0.18256	0.34368	0.55963	0.79686
Panel 1. Higher root selected if less than 1, otherwise lower root						
q	Part 1	0.19784	0.33708	0.62337	0.89003	1.00000
	Part 2	0.22867	0.40530	0.57726	0.76608	0.92879
$(1 - C) - q$	Part 1	−0.38813	−0.25546	−0.15754	−0.06181	0.02347
	Part 2	−0.42608	−0.26599	−0.15717	−0.09045	−0.02830
Panel 2. Lower root selected if greater than 0, otherwise higher root						
q	Part 1	0.08675	0.18713	0.39046	0.78051	1.00624
	Part 2	0.08937	0.18629	0.38071	0.62468	0.89691
$(1 - C) - q$	Part 1	−0.38813	−0.23840	0.02819	0.14755	0.26932
	Part 2	−0.45623	−0.26860	0.06851	0.16924	0.26545
Panel 3. Higher root selected if D_t/R_t increasing, otherwise lower root						
q	Part 1	0.11726	0.28606	0.55456	0.87979	1.00624
	Part 2	0.12116	0.25347	0.45110	0.68320	0.93688
$(1 - C) - q$	Part 1	−0.42036	−0.26083	−0.13418	0.00607	0.15660
	Part 2	−0.45931	−0.27150	−0.11431	0.10836	0.21140

Legend: Part 2 = Estimates of g and ROI are both positive, Part 1 = other firms. Correlation (D_t, R_t) = Correlation coefficient between D_t and R_t.

expenditures are expired as expenses faster than expenditures generate revenues. This result is consistent with prior research (Laitinen, 2018).

ROI method of estimation. The ROI method is based on the relationship between q and r according to Equation (14). This equation makes it possible to assess the value of the lag parameter q if IRR (r) can be approximated properly. The usual approximation of r (r^*) used in different situations is ROI (accounting rate of return, ARR) that is calculated from the financial statements. The relationship between ROI and r is not simple and ROI may be biased and unstable measure of r. The relationship between ROI and r depends on the expense (depreciation) method used by the firm. In this research, three kinds of expense methods are used to depict the relationship (Laitinen, 2018). Firstly, it is assumed that the firm uses a neutral declining-balance method and selects the proportion of expense C so that $1 - C = q$. Secondly, it is assumed that the firm makes use of the annuity (compound interest) expense method. Thirdly, the rate-of-return (realization) expense method is assumed.

Table 3 in Panels 1 to 3 presents the estimation results for these three expense methods, when r is approximated by ROI in different ways. The results indicate that for each expense method in both part 1 and 2 the approximations tends to

Table 3. Results of the *ROI* method of estimation (697 firms).

Panel 1. Assuming $ROI = r(1 + g)/(1 + r)$ (declining-balance expense for $1 - C = q$)						
Estimate of	Sample	Percentiles:				
		10	25	50	75	90
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05851	0.18256	0.34367	0.55962	0.79686
q	Part 1	0.03175	0.11841	0.23300	0.35103	0.61049
	Part 2	−0.00520	0.08901	0.20449	0.34512	0.67673
$(1 - C) - q$	Part 1	−0.12330	−0.02266	0.03807	0.29137	0.87987
	Part 2	−0.21477	−0.04396	0.09092	0.31696	0.66594
Panel 2. Assuming $ROI = r$ (the compound interest (annuity) expense)						
Estimate of	Sample	Percentiles:				
		10	25	50	75	90
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05851	0.18256	0.34367	0.55962	0.79686
q	Part 1	0.02720	0.12279	0.24787	0.38842	0.61820
	Part 2	−0.00543	0.12552	0.24853	0.40325	0.70845
$(1 - C) - q$	Part 1	−0.13130	−0.03792	0.02849	0.28337	0.85969
	Part 2	−0.28984	−0.11683	0.04448	0.29725	0.64846
Panel 3. Assuming $ROI = r\,R/M$ (rate-of-return (realization) expense)						
Estimate of	Sample	Percentiles:				
		10	25	50	75	90
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05851	0.18256	0.34367	0.55962	0.79686
q	Part 1	0.02730	0.12276	0.24390	0.36898	0.62189
	Part 2	−0.00544	0.12339	0.24156	0.38466	0.69081
$(1 - C) - q$	Part 1	−0.13113	−0.03189	0.03419	0.30576	0.86623
	Part 2	−0.26210	−0.10157	0.05089	0.30284	0.65036

