Tommi P. Laiho

MA - Industrial Design

The role of luck at Wall Street

What if I told you that Warren Buffet is merely lucky?

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Author:	Tommi P. Laiho
Authors' student number:	b71362
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TIIVISTELMÄ:

Tämä pro gradu työ etsii tuurin merkitystä sijoitustoiminnassa. Työ juontaa aiheensa siitä väittämästä, että markkina indeksien käyttäytymistä voidaan kuvailla humalaisen ihmisen satunnaiskävelyllä. Koskaan et siis voi olla täysin varma, ottaako humalainen askeleen oikealle vai vasemmalle vai pysyykö yhtäkkisesti paikoillaan. Samalla tavalla voidaan ns. Random walk hypoteesin vallitessa katsoa, että pörssi-indeksi joko nousee, laskee tai pysyy paikoillaan ennalta arvaamattomasti kuin vedonlyönti rulettipöydän ääressä kasinolla.

Näin ollen voidaan kauaskantoisesti olettaa, että on olemassa vain tuurilla menestykseen nousseita osakesuursijoittajia, koska sijoitustaidolla ei ole Eugene Faman tehokkaiden markkinoiden hypoteesin vallitessa mitään käytännön merkitystä. Nobelisti Eugene Faman mukaan fundamentalistianalyysilla tai teknisellä analyysillä ei pitäisi olla mitään käytännön merkitystä kenellekään sijoittajalle. Random walk - hypoteesi nauttii akateemisissa piireissä suurehkoa luottamusta.

Tutkimalla Standard & Poor 500 indeksin jakaumaa ja eri osakkeiden jakaumia suhteessa indeksin jakaumaan voidaan Mann-Whitney U-testillä mitata, että ovatko mitattavat jakaumat samanlaiset. Jos mitattavat jakaumat ovat samanlaiset, on olemassa random walk ilmiö. Jos mitattavat jakaumat ovat erilaiset, ei ole olemassa random walk ilmiötä osakemarkkinoilla. Tutkimus tulee siihen lopputulokseen, että markkinat ovat ainakin kohtuullisen tehokkaat, mutta eivät ole aina normaalisti jakautuneet. Mitään autokorrelaatioon verrattavissa olevaa ilmiötä ei ole juurikaan havaittavissa tällä havaintoaineistolla.

AVAINSANAT

Random walk hypothesis, EMH, Nobelist Eugene Fama, Mann-Whitney U-test,

normality assumption on the stock markets, Benoit Mandelbrot, Warren Buffet.

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1 Introduction – the role of luck at Wall Street

Are Wall Street's best companies led by luck or with skill? I will concentrate on the years 2000 – 2022 and the simple arithmetic returns of the stock returns **of the most successful companies** during that period. (Roberts, 1959, p. 8) The five investment companies involved are said to be the **best of the best**, and the other five stocks are industrial and service companies, which are top performers of the S&P 500 stock index of the **last two decades**. Too young companies like Google and Tesla were rejected. The mathematics here is very simple and is a "percentage calculation" between older and younger price data. Later things get more complicated with normality tests and Mann-Whitney U-tests.

There will be five investment companies, and the rest are industrial and service sectors. The zero hypothesis is that the S&P 500 stock index and its financial return distribution should not differ from the investigated return of underlying stock distribution. In other words, if the distributions are the same between the S&P 500 and between – say, NVIDIA or Warren Buffet's Berkshire Hathaway investment company – the company's success is merely based on luck only. And why not – it is so difficult to see to the future as an investment strategist that the winners are probably more lucky than skillful in some evolutionary **survival of the fittest game** based on blind luck. However, some of the investigated companies are led by skill, not by chance, based on Mann - Whitney U - test analyses.

Why did I decide to do a master's thesis on this subject? The vital issue of becoming rich in the stock markets may have bothered me when finding a simple way to make fortunes. It was one of the reasons why I came to the University of Vaasa. I had seen all the happenings of the 1980s and the easy money coming from stock investing and money markets, all very quickly and easily financed by commercial banks. But unfortunately, I also saw the collapse of the 1980s yuppie dream to the disastrous 1990s brutal recession. That was a life lesson.

All this confused me a lot. It seemed that the economy was some power-play of money, and lots of serious money could be burned without almost any kind of feeling of responsibility. No wonder this foolish gambling was called “the casino economy” by the press. It felt stupid that grown-up people behaved as they did in the economic boom of the 1980s. When I was a kid, I thought that almost whatever company was a severe production factory meant to stay here for a long time. Maybe I was seeking some safety from the plans of the grown-up people. However, the situation of the 1990s recession was a disappointing time for me.

1.2 To boldly go there, where very few of us wanted to go

However, like many other finance students at the University of Vaasa, I wanted to find a way for at least minor economic success in real life. Although the unemployment of the masters of finance students at the beginning of the 1990s was massive, there was perhaps some hope in the future. I also sought to find becoming victorious shares of the subsequent economic recovery. In other words, the idea of an investment company was in my eyes, although nobody wanted to talk about investing in shares during that 1990s brutal recession. Nevertheless, I did not give up on my dream.

Later the disappointment was huge when I found the “EMH thinking”, Eugene Fama’s influential Efficient Markets Hypothesis, and realized that this dream of my investing company was perhaps entirely naive. Later abbreviation EMH is used for this hypothesis. The Efficient Markets Hypothesis shows there is no sense in establishing an investment company if you want to succeed seriously. All the intellectual ability was meaningless; basically, sheer luck determines why somebody becomes the following stock investing tycoon. Many investors dislike this idea because it makes their competence as an investor relatively futile. People also think wealthy investors must have earned their money based on skill more than luck. Therefore, random winning is seen as an almost impossible idea.

1.3 Yes – but what if success in life is based on luck?

However, according to Fama and many other researchers, random winning is not impossible. Sometimes sheer luck determines success based on money, especially in investing. Luck is the essence of the random walk theory, and it holds in real life very well. An arbitrary hamster as an investor may beat a human investor without any problems. (Molloy, 2021, p. 1).

According to Eugene Fama and the Efficient Market Hypothesis, your chances of beating the markets and thus making severe money are very small. Nevertheless, of course, you can get perhaps that average S&P 500 annual 10% return on investment relatively safely, and compound interest will be huge in the long run. That is an almost sure thing (Berkshire Hathaway, 2021, p. 2).

But who discovered first the random walk of the Stock exchange markets? Some sources say that Louis Bachelier is one of the first people to think random walk behavior of the stock markets. Louis Bachelier's "Théorie de la spéculation," is said to be forerunner of the modern financial theory. However, Louis does have a rival in this issue, and his name is Jules Regnault. Jules wrote a book called Calcul des chances et philosophie de la Bourse. It was Published in 1863 in Paris (Preda, 2004, pp. 351–353). For example, Franck Jovanovic and Philippe Le Gall have recently argued that the key elements of the random walk hypothesis were first formulated in a book by Jules Regnault

2 What is the random walk hypothesis?

Nevertheless, what is the random walk theory, and why should it bother you as an investor? As the name suggests, the random walk name comes from the unpredictable walking style of a heavily drunk man. (Pearson, 1905, p. 342) When we observe this kind of walking style, we do not know whether the drunk man will take a step to the right or the left. That is the same in the stock index behavior: We do not know whether the index will lift or drop down for the next. In general, we can say that the stock exchange Monday, Tuesday, or Wednesday will not affect each other's behavior in any way. All stock exchange days are independent of each other (Roberts, 1959, p. 9).

Furthermore, this is a required notice in general. That is because no matter how intelligent or competent an investor is, you will not have any way to predict the random walk behavior of the stock markets. Your stock portfolio's financial performance is not based on any intelligent prediction but sheer luck only, and there is no way to circulate this. The monkey is as good as an investor as you are. Quite few people will find this end conclusion disturbing (Dittrich & Srbek, 2020, pp. 352–355).

However, why would this all happen? Nobelist Eugene Fama, who developed an efficient market hypothesis, explained why. He assumes that the market's problem is that the competition is bloody. As a result, there is no single moment when the Wall Street stock index is in a rest position. Instead, the index is constantly updated with new info on the markets. So, according to Fama, it is practically impossible to have such info from the past which is not already considered in the underlying stock pricing. This information flood is massive, effective, and extremely fast. As a result, the stock price is relatively correct all the time, and it is nearly impossible to find shares that are not already priced correctly. Little are your chances to beat the expected incomes of the markets, then (Malkiel & Fama, 1970, p. 1).

3 Eugene Fama and efficient market hypothesis

"On the other hand, ‘The "pragmatic school" of indexing simply amassed vast statistical evidence showing that the returns earned by active managers seldom outpace the S&P 500 Index" (Henderson, 2013, p. 1).

Eugene Fama discovered Efficient Market Hypothesis in 1965 in his doctoral thesis. Later this finding turned out to be a cornerstone of the modern finance theory. The efficient market hypothesis means that there is very little hope for an investor to beat the average income of the markets. Thus, it is one of the **most hopeless and cynical** economic theories modern times have ever produced. Maybe that is why **I liked it a lot** when I realized it. It says there is little sense in establishing an investment company unless you are playing with other people's money and be only a stock broker who advises gamblers. This stockbroker does not take part in clients' losses but profits only. Therefore, the house will always win, as they say in the casinos (Malkiel & Fama, 1970, p. 1).

The problem of the markets, according to EMH, is ultra-high competition in the information markets. Therefore, if something new and significant news in the markets affects the underlying stock, that news will immediately influence the stock price. This process is almost as quick as the speed of light because radio signals proceed at a light speed. Therefore, there is no chance of being faster than light in the pricing process according to Albert Einstein (Einstein, 1905, p. 1). However, the “spooky action at a distant effect” and perhaps recent findings in quantum mechanics may or may not disproof this “universal speed limit c” of Albert Einstein some day (published, 2017, p. 1).

In real life, there are many levels of competitive markets. These conditions are weak market efficiency, semi-strong, and strong market efficiency. The strong market efficiency means that all possible info is already in the prices, and there is absolute no way to beat the average market income except with sheer luck. This setup is sporadic in real life, meaning that even illegal inside information is useless. Usually, markets are assumed to be semi-strong markets, which means that all public information is reflected in the price of the shares. This kind of low effectiveness leaves illegal inside information many chances to be beneficial (Laird, 1995, p. 22).

Finally, there is a weak form of an efficient market, which means that all historical prices are reflected in asset prices. The weak form of market efficiency should be evident in almost any stock market on Earth (Peón et al., 2019, p. 269).

3.2 Evidence against Efficient Market Hypothesis

Is there any evidence against EMH? Yes, there is (Caporale & Plastun, 2020, p. 253). The most typical counterargument is anomalies found in the financial markets. For example, almost every investor has heard that the stock index may fall in December and lift slightly in January. That is called the January phenomenon, and this phenomenon is still alive. Another very typical anomaly is the so-called overreaction anomaly, which states that the meaning of the recent info of the stock is overvalued (Caporale & Plastun, 2020, p. 252).

There are plenty of anomalies in the markets, and new findings have been made all the time. These anomalies may be some opportunity to make money on the financial markets, but it requires careful active monitoring of the financial markets and significant capital of money. It is also rather typical that an anomaly may exist, but its meaning it may be difficult to make money with that. In such a case, the anomaly has no practical meaning (Caporale & Plastun, 2020, p. 252).

However, stock options could also be used if more leverage is wanted for the smaller capital. According to Eugene Fama's logic, these anomalies should disappear when recognized, but some are 100 years old in the long run. It does not mean that these old anomalies will continue to work in the future. Therefore, there is always some risk involved in investing, which is obvious — nothing new under the sun.

3.3 “We do not believe the market is efficient”

The title of this chapter is a comment by the people, who are working on forecasting the industry of the stock markets (Mandelbrot & Hudson, 2004, p. 192).

One of the main points of the author of the book “The misbehavior of the markets” by Benoit Mandelbrot is, that this kind of forecasting industry of the stock markets should soon vanish if it was an entirely profitless and futile attempt to make money than the typical normal income 10% pa. Lots of money and attempts to buy better and better computers and code more improved software to beat the normal income should be fully wasted if it was a pointless attempt to beat the markets. And according to Benoit Mandelbrot, this kind of industry is not a bad or hopeless money-making machine. We almost all know this, who operate in the field that something is very wrong when we are talking about efficient market hypothesis. Just like there is no perfect competition of the economic theories in the real life, there is no efficient market as well. Therefore in practice, the EMH is a fairy tale according to Mandelbrot (Mandelbrot & Hudson, 2004, p. 194).

However, there is always a possibility to be lucky and there can be surprisingly many lottery winners among us?

But there is more critics by Benoit Mandelbrot against EMH. Let us look at for example Black & Scholes option pricing models. There are some variations of them, but they are all based on the idea that the prices of the markets are normally distributed and thus follow a random walk hypothesis. One of the main findings of this thesis is, that markets are not normally distributed well. Only at the yearly level S&P 500 was normally distributed, see table 1 page 34 in this thesis. However, It is typically normal to assume non-normality with stock markets. And so be it, but sometimes some behavior like normality can be seen especially at the annual level (Mandelbrot & Hudson, 2004, p. 193).

4 Research methodology and tools

The goal of this chapter is to study the nature and character of the S&P 500 stock index from a statistical point of view. There are many criteria for a proper test that could find out the nature of the sample. That is why the study starts with a QQ plot and histogram analyses continuing with numerical analyses like the Bartlett test and Shapiro – Wilkinson test. Bartlett test is used to study if variances of the sample are the same and Shapiro – Wilkinson is used to find the normality of the sample. However, Shapiro – Wilkinson test does not measure samples whose size is over 5000 according to R software. That is why I will also use two other tests to find out the normality of the samples. These tests to see the goodness of fit of the Gauss normality hypothesis are Anderson – Darling test and Jarque - Bera test.

We will end up with the conclusion, that the sample of the S&P 500 is not always normally distributed but this sample has equal variances at least. However, Student's t-test should not be used at all, because both of the samples are not always normally distributed. We will end up using Mann-Whitney U – test.

The tool for the task is R-Studio and R-software, which is said to be the gold standard of all statistical software currently. I am using also Ubuntu Linux 22.04 with a custom kernel 6.0.6 as an OS, which creates its challenges to treat R – software properly. For example with a new custom kernel you will need also the very latest GCC 12 compilers and also G++ 12 libraries to successfully compile R – packages at this moment this thesis is written. The main reason why I used R and Yahoo finance data was a lack of money for something better and also a willing to learn R, which is notorious for its usability, but is otherwise the best of all tools there is.

4.2 R-statistics software as a chosen tool for this thesis

There is nothing wrong with R and R-Studio software, although R itself is **free and open-source** software. Furthermore, R lacks almost all forms of a graphical interface which in this case is not a weakness but its strength. The reason for a console-based interface is that programming ability with R-language makes it possible to get what you want from the software.

Also, the R- community is extensive, and it seems there is plenty of support on YouTube and the internet overall, and it is relatively easy to find tutorials and help with simple Google search terms only. A good researcher in the financial field might do him or herself a big favor if he studies Python and R- language with an interest in mastering both of these programming languages. Most of the tests and visualizations of this thesis are done with R-scripts, which are slightly or heavily edited based on the ready-made scripts found by the users made for internet tutorials. These free tutorials are a significant asset for every novice R-user like me. Also, it is an excellent asset that you can download many types of financial data with R from Yahoo or FRED for free, and there are tons of it for every user. I hope these services will stay free in the future too.

4.3 Are all the samples normally distributed in this study?

I have selected 10 best stocks for this study based on their performance. I am not interested of mediocre performance at all in this small study. However, before we enter the task of measuring the research question between skill against luck, it must be measured if the study samples are normally distributed. In case the sample is normally distributed, different methods should be used to find answers to the research question. First, there will be presented ten outstanding stocks and then two groups of them. The first five stocks are all investment companies **GROUP A** stocks, and the rest 5 are best general stocks and belong to **GROUP B.** Also, Warren Buffet is studied as a case of his own against the S&P 500 index. The stocks of the top investors' GROUP A are as:

1A. Warren Buffet and Berkshire Hathaway (BRK-A, BRK-B)

2A. Morgan Stanley (MS)

3A. Goldman Sachs Group (GS)

4A. Black Rock (BLK)

5A. Markel (MKL)

The best industrial and service sector companies of GROUP B are:

1B. Apple (AAPL)

2B. Microsoft (MSFT)

3B. Amazon (AMZN)

4B. Nvidia Corp. (NVDA)

5B. United Health Group Inc. (UNH)

4.4 S&P 500 index's normality histogram and QQ plot

Standard & Poor's 500 can be said to be a list of the most significant US enterprises there is. As the name suggests, the index is a list of 500 companies. The index was introduced in 1957 as a stock market index to track the value of the 500 giant corporations listed on the New York Stock exchange (*What Is the History of the S&P 500?*, 2022, p. 1).

During the high inflation and stagnancy growth from 1969 to 1981, the index gradually declined under the pressure of inflation and slow growth. Also, in 2020 the Coronavirus sent its mark to this index. In other words, it is a wonderful measurement of the condition of the US economy (*What Is the History of the S&P 500?*, 2022, p. 1). Two significant components, Tesla and Google, have been left out of this study because they have not been there since 2000 but are younger companies.

