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Research Article

An Extended Interval Type-2 Fuzzy VIKOR Technique with Equitable Linguistic Scales and Z-Numbers for Solving Water Security Problems in Malaysia

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Interval Type-2 Fuzzy VIseKriterijumska Optimizacija I Kompromisno Resenje (IT2FVIKOR) technique is one of the techniques of Interval Type-2 Fuzzy Multi-Criteria Decision Making (IT2FMCDM), which was developed to solve problems involving conflicting and multiple objectives. Most of the IT2FVIKOR methods are created from linguistic variables based on Interval Type-2 Fuzzy Set (IT2FS) and its generalization, such as Interval Type-2 Fuzzy Numbers (IT2FNs). Recent literature suggests that equitable linguistic scales can offer a better alternative, particularly when IT2FSs have some limitations in handling uncertainty and imbalance. This paper proposes the extended IT2FVIKOR with an equitable linguistic scale and Z-Numbers, where its linguistic scale introduces an equitable balance of positive and negative scales added to the restriction and reliability approach. Different from the typical IT2FVIKOR, which directly utilizes IT2FNs with a positive membership, the proposed method introduces positive and negative membership where each side considers a restriction and reliability approach. Besides, this paper also offers objective weights using fuzzy entropy-based IT2FS to calculate the weights of the extended IT2FVIKOR. The obtained solutions would help decision makers (DMs) identify the best solution to enhance water security projects in terms of finding the best strategies for water supply security in Malaysia.

1. Introduction

Fuzzy Multi-Criteria Decision Making (FMCDM) is an extension of Multi-Criteria Decision Making (MCDM), where it is the study of evaluating typically multiple conflicting criteria with the added of uncertainty issues using the concept of Type-1 Fuzzy Sets (T1FSs). In this approach, the decision makers (DMs) should provide subjective and objective measurements in order to verify the performance of each alternative based on the specific criteria for each problem. Various research works have discussed FMCDM such as (1) Fuzzy Analytic Hierarchy Process (FAHP) [1], in which a pair-wise comparison was applied to estimate the relative magnitudes of factors based on DMs preferences; (2) Fuzzy Techniques for Order Preference by Similarity to Ideal Solution (FTOPSIS) [2] used DM decision information about weights and values of attributes to identify the utmost required alternatives from an array of n feasible alternatives;

(3) a structured communication technique using fuzzy Delphi [3] was originally developed as an interactive, systematic forecasting method which relies on DMs; (4) Fuzzy ELimination and Choice Expressing REality (FELECTRE) [4] was introduced in order to overcome uncertainty in linguistic judgment, the assertion was developed that a concordance of attributes was present in favour of the proclamation that an alternative was just as good as another, and that strong discordance was detected amongst the score values, thereby refuting the previous statement that did not exist; (5) Fuzzy Decision-Making Trial and Evaluation Laboratory (FDEMATEL) [5] was developed to visualize and construct interrelations between criteria and subcriteria. Fuzzy VIseKriterijumska Optimizacija I Kompromisno Resenje (FVIKOR), which is the method chosen in this study, is developed based on the initial (given) weights, then determines a compromise ranking list for decision problems with conflicting criteria, and next determines a compromise solution with such a ranking list [6]. FVIKOR seems to have an ability to solve uncertainty issues due to its ability to combine the solution into a maximum group utility and a minimum individual regret of the opponent. Besides, FVIKOR would discover the best solution and a compromise solution, by prioritizing important factors [7]. However, FVIKOR still used T1FS. Interval Type-2 Fuzzy Sets (IT2FSs) [8, 9] were developed to overcome the lack of defining the level of uncertainty in T1FS.

Interval Type-2 Fuzzy VIKOR (IT2FVIKOR) was first created by Qin et al. [10]. IT2FVIKOR seeks to discover a compromise solution that can result in an agreement of mutual concessions, which is the nearest to the ideal solution. A further benefit of this approach is that it incorporates the DM's behaviour preference into the MCDM, which will allow for more realistic results that reflect both their preference and actual needs. Moreover, the IT2FVIKOR method requires the parameter m, which in this context can be observed as a quantity of the DM's behaviour preference, so that the DM can adjust m to achieve a compromise with respect to his/her preferences. Due to many benefits from IT2FVIKOR, many studies have discussed this method with different extended methods and in various applications. Gul et al. [11] enhanced the Fine–Kinney occupational risk evaluation method with IT2FVIKOR. Then, they applied this technique to solve occupational health and safety risk evaluation. Qin and Liu [12] developed IT2FVIKOR based on the prospect theory. Then, they applied their proposed method to solve the high-tech investment evaluation. Wu et al. [13] proposed IT2FVIKOR with the extended Fuzzy Best-Worst Method (FBWM) to apply it in the green supply selection. Next, Liu et al. [14] combined Interval Type-2 Fuzzy Analytical Network Process (IT2FANP) with IT2F-VIKOR to figure out the supplier selection issue in Sustainable Supplier Chain Management (SSCM). Soner et al. [15] integrated IT2FAHP with IT2FVIKOR to solve MCDM problems in the maritime transportation industry.

However, up to now, the preference scale in IT2FVIKOR has received little attention. The previous ITFVIKOR only used general preference scales generated from Interval Type-2 Fuzzy Number (IT2FN) linguistic scales. Existing IT2FVIKOR methods do not consider the equitable linguistic scale of DMs' behaviour preferences, thus exhibiting a common shortcoming [16]. Besides, they also lack in considering the reliability of preference information [16, 17]. A DM's behaviour portrays a major role in the final decision result in many real-life decision conditions. The equitable linguistic scale is useful to cope with the subjective judgments of DMs, where both the lowest and highest scale members are equally strong, where it proposes an equitable positive and negative scale [16, 18]. Inspired by the theory of the equilibrium concept (i.e., the balance between two sides of a matter), an equitable linguistic scale is proposed where positive and negative scales share equilibrium. The low and high scores of this linguistic variable are equally strong in relation to subjective judgments from the DMs. Furthermore, all correctly classified examples are represented by positive data, while all negative examples are represented by negative data. In this study, negative data are not indicative of faulty or corrupt data. There is a hypothesis that makes the negative data well separated [19]. The positive and negative are relative, and it is equitable.