Legend: Part 2 = Estimates of g and ROI are both positive, Part 1 = other firms. Correlation (D_t, R_t) = Correlation coefficient between D_t and R_t. C (steady) = $(D/M)\cdot g/(1 + g - D/M)$.

lead intuitively to too low estimates of q. The lowest estimates for each expense method are negative. In fact, the differences in the estimates between the expense methods are small reflecting the small differences in approximations. These estimates show that ROI on these assumptions is not a proper approximation of r (r^*). The median level of q corresponds to the median of $1 - C$ if r is approximated in the sample by $r = 0.65 \cdot ROI$ indicating that ROI significantly overestimates r. In conclusion, this ROI method is not efficient without a quite exact approximation of r.

Logarithmic difference method of estimation. The third method of estimation extracts the estimates of the lag parameters q and $1 - C$ from the logarithmic first-order differences of the discounted time series of D_t and R_t. Table 4 shows the results for this estimation method. The correlation coefficients of the logarithmic differences with time t are quite low and, against assumptions, positive for about 20% - 25% of the firms in part 1 and 2. Positive correlations mean that the estimates for $1 - C$ or q exceed unity. These kinds of estimates are more common in part 1(where g and ROI can get negative values) than in part 2. The estimates of $1 - C$ and q are on average higher than those got by the correlation method are and refer to time lags of 0.85 - 1.12 years.

At the level of median, the differences between $1 - C$ and q are close to zero referring to a good quality of timing. However, for about 20% of the firms these differences are very large due to values of the lag parameters exceeding unity. The differences in the tails of the distributions are larger for part 1 than part 2. Thus, the results imply that about for half of the firms expenditures are expensed

Table 4. Results of the logarithmic difference method of estimation (697 firms).

Estimate of	Sample	Percentiles:				
		10	25	50	75	90
Correlation ($\log\Delta D_t$, t)	Part 1	−0.62933	−0.47181	−0.24113	−0.00298	0.28927
	Part 2	−0.63863	−0.50194	−0.26094	−0.03635	0.19121
$1 - C$	Part 1	0.12036	0.26085	0.45795	0.94878	2.27895
	Part 2	0.13533	0.25493	0.50515	0.94460	1.79144
Correlation ($\log\Delta R_t$, t)	Part 1	−0.62989	−0.43587	−0.20841	0.02672	0.24986
	Part 2	−0.63506	−0.50346	−0.27972	−0.04633	0.17165
q	Part 1	0.12158	0.22995	0.53011	1.04047	2.10614
	Part 2	0.13475	0.24978	0.47760	0.94273	1.74323
$(1 - C) - q$	Part 1	−0.42802	−0.11648	0.00125	0.11390	0.43862
	Part 2	−0.24771	−0.05804	0.00423	0.07994	0.35360

Legend: Part 2 = Estimates of g and ROI are both positive, Part 1 = other firms. Correlation ($\log\Delta D_t$, t) = Correlation coefficient between logarithmic difference of D_t and t. Correlation ($\log\Delta R_t$, t) = Correlation coefficient between logarithmic difference of R_t and t. Logarithms of $-X$ are substituted by $-\log(X)$.

faster than expenditures generate revenues referring to an accelerated expense method. The results should however be interpreted cautiously due to the low correlations and to the potential effect of the negative value transformation $\log(-X) = -\log(X)$. If only positive differences are used in (partial) estimation, the results as expected significantly overestimate the lag parameters $1 - C$ and q.