The first phase to find normality in the whole sample is to check Q-Q plots. There are also purely numerical ways to find the normality behavior, but for example, Shapiro – Wilkinson test will not work if the sample is smaller than three or more extensive than 5000. That is what R statistical software has claimed many times. Therefore the only way to see whether a daily-based sample is behaving is to check the Q-Q plot first. In figure 3.1, The Q-Q plots show no normality in the daily data. The straighter the petrol blue diagonal piercing the Q-Q box - typically in about 45 degrees of angle - the better chances for normality. So Q-Q plots show that the sample is not behaving like normal distribution because the dots' tails are not following a direct line.

In figures 3.1-3.6, the S&P 500 histograms and QQ plots do not show evidence of normal distribution – all but that. The histograms and QQ plots are based on R analyses and plotted from the year 2000 to 2022 which is our period of investigation. The petrol blue direct line in QQ plots presents an ideal line for the dots to follow to show normality, which does not exist in this case at all. To avoid critics of overfitting the data I have taken daily, weekly, monthly and annually-based data of returns. The data which I used was not actually closed stock price but the returns of these stocks. The difference between the sequence of the time series was based on simple arithmetic "percentage calculation" only. This is a "delt" function of quantmod library in R statistical software, see appendix 7 and the following equation 1 (*Delt Function - RDocumentation*, n.d., p. 1):

$$Lag = \frac{\alpha_2 - \alpha_1}{\alpha_1} \tag{1}$$

So there is no normality in the behavior of the S&P 500 stock index in this analysis and at this time except on the annual level – in all forms of testing. Otherwise, a student's t-test should be used.

The snippet to calculate the returns of the shares' incomes for R – software is as :

```
install.packages("quantmod")
library("quantmod")
stock <- read_excel("Wall_Street_20_Weekly.xlsx")
model <- Delt(stock$'BRK-B')
print(model)
```

This snippet reads the weekly files of the Berkshire Hathaway company's returns for the B-share and finally prints it. You will also need excel file library for R ("Calculate Price Return in R (2 Examples) | Returns from Vector of Prices," n.d., p. 1).

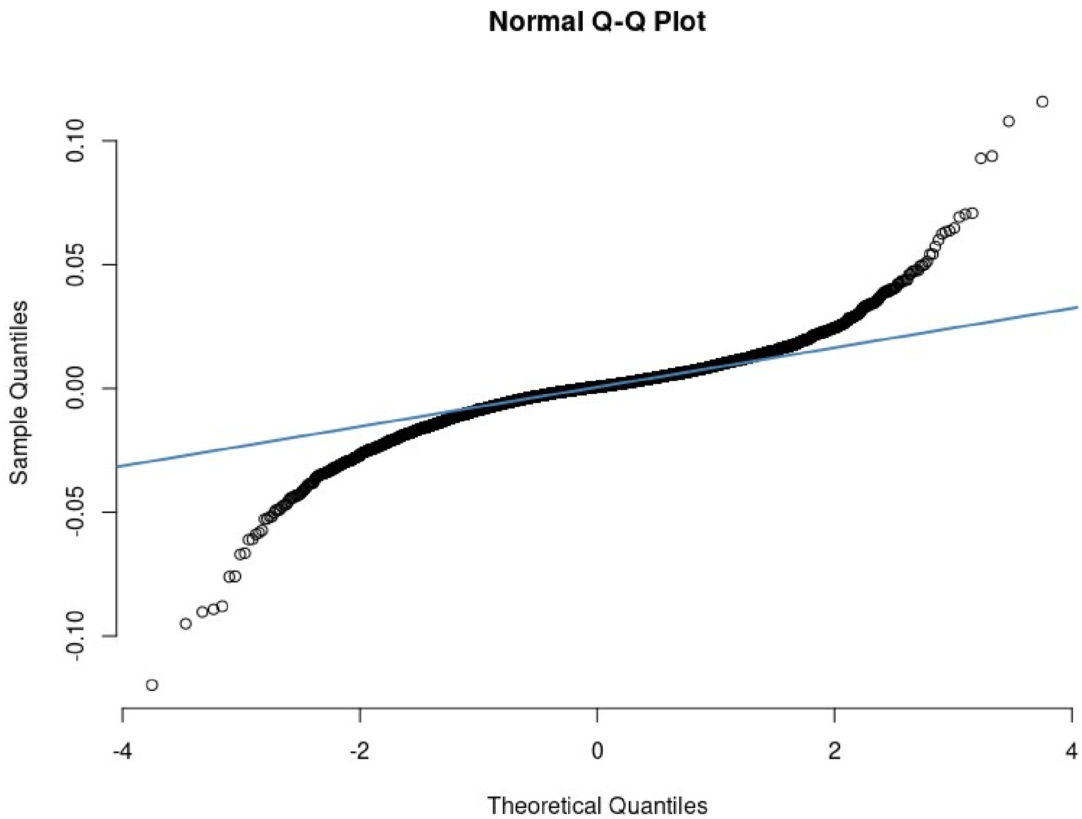

Figure 4.1: The QQ plot of the S&P 500 index based on daily returns of the index. Author's image 2022.

Furthermore, the snippet to print the QQ – plot is as follows (*Delt Function - RDocumentation*, n.d., p. 1):

```
stock <- read_excel("Wall_Street_20_Daily.xlsx")
model <- Delt(stock$SP_500)
qqnorm(model, pch = 1, frame = FALSE
qqline(model, col = "steelblue", lwd = 2)
```

("Quantile-Quantile Plot in R | Qqplot, Qqnorm, Qqline Functions & Ggplot2," n.d., p. 1)

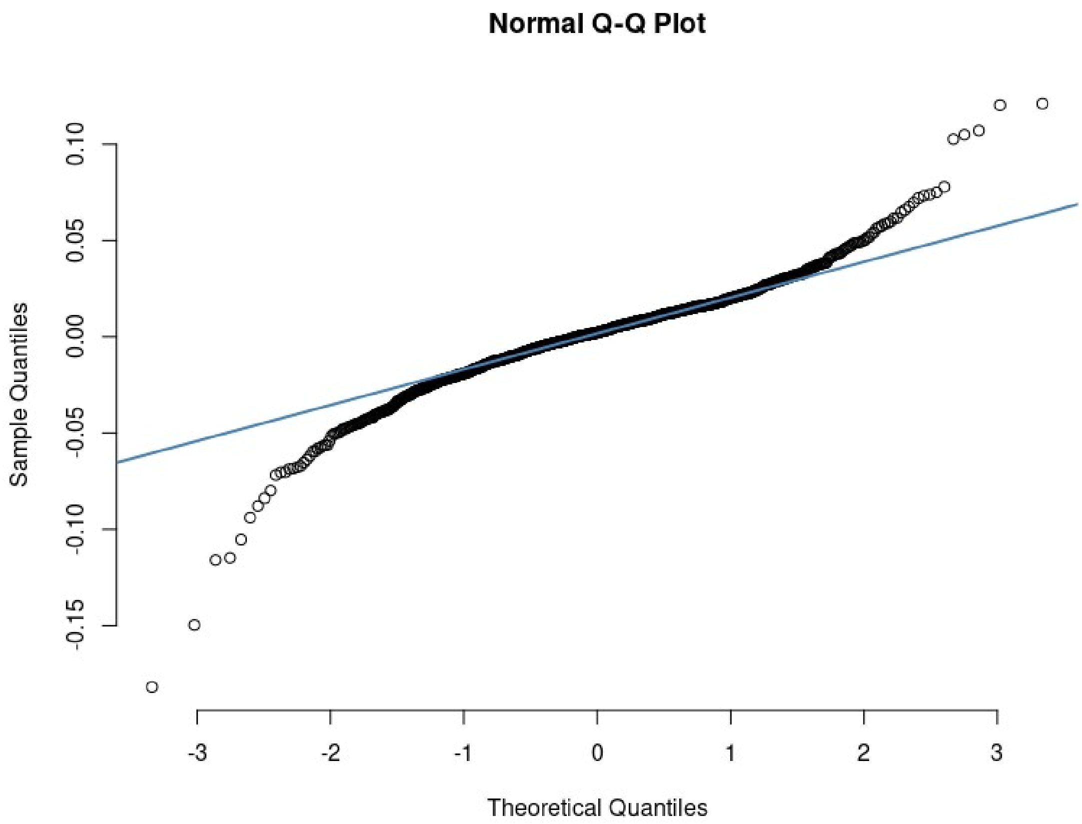

Figure 4.2: The QQ plot of S&P 500 stock index based on weekly return data. Author's image 2022.

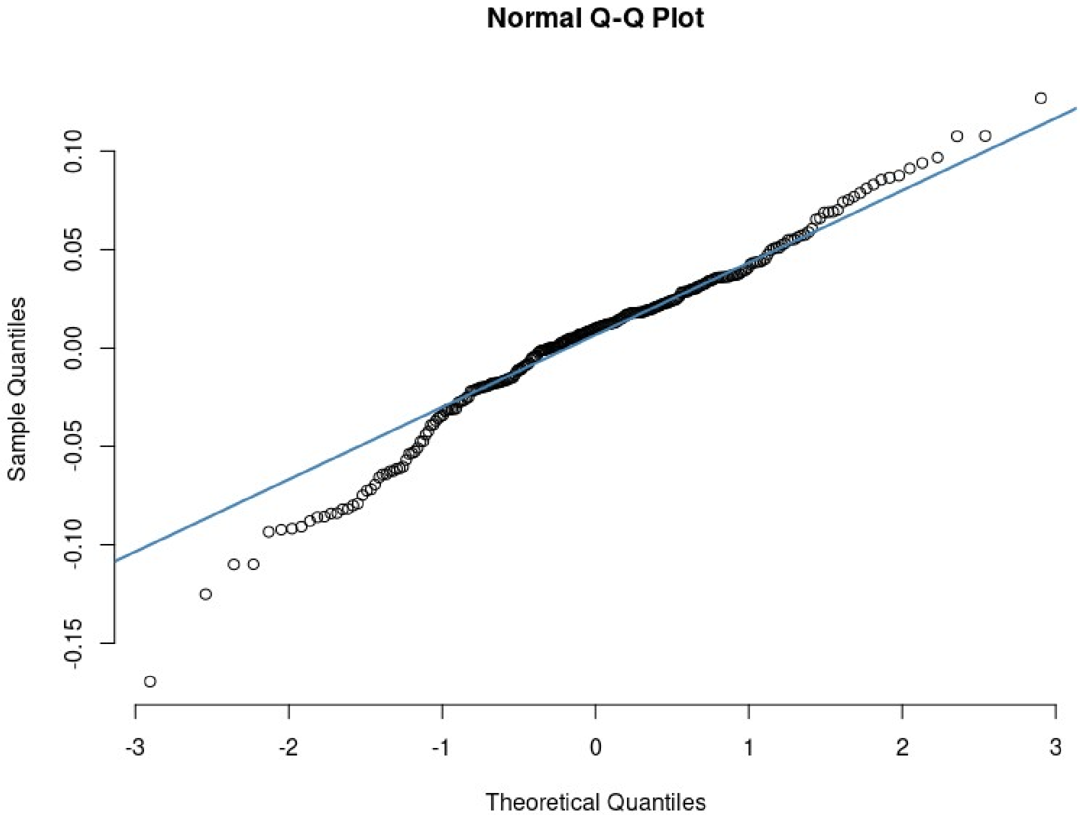

Figure 4.3: The QQ plot of S&P 500 stock index based on monthly return data. Author's image 2022.

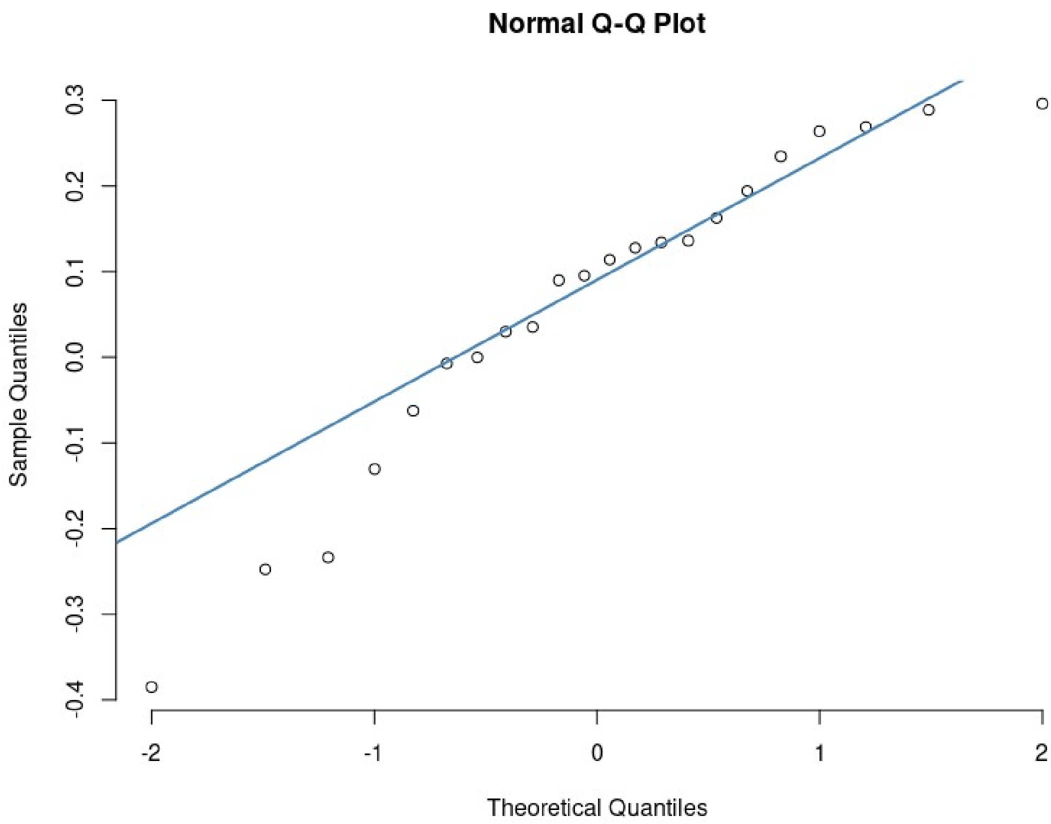

Figure 4.4: The QQ plot of S&P 500 stock index based on annual return data. Author's image 2022.

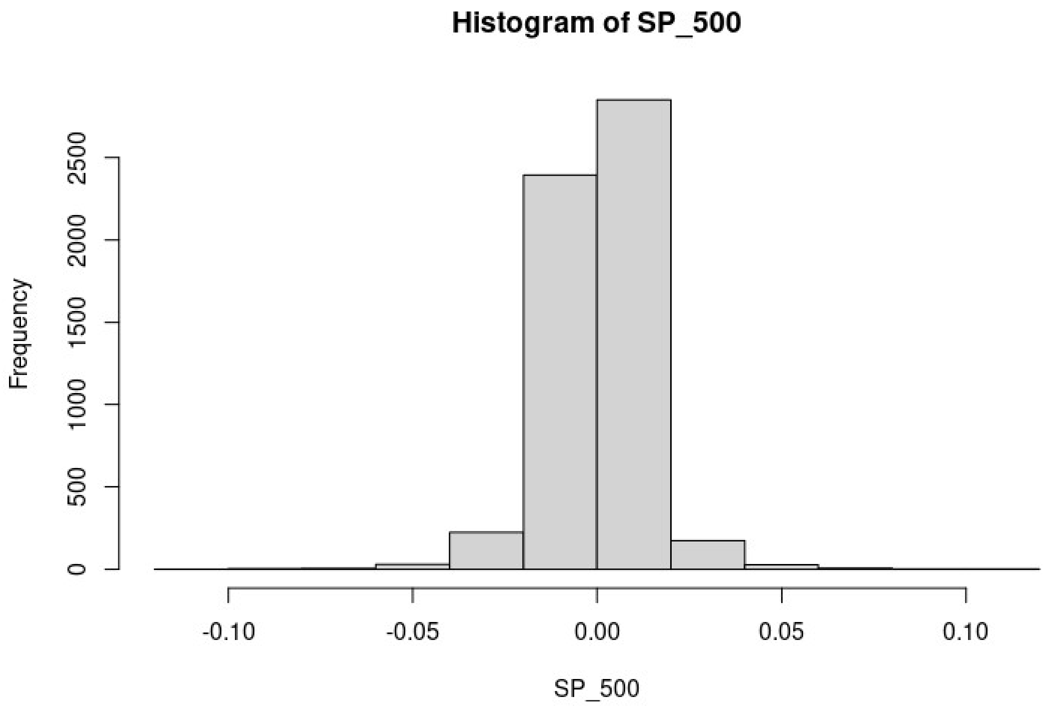

Figure 4.5: Histogram of S&P 500 returns from the year 2000 at daily level. Author's image 2022.