Moreover, this study also considers restriction and reliability. The concept of restriction and reliability comes from the idea of Z-Numbers proposed by Zadeh [17]. Through Z-Numbers, DMs are able to incorporate their reasonable evaluations into the language used to represent answers. It aids in the resolution of some issues and would undoubtedly make for intriguing research on information capture and interpretation, as a guide for the expertise's interest [20]. Additionally, because Z-Numbers take reliability and constraint into account, they provide us more latitude to depict the fuzziness and ambiguity of real-world circumstances [20]. We therefore construct an expanded IT2FVIKOR technique based on the equitable linguistic scale and the Z-Numbers to handle MCDM within IT2FSs, prompted by the idea of the IT2FVIKOR method, equitable linguistic scale, and the Z-Numbers. Apart from the linguistic scale, this study also assigns entropy weight to obtain the weight of the decision matrix. Besides, we employed the idea of objective weight to establish weight for criteria and alternatives in decision-making situations. Because of this, our suggested method can offer substantial objective weights to confirm that the evaluation outcome is not impacted by the interdependence of criteria and inconsistent subjective weights [21]. Additionally, it can confirm objectivity while avoiding subjectivity due to the DM's personal bias. Therefore, using an entropy weight for our extended IT2FVIKOR is more adaptable and effective since it accepts superior flexibility in the presentation of uncertainties.

The enhanced IT2FVIKOR could be used in a variety of real-world FMCDM applications. Using a practical application to find the best ways to improve the security of Malaysia's water supply, the methods and viability of the suggestions are demonstrated. Six alternatives with five criteria are constructed to evaluate five different backgrounds of DMs. To check its efficiency of this proposed method with a real application, sensitivity analysis is applied in this study. Sensitivity analysis is a good platform for the extended IT2FVIKOR method to check how sensitive this method is towards the different weights' value. The remainder of this paper is structured as follows. Section 2 describes the related work of the study. Section 3 discusses the theoretical background of this study. Section 4 elaborates the proposed IT2FVIKOR including the construction of a new linguistic scale and weighting process. Section 5 applies the real application of water security towards the proposed method. Section 6 makes some analysis and comparison between the proposed method with the previous method. Lastly, the conclusion is drawn in Section 7.

2. Related Work

Most of the FMCDM methods can be separated into four main phases which are rating phase, weighting phase [22], aggregating phase [23], and ranking phase [22]. The rating phase focuses on constructing a decision matrix, where the preference scale with Fuzzy Numbers (FNs) is used to measure rating alternatives towards attributes. The weighting phase focuses on constructing the weighting matrix. FNs are used for rating the weight of each alternative. The weight can be used as a parameter for ranking the alternatives to the problem and lead the experts in making choices. In the aggregating phase, the ranking values are calculated to achieve crisp numbers. The ranking phase calculates the distance between each alternative to find the best relative degree. Lastly, it sorts the values for all alternatives.

Due to these different phases, numerous publications have emerged in FMCDM to discuss each of these phases. For example, for the rating phase, Keshavarz-Ghorabaee et al. [24] proposed a new technique of Evaluation by an Area-based Method of Ranking Interval Type-2 Fuzzy Sets (EAMRIT-2F) for ranking IT2FSs based on the area under the upper and lower memberships. Then, they applied EAMRIT-2F to select the facility location. Results of the EAMRIT-2F showed consistency and were comparable with five other methods. Chen et al. [25] encoded Proportional Hesitant Fuzzy Linguistic Term Sets (PHFLTSs) based on the idea of a Proportional Interval Type-2 Hesitant Fuzzy Set (PIT2 HFS). Based on Archimedean t-norms and s-norms, fundamental operations fulfilling the closure property were identified for PIT2 HFSs. By using a second-generation fuzzy logic technique based on IT2FSs to convey linguistic phrases in numbers, a process known as computing with words, Hong et al. [26] concentrated on transforming linguistic graded qualitative risk matrices. This converting linguistic was later applied to support risk management decision making. Liu et al. [27] extended Hesitant Fuzzy Linguistic Term Sets (EHFLTSs) to overcome the inadequacy of Multi-Criteria Group Decision Making (MCGDM) to decrease information loss in computing with words. Their proposed method was applied in a cross-border e-commerce selection situation. Sajjad et al. [28] proposed a formula for the correlation coefficient based on intuitionistic 2-tuple fuzzy linguistic (I2TFL) using the best-worst method (BWM). Then, illustrated IT2L BWM using two numerical examples. Then, they illustrated it using two numerical examples.

Later, for the weighting phase, Keshavarz-Ghorabaee et al. [29] suggested an expanded Stepwise Weight Assessment Ratio Analysis (SWARA) with symmetric IT2FSs to estimate the weights of criteria based on the views of a group of DMs. The value of rational capital measurements and components in a corporation was then assessed using the suggested method. Their findings demonstrated how effective the suggested method was at capturing the information ambiguity and defining the subjective weights of criteria. Based on novel entropy and evidential reasoning, Yuan and Luo [30] proposed a novel intuitionistic fuzzy entropy (IFE) method. The proposed method should next be put to the test using actual cases in Beijing to show its superiority and efficacy. By utilizing a newly developed divergence-based cross entropy measure of Atanassov's intuitionistic fuzzy sets, Song et al. [31] introduced uncertainty metrics (AIFSs). To identify attribute weights in Multi-Attribute Group Decision-Making (MAGDM) issues, they then used the cross entropy and uncertainty measurements into an optimal model. Two MCDM techniques were proposed by Keshavarz-Ghorabaee et al. [32]: Simultaneous Evaluation of Criteria and Alternatives (SECA)-based IT2FSs and Weighted Aggregated Sum Product Assessment (WASPAS). They then employed the suggested method to assess sustainable manufacturing strategies, and the results revealed that the devised model's effectiveness was based on "Eco-efficiency." The removal effects of criteria-based intuitionistic fuzzy method (Intuitionistic Fuzzy MEREC) and ranking sum (RS) were studied by Hezam et al. [33] to evaluate the objective and subjective weighting values of numerous parameters for alternative fuel cars (AFV). Their findings indicated that societal benefits, fueling/charging infrastructure, and financial incentives, in that order, are the most important factors for AFV appraisal. For the best of the FCH supplier, Alipour et al. [34] combined a strategy based on entropy, Stepwise Weight Assessment Ratio Analysis (SWARA), and Complex Proportional Assessment (COPRAS) methodologies in a Pythagorean fuzzy environment. This includes integrating the subjective weights from the SWARA method with the objective weights established by the entropy-based technique to create criteria weights.