Unrestricted OLS method of estimation. Fourthly, an unrestricted OLS method is used in the estimation of the lag parameter q. Table 5 shows the results and the steady values of $1 - C$ to be compared with q. In general, the multiple correlations squared in the lagged estimation equation are very high for most firms implying a good fit with the data. However, high multiple correlations squared are typical for distributed lag models and do not ensure that the estimates of the parameters are reliable. In the estimation equation, the estimates of the constant a do not deviate from zero which is also theoretically justified. However, the estimates of K are statistically significant. These estimates tend to be higher in part 1 than part 2.

Table 5. OLS results of the unrestricted distributed lag model (697 firms).

Estimate of	Sample	Percentiles:				
		10	25	50	75	90
Constant a	Part 1	−8510.3	−2361.9	1090.5	8785.3	9223.4
	Part 2	−5961.9	−82.9	2667.6	9223.4	9223.4
t-value of constant a	Part 1	−0.00045	−0.00010	0.00002	0.00016	0.00070
	Part 2	−0.00017	0.00000	0.00008	0.00036	0.00157
K	Part 1	0.28938	0.64136	0.86466	1.00258	1.12081
	Part 2	0.20684	0.49737	0.76934	0.94916	1.09116
t-value of coefficient K	Part 1	6.22116	19.62353	49.34928	154.45664	376.61128
	Part 2	5.32221	14.59641	38.79490	101.58321	266.61773
q	Part 1	−0.09756	0.01757	0.10868	0.26881	0.45276
	Part 2	−0.08519	0.02952	0.18677	0.39920	0.63103
t-value of coefficient q	Part 1	−4.45864	0.82381	5.91334	18.98868	50.73440
	Part 2	−5.43711	1.29106	8.86826	27.15492	61.85473
r (calculated)	Part 1	−1.91919	−0.95307	−0.33320	0.20565	1.08154
	Part 2	−1.50085	−0.74743	−0.18397	0.09632	0.80535
R^2 of estimation model	Part 1	0.57757	0.80048	0.93563	0.97767	0.99136
	Part 2	0.65608	0.86306	0.95383	0.98180	0.99339
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05850	0.18256	0.34368	0.55963	0.79686
$(1 - C) - q$	Part 1	−0.13141	0.03038	0.21544	0.50953	0.95686
	Part 2	−0.24224	−0.05920	0.12299	0.35275	0.60126

Legend: Part 2 = Estimates of g and ROI are both positive, Part 1 = other firms. r (calculated) = $(1 - K - q)/(K - 1)$. C (steady) = $(D/M)\cdot g/(1 + g - D/M)$.

The estimates of q are generally significant but intuitively too low, especially in part 1. The median estimate of q in part 1 is about 0.11 referring to a time lag of only 0.12 years. In part 2, the median estimate is about 0.19 that refers to a time lag of 0.23 years. Intuitively, also this lag is too low. Since these estimates are used to calculate r, the low estimates of q lead to very low estimates of *IRR* or r. For at least half of the firms in both parts of the sample, the estimates of r are heavily negative. Consequently, the differences between the steady values of $1 - C$ and q are very large at the median level. In conclusion, an unrestricted OLS models seems to be an unreliable method to estimate q.

Restricted OLS method of estimation. **Table 6** reports the results for the fifth estimation procedure that is the restricted OLS method. It is expected to improve some deficiencies in the unrestricted OLS. However, due to the restrictions, the multiple correlations squared are generally lower than for the unrestricted model. The estimates of K are for most firms very significant and generally, close to the estimates got for the unrestricted model. In part 2, the estimates tend to be lower than in part 1. For both parts, more than 10% of firms have a negative estimate for q that indicates that the model tends to give intuitively too

Table 6. OLS results of the restricted distributed lag model (697 firms).