The snippet to make all these histograms to describe returns of the S&P 500 index is (*Delt Function - RDocumentation*, n.d., p. 1):

```
stock <- read_excel("Wall_Street_20_Daily.xlsx")
model <- Delt(stock$SP_500)
hist(model)
```

("R Hist() to Create Histograms (With Numerous Examples)," 2017, p. 1)

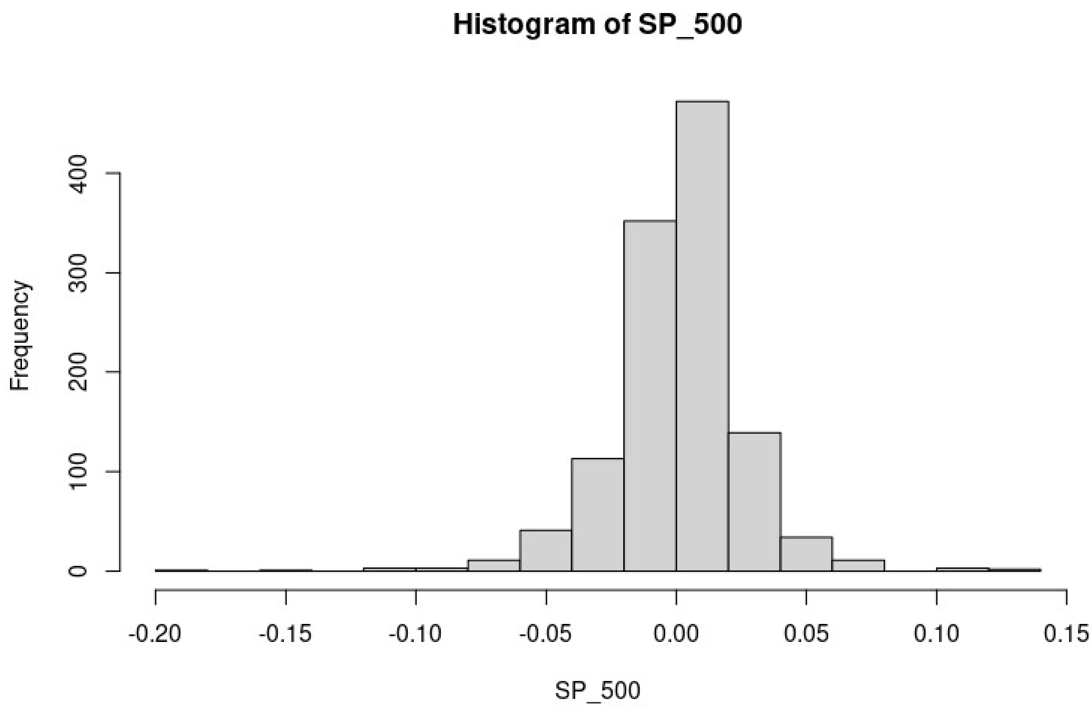

Figure 4.6: Histogram of S&P 500 returns from the year 2000 on weekly level. Author's image 2022.

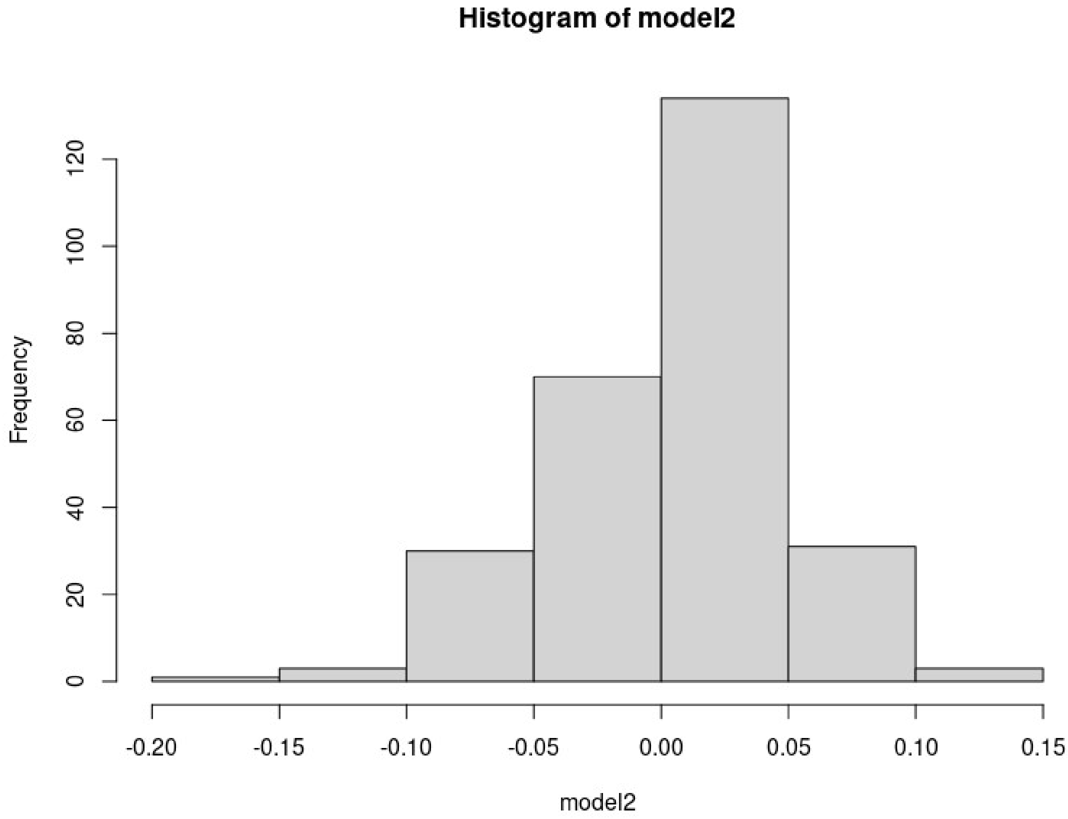

Figure 4.7: The S&P 500 stock index based on monthly returns from 2000 to 2022. Author's image 2022.

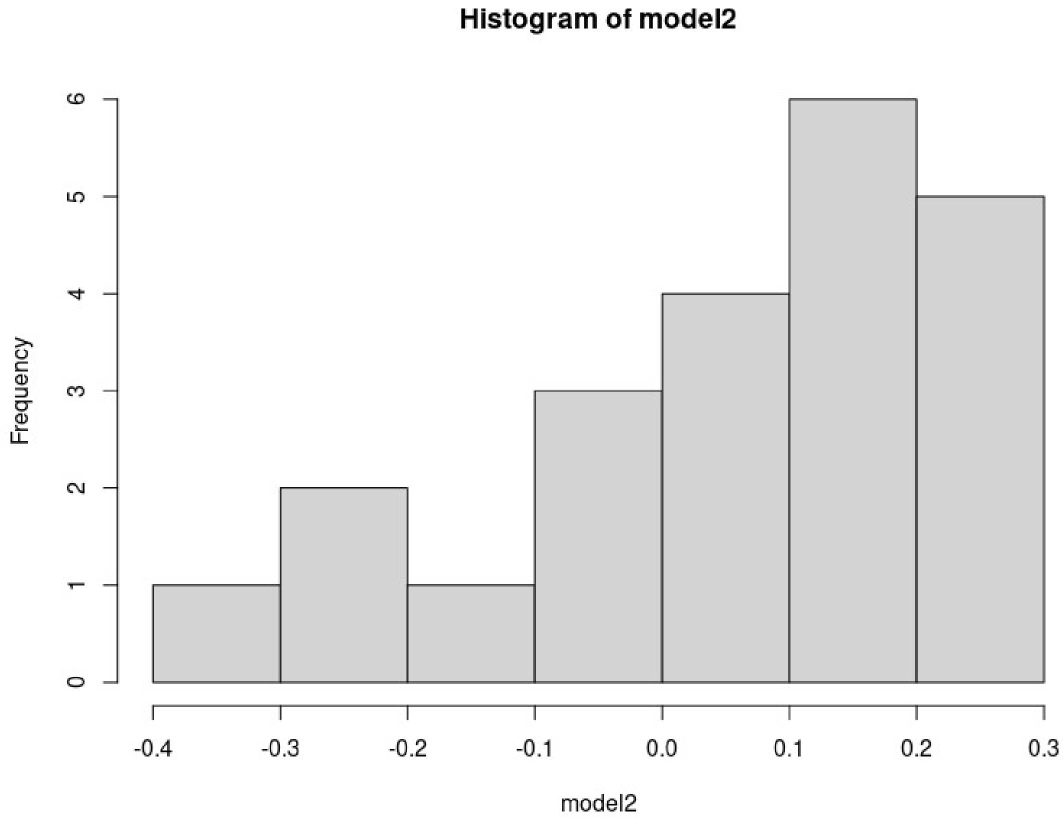

Figure 4.8: The S&P 500 stock index based on annual returns from 2000 to 2022. Author's image 2022.

4.5 Shapiro-Wilkinson, Anderson-Darling, and Jarque-Bera tests of the normality of the S&P 500

We will use **Shapiro-Wilkinson, Anderson-Darling, and Jarque-Bera tests** to find the possible normality of the S&P 500 stock index numerically. The reason for so many angles is, that for example Shapiro – Wilkinson test cannot handle really large samples like Anderson-Darling and Jarque-Bera tests do. Furthermore, the issue is the S&P 500 normally or otherwise distributed is very interesting as a scientific curiosity.

4.5.3 Shapiro-Wilkinson test for S&P 500

Shapiro – Wilkinson normality test in R software confirms the visual intuition, and it says that typically on a weekly and monthly level S&P 500 is clearly not normally distributed, but on the other hand on the annual level the S&P 500 is normally distributed. Furthermore, on the daily level, we can't say too much about the "truth" here because the sample size is far above 5000. Typically such large samples are assumed to be normally distributed. The sample is based on the returns of the stocks, not on the actual closed prices of the stocks. The W value works between zero and one; the closer it is to number one, the closer the sample is to a normal distribution and, vice versa. Furthermore, the small p-value leads to rejection.

What kind of formula is Shapiro Wilkinson's model as mathematically presented? Next Formula 2 expresses the main lines of the Shapiro Wilkinson formula(“Shapiro–Wilk Test in R Programming,” 2020, p. 1) :

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\left(\sum_{i=1}^{n} x_i - \bar{x}\right)^2} \tag{2}$$

where,

1. $x_{(i)}$ is the ith smallest number in the used sample
2. the mean (x) is ($x_{(1)}$ + x(2)+… + $x_{(n)}$) / n
3. $a_{(i)}$ is a coefficient, which can be calculated like ($a_{(1)}$+ $a_{(2)}$ … $a_{(n)}$) = (mTV-1)/C. The V is a covariance matrix, and the m and C are vector norms. These vector norms can be calculated as C= || V-1 m || and m = ($m_{(1)}$, $m_{(2)}$,..., $m_{(n)}$).

The used R library in this Shapiro Wilkinson test was “dplyr” (“Shapiro–Wilk Test in R Programming,” 2020, p. 1).

The snippet to make this Shapiro Wilkinson test is as:

```
stock <- read_excel("Wall_Street_20_Weekly.xlsx")
model <- Delt(stock$SP_500)
shapiro.test(SP_500)
```

So by using this formula 2 and R dplyr package we will get results, which can be seen in table 1. Surprisingly enough, the S&P 500 is normally distributed at the annual level but not otherwise. We can also see a peculiar trend in the p – value: The wider the measuring period the better the probability of the normality of the sample also becomes. In other words, there is no hope to see any normality on measuring on the daily level but the probability towards normality becomes significantly better when we use the weekly level to the final annual level, where the normality of the sample is obvious. The following graph shows the trend in the p - value:

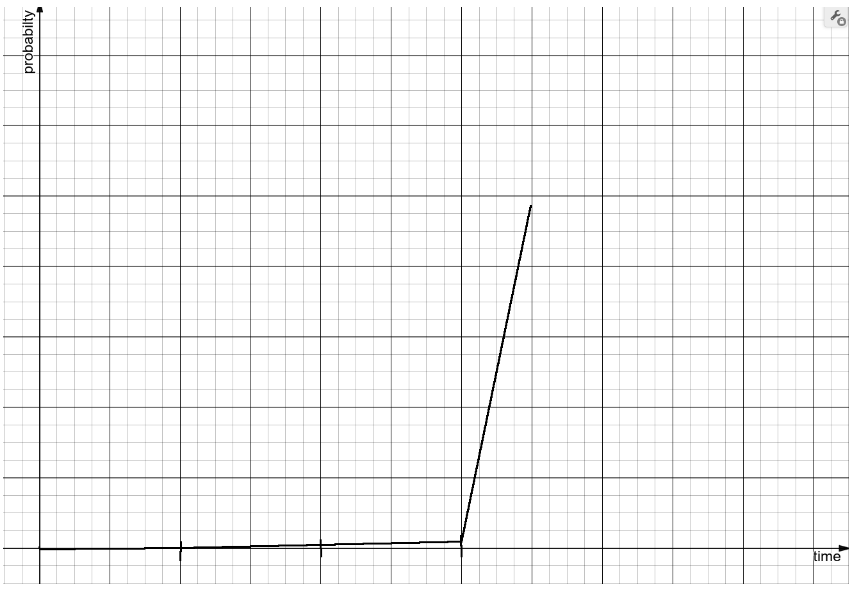

Figure 4.9: The probability of the normality increases as a function of time. Author's image 2022.

This odd rule continues in other tests of the normality of the samples and it raises a question: “What if the time period is over fitted if we use a shorter than annual time period?”. However, to find a decent answer to this more research work should be done and it is beyond of the scope this master’s thesis. The figure 4.9 is not in scale.

Shapiro – Wilkinson normality test results of the S&P 500 based on returns.				
H_0 hypothesis NA	α – value NA	p - value NA	Error in shapiro.test(S&P 500) : sample size must be between 3 and 5000	NA
H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 2.2e-16	W 0.93264	The sample is not normally distributed.
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 0.0001481	W 0.97591	The sample is not normally distributed.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.09948	W 0.92563	The sample **is** normally distributed.

Table 1: The normality of the S&P 500 is hard to see. Different time periods give different results. Author's table 2022.

4.5.4 Anderson-Darling test for S&P 500

Formula 3 is Anderson – Darling test, which is yet another normality test and it is written as follows(Stephanie, 2014, p. 1):

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[lnF\left(x_i\right) + ln(1 - F\left(X_{n-i+1}\right))\right] \tag{3}$$

In this formula, n is the sample size and F(x) is so called CDF = " Cumulative Distribution Function. This CDF function is also called the distribution function. It gives you the additive probability which is associated with the function (Macsin, 2020, p. 1).

The hypothesis of the Anderson-Darling test is:

H_0 = The data of the sample indeed comes from the distribution.

H_1 = The data of the sample does not come from the distribution.

Therefore, if the probability is really low and below, say the alpha level is 0.05, we may reject the H_0 hypothesis. That means the sample is not normally distributed in our case (Stephanie, 2014, p. 1).

The snippet to make this test for returns of the S&P 500 stock index is ("Calculate Price Return in R (2 Examples) | Returns from Vector of Prices," n.d., p. 1):

```
stock <- read_excel("Wall_Street_20_Weekly.xlsx")
model <- Delt(stock$SP_500)
ad.test(model)
```

(*Nortest.Pdf*, n.d., p. 2)

We can again see that on the annual level the S&P 500 indeed is normally distributed but not otherwise. The probability for the normality increases as a function of the time just like in figure 4.9 in chapter 4.5.3. earlier. We can assume at the 95% certainty level that S&P 500 is normally distributed on an annual measuring level.

Anderson - Darling normality test results of the S&P 500 based on returns.				
H_0 hypothesis rejected, α > p daily test sample	α – value 0.05	p - value 2.2e-16	A 113.33	The sample is not normally distributed
H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 2.2e-16	A 14.767	The sample is not normally distributed
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 0.0001481	A 4.048e-06	The sample is not normally distributed
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.1635	A 0.52243	The sample is normally distributed

Table 2: Anderson - Darling test for the samples. Author's table 2022.

4.5.5 Jarque-Bera test for S&P 500

Formula 4 is used Jarque-Bera test to test the normality of the S&P 500 stock exchange index(Stephanie, 2016a, p. 1) :

$$JB = n\left[\left(\sqrt{b1}\right)^2 \diagup 6 + (b_2 - 3)^2 \diagup 24\right] \qquad (4)$$

where,
n = size of the sample,
$\sqrt{b1}$ = sample skewness coefficient,
b_2 = kurtosis coefficient.

(Stephanie, 2016a, p. 1)

We can again see, that S&P 500 is normally distributed on the annual level, but not otherwise. Furthermore, the probability of normality will increase as a function of the time. The longer the measured time period the better probability of normality. This rule exists well for the investigated 2000-2022 time period. The snippet to calculate values for returns of the S&P 500 index was like:

```
library(tidyquant)

library(quantmod)

library(nortest)

library (tseries)

stock <- read_excel("Wall_Street_20_Weekly.xlsx")

SP_500 <- Delt(stock$GSPC)

veijo <- na.omit(SP_500)

jarque.bera.test(veijo)
```

(Zach, 2019, p. 1)

Jarque - Bera normality test results of the S&P 500 based on returns.				
H_0 hypothesis rejected, α > p daily test sample	α – value 0.05	p - value 2.2e-16	X-Squared 24719	The sample is not normally distributed
H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 2.2e-16	X-Squared 1857.2	The sample is not normally distributed
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 5.615e-05	X-Squared 19.575	The sample is not normally distributed
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.2815	X-Squared 2.5354	The sample is normally distributed

Table 3: The results of the S&P 500 normality test. Author's image 2022.