For the aggregating phase, Keshavarz-Ghorabaee et al. [35] recommended EDAS (Evaluation based on Distance from Average Solution) and IT2FSs for assessing providers with regard to environmental criteria. They then used their integrated model to evaluate suppliers and allocate orders while considering economic and environmental factors. Their findings demonstrated how effective and practical their suggested model was for solving real-world issues. To define the weights of criterion, Tian et al. [36] suggested a mathematical programming model based on the Shapley fuzzy measure. The weighted picture fuzzy power Choquet ordered geometric (WPFPSCOG) operator was then used to aggregate the evaluation value of each alternative. To determine the best breed of horsegram, Janani et al. [37] developed aggregation operators called the complex Pythagorean fuzzy Einstein ordered weighted arithmetic aggregating operator (CPFEOWA), complex Pythagorean fuzzy Einstein weighted arithmetic aggregating operator (CPFEWA), complex Pythagorean fuzzy Einstein weighted ordered geometric aggregating operator (CPFEOWG), and complex Pythagorean fuzzy Einstein weighted geometric aggregating operator (CPFEWG) to select the best breed of horsegram. Jia and Wang [38] enhanced the Choquet integral-based intuitionistic fuzzy arithmetic aggregation (CIIFAA) operator used in MCDM approaches to pick the best option(s) in an intuitionistic fuzzy environment. Taking into account both the lowest distance from the positive ideal point and the greatest distance from the negative ideal point, Shang et al. [39] incorporated fuzzy Shannon entropy, fuzzy MULTIMOORA, and BWM in 2022. Their suggested method's viability and efficacy were confirmed in an descriptive application at Company L, a renowned Chinese forklift truck manufacturer on a global scale.

Finally, to achieve more convincing and useful weights for the criteria, Keshavarz-Ghorabaee et al. [35] combined the subjective weights conveyed by DMs with the objective weights computed using the deviation-based method for the ranking phase. This new method used IT2FSs based on Fuzzy Ranking and Aggregated Weights (AFRAW). Additionally, their suggested solution made use of the aggregated weights. They then showed how their suggested strategy might evaluate vendors in a supply chain. The interval-valued intuitionistic hesitant fuzzy entropy and interval-valued intuitionistic hesitant fuzzy VIKOR methods for ranking the alternatives and the importance of the criteria, respectively, were proposed by Narayanamoorthy et al. [40]. Their suggested approach was then used to choose industrial robots. In order to rank the eight Supply Chain Analytics (SCA) tool alternatives from various companies, Büyüközkan and Güler [41] proposed Hesitant Fuzzy Linguistic Multi-Objective Optimization by Ratio Analysis, where the Full Multiplicative (HFL MULTIMOORA) method is combined with the fuzzy envelope technique. They then used a case study of a logistics company to apply their suggested methodology. Fuzzy-Combined Compromise Solution (Fuzzy-CoCoSo), developed by Alao et al. [42], is a method for choosing the best prime movers (PMs) for situations involving combined heat and power (CHP). Their findings showed that the greatest and least important factors for selecting PMs were, respectively, the main financing cost in the economic group and the social imprint. Our suggested approach is mostly concerned with the rating phase and weighting phase.

3. Theoretical Background

This section briefly overviews of the main concepts and basic relations on Type-2 Fuzzy Sets (T2FSs), Interval Type-2 Fuzzy Sets (IT2FSs), Arithmetic operations between trapezoidal IT2FSs, Z-Numbers, and entropy weight. These definitions can be used in subsequent sections.

3.1. Type-2 Fuzzy Sets (T2FSs)

Definition 1 (see [43]). Let X be the universe of discourse, and a type-2 set (T2FS) $\widetilde{B}$ can be illustrated as $B=\{((y,\ d),\ \mu_{\widetilde{B}}(y,\ d))|\forall x\in X,\ \ \forall d\in K_y\subseteq[0,\ 1]\}$, where y is the primary variable, d is the secondary variable, and $K_y\ \in\ [0,\ 1]$ is the primary membership function of y.

The T2FS can be equivalently rewritten as $\widetilde{B}=\int_{y\in X}\int u\in K_y\mu_{\widetilde{B}}(y,d)/y,\mathrm{d}=\int_{x\in X}(\int d\in K_y u_{\widetilde{B}}(y,d)/\mathrm{d}y)$, where $\int d\in K_y u_{\widetilde{B}}(y,d)/d$ is the second membership function at y, and the sign denotes the traversal of all y and d.

Definition 2 (see [44]). Consider $\tilde{\widetilde{B}}$ as a T2FS in the reference set Y corresponding to the type-2 membership function $\mu_{\tilde{\widetilde{B}}}$. $\tilde{\widetilde{B}}$ could be called as an IT2FS, if all $\mu_{\tilde{\widetilde{B}}}(Y,d)=1$. As a special case of a T2FS, the IT2FS $\tilde{\widetilde{B}}$ can be written as follows:

$$\tilde{\widetilde{B}}=\iint\frac{1}{(Y,D)}=\frac{\int\left[\int(1/D)\right]}{y},\tag{1}$$

where y is the primary variable and K_y is the primary membership of y specified as an interval in [0, 1]. Additionally, the secondary membership function (MF) at y is represented by $\int 1/D$ and the secondary variable is indicated by d.