Estimate of	Sample	Percentiles:				
		10	25	50	75	90
K	Part 1	0.45372	0.69587	0.87361	0.99040	1.09030
	Part 2	0.30978	0.56901	0.80991	0.96353	1.06455
t-value of coefficient K	Part 1	11.05002	31.56482	89.69661	234.54489	556.88209
	Part 2	10.44087	25.42037	68.47945	167.46149	430.53420
q	Part 1	−0.03059	0.04273	0.15746	0.32360	0.54402
	Part 2	−0.01532	0.07884	0.22323	0.47691	0.71690
t-value of coefficient q	Part 1	−3.43128	4.50373	18.25859	42.37151	118.78322
	Part 2	−1.52260	5.55261	18.15828	44.41102	102.34282
r (calculated)	Part 1	−1.28853	−0.21657	0.01027	0.13427	0.60802
	Part 2	−0.71700	0.00975	0.08101	0.22566	0.67175
R^2 of estimation model	Part 1	0.52903	0.78287	0.92913	0.97568	0.99014
	Part 2	0.63277	0.85090	0.94961	0.98072	0.99206
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05850	0.18256	0.34368	0.55963	0.79686
$(1 - C) - q$	Part 1	−0.12712	0.00771	0.16047	0.42360	0.83481
	Part 2	−0.28239	−0.09095	0.07781	0.28882	0.51344

Legend: Part 2 = Estimates of g and *ROI* are both positive, Part 1 = other firms. r (calculated) = $(1 - K - q)/(K - 1)$. C (steady) = $(D/M) \cdot g/(1 + g - D/M)$.

low estimates similarly to the unrestricted model. In fact, the median estimate of q is for part 1 about 0.16 corresponding to a time lag of 0.19 years and for part 2 about 0.22 referring to a time lag of 0.29 years.

Consequently, the estimates of r are in general very low except for the upper 25% of the sample firms. In part 1, the median estimate of r is only −0.33 that obviously significantly underestimates the actual *IRR*. The absolute differences between the steady value of $1 - C$ and the estimate of q are large. The differences are positive for about 50% of the firms indicating that, in the sample, both accelerated and deferred expense methods are used frequently. Thus, in spite of high multiple correlations squared the estimates provided by the restricted OLS are not reliable. The statistical problems with both OLS models may be due to multicollinearity, since the correlation coefficients between R_t and M_t are very high. The median correlation coefficient in the whole sample is 0.95 indicating strong multicollinearity.

Excel Solver optimization method of estimation. Table 7 presents the results for the sixth estimation method that is based on Excel Solver solution of a model including restrictions to r ($r < 1$ and $r \geq -0.5$) and q ($q < 1$ and $q \geq 0$) to improve the quality of estimates. This method suggests that the median estimates of r are positive and close to 2% in each part. However, in part 2 at least 25% of the optimal values of r are equal to −0.5 locating on the lower boundary of the estimate. In general, the estimates of r are low in both parts and 90% of the firms have an estimate lower than 0.10. The median estimates of q are comparable with the estimates given by the correlation method. For part 1, the median equals 0.41 referring to a time lag of 0.70 years whereas for part 2 the median is 0.47 indicating a time lag up to 0.90 years.

Table 7. Results of the Excel Solver optimization (GRG) model (697 firms).

Estimate of	Sample	Percentiles:				
		10	25	50	75	90
r (optimal)	Part 1	−0.02456	0.00170	0.01978	0.04244	0.09204
	Part 2	−0.50000	−0.50000	0.02111	0.06322	0.09429
q (optimal)	Part 1	0.06297	0.26539	0.41280	0.55956	0.69618
	Part 2	−0.02100	−0.00568	0.47444	0.77460	0.90667
$1 - C$ (steady)	Part 1	0.06103	0.14648	0.33635	0.69659	1.04849
	Part 2	0.05850	0.18256	0.34368	0.55963	0.79686
$(1 - C) - q$	Part 1	−0.32767	−0.16845	−0.00703	0.21464	0.56111
	Part 2	−0.62561	−0.37211	0.06095	0.28365	0.46670

Legend: Part 2 = Estimates of g and *ROI* are both positive, Part 1 = other firms. GRG = Generalized Reduced Gradient method. Model is based on minimization of the sum of squared errors. Restrictions: $r < 1$, $r \geq -0, 5$, $q < 1$, $q \geq 0$.