4.6 Durbin-Watson autocorrelation test of the S&P 500

Formula 5 presents the Durbin-Watson autocorrelation test for residuals for example in time series and also in regression analysis (Stephanie, 2016b, p. 1):

$$DW = \frac{\sum_{t=2}^{T} \left(\bar{e}_t - \bar{e}_{t-1}\right)^2}{\sum_{t=1}^{T} \bar{e}_t^2} \tag{5}$$

The $\bar{e}_t$:s are residuals of the regression model of ordinary least squares. If there is no autocorrelation of the returns of the stock prices the predictability of the prices is very difficult to forecast indeed. We can surely say that there is no autocorrelation in the pricing of the returns of the S&P 500. This means that statistical methods to predict it are hard to find (Stephanie, 2016b, p. 1).

The snippet used in this master's thesis was:

```
stock <- read_excel("Wall_Street_20_Weekly.xlsx")
model <- Delt(stock$GSPC)
DW <- lm(model ~ ref_date, data = stock)
dwtest(DW)
```

(Bedre, 2019, p. 1)

Again we find a somewhat interesting trend in the probability of the autocorrelation as a function of time. The longer the studied period the lower is the probability to autocorrelation. Table 4 says that on the annual level there are less chances for autocorrelation than on the daily level. Please notice that on daily level the chances to autocorrelation are exactly zero according to R statistical software in this sample. This means that on a daily level investing in stocks is simply lottery-like gambling at some weird casino.

Durbin – Watson autocorrelation test of the S&P 500 index based on returns of the stocks on daily, weekly, monthly and annual basis.				
H_0 hypothesis accepted α < p. daily	α - value 0.05	p - value 1	DW 2.2131	No autocorrelation
H_0 hypothesis accepted, α < p. weekly	α - value 0.05	p - value 0.9963	DW 2.1572	No autocorrelation
H_0 hypothesis accepted, α < p. monthly	α - value 0.05	p - value 0.2736	DW 1.9347	No autocorrelation
H_0 hypothesis accepted α < p. annually	α - value 0.05	p - value 0.6031	DW 2.2101	No autocorrelation

Table 4: Autocorrelation tests for S&P 500 shows no sign of entirely inefficient markets. Author's table 2022.

4.7 Bartlett test results to see the need for a final test

The Bartlett test should be done to see the need for a proper test. In our case to see the meaning of luck between a Student's t-test should be used if the sample was normal. However, from many angles, our sample is not normally distributed according to the Gauss hypothesis. No matter how hard I try, I can't see the S&P 500 stock index or some stocks related to it behaving in a normally distributed manner except on an annual level.

The Bartlett test is written as follows (Zach, 2021a, p. 1):

$$B = (n - k)\, lns^2 - \sum (n_j - 1)\, Ins_j^2 / c \tag{6}$$

where,

n = total number of all observations in all sample groups

k = total amount of groups

ln = natural log

s = pooled variance.

n_j = amount of observations in a group j

sj_2 = the group j's variance

Again the snippet example to make this test in R is as:

```
stock <- read_excel("Wall_Street_20_Weekly.xlsx")

model <- Delt(stock$'BRK-B')

SP_500 <- Delt(stock$GSPC)

bartlett.test(list(SP_500,model), centre = mean)
```

(Zach, 2021b, p. 1)

So Bartlett test should be used to find the variance of the sample groups. If the variance is similar between the groups and homo-genic, the p-value of the test will be very small and below the alpha-value, which is 0.05 in our case. Because our test results show that p-value is very small and almost zero, there is no difference in variance between the groups. Many statistical tests require exactly this kind of property, like one-way ANOVA. The following image 4.1 was free to use in a fare manner:

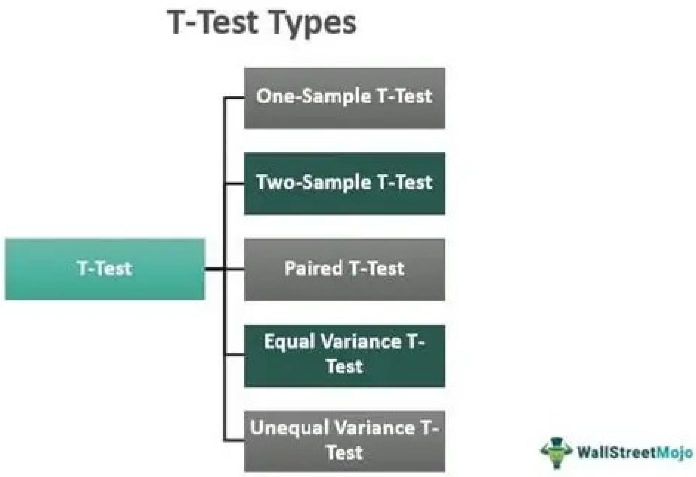

Figure 4.10: Most forms of Student's t-test. ***Any of these require randomness and normality to work from all samples.*** *Image by wallstreetmojo.com 2022*

However, the Student's t-test could be used based on Bartlett test, but because the normality in the case of the S&P 500 is weak or non-existing, we can not use any form of Student's t-test including all of its forms described in image 4.10. This should be clear and obvious statement. Therefore we will end up to using **the Mann-Whitney U - test.** The Mann – Whitney U - test is crude and simple but very effective in our case.

5 So why use the Mann-Whitney U test?

The world's first Mann-Whitney U - test was a rat test. So, there was an issue of whether a new drug could save rats from bacterial infection. The basic idea was to test first the distribution of the death of the rats with the drug and then test the distribution of the rats without that drug (Mann & Whitney, 1947, p. 1).

$$U_1 = R_1 \frac{n_1(n_1+1)}{2} \qquad or \qquad U_2 = R_2 \frac{n_2(n_2+1)}{2} \tag{7}$$

where,

R_x = the ranks of the sample

n_x = amount of the items in the sample

Both versions can be used similarly (Stephanie, 2021, p. 1).

The snippet example used for this master's thesis was as (mridul7719, 2020, p. 1):

```
stock <- read_excel("Wall_Street_20_Weekly.xlsx")
model <- Delt(stock$'BRK-B')
SP_500 <- Delt(stock$GSPC)
wilcox.test(SP_500,model, alternative = "two.sided", paired
= FALSE, exact = FALSE, correct = TRUE, conf.level = 0.95)
```

The basic realization was that if the distributions are the same when tested by reason X in the comparison case of two distributions, there has not been any effect for that reason X. if the distributions are different, the X has affected the distributions of the samples. Mann-Whitney U test is now an excellent way to determine whether the distribution incomes of Berkshire Hathaway are different from the S&P 500 incomes.

Once we can not be exactly sure if S&P 500 is normally distributed or not due to sake of certainty we have to use the Mann-Whitney U – test which works even in case distribution is not normally distributed (Mann & Whitney, 1947, p. 1).

It is possible to use Mann-Whitney U - test in many ways. I think that there are countless ways to use it in almost whatever situation, where we need to compare two not normally distributions. But, of course, there are some limitations too (Karch, 2021, p. 10), so this method should not be used blindly.

For example, in the paper “Comparison of Customer Satisfaction in SBI and ICICI-Application of Mann-Whitney Rank Sum U-test”, this statistical test has been used to determine whether customer service is better in a private bank or a public bank. Surprisingly, the study finds no significant difference between private and public banking in India. Moreover, it isn't easy to search the situation in many banks, so the usage of Mann-Whitney U - test enables researchers to rely on this small sample only and generalize the results to a larger population successfully. We are looking forward to finding out similar kind of effect with the case of Warren Buffet and Berkshire Hathaway investment company and 8 other successful stocks of the S&P 500 (Marimuthu et al., 2018, p. 17).

6 Constructing the research hypothesis

Now It seems that the samples **are generally not normally distributed,** so that we will use different methods like the usual student's t-test. The method to seek the essential difference between skill and luck is the Mann - Whitney U – test. Both samples, the S&P 500 which is our basis sample and the comparable sample of some stock should be normally distributed if Student's t-test is wanted to be used. In our study this is never the case (Stephanie, 2021, p. 1).

Somebody may think that there could well be lottery millionaires among us in the business of investing, but it is much more likely that these enormously lucky people are rare and not a rule. Now it is good to ask how many Warren Buffet like persons there are in this world? I must say that only one among approx eight billion. So why not to assume, that precisely Warren Buffet is most lucky guy in the world of Wall Street? The incomes drop heavily among the extremely rich people right after this. The author presents a model of option markets in his simple model, showing how a fortunate person made huge profits at the options markets (Laiho, 2019, pp. 41–43).

$$E = cm^{\frac{ln\ (P1)}{ln\ (P2)}} \tag{8}$$

Where,

E = Expansion of the Wealth

c = capital at use

m = multiplier of the capital

ln = logaritmus naturalis

P1 = *Probability based on the population size*

P2 = Probability based on the success

The model of mine is based on population, which is typical in a small 5.6 million people like Finland is. For example we can double the $ 1, 000 capital approximately 24,152 times in the option markets if we just assume a lottery head winners luck. The result is then $18,643,560,000. The possibilities to find much more luck is easy to find in the population of the **whole world with 8 billion habitants**.

Therefore the **H_0 hypothesis** is that if there is **no difference** between the distributions of incomes between the investment corporations or general corporations X and S&P 500, an investor or board of directors is merely lucky and not so skilful. On the other hand, the **H_1 hypothesis** is that X is not lucky and **very skillful.** So, if not lucky, then skillful. So simple is that. **How could it be otherwise? We are studying the group of the best, so assuming high skill level is a must.**

In this case, by using Mann-Whitney U test, it is possible to compare the distribution with S&P 500 and some investment or manufacturing companies' distribution. If these distributions are similar, the H_0 hypothesis will be satisfied, and we can assume that investor X is only lucky and his investment company's daily stock exchange quotations do not differ too much from the behavior of the S&P 500 index. On the other hand, if H_0 must be rejected, there are reasons to assume that investor X is not investing by luck only, but he or she makes investment decisions which are more rational and well made with skill.

7 Tests for GROUP A

There will be presented all needed tests here for the GROUP A. Due to limitations of page number and used time most of these tests are done only by **weekly level,** not otherwise. These tests are:

1. Normality test by Shapiro – Wilkinson

2. histogram

3. QQ - plot

4. Durbin – Watson test for autocorrelation.

5. Bartlett test to see proper test case by case.

5. Mann-Whitney U – tests or Student's t-tests.

Furthermore there will be short explanation of the company itself and a stock quotation chart. For next the focus will be in the group of five of the best investors of the S&P 500 index.

7.2 Berkshire Hathaway A – share (BRK-A)

Figure 7.1: Warren Buffet's Berkshire Hathaway company has steadily grown while it suffers from the European war in the year 2022. Author's image 2022

Berkshire Hathaway was established in the 19th century, and Warren Buffet wanted to invest in this company in the mid-1960s. His purpose was to turn Berkshire Hathaway into a conglomerate with several other insurance companies, including National Indemnity. By doing this, Buffet gained a chance to use unpaid premiums by insurance companies to acquire more investment opportunities for Berkshire Hathaway. It is typical for Warren Buffet to seek companies that are not well maintained to a new upward surge by rationalizing their activities.

However, Warren Buffet was not eager to pay dividends but instead aimed for new opportunities to invest the gained money in a new profitable way. This approach has been widely accepted by shareholders of Berkshire Hathaway owners who tend to rely on Warren Buffet's skills to make more money with money. It is also good to bear in mind that Warren Buffet's shareholders tend to keep their investments for a long time and Warren Buffet is not eager to make splittings of the share due to this fact. The value of the Berkshire Hathaway share has risen from $275 in 1980 to $308,530 in 2018 (Salzar, 2019, p. 3).

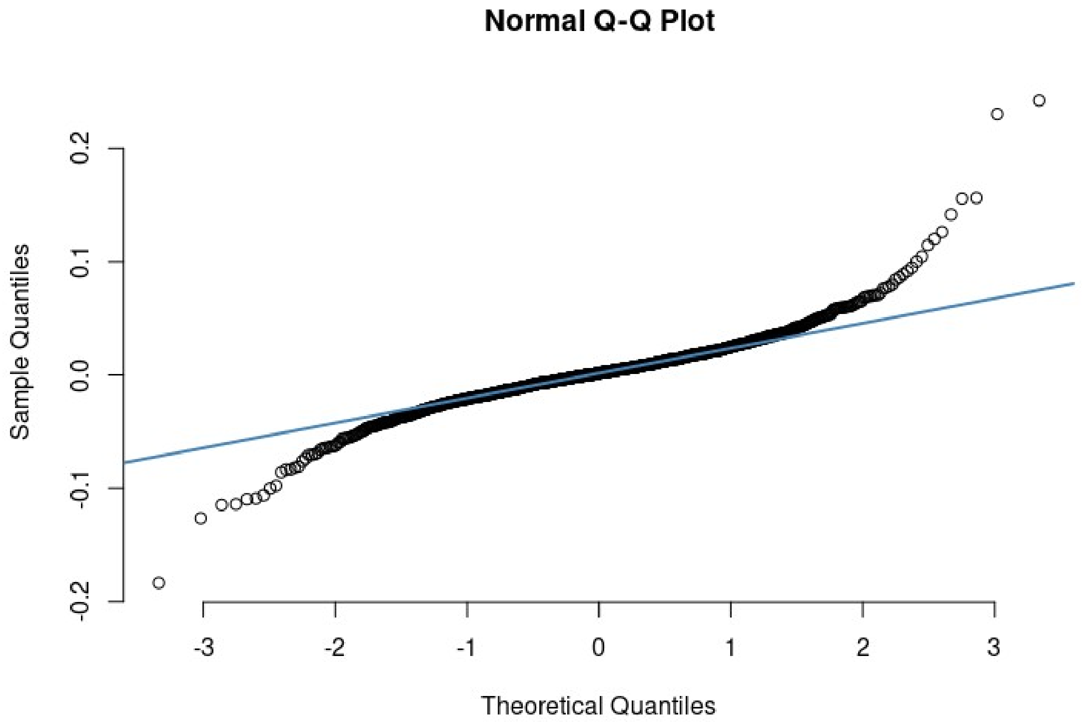

Figure 7.2: The Q-Q plot of Berkshire Hathaway weekly returns since the 2000 to this day. Author's image 2022.

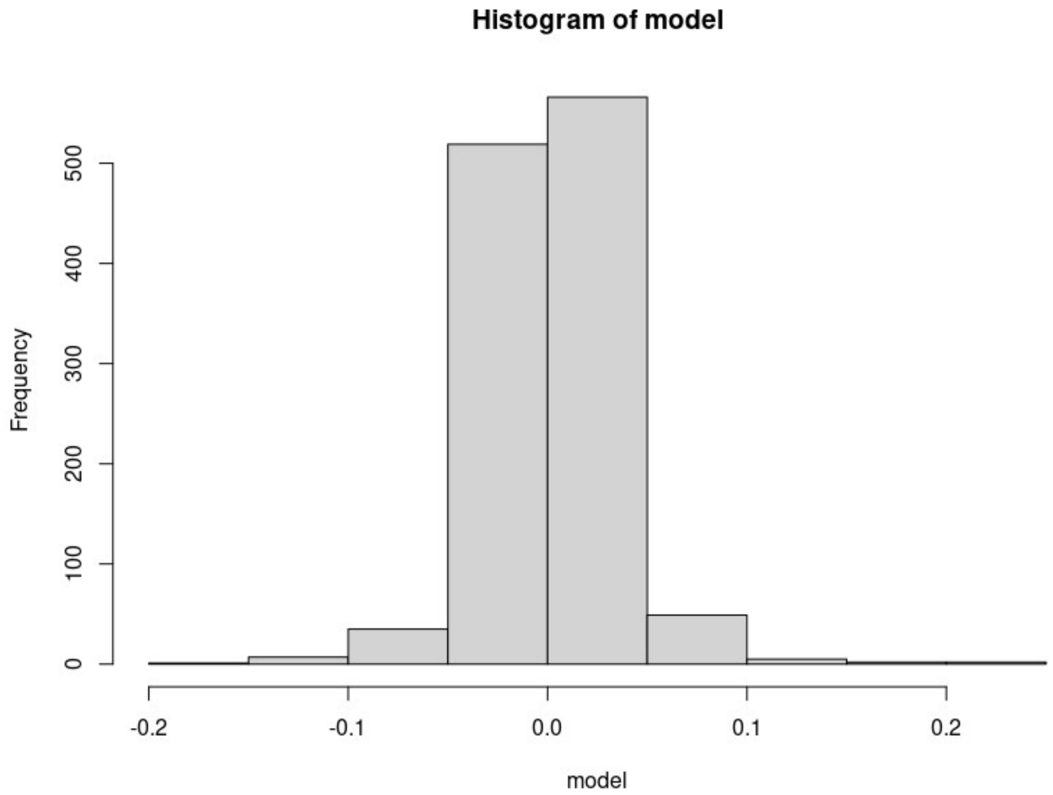

Figure 7.3: Berkshire Hathaway's A-share shows no sign of normality. Author's image 2022.