3.2. Interval Type-2 Fuzzy Sets (IT2FSs).

This section briefly analyzes some descriptions of T2FSs and IT2FSs from Mendel et al. [44].

Definition 3 (see [44]). A T2FS $\tilde{\widetilde{B}}$ in the universe of discourse Y can be presented by a type-2 membership function $\mu_{\tilde{\widetilde{B}}}$, displayed as follows:

$$\tilde{\widetilde{B}}=\left\{\left((y,d),\mu_{\tilde{\widetilde{B}}}(y,d)\right)|\forall y\in Y,\quad \forall d\in K_y\subseteq[0,1],\ 0\le\mu_{\tilde{\widetilde{B}}}(y,d)\le1\right\},\tag{2}$$

where J_x indicates an interval in $[0,1]$. Moreover, the T2FS $\widetilde{A}$ also can be characterized as follows:

$$\tilde{\widetilde{B}}=\int_{y\in Y}\int_{\mu\in J_y}\frac{\mu_{\tilde{\widetilde{B}}}(y,d)}{(y,d)},\tag{3}$$

where $K_y\subseteq[0,1]$ and $\iint$ indicates the union over all admissible y and d.

Definition 4 (see [44]). Let $\tilde{\widetilde{B}}$ be a T2FS in the universe of discourse Y characterized by the type-2 membership function $\mu_{\tilde{\widetilde{B}}}$. If all $\mu_{\tilde{\widetilde{B}}}=1$, then B is called an IT2FS. An IT2FS $\tilde{\widetilde{B}}$ can be regarded as a special case of a T2FS, represented as follows:

$$\tilde{\widetilde{B}}=\int_{y\in Y}\int_{\mu\in J_y}\frac{1}{(y,d)},\tag{4}$$

where $J_y \subseteq [0,1]$.

3.3. Arithmetic Operations between Trapezoidal IT2FSs. The evaluations of arithmetic operations between trapezoidal IT2FS are defined in Lee and Chen [45].

Figure 1 shows a trapezoidal IT2FS $\tilde{\tilde{B}}_i = (\tilde{B}_i^U, \tilde{B}_i^L) = ((b_{i1}^U, b_{i2}^U, b_{i3}^U, b_{i4}^U; G_1(\tilde{B}_i^U), G_2(\tilde{B}_i^U)), (b_{i1}^L, b_{i2}^L, b_{i3}^L, b_{i4}^L; G_1(\tilde{B}_i^L), G_2(\tilde{B}_i^L)))$ [45], where $\tilde{B}_i^U$ and $\tilde{B}_i^L$ are T1FSs, $b_{i1}^U, b_{i2}^U, b_{i3}^U, b_{i4}^U, b_{i1}^L, b_{i2}^L, b_{i3}^L$, and b_{i4}^L are the reference points of the T1FS $\tilde{\tilde{B}}_I$, $G_k(\tilde{B}_i^U)$ indicates the membership value of the element $b_{i(k+1)}^U$ in the upper trapezoidal membership function $\tilde{B}_i^U$, $1 \le k \le 2, G_k(\tilde{B}_i^L)$ indicates the membership value of the element $b_{i(k+1)}^L$ in the lower trapezoidal membership function $\tilde{B}_i^L$, $1 \le k \le 2, G_1(\tilde{B}_i^U) \in [0,1], G_2(\tilde{B}_i^U) \in [0,1], G_1(\tilde{B}_i^L) \in [0,1], G_2(B_i^L) \in [0,1]$, and $1 \le i \le n$.

Definition 5 (see [45]). The addition operation between the trapezoidal IT2FSs $\tilde{\tilde{B}}_1 = (\tilde{B}_1^U, \tilde{B}_1^L) = ((b_{11}^U, b_{12}^U, b_{13}^U, b_{14}^U; G_1(\tilde{B}_1^U), G_2(\tilde{B}_1^U)), (b_{11}^L, b_{12}^L, b_{13}^L, b_{14}^L; G_1(\tilde{B}_1^L), G_2(\tilde{B}_1^L)))$ and $\tilde{\tilde{B}}_2 = (\tilde{B}_2^U, \tilde{B}_2^L) = ((b_{21}^U, b_{22}^U, b_{23}^U, b_{24}^U; G_1(\tilde{B}_2^U), G_2(\tilde{B}_2^U)), (b_{21}^L, b_{22}^L, b_{23}^L, a_{24}^L; G_1(\tilde{B}_2^L), G_2(\tilde{B}_2^L)))$ is defined as follows:

$$\tilde{\tilde{B}}_1 \oplus \tilde{\tilde{B}}_2 = \left(\tilde{B}_1^U, \tilde{B}_1^L\right) \oplus \left(\tilde{B}_2^U, \tilde{B}_2^L\right) \begin{pmatrix} \left(b_{11}^U + b_{21}^U, b_{12}^U + b_{22}^U, b_{13}^U + b_{23}^U, b_{14}^U + b_{24}^U; \min\left(G_1\left(\tilde{B}_1^U\right), G_1\left(\tilde{B}_2^U\right)\right), \min\left(G_2\left(\tilde{B}_1^U\right), G_2\left(\tilde{B}_2^U\right)\right)\right), \\ \left(b_{11}^L + b_{21}^L, b_{12}^L + b_{22}^L, b_{13}^L + b_{23}^L, b_{14}^L + b_{24}^L; \min\left(G_1\left(\tilde{B}_1^L\right), G_1\left(\tilde{B}_2^L\right)\right), \min\left(G_2\left(\tilde{B}_1^L\right), G_2\left(\tilde{B}_2^L\right)\right)\right) \end{pmatrix}. \tag{5}$$