In spite of the lower boundary for q, at least 25% of the estimates in part 2 are slightly negative (being approximate values around 0). The median values of the differences between the steady value of $1 - C$ and q are close to zero, especially in part 1. The absolute differences are larger in part 1 than in part 2, except for the highest values of q. The results imply that about half of the sample firms use an accelerated expense method and half a deferred one. The results are satisfactory but not good due to the surprisingly low estimates of r and especially to the large number of boundary solutions in part 2.

5. Conclusion

The specific purpose of the study was to analyze the performance of six different approaches to estimate the lag parameters $1 - C$ and especially q. Mostly, the steady value of $1 - C$ was used as the estimate whereas q was estimated using a statistical modeling. The most challenging task in this issue is to get a valid method to estimate q. The first approach was the correlation method that gave intuitively acceptable results. However, if the firm lives at the steady state, correlation between revenues and expenses is approaching unity regardless of the values of C and q. Thus, the correlations on the steady-state circumstances do not factually include information about $1 - C$ and q to assess timing. However, in the present sample of Finnish firms, only few firms grew at steady rates. The results of the correlation analysis in the sample implied that the majority of Finnish firms tend to use an accelerated depreciation method where expenditures are expensed faster than expenditures generate revenues.

In the second approach, a modified version of *ROI* was employed as an approximation of *IRR* to give an estimate of q. The results showed that different expense methods used in this estimation lead to very similar approximations of *IRR* and therefore very similar estimates of q, although q is sensitive to *IRR*. Consequently, this approach gave intuitively too low estimates for q indicating that firms tend to use a decelerated expense method. In conclusion, this approach is not efficient without a more exact approximation of *IRR*. The third approach of estimation was the logarithmic difference method. This method extracts estimates for $1 - C$ and q from the logarithmic first-order differences of the discounted time series of expenses and revenues. The method performed satisfactorily but suffered from low negative or even positive correlations between the logarithmic time series and time. Moreover, there is an issue how to replace negative observations (differences) to be able to take logarithms. In this study, a simple transformation was used and $\log(-X)$ was replaced by $-\log(X)$. Therefore, the results should be interpreted cautiously. However, this approach is, in spite of its deficiencies, a promising method to estimate separately both q and $1 - C$.

The fourth approach was the unrestricted OLS model applying Koyck transformation for the infinite geometric lag distribution. However, this approach was considered unreliable giving obviously too low values for q and, consequently, for *IRR*. In the fifth model, equality restrictions were incorporated to the OLS model using the steady revenue-expenditure ratio as a restriction. Unfortunately, also this method was unreliable giving intuitively too low, and even negative, estimates for q. In the sixth approach, inequality restrictions were incorporated in the estimation and an Excel Solver algorithm was used to search for the optimal estimates of *IRR* and q. The results were largely satisfactory but not good due to the many very low estimates of *IRR* and especially to the large number of boundary solutions.

The present study showed that estimating of the time lag parameters is a challenging issue. It revealed many deficiencies in estimation methods. Thus, more research is called for. The logarithmic difference method provided one promising way to estimate the lags. However, more effort should be put to improve estimation and to develop new non-linear methods to avoid the problem with logarithmic values. The validity of the Koyck models should be improved by methods avoiding multicollinearity, such as the ridge regression. In the assessment of the approaches, a problem is the absence of the correct values of $1 - C$ and q, which makes it difficult to measure the bias associated with the estimates and, thus, the validity of the models. Therefore, experimental designs, where $1 - C$ and q are given and the effects of different factors on the estimates can be controlled, are welcome. To conclude, it is obvious that the estimation of the time lag parameters to assess timing of expenses and revenues needs a lot of further research in the future.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Basu, S., Cready, W. M., & Paek, W. (2016). *Measuring Matching Using Reported Revenues and Expenses.* https://pdfs.semanticscholar.org/8b4f/b588db049125e6bf526829b1d4046ab43d99.pdf

Bauwhede, H. V. A., & Willekens, M. W. (2003). Earnings Management in Belgium: a Review of the Empirical Evidence. *Tijdschrift voor Economie en Management, 48,* 199-218.