Berkshire Hathaway (BRK-A)				
Durbin -Watson test: H_0 hypothesis accepted, $\alpha < p$	α - value 0.05	p - value 1	DW 2.3207	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.05	p - value 2.2e-16	W 0.90927	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.05	p - value 4.858e-13	Bartlett's K-squared = 52.262 df = 1	The testee has equal variances.

Table 5: This table gives us more info that Mann-Whitney U-test is a correct choise. All results are based on weekly returns of the BRK-A share. Author's image 2022.

Warren Buffet A -share (BRK-A)				
H_0 hypothesis accepted, $\alpha < p$ daily	α - value 0.05	p - value 0.08612	W 16679718	The testee is merely lucky.
H_0 hypothesis accepted, $\alpha < p$. weekly	α - value 0.05	p - value 0.9161	W 705056	The testee is merely lucky.
H_0 hypothesis accepted, $\alpha < p$ monthly	α - value 0.05	p - value 0.7819	W 36484	The testee is merely lucky.
H_0 hypothesis accepted, $\alpha < p$ annual	α - value 0.05	p - value 0.6985	W 225	The testee is merely lucky.

Table 6: Two sided Mann-Whitney U test results for Warren Buffet A -share based on closed pricing returns on daily, weekly, monthly and annual basis. Author's table 2022.

7.3 Berkshire Hathaway B – share (BRK-B)

Figure 7.4: The Warren Buffet's Berkshire Hathaway stock prices since the year 2000.

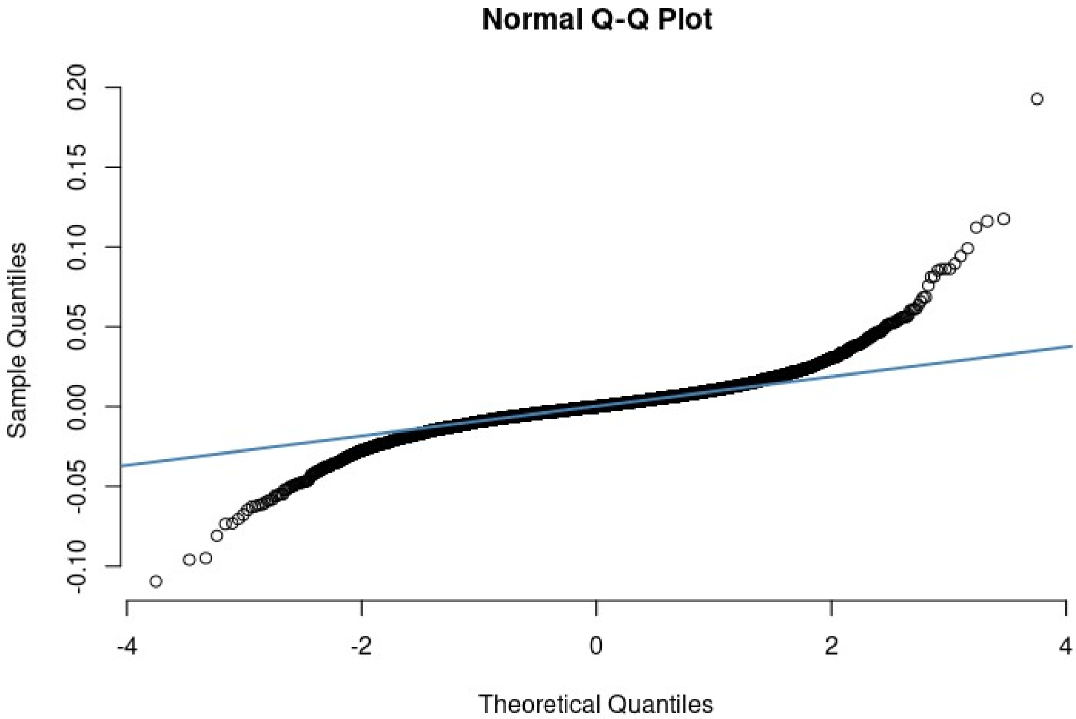

Figure 7.5: The Berkshire Hathaway QQ-plot. Author's image 2022.

Berkshire Hathaway (BRK-B)				
Durbin -Watson test: H_0 hypothesis accepted, $\alpha < p$	α - value 0.05	p - value 0.9941	DW 2.066903	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.05	p - value 2.2e-16	W 0.90927	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.05	p - value 1.586e-14	Bartlett's K-squared = 58.988 df = 1	The testee has equal variances.

Table 7: The B-share of Warren Buffet. Author's table 2022.

Warren Buffet B -share (BRK-B)				
H_0 hypothesis accepted, α < p daily	α - value 0.05	p - value 0.1503	W 16630656	The testee is merely lucky.
H_0 hypothesis accepted, α < p. weekly	α - value 0.05	p - value 0.8578	W 706287	The testee is merely lucky.
H_0 hypothesis accepted, α < p monthly	α - value 0.05	p - value 0.7198	W 36334	The testee is merely lucky.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.6985	W 225	The testee is merely lucky.

Table 8: Two sided Mann-Whitney U test results for Warren Buffet B -share based on closed pricing returns on daily, weekly, monthly and annual basis. Author's table 2022.

7.4 Black Rock (BLK)

Figure 7.6: Black Rock stock quotations since 2000 to present day. Author's image 2022.

The investment company Black Rock was established in 1988 by eight people. The critical point in establishing this company was understanding and managing risk and its meaning to the customers. In 2000, Black Rock solutions were established, which meant a new era as a technology provider for this company (*History*, 2022, p. 1).

In 2006 Black Rock bought Merrill Lynch Investment Management, which improves its international presence. In 2008 Black Rock played a role as one solver in the financial crisis. It has a meaning as an adviser and holds credits as one of the companies that helped government navigate this crisis (*History*, 2022, p. 1).

Currently, the focus in trading is on AI solutions and machine learning data. This move will improve the company's ability to serve its clients modern and practical. (History, 2022, p. 1) .The annual turnover of Black Rock was about $15 billion in the year 2019 (*BlackRock-2019-Annual-Report.Pdf*, n.d., p. 2).

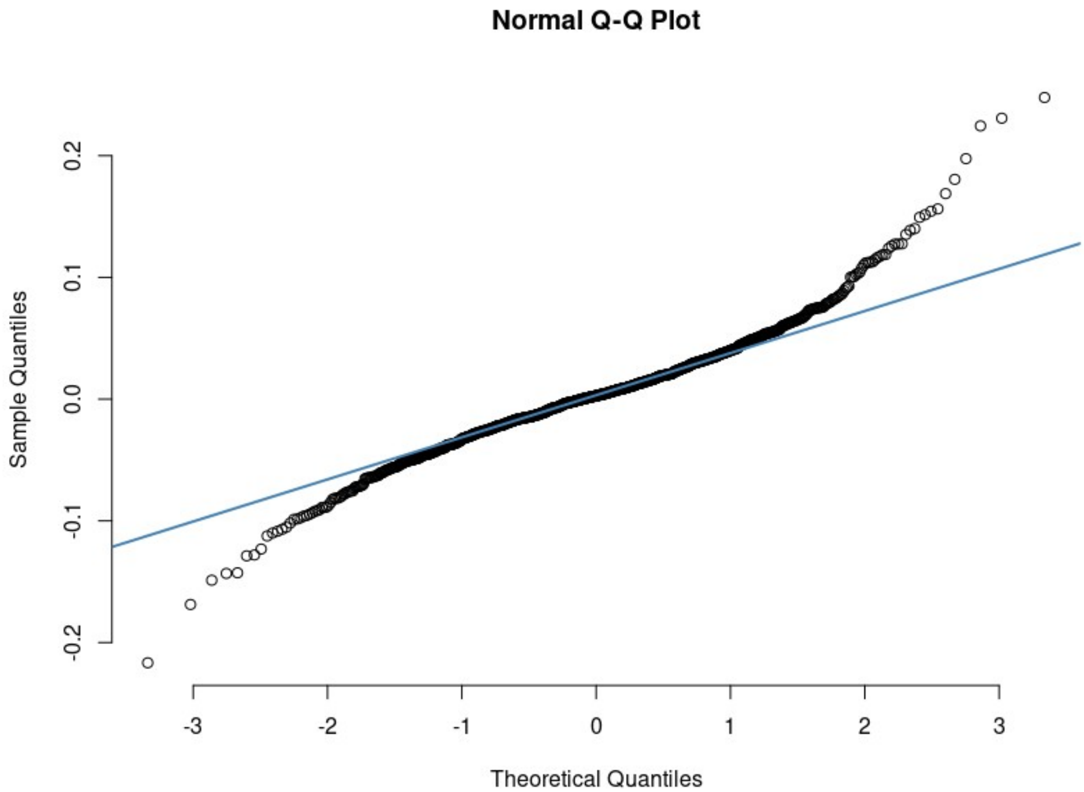

Figure 7.7: Black rock QQ plot shows no sign of normality based on returns at weekly level.

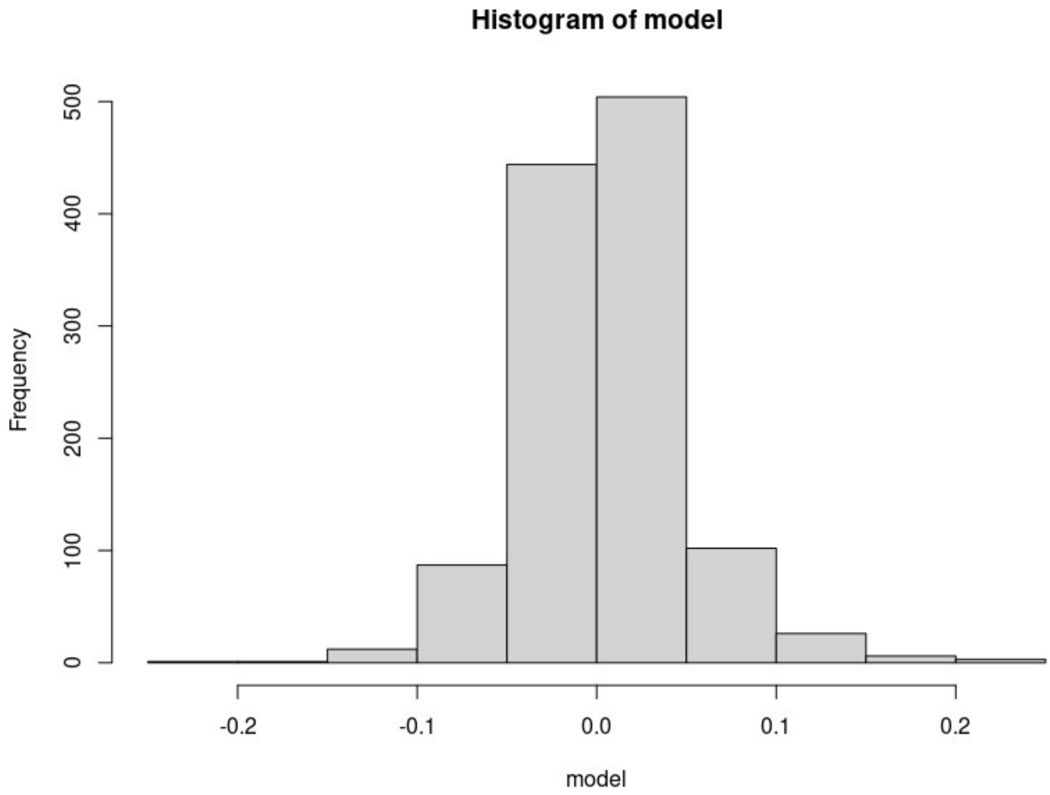

Figure 7.8: Black rock histogram is not normally distributed figure. Author's image 2022.

Black Rock (BLK)				
Durbin -Watson test: H_0 hypothesis accepted, α < p	α - value 0.05	p - value 0.9999	DW 2.2194	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 2.2e-16	W 0.95902	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 2.2e-16	Bartlett's K-squared = 370.62, df = 1,	The testee has equal variances.

Table 9: Test results for the Black Rock company. Author's table 2022.

Black Rock (BLK)				
H_0 hypothesis accepted, α < p daily	α - value 0.05	p - value 0.3682	W 16217312	The testee is merely lucky.
H_0 hypothesis accepted, α < p. weekly	α - value 0.05	p - value 0.3231	W 686818	The testee is merely lucky.
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 0.04601	W 33334	The testee is not lucky.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.1556	W 181	The testee is merely lucky.

Table 10: Two sided Mann-Whitney U test results for Black Rock share based on closed pricing returns on daily, weekly, monthly and annual basis. Author's table 2022.

7.5 Markel (MKL)

Figure 7.9: Markel stock quotations since the year 2000. Author's image 2022.

In 1930 Sam Markel established a new insurance company in the United States. Today it belongs to the Fortune 500 list (*The Markel Story*, 2022, p. 1). Soon his relatives joined this company in 1935, and a new family company was born. Mr Markel's interest is safety, and he has even been giving help to establish the National Motor Carrier Act of 1935 (*The Markel Story*, 2022, p. 2) .

In 1940 Markel became the industry leader in insurance companies. Markel's speciality is covering insurance of trucks and buses, which was a way to success (*The Markel Story*, 2022, p. 3).

The heavy expansion growth continues, and Markel wants to enlarge its activities on the international level in Canada at first. Therefore, a new headquarter is established in the Toronto area. This all became true in 1951 (*The Markel Story*, 2022, p. 4).

In 1986 Markel goes public and their IPO is offered $8.33 per share and market capitalization is nearly $15 mil (*The Markel Story*, 2022, p. 5). Today the same numbers are $ 1181 and $15,98 Mrd (Markel Corporation (MKL) Osakekurssit ja uutiset – Google Finance, 2022, p. 1.

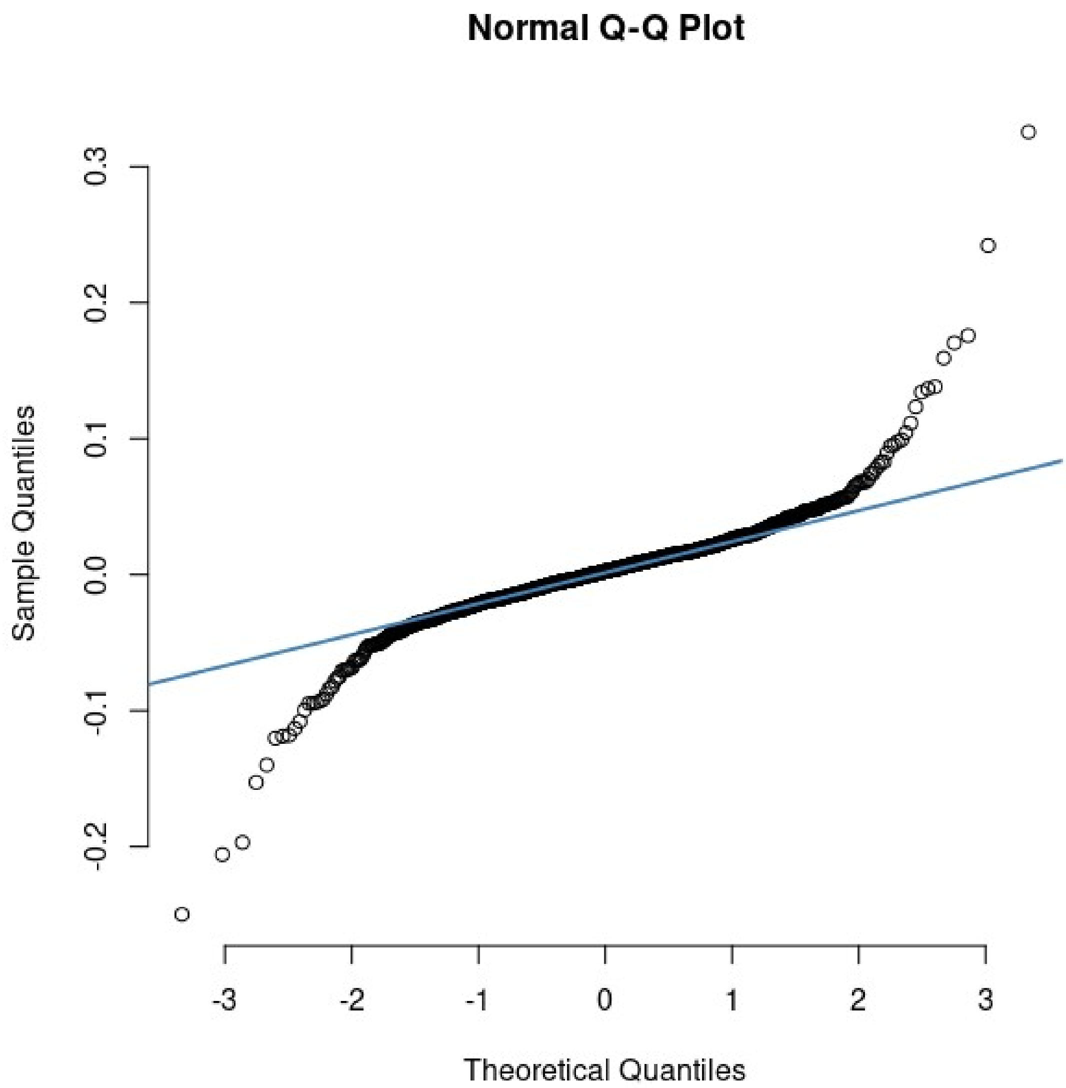

Figure 7.10: Markel's QQ plot shows no signs of normal distribution. Author's image 2022.