Definition 6 (see [45]). The arithmetic operation between the trapezoidal IT2FS $\tilde{\tilde{B}}_1 = (\tilde{B}_1^U, \tilde{B}_1^L) = ((b_{11}^U, b_{12}^U, b_{13}^U, b_{14}^U; G_1(\tilde{B}_1^U), G_2(\tilde{B}_1^U)), b_{11}^L, b_{12}^L, b_{13}^L, b_{14}^L; G_1(\tilde{B}_1^L), G_2(\tilde{B}_1^L))$ and the crisp value t is specified as follows:

$$\begin{aligned} t\tilde{\tilde{B}}_1 &= \begin{pmatrix} \left(\frac{1}{t} \times b_{11}^U, \frac{1}{t} \times b_{12}^U, \frac{1}{t} \times b_{13}^U, \frac{1}{t} \times b_{14}^U; G_1\left(\tilde{B}_1^U\right), G_2\left(\tilde{B}_1^U\right)\right), \\ \left(\frac{1}{t} \times b_{11}^L, \frac{1}{t} \times b_{12}^L, \frac{1}{t} \times b_{13}^L, \frac{1}{t} \times b_{14}^L; G_1\left(\tilde{B}_1^L\right), G_2\left(\tilde{B}_1^L\right)\right) \end{pmatrix}, \\ \frac{\tilde{\tilde{B}}_1}{t} &= \begin{pmatrix} \left(\frac{1}{t} \times b_{11}^U, \frac{1}{t} \times b_{12}^U, \frac{1}{t} \times b_{13}^U, \frac{1}{t} \times b_{14}^U; G_1\left(\tilde{B}_1^U\right), G_2\left(\tilde{B}_1^U\right)\right), \\ \left(\frac{1}{t} \times b_{11}^L, \frac{1}{t} \times b_{12}^L, \frac{1}{t} \times b_{13}^L, \frac{1}{t} \times b_{14}^L; G_1\left(\tilde{B}_1^L\right), G_2\left(\tilde{B}_1^L\right)\right) \end{pmatrix}, \end{aligned} \tag{6}$$

where $k > 0$.

3.4. Z-Numbers

Definition 7 (see [17]). It is sufficient to formalise information from the real-world using Z-Numbers, which should be approximately weighed in terms of reliability. The major problem is that the accuracy of the information is not properly considered. A novel idea called Z-Numbers has been developed by Zadeh to better capture the ambiguity. Z-Numbers demand reliability and moderation. Compared to the traditional fuzzy number, Z-Numbers are better able to describe the actual information that humans have [46]. One of the Z-Numbers' primary objectives is to generate fuzzily confident numbers so as to know the true information. The knowledge of humans can be better represented by the Z-Numbers [47].

A Z-Number $Z = (\tilde{A}, \tilde{R})$ is generated from fuzzy number in an ordered pair. The fuzzy restriction is a real-valued uncertain variable of x and signified as $\tilde{A}$. The fuzzy reliability is a measure of the second component of $\tilde{R}$. $\tilde{R}$ signifies the idea of certainty or other related concepts, like certainty, confidence, true intensity, probability, or measure of capability to the first component [48].

3.5. Entropy Weight

Definition 8 (see [49]). The term "entropy weight" refers to a parameter that quantifies how closely various options are related to one another. On the other hand, a system with low information entropy is one that is well ordered. In information theory, the entropy value can be determined as the equation below:

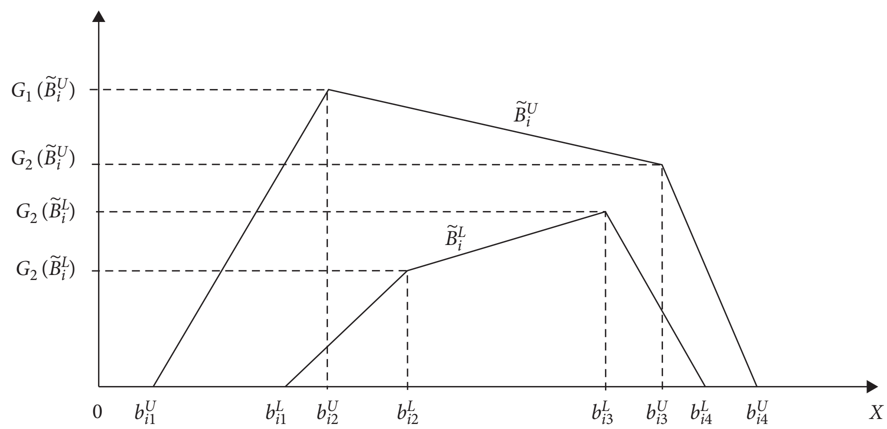

Figure 1: The upper trapezoidal membership function $\tilde{B}_i^U$ and the lower trapezoidal membership function $\tilde{B}_i^L$ of the T1FS $\tilde{\tilde{B}}_i$.

$$H(p_1, p_2, \ldots, p_n) = -\sum_{j=1}^{n} p_j \ln p_j, \quad (7)$$

where H is the level of entropy and p_j is the probability of event occurrence.

Definition 9 (see [50]). In their paper, they suggested entropy as a ratio of distances between (L, L_{near}) and (L, L_{far}). The statement is provided as follows:

$$E_{\text{SK}}(L) = \frac{(L, L_{\text{near}})}{(L, L_{\text{far}})}, \quad (8)$$

where (L, L_{near}) is the distance from L to the nearer point L_{near}, and (L, L_{far}) is the distance from L to the farther point Lfar. De Luca and Termini [51] have already suggested the axioms of entropy for FSs.

These preliminaries are being used in shaping a new rating of IT2E FVIKOR with Z-Numbers and weighting of the IT2FVIKOR.

4. The Extended IT2FVIKOR

To be completely operationalized, the enhanced IT2FVIKOR with equitable language scales and Z-Number requires a newly identified linguistic scale, weighting value, and rank values. Before proposing the new approach, this part is intentionally separated into three subdivisions to describe the predetermined rating phases: linguistic scale, weighting phase using weighting values, and ranking phase using rank values. The creation of linguistic scales using Z-Number-based IT2FS and equitable linguistic scales is discussed in the first subsection. The enhanced IT2FVIKOR technique, which incorporates rank and entropy weight weighting values, is presented in the subsection that follows.