Beneish, M. D., Lee, C. M. C., & Nichols, G. D. (2013). Earnings Manipulation and Expected Returns. *Financial Analysts Journal, 69,* 57-82. https://doi.org/10.2469/faj.v69.n2.1

Brief, R. P. (Ed.) (2013). *Estimating the Economic Rate of Return from Accounting Data (RLE Accounting)* (Vol. 16). Routledge. https://doi.org/10.4324/9781315886404

Bushman, R. M., Lerman, A., & Zhang, X. F. (2016). The Changing Landscape of Accrual Accounting. *Journal of Accounting Research, 54,* 41-78. https://doi.org/10.1111/1475-679X.12100

DeAngelo, L. E. (1986). Accounting Numbers as Market Valuation Substitutes: A Study of Management Buyouts of Public Stockholders. *The Accounting Review, 61,* 400-420.

Dechow, P., Sloan, R., & Sweeney, A. (1995). Detecting Earnings Management. *The Ac-*

counting Review, 70, 193-225.

Dichev, I., & Tang, V. W. (2008). Matching the Changing Properties of Accounting Earnings over the Last 40 Years. *The Accounting Review, 83,* 1425-1460. https://doi.org/10.2308/accr.2008.83.6.1425

Donelson, D. C., Jennings, R., & McInnis, J. (2011). Changes over Time in the Revenue-Expense Relation: Accounting or Economics? *The Accounting Review, 86,* 945-974. https://doi.org/10.2308/accr.00000046

Feenstra, D. W., & Wang, H. (2000). *Economic and Accounting Rates of Return.* Research Report 00E42, University of Groningen, Research Institute SOM, SOM-Theme E Financial Markets and Institutions.

Fisher, F., & McGowan, J. J. (1983). On the Misuse of Accounting Rate of Return to Infer Monopoly Profits. *American Economic Review, 73,* 82-97.

Fomby, T. B., Hill, C. R., & Johnson, S. R. (1984). *Advanced Econometric Methods.* Springer Verlag. https://doi.org/10.1007/978-1-4419-8746-4

Franses, P. H., & van Oest, R. (2004). *On the Econometrics of the Koyck Model.* Econometric Institute Research Papers EI 2004-07, Erasmus University Rotterdam.

Gerakos, J. (2012). Discussion of Detecting Earnings Management: A New Approach. *Journal of Accounting Research, 50,* 335-347. https://doi.org/10.1111/j.1475-679X.2012.00452.x

Graham, J. R., Harvey, C. R., & Rajgopal, S. (2005). The Economic Implications of Corporate Financial Reporting. *Journal of Accounting and Economics, 40,* 3-73. https://doi.org/10.1016/j.jacceco.2005.01.002

Hall, B. H. (2007). *Measuring the Returns to R&D: The Depreciation Problem.* Working Paper 13473, National Bureau of Economic Research. https://doi.org/10.3386/w13473

Harcourt, G. C. (1965). The Accountant in a Golden Age. *Oxford Economic Papers, 17,* 66-80. https://doi.org/10.1093/oxfordjournals.oep.a040979

He, W., & Shan, Y. (2016). International Evidence on the Matching between Revenues and Expenses. *Contemporary Accounting Research, 33,* 1267-1297. https://doi.org/10.1111/1911-3846.12183

Healy, P. M. (1985). The Effect of Bonus Schemes on Accounting Decisions. *Journal of Accounting and Economics, 7,* 85-107. https://doi.org/10.1016/0165-4101(85)90029-1

Healy, P. M., & Wahlen, J. M. (1999). A Review of the Earnings Management Literature and Its Implications for Standard Setting. *Accounting Horizons, 13,* 365-383. https://doi.org/10.2308/acch.1999.13.4.365

Johnston, J. (1972). *Econometric Methods* (2nd ed.). McGraw-Hill, Inc.