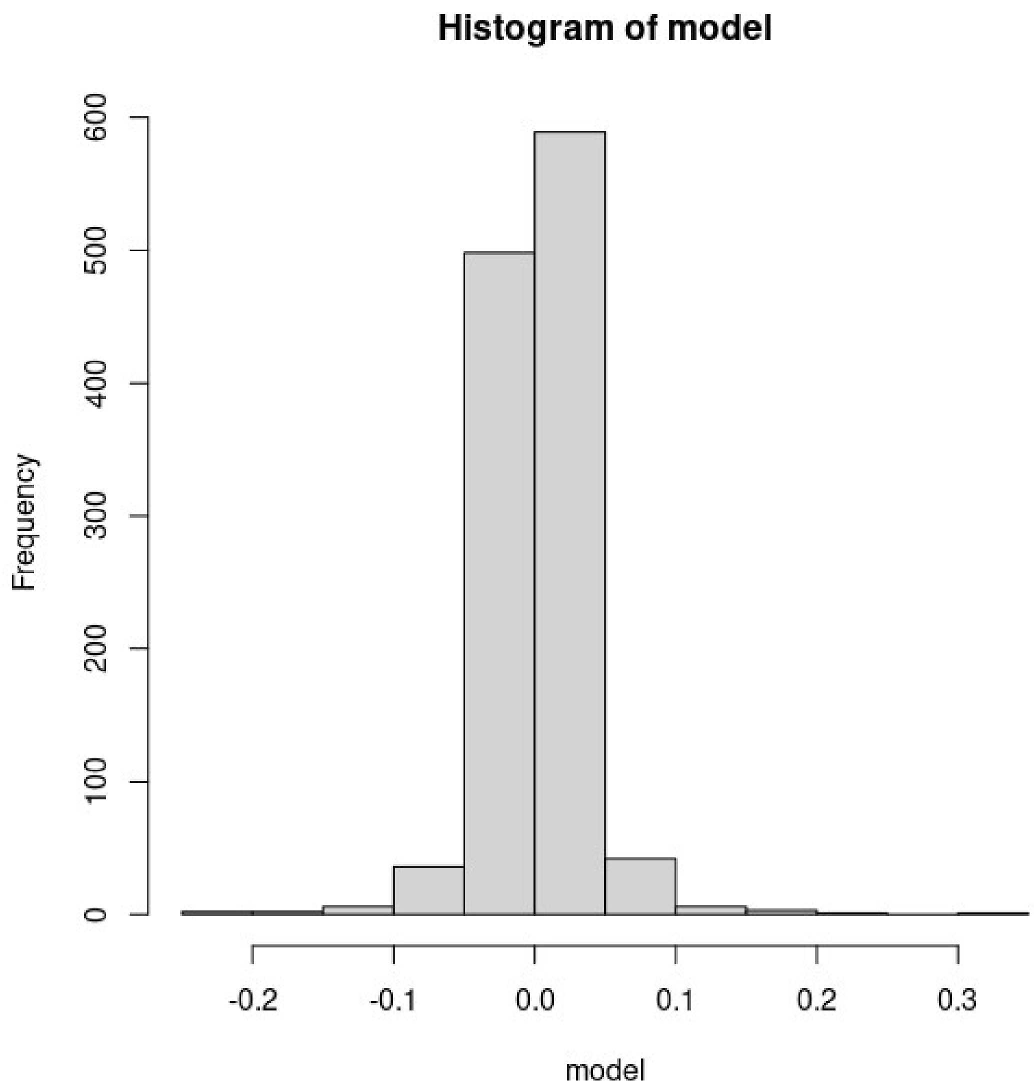

Figure 7.11: Markel histogram shows no sign of normal distribution. Author's image 2022.

Markel (MLK)				
Durbin -Watson test: H_0 hypothesis accepted, $\alpha < p$	α - value 0.05	p - value 1	DW 2.2409	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.05	p - value 2.2e-16	W 0.85717	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.05	p - value 2.2e-16	Bartlett's K-squared = 117.29, df = 1	The testee has equal variances.

Table 11: Markel test results. Author's table 2022.

Markel (MKL)				
H_0 hypothesis accepted, α < p daily	α - value 0.05	p - value 0.7419	W 16428831	The testee is merely lucky.
H_0 hypothesis accepted, α < p. weekly	α - value 0.05	p - value 0.5857	W 694207	The testee is merely lucky.
H_0 hypothesis accepted, α < p monthly	α - value 0.05	p - value 0.5281	W 35835	The testee is merely lucky.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.6472	W 222	The testee is merely lucky.

Table 12: Mann - Whitney U - test results for the returns of the Markel stock. Author's table 2022.

7.6 Goldman Sachs Group (GS)

Goldman Sachs was established in 1885 and joined New York Stock Exchange in 1896. It soon became a national company, opening many other offices at the beginning of 1900. In addition, the company soon established new relationships with European capital in the early 1900s. Goldman Sachs also took a unique approach to financial assets by evaluating intangible values in the financial statements. This raised awareness of lots of new capital, which otherwise could have been ignored (A-Brief-History-of-Gs.Pdf, n.d., p. 1).

Goldman Sachs has always been a very international company. This probably is one of its vital success features. It companies exist in Japan and almost anywhere in the world, which is worth keeping an office. Also, new personnel is soon educated to meet its core business values and strategy and aims to help its personnel all over the globe. (A-Brief-History-of-Gs.Pdf, n.d., p. 4).

Today Goldman Sachs has about $36.6 billion annual turnover, according to 2018 financial statements and personnel of about 36600 people (*2018-Q4-Results.Pdf*, n.d.).

7.7 Morgan Stanley (MS) test results

Figure 7.12: Morgan Stanley stock quotations since the year 2000. Author's image 2022.

Morgan Stanley is an investment banking house which is established in 1935. In its first year, the company is a huge success. It also shows some philanthropic signs in the year 1940, when Harold Stanley raised 1.5 million for the committee of European children, which is today's 2014 money of 24 million dollars (Stanley, 1922, p. 1).

In 1941 and 1942, the company underwent a demanding reorganization phase. The target was to join the New York Stock exchange while increasing securities business for the stock markets. In 1952 the company helped World Bank manage its 50 million dollars equity to reconstruct Europe after World War II. Regarding pioneering computer technology, the Morgan Stanley company seems to have relatively extensive credits. It was among the first developers of a computer model for analysts in 1961 (Stanley, 1922, p. 1).

Today the Morgan Stanley investment company has 45000 people at personnel and $ 85.3 billions annual turnover (Google, 2022, p. 1).

Morgan Stanley (MS)				
H_0 hypothesis accepted, $\alpha < p$ daily	α - value 0.05	p - value 0.7747	W 16426954	The testee is merely lucky.
H_0 hypothesis accepted, $\alpha < p$ weekly	α - value 0.05	p - value 0.5399	W 713522	The testee is merely lucky.
H_0 hypothesis accepted, $\alpha < p$ monthly	α - value 0.05	p - value 0.3919	W 35422	The testee is merely lucky.
H_0 hypothesis accepted, $\alpha < p$ annual	α - value 0.05	p - value 0.7513	W 228	The testee is merely lucky.

Table 13: Mann Whitney test results for the Morgan Stanley shares returns since the year 2000. Author's table 2022.

8 The Mann – Whitney U test with GROUP B of S&P 500

8.2 Apple (AAPL)

Apple Inc. is one of the most profitable companies that has existed on the planet Earth. Its products carry a highly prestige styling and form of industrial design, which has been seen as impossible to copy by its rivals. Its customers are as well as private citizens as they are business users. Apple Inc. has also started a trend where business angels finance start-up companies while wishing that these start-up companies will produce a similar type of success in their home country. That is because Apple Inc. was a modest company, and it started having only a few members: Steve Jobs, Mike Mirkkula and Steve Wozniak. The concept of the Apple personal computer was initially designed by Steve Wozniak, while Steve Jobs and Mike Mirkkula were more like marketing and management people (Kubilay, 2015, p. 2019,2020).

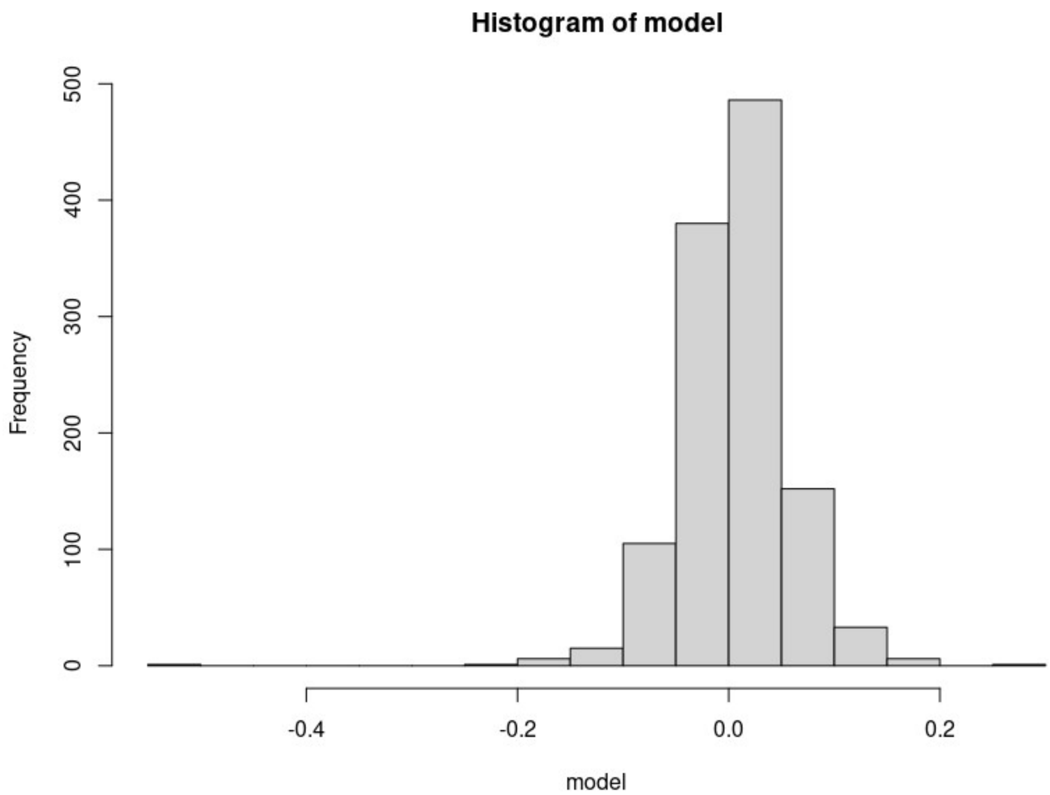

Figure 8.1: Apple histogram of weekly incomes of the shares 2000-2022. Author's image 2022.

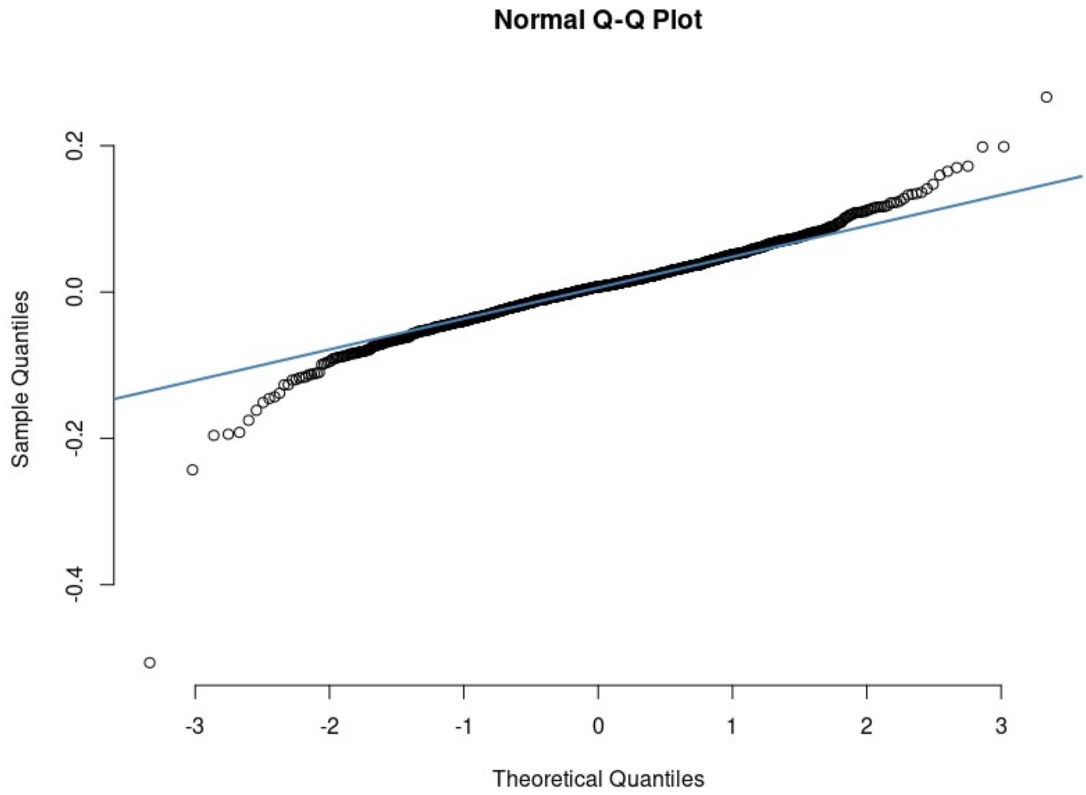

Figure 8.2: Apple QQ plot based on weekly returns of the shares 2000-2022. Author's image 2022.

Apple Inc. (AAPL)				
Durbin -Watson test: H_0 hypothesis accepted, α < p	α - value 0.05	p - value 0.05346	DW 1.9081	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 2.2e-16	W 0.94368	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 2.2e-16	Bartlett's K-squared = 581.67, df = 1	The testee has equal variances.

Table 14: Different tests based on weekly returns of shares. Author's table 2022.

Apple Inc. (AAPL)				
H_0 hypothesis rejected, $\alpha > p$ daily	α - value 0.05	p - value 0.01254	W 15909857	The testee is not lucky.
H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.05	p - value 0.0005183	W 645406	The testee is not lucky.
H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.05	p - value 0.0005112	W 30622	The testee is not lucky.
H_0 hypothesis rejected, $\alpha > p$ annual	α - value 0.05	p - value 0.009494	W 131	The testee is not lucky.

Table 15: Mann - Whitney U-test for Apple inc. Author's image 2022.

8.3 Microsoft (MSFT

Figure 8.3: Microsoft stock quotations 2000-2022. Author's image plotted by R 2022.

The Microsoft founders Paul Allen and Bill Gates met a few years earlier, in 1972, when they founded a company named Traf-O-Data. That was a crude computer which handled simple traffic data. In 1975 Bill Gates and Paul Allen wrote the BASIC language for that device. Later that year, Gates left to study in college and develop computer languages for the Altair company. Finally, at the end of 1975, Bill Gates and Paul Allen established Microsoft (Nesterov, 2015, p. 78).

In 2022, Microsoft will be a giant software development company with Windows 11 and Microsoft Office 365 packages as major sales articles. Before that, one of the best-selling articles of this software company was Windows XP (Nesterov, 2015, p. 78).

It can be said that Microsoft has a natural monopoly when it comes to operating system issues. There is very little sense in developing too many operating systems for this world once it could only lead to a situation where all software developers should develop their software releases to meet the needs of all these different operating systems, which otherwise could exist. Therefore there should be only one serious OS on the markets, and let Bill Gates keep his money once he was one of the first but perhaps not the best option for the markets and our civilization (Nesterov, 2015, p. 78).