4.1. Construction of a New Linguistic Scale. The term linguistic variable refers to a condition that has an inherent value inherent to the language phase and is not adequately described by a conventional quantitative expression due to the complexity of the situation [52]. It is a variable whose values are either natural or artificial language words or phrases rather than numbers [8]. In the IT2FS context, the earlier IT2FVIKOR-based IT2FMCDM was seen as a compromise option with maximum group utility and minimal regret of specialists with incommensurable and contradictory qualities [53]. Its linguistic scale disregards the idea of equilibrium, which considers both the positive and the negative aspects of a situation under the theory of equitable linguistics [16]. Besides, it also neglected the concept of restriction and reliability-based Z-Numbers [17].

As a result, the proposed language for the enlarged IT2FVIKOR takes into account both the good and negative aspects, as well as the implementation of the ideas of restriction and reliability. In this case, we have used seven scales of the new linguistic scales, where Very Poor is the lowest scale at the lowest negative scale and Very Good is the highest scale at the highest positive scales for restriction (Table 1 and Figure 2). Additionally, we used seven scales of new language scales for dependability, with Strongly Unlikely characterized as the lowest scale at the lowest negative scale and Strongly Likely defined as the highest scale at the highest positive scale (Table 2 and Figure 3).

For detailed explanation on these linguistic scales, let us take one example of the linguistic scale with restriction for Very Poor, Medium, and Very Good. Very Poor value can be stated as $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = ((-b_{11}^U, -b_{12}^U, -b_{13}^U, -b_{14}^U;\ G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), (-b_{11}^L, -b_{12}^L, -b_{13}^L, -b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)))$, where $\tilde{\tilde{B}}_i^U$ and $\tilde{\tilde{B}}_i^L$ are positive and negative equitable linguistic scales, $-b_{i1}^U, -b_{i2}^U, -b_{i3}^U, -b_{i4}^U, -b_{i1}^L, -b_{i2}^L, -b_{i3}^L$, and $-b_{i4}^L$ are the reference points of $\tilde{\tilde{B}}_i$, $G_k(\tilde{\tilde{B}}_i^U)$ indicates the membership value of the element $-b_{i(j+1)}^U$ in the upper trapezoidal membership function $\tilde{\tilde{B}}_i^U$, $1 \le k \le 2,$ $G_k(\tilde{\tilde{B}}_i^L)$ indicates the membership value of the element $-b_{i(k+1)}^L$ in the lower trapezoidal membership function $\tilde{\tilde{B}}_i^L, 1 \le k \le 2,, G_1(\tilde{\tilde{B}}_i^U) \in [0,1], G_2(\tilde{\tilde{B}}_i^U) \in [0,1], G_1(\tilde{\tilde{B}}_i^L) \in [0,1], G_2(\tilde{\tilde{A}}_i^L) \in [0,1]$, and $1 \le i \le n$.

Table 1: Linguistic scale of restriction for the extended IT2FVIKOR.

Linguistic terms	Linguistic scale
Very Poor (VP)	((−1.0, −0.9, −0.8, −0.7; 0.8, 0.8), (−1.0, −1.0, −0.8, −0.6; 1, 1))
Poor (P)	((−0.8, −0.7, −0.5, −0.4; 0.8, 0.8), (−0.9, −0.7, −0.5, −0.3; 1, 1))
Medium Poor (MP)	((−0.5, −0.4, −0.2, −0.1; 0.8, 0.8), (−0.6, −0.4, −0.2, 0; 1, 1))
Medium (M)	((−0.2, −0.1, 0.1, 0.2; 0.8, 0.8), (−0.3, −0.2, 0.2, 0.3; 1, 1))
Medium Good (MG)	((0.1, 0.2, 0.4, 0.5; 0.8, 0.8), (0, 0.2, 0.4, 0.6; 1, 1))
Good (G)	((0.4, 0.5, 0.7, 0.8; 0.8, 0.8), (0.3, 0.5, 0.7, 0.9; 1, 1))
Very Good (VG)	((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1))

Medium value can be stated as $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = ((-b_{11}^U, -b_{12}^U, b_{13}^U, b_{14}^U; G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), (-b_{11}^L, -b_{12}^L, b_{13}^L, b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)))$, where $\tilde{\tilde{B}}_i^U$ and $\tilde{\tilde{B}}_1^L$ are positive and negative equitable linguistic scales, $-b_{i1}^U$, $-b_{i2}^U$, b_{i3}^U, b_{i4}^U, $-b_{i1}^L$, $-b_{i2}^L$, b_{i3}^L, and b_{i4}^L are the reference points of $\tilde{\tilde{B}}_i, G_k(\tilde{\tilde{B}}_i^U)$ implies the membership value of the element $-b_{11}^U, -b_{12}^U, b_{13}^U, b_{14}^U$ in the upper trapezoidal membership function $\tilde{\tilde{B}}_i^U, 1 \le k \le 2, G_k(\tilde{\tilde{B}}_i^L)$ implies the membership value of the element $-b_{11}^L, -b_{12}^L, b_{13}^L, b_{14}^L$ in the lower trapezoidal membership function $\tilde{\tilde{B}}_i^L, 1 \le k \le 2,, G_1(\tilde{\tilde{B}}_i^U) \in [0,1], G_2(\tilde{\tilde{B}}_i^U) \in [0,1], G_1(\tilde{\tilde{B}}_i^L) \in [0,1], G_2(\tilde{\tilde{A}}_i^L) \in [0,1]$, and $1 \le i \le n$.