Jones, J. (1991). Earnings Management during Import Relief Investigations. *Journal of Accounting Research, 29,* 193-228. https://doi.org/10.2307/2491047

Kay, J. A. (1976). Accountants, Too, Could Be Happy in the Golden Age: The Accountants' Rate of Profit and the Internal Rate of Return. *Oxford Economic Papers, 28,* 447-460. https://doi.org/10.1093/oxfordjournals.oep.a041353

Keating, A. S., & Zimmerman, J. L. (1999). Depreciation-Policy Changes: Tax, Earnings Management, and Investment Opportunity Incentives. *Journal of Accounting and Economics, 28,* 359-389. https://doi.org/10.1016/S0165-4101(00)00004-5

Koyck, L. M. (1954). *Distributed Lags and Investment Analysis.* North-Holland Pub. Co.

Laitinen, E. K. (2017). Profitability Ratios in the Early Stages of a Startup. *The Journal of Entrepreneurial Finance, 19,* 1-28.

Laitinen, E. K. (2018). Implied Expense Theory in Financial Reporting: A Steady-State Approach. *Journal of Financial Reporting and Accounting, 16,* 49-83. https://doi.org/10.1108/JFRA-05-2016-0032

Laitinen, E. K. (2020). Matching of Expenses in Financial Reporting: A Matching Function Approach. *Journal of Financial Reporting and Accounting, 18,* 19-50. https://doi.org/10.1108/JFRA-01-2019-0009

Lasdon, L. S., Fox, R. L., & Ratner, M. W. (1973). *Nonlinear Optimization Using the Generalized Reduced Gradient Method.* Technical Memorandum No. 325, Department of Operations Research, Case Western Reserve University.

Luckett, P. F. (1984). ARR vs. IRR: A Review and an Analysis. *Journal of Business Finance & Accounting, 11,* 213-231. https://doi.org/10.1111/j.1468-5957.1984.tb00070.x

Paton, W. A., & Littleton, A. C. (1940). *An Introduction to Corporate Accounting Standards. Monograph No. 3.* Sarasota: American Accounting Association.

Peasnell, K. V. (1982). Some Formal Connections between Economic Values and Yields and Accounting Numbers. *Journal of Business Finance & Accounting, 9,* 361-381. https://doi.org/10.1111/j.1468-5957.1982.tb01001.x

Peasnell, K. V. (1996). Using Accounting Data to Measure the Economic Performance of Firms. *Journal of Accounting and Public Policy, 15,* 291-303. https://doi.org/10.1016/S0278-4254(96)00041-5

Sivakumar, K., & Waymire, G. (2003). Enforceable Accounting Rules and Income Measurement by Early 20th Century Railroads. *Journal of Accounting Research, 41,* 397-432. https://doi.org/10.1111/1475-679X.00110

Solomon, E. (1966). Return on Investment: the Relation of Book Yield to True Yield. In R. K. Jaedicke, Y. Ijiri, & O. Nielsen (Eds.), *Research in Accounting Measurement* (pp. 232-244). American Accounting Association.

Spohr, J. (2005). *Essays on Earnings Management.* Doctoral Thesis, Swedish School of Economics and Business Administration.

Srivastava, A. (2011). Why Has Matching Declined? *SSRN Electronic Journal.* https://doi.org/10.2139/ssrn.1932272

Whittington, G. (1988). The Usefulness of Accounting Data in Measuring the Economic Performance of Firms. *Journal of Accounting and Public Policy, 7,* 261-266. https://doi.org/10.1016/0278-4254(88)90037-3