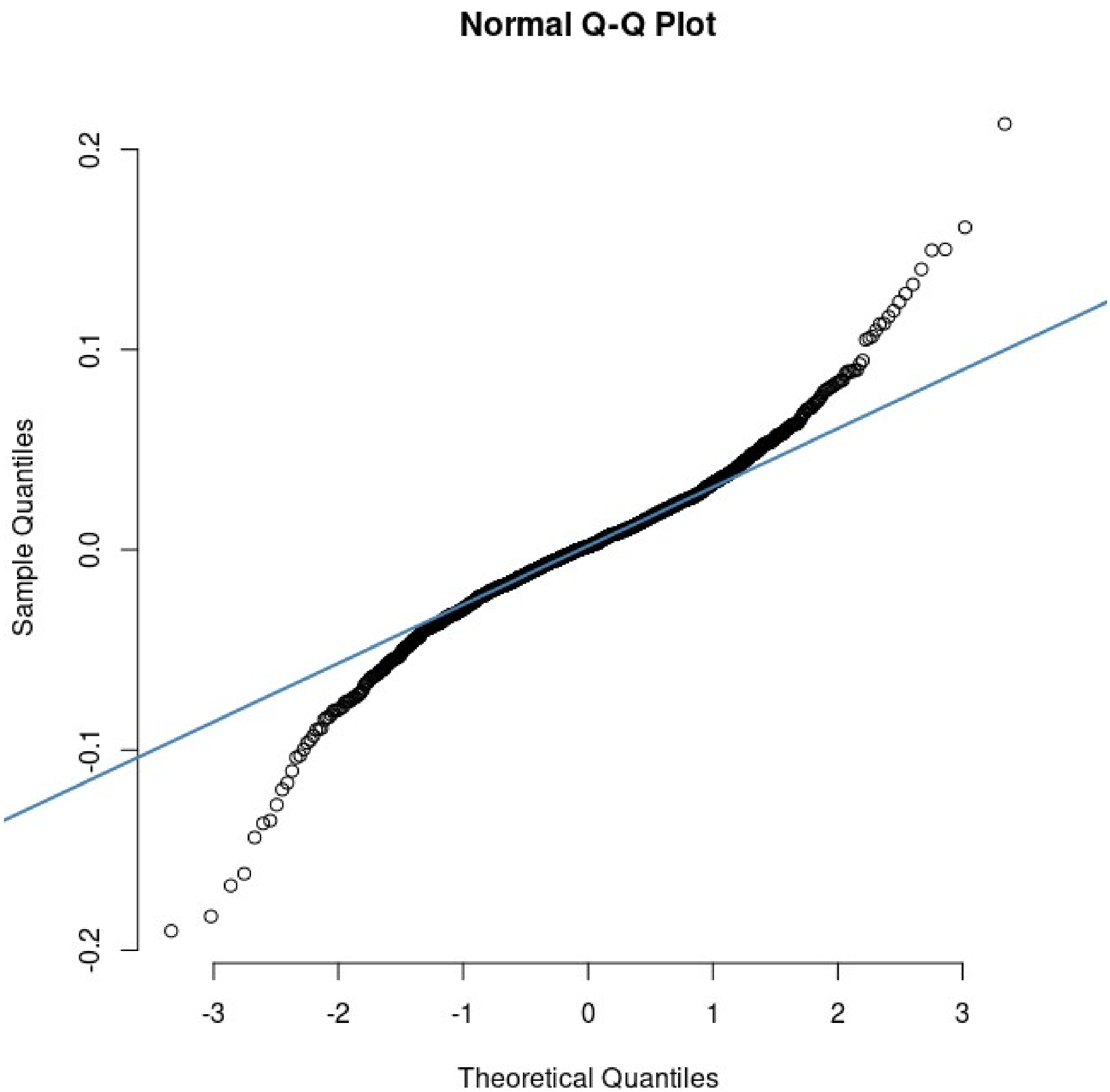

Figure 8.4: Microsoft's stock QQ plot to seek normality of the sample. Author's image 2022.

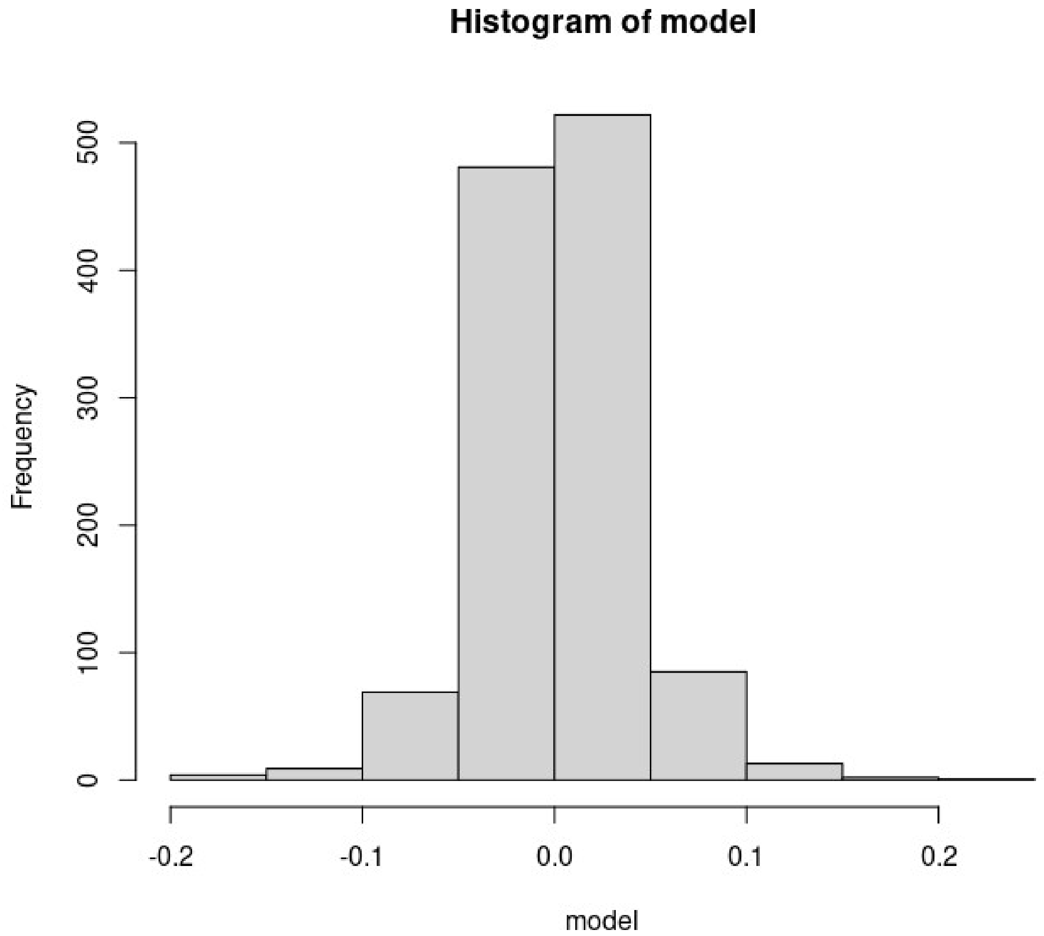

Figure 8.5: Histogram of Microsoft's stock quotations returns. Author's image 2022.

Microsoft (MSFT)				
Durbin -Watson test: H_0 hypothesis accepted, α < p	α - value 0.05	p - value 0.8712	DW 2.0674	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 2.2e-16	W 0.95885	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 2.2e-16	Bartlett's K-squared = 212.34, df = 1	The testee has equal variances.

Table 16: Test results for Microsoft's share. Author's table 2022.

Microsoft (MSFT)				
H_0 hypothesis accepted, α < p daily	α - value 0.05	p - value 0.8103	W 16418805	The testee is merely lucky.
H_0 hypothesis accepted, α < p weekly	α - value 0.05	p - value 0.7842	W 698731	The testee is merely lucky.
H_0 hypothesis accepted, α < p monthly	α - value 0.05	p - value 0.3699	W 35348	The testee is merely lucky.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.3072	W 198	The testee is merely lucky.

Table 17: Mann Whitney U -test results for Microsoft stock's returns. Author's image 2022.

8.4 Nvidia Corp. (NVDA)

Figure 8.6: NVIDIA stock quotations since the year 2000. Author's image 2022.

Nvidia Corporation is among the top 9 of the S&P 500 because it is one of the foremost manufacturers of video cards which are essential PC parts. This is true, especially for gamers, engineers and industrial designers of computer-aided design and scientific users' needs. The company of Nvidia was established in 1993 by Jensen Huang, Chris

Malachowsky and Curtis Priem. The best hit product for Nvidia has been Geforce graphics cards since 1999 until this day (*NVIDIA*, 2022, p. 1).

The other hit product has been the costly Quadro workstation video cards made for engineers and 3D artists of the Hollywood film industry at its best. Many visual effects of current sci-fiction movies are realized using NVIDIA products and high-end 3D CAD software like Autodesk Maya. Since 1997 huge improvements have happened in the memory amount and clock speed of the Geforce and Quadro video cards. What was impossible to do with early Geforce's and Quadro's has become a reality (NVIDIA, 2022, p. 1).

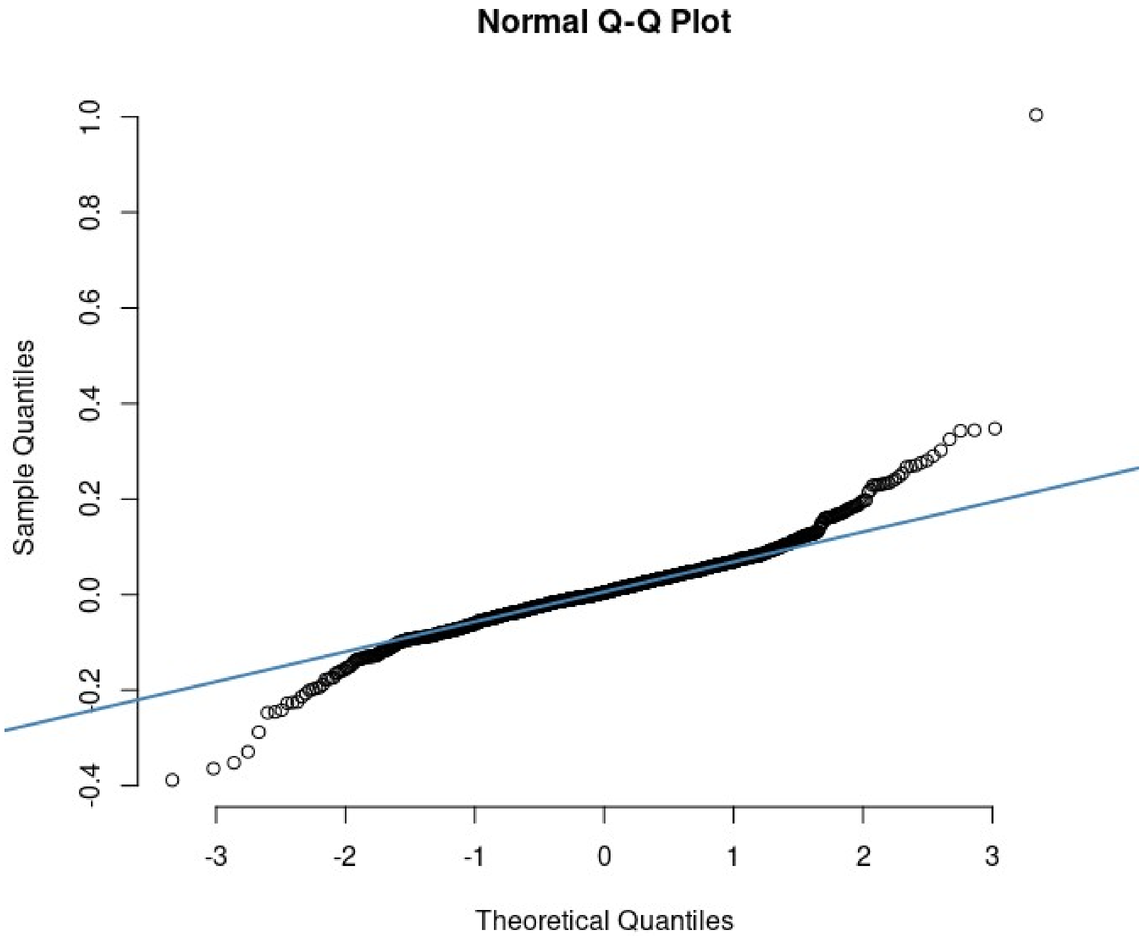

Figure 8.7: Histogram of the returns of the NVIDIA share. Author's image 2022.

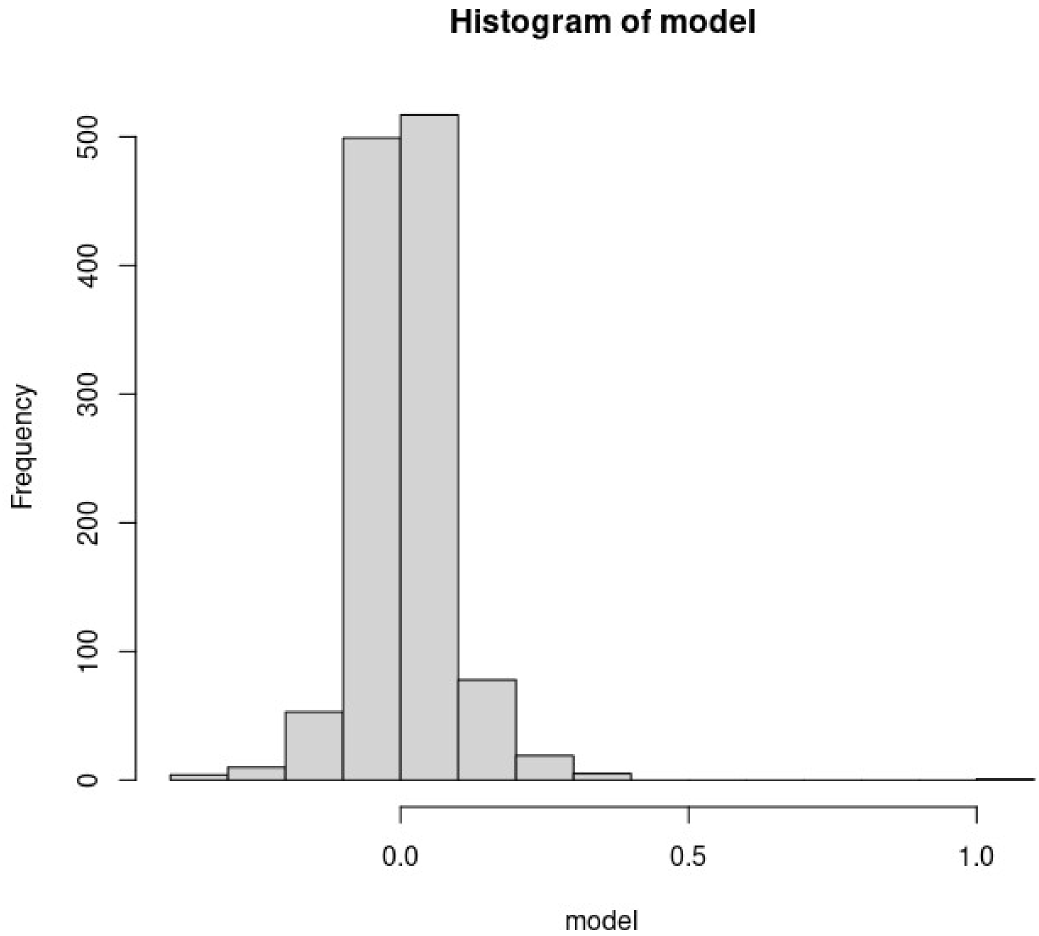

Figure 8.8: Histogram of the returns of NVIDIA share. Author's image 2022.

NVIDIA (NVDA)				
Durbin -Watson test: H_0 hypothesis accepted, $\alpha < p$	α - value 0.05	p - value 0.5911	DW 2.0151	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.05	p - value 2.2e-16	W 0.89897	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.05	p - value 2.2e-16	Bartlett's K-squared = 1452.5, df = 1	The testee has equal variances.

Table 18: Test results of the NVIDIA share. Author's image 2022.

NVIDIA (NVDA)				
H_0 hypothesis accepted, α < p daily	α - value 0.05	p - value 0.07188	W 16058222	The testee is merely lucky.
H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 0.04209	W 669398	The testee is not lucky.
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 0.008865	W 32194	The testee is not lucky.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.05882	W 161	The testee is merely lucky.

Table 19: Two sided Mann-Whitney U test results for NVIDIA share based on closed pricing returns on daily, weekly, monthly and annual basis. Author's table 2022.

8.5 Amazon (AMZN)

Figure 8.9: Amazon's returns for its share since the year 2000. Author's image 2022.

Amazon.com as numbers a vast number crushing. In 2019, it was assumed that Amazon.com would have almost half of all online retail dollars used by all the e-buying power of the US market. The key to all this success has been estimated to be the solid logistics and distribution arrangements of Amazon.com and its e-buying culture (Kantar, 2019, p. 1).

As Amazon.com has become a gigantic and powerful online retail marketer in the whole US, environmental issues are more or less a concern. Amazon.com uses about 40% of US mail, and retail stores are everywhere. This level of consumption is against all green imperatives, and therefore Amazon.com is considering establishing its delivery systems independent of current US mail with mail and drone technology (Kantar, 2019, p. 1).