Lastly, Very Good value can be stated as $\tilde{\tilde{B}}_i = (\tilde{\tilde{B}}_i^U, \tilde{\tilde{B}}_i^L) = ((b_{i1}^U, b_{i2}^U, b_{i3}^U, b_{i4}^U; G_1(\tilde{\tilde{B}}_i^U), G_2(\tilde{\tilde{B}}_i^U)), (b_{i1}^L, b_{i2}^L, b_{i3}^L, b_{i4}^L; G_1(\tilde{\tilde{B}}_i^L), G_2(\tilde{\tilde{B}}_i^L)))$, where $\tilde{\tilde{B}}_i^U$ and $\tilde{\tilde{B}}_i^L$ are positive and negative equitable linguistic scales, $b_{i1}^U, b_{i2}^U, b_{i3}^U, b_{i4}^U, b_{i1}^L, b_{i2}^L, b_{i3}^L$, and b_{i4}^L are the reference points of $\tilde{\tilde{B}}_i, G_k(\tilde{\tilde{B}}_i^U)$ indicates the membership value of the element $b_{i(k+1)}^U$ in the upper trapezoidal membership function $\tilde{\tilde{B}}_i^U$, $1 \le k \le 2, G_k(\tilde{\tilde{B}}_i^L)$ indicates the membership value of the element $b_{i(k+1)}^L$ in the lower trapezoidal membership function $\tilde{\tilde{B}}_i^L, 1 \le k \le 2,, G_1(\tilde{\tilde{B}}_i^U) \in [0,1], G_2(\tilde{\tilde{B}}_i^U) \in [0,1], G_1(\tilde{\tilde{B}}_i^L) \in [0,1], G_2(\tilde{\tilde{A}}_i^L) \in [0,1]$, and $1 \le i \le n$.

Next, the arithmetic operations for different types of linguistic scales are described as follows.

The addition operation between the linguistic scales $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = ((-b_{11}^U, -b_{12}^U, -b_{13}^U, -b_{14}^U; G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), (-b_{11}^L, -b_{12}^L, -b_{13}^L, -b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)))$ and $\tilde{\tilde{B}}_2 = (\tilde{\tilde{B}}_2^U, \tilde{\tilde{B}}_2^L) = ((-b_{21}^U, -b_{22}^U, -b_{23}^U, -b_{24}^U; G_1(\tilde{\tilde{B}}_2^U), G_2(\tilde{\tilde{B}}_2^U)), (-b_{21}^L, -b_{22}^L, -b_{23}^L, -b_{24}^L; G_1(\tilde{\tilde{B}}_2^L), G_2(\tilde{\tilde{B}}_2^L)))$ is defined as follows:

$$\begin{aligned} \tilde{\tilde{B}}_1 \oplus \tilde{\tilde{B}}_2 &= \left(\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L\right) \oplus \left(\tilde{\tilde{B}}_2^U, \tilde{\tilde{B}}_2^L\right) \\ &= \left(\begin{array}{c} \left(\begin{array}{c} \left(-b_{11}^U\right) + \left(-b_{21}^U\right), \left(-b_{12}^U\right) + \left(-b_{22}^U\right), \left(-b_{13}^U\right) + \left(-b_{23}^U\right), \left(-b_{14}^U\right) + \left(-b_{24}^U\right); \\ \min\left(G_1\left(\tilde{\tilde{B}}_1^U\right), G_1\left(\tilde{\tilde{B}}_2^U\right)\right), \min\left(G_2\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_2^U\right)\right) \end{array} \right), \\ \left(\begin{array}{c} \left(-b_{11}^L\right) + \left(-b_{21}^L\right), \left(-b_{12}^L\right) + \left(-b_{22}^L\right), \left(-b_{13}^L\right) + \left(-b_{23}^L\right), \left(-b_{14}^L\right) + \left(-b_{24}^L\right); \\ \min\left(G_1\left(\tilde{\tilde{B}}_1^L\right), G_1\left(\tilde{\tilde{B}}_2^L\right)\right), \min\left(G_2\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_2^L\right)\right) \end{array} \right) \end{array} \right). \end{aligned} \tag{9}$$

The subtraction operation between the linguistic scales $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = ((b_{11}^U, b_{12}^U, b_{13}^U, b_{14}^U; G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), (b_{11}^L, b_{12}^L, b_{13}^L, b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)))$ and $\tilde{\tilde{B}}_2 = (\tilde{\tilde{B}}_2^U, \tilde{\tilde{B}}_2^L) = ((-b_{21}^U, -b_{22}^U, -b_{23}^U, -b_{24}^U; G_1(\tilde{\tilde{B}}_2^U), G_2(\tilde{\tilde{B}}_2^U)), (-b_{21}^L, -b_{22}^L, -b_{23}^L, -b_{24}^L; G_1(\tilde{\tilde{B}}_2^L), G_2(\tilde{\tilde{B}}_2^L)))$ is defined as follows:

$$
\begin{aligned}
\tilde{\tilde{B}}_1 \ominus \tilde{\tilde{B}}_2 &= \left(\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L\right) \ominus \left(\tilde{\tilde{B}}_2^U, \tilde{\tilde{B}}_2^L\right) \\
&= \begin{pmatrix} \begin{pmatrix} b_{11}^U - \left(-b_{21}^U\right), b_{12}^U - \left(-b_{22}^U\right), b_{13}^U - \left(-b_{23}^U\right), b_{14}^U - \left(-b_{24}^U\right); \\ \min\left(G_1\left(\tilde{\tilde{B}}_1^U\right), G_1\left(\tilde{\tilde{B}}_2^U\right)\right), \min\left(G_2\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_2^U\right)\right) \end{pmatrix} \\ \begin{pmatrix} b_{11}^L - \left(-b_{21}^L\right), b_{12}^L - \left(-b_{22}^L\right), b_{13}^L - \left(-b_{23}^L\right), b_{14}^L - \left(-b_{24}^L\right); \\ \min\left(G_1\left(\tilde{\tilde{B}}_1^L\right), G_1\left(\tilde{\tilde{B}}_2^L\right)\right), \min\left(G_2\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_2^L\right)\right) \end{pmatrix} \end{pmatrix}.
\end{aligned}
\tag{10}
$$