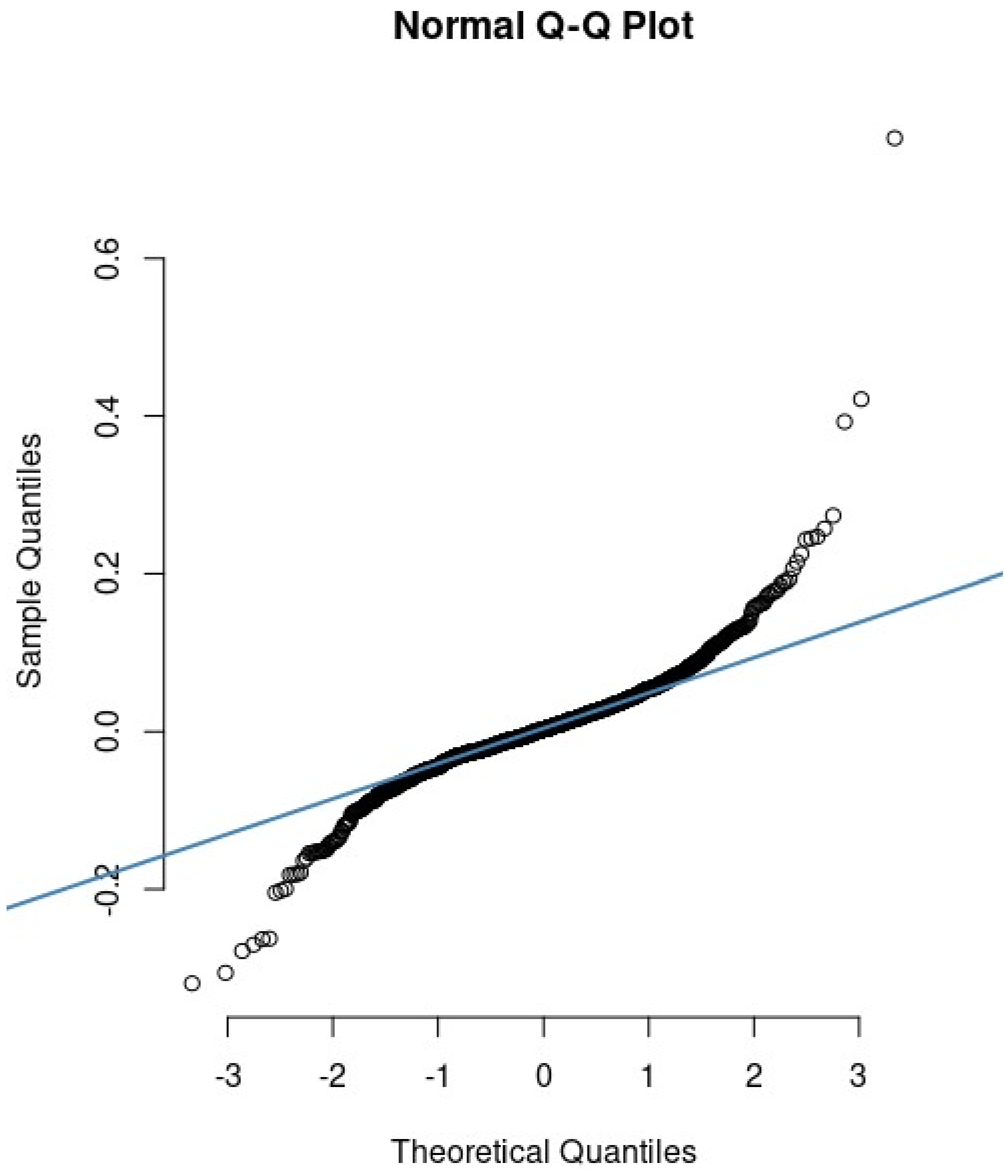

Figure 8.10: QQ plot of the returns of Amazon stock. Author's image 2022.

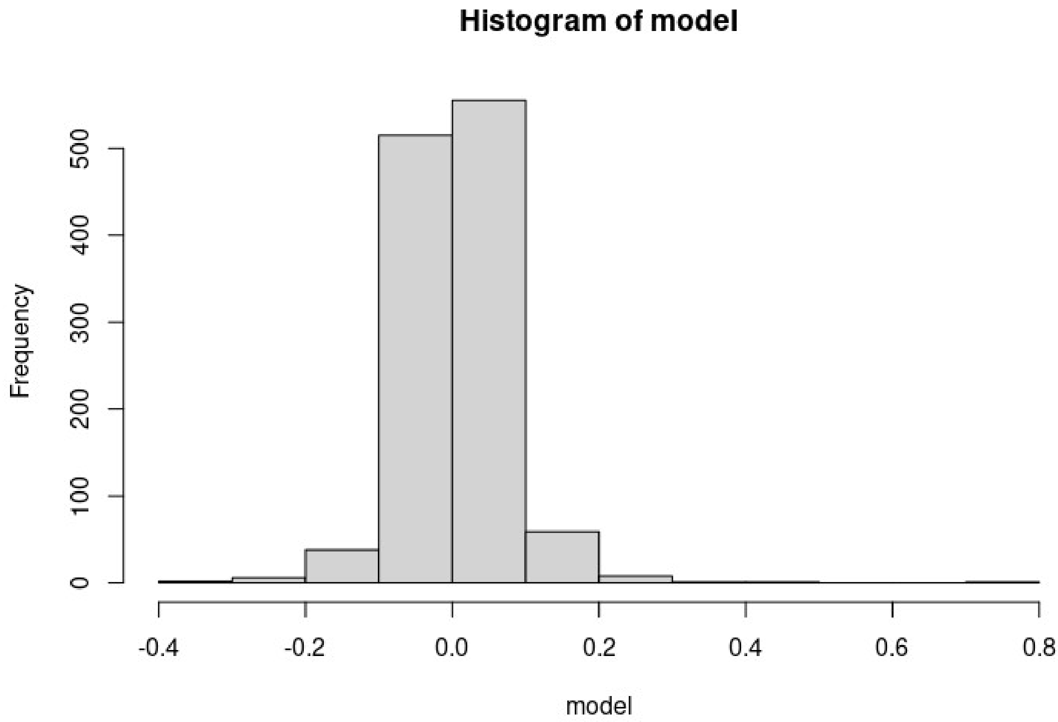

Figure 8.11: Amazon histogram of returns on a weekly level. Author's image 2022.

Amazon (AMZN)				
Durbin -Watson test: H_0 hypothesis accepted, $\alpha < p$	α - value 0.01	p - value 0.8367	DW 2.0586	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.01	p - value 2.2e-16	W 0.8827	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.01	p - value 2.2e-16	Bartlett's K-squared = 1028.3, df = 1	The testee has equal variances.

Table 20: Test results of the AMAZON share. Author's image 2022.

Amazon (AMZN)				
H_0 hypothesis accepted, α < p daily	α - value 0.05	p - value 0.4683	W 16242488	The testee is merely lucky.
H_0 hypothesis accepted, α < p. weekly	α - value 0.05	p - value 0.2806	W 685303	The testee is merely lucky.
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 0.0263	W 32919	The testee is not lucky.
H_0 hypothesis accepted, α < p annual	α - value 0.05	p - value 0.1771	W 184	The testee is merely lucky.

Table 21: The Mann - Whitney U - test results for the AMAZON share. Author's table 2022.

8.6 United Health Group Inc. (UNH)

Figure 8.12: Stock Quotations of the United Health Group since the year 2000. Author's image 2022.

According to its revenue, the American United Health Group Inc. is the world's most extensive health care system, which is about 240 billion US dollars large (McFarlane, 2019, p. 1).

The United Health Group Inc. was established in the year 1977. During the years the company has started to expand by many acquisitions, which most of them has been successful business opportunities. There are plenty of them and mentioning each deal is beyond the scope of this kind of short historical report (*United Health Group History*, 2020, p. 1).

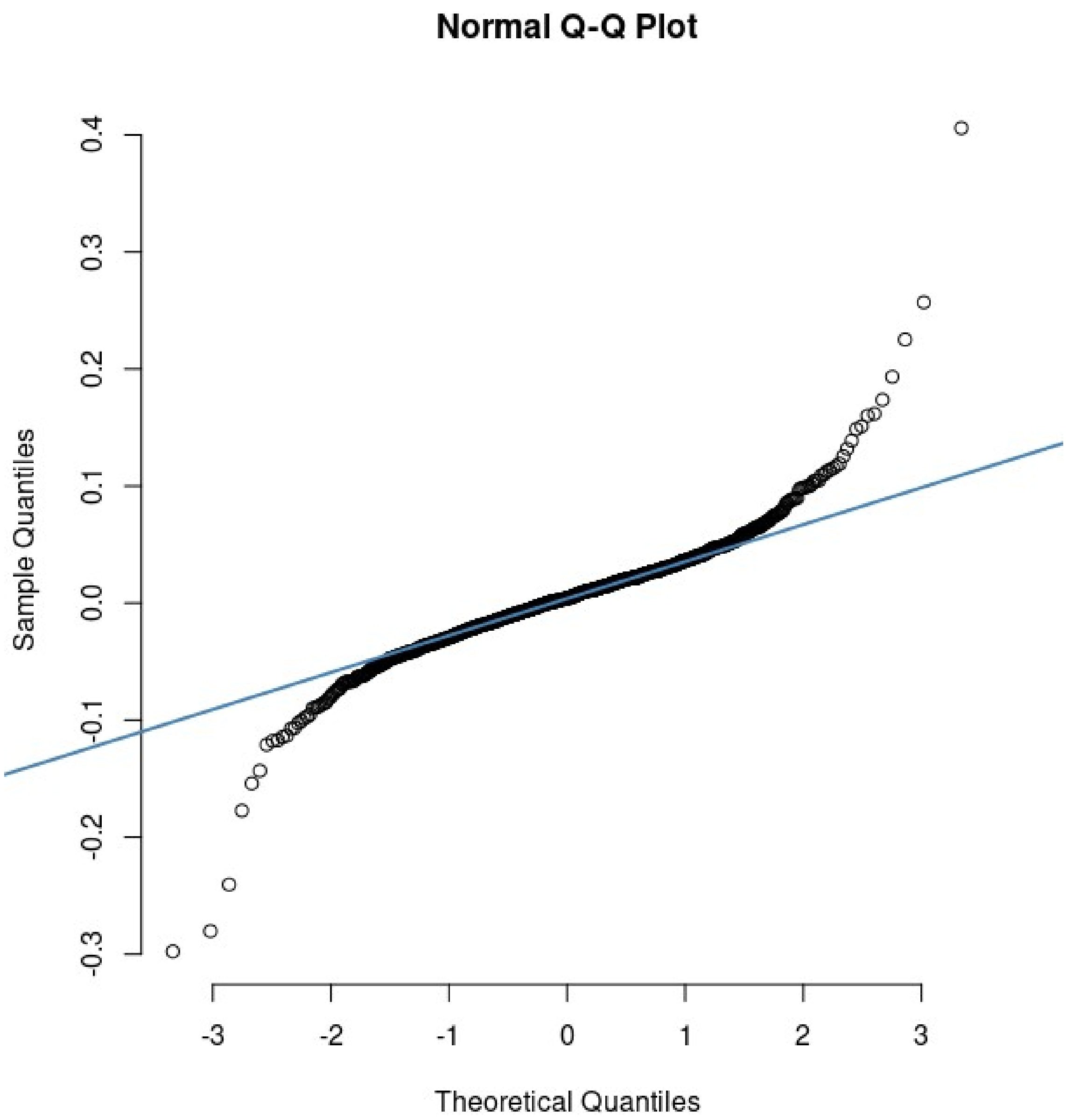

Figure 8.13: QQ plot of the returns of UNH share. Author's image 2022.

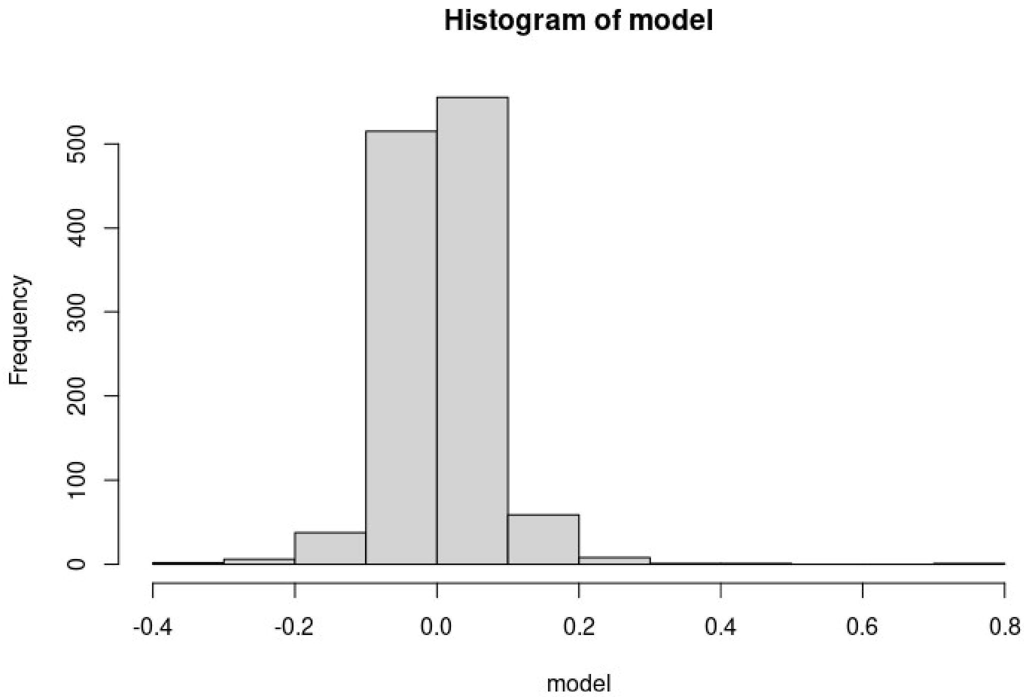

Figure 8.14: A histogram of United Health Group Inc's weekly returns since 2000 to this day. Author's image 2022.

United Health Group Inc. (UNH)				
Durbin -Watson test: H_0 hypothesis accepted, $\alpha < p$	α - value 0.05	p - value 1	DW 2.242	The testee is not auto-correlated.
Shapiro-Wilkinson normality test: H_0 hypothesis rejected, $\alpha > p$ weekly	α - value 0.05	p - value 2.2e-16	W 0.89448	The testee is not normally distributed.
Bartlett test of homogeneity of variances: H_0 hypothesis rejected, $\alpha > p$ monthly	α - value 0.05	p - value 2.2e-16	Bartlett's K-squared = 348.18, df = 1	The testee has equal variances.

Table 22: Test results for the returns of the UNH share. Author's table 2022.

United Health Group Inc. (UNH)				
H_0 hypothesis rejected, α > p daily	α - value 0.05	p - value 0.04735	W 16020192	The testee is not lucky.
H_0 hypothesis rejected, α > p weekly	α - value 0.05	p - value 0.01758	W 663698	The testee is not lucky.
H_0 hypothesis rejected, α > p monthly	α - value 0.05	p - value 0.001071	W 30995	The testee is not lucky.
H_0 hypothesis rejected, α > p annual	α - value 0.05	p - value 0.01243	W 135	The testee is not lucky.

Table 23: The Mann - Whitney U - test results for the UNH shares incomes contra S&P 500 returns. Author's table 2022.

9 Conclusions – there is random walk at Wall Street?

The H_0 hypothesis is, that there is no difference between the distributions of the S&P 500 stock index returns and the studied returns of the stock X. This means that returns are too similar to be due to skill. Moreover, the hypothesis H_1 is that there is a significant difference between the underlying S&P 500 stock index returns and the studied returns of the stock X. This means that stock X has then so good returns that it can not be due to luck only.

Based on the over hundred of thousands measurements of the study, there are signs of the random walk on Wall Street. At the 95% confidence interval we can say, that only couple of stocks has such a returns, that it can not be explained with luck only. The meaning of luck seems to be there even when big money is invested or when a highly successful company is run by competent personnel. This emphasis my earlier intuition, that investment companies are more or less like casinos.

In this group there are only two obvious winners, which are not accidents, but signs that something exceptionally skillful is leading these companies. The H_0 hypothesis is in these two cases rejected. The two winning companies are **Apple, which was lead by Mr. Steve Jobs and United Health Group Inc. The Mann Whitney U – test clearly states in all samples, that there must be something extraordinary in these two companies. The world's richest investor, Mr. Warren Buffet seems to be only lucky. Another disappointment was Mr. Bill Gates with his Microsoft company.**

I am mainly concentrating on **exceptional winners** beyond the mediocre. The Mann – Whitney U – test is very merciless against all selected candidates and maybe a 90 percent confidential interval could have been a bit better limitation for the alpha number. However, I wanted to be very selective when I am dealing with the best of the best. With 90% confidence interval also NVIDIA must be added to the group of the winners.

If the underlying distributions are similar, the random walk hypothesis could well explain the phenomenon. However, now in 8 cases, the underlying distributions are almost similar resulting in random walk on Wall Street. Once there is evidence of random walk, what causes the exceptional success at Wall Street is difficult to explain with anything other than unusual skill, "business hunch", and hard work. This was all related to Steve Jobs in the case of Apple Inc. and also in the case of United Health Group Inc. With 90% confidence interval the NVIDIA must also be included to the skillful club.

All this testing aims to find at least some trace of the random walk on Wall Street among the Standard & Poor's 500 indexes. During the whole testing, there are some signs of random walk found. There was no autocorrelation at all in any sample and at annual level the distribution of the S&P 500 was normally distributed. Based on this study, the best investors and investing companies are like big casino players. These companies are not doing their work well above random walk assumptions. Who really knows what will happen on the planet Earth for the following 10 years before hand? I think no one, but somebody surely may be able to guess correctly among 8 billion people like lottery winners are able to do once in a life time.

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