The multiplication operation between the linguistic scales $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = \begin{pmatrix} ((-b_{11}^U), (-b_{12}^U), b_{13}^U, b_{14}^U; G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), \\ ((-b_{11}^L), (-b_{12}^L), b_{13}^L, b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)) \end{pmatrix}$ and $\tilde{\tilde{B}}_2 = (\tilde{\tilde{B}}_2^U, \tilde{\tilde{B}}_2^L) = \begin{pmatrix} ((-b_{21}^U), (-b_{22}^U), b_{23}^U, b_{24}^U; G_1(\tilde{\tilde{B}}_2^U), G_2(\tilde{\tilde{B}}_2^U)), \\ ((-b_{21}^L), (-b_{22}^L), b_{23}^L, b_{24}^L; G_1(\tilde{\tilde{B}}_2^L), G_2(\tilde{\tilde{B}}_2^L)) \end{pmatrix}$ is defined as follows:

$$
\begin{aligned}
\tilde{\tilde{B}}_1 \otimes \tilde{\tilde{B}}_2 &= \left(\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L\right) \otimes \left(\tilde{\tilde{B}}_2^U, \tilde{\tilde{B}}_2^L\right) \\
&= \begin{pmatrix} \begin{pmatrix} \left(-b_{11}^U\right) \times \left(-b_{21}^U\right), \left(-b_{12}^U\right) \times \left(-b_{22}^U\right), b_{13}^U \times b_{23}^U, b_{14}^U \times b_{24}^U; \\ \min\left(G_1\left(\tilde{\tilde{B}}_1^U\right), G_1\left(\tilde{\tilde{B}}_2^U\right)\right), \min\left(G_2\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_2^U\right)\right) \end{pmatrix} \\ \begin{pmatrix} \left(-b_{11}^L\right) \times \left(-b_{21}^L\right), \left(-b_{12}^L\right) \times \left(-b_{22}^L\right), b_{13}^L \times b_{23}^L, b_{14}^L \times b_{24}^L; \\ \min\left(G_1\left(\tilde{\tilde{B}}_1^L\right), G_1\left(\tilde{\tilde{B}}_2^L\right)\right), \min\left(G_2\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_2^L\right)\right) \end{pmatrix} \end{pmatrix}.
\end{aligned}
\tag{11}
$$

The arithmetic operation between the linguistic scale $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = \begin{pmatrix} (-b_{11}^U, -b_{12}^U, -b_{13}^U, -b_{14}^U; G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), \\ (-b_{11}^L, -b_{12}^L, -b_{13}^L, -b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)) \end{pmatrix}$ and the crisp value t is described as follows:

$$
\begin{aligned}
t\tilde{\tilde{B}}_1 &= \begin{pmatrix} \left(t \times \left(-b_{11}^U\right), t \times \left(-b_{12}^U\right), t \times \left(-b_{13}^U\right), t \times \left(-b_{14}^U\right); G_1\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_1^U\right)\right) \\ \left(t \times \left(-b_{11}^L\right), t \times \left(-b_{12}^L\right), t \times \left(-b_{13}^L\right), t \times \left(-b_{14}^L\right); G_1\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_1^L\right)\right) \end{pmatrix}, \\
\frac{\tilde{\tilde{B}}_1}{t} &= \begin{pmatrix} \left(\frac{1}{t} \times \left(-b_{11}^U\right), \frac{1}{t} \times \left(-b_{12}^U\right), \frac{1}{t} \times \left(-b_{13}^U\right), \frac{1}{t} \times \left(-b_{14}^U\right); G_1\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_1^U\right)\right) \\ \left(\frac{1}{t} \times \left(-b_{11}^L\right), \frac{1}{t} \times \left(-b_{12}^L\right), \frac{1}{t} \times \left(-b_{13}^U\right), \frac{1}{t} \times \left(-b_{14}^U\right); G_1\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_1^L\right)\right) \end{pmatrix},
\end{aligned}
\tag{12}
$$

where $t > 0$.

The arithmetic operation between the linguistic scale $\tilde{\tilde{B}}_1 = (\tilde{\tilde{B}}_1^U, \tilde{\tilde{B}}_1^L) = \begin{pmatrix} ((-b_{11}^U), (-b_{12}^U), b_{13}^U, b_{14}^U; G_1(\tilde{\tilde{B}}_1^U), G_2(\tilde{\tilde{B}}_1^U)), \\ ((-b_{11}^L), (-b_{12}^L), b_{13}^L, b_{14}^L; G_1(\tilde{\tilde{B}}_1^L), G_2(\tilde{\tilde{B}}_1^L)) \end{pmatrix}$ and the crisp value t is described as follows:

$$t\tilde{\tilde{B}}_1 = \begin{pmatrix} \left(t \times \left(-b_{11}^U\right), t \times \left(-b_{12}^U\right), t \times b_{13}^U, t \times b_{14}^U; G_1\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_1^U\right)\right) \\ \left(t \times \left(-b_{11}^L\right), t \times \left(-b_{12}^L\right), t \times b_{13}^L, t \times b_{14}^L; G_1\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_1^L\right)\right) \end{pmatrix},$$
$$\frac{\tilde{\tilde{B}}_1}{t} = \begin{pmatrix} \left(\frac{1}{t} \times \left(-b_{11}^U\right), \frac{1}{t} \times \left(-b_{12}^U\right), \frac{1}{t} \times b_{13}^U, \frac{1}{t} \times b_{14}^U; G_1\left(\tilde{\tilde{B}}_1^U\right), G_2\left(\tilde{\tilde{B}}_1^U\right)\right) \\ \left(\frac{1}{t} \times \left(-b_{11}^L\right), \frac{1}{t} \times \left(-b_{12}^L\right), \frac{1}{t} \times b_{13}^L, \frac{1}{t} \times b_{14}^L; G_1\left(\tilde{\tilde{B}}_1^L\right), G_2\left(\tilde{\tilde{B}}_1^L\right)\right) \end{pmatrix}, \tag{13}$$

where $t > 0$.

These linguistic scales are considered an analogous concept of the extended IT2FVIKOR with equitable linguistic and Z-Number methods. The enhanced IT2FVIKOR method's weighting phase, which employs fuzzy entropy-based IT2FS to capture the linguistic scale, is described in the following section.

4.2. The Extended IT2FVIKOR Procedures. A method for resolving the IT2FMCDM problem using the extended IT2FVIKOR is proposed based on the preceding theoretical study. The IT2FVIKOR with equitable linguistic scale and Z-Number approach is designed to attain a higher level of rational, systematic decision making through which one may identify the ideal answer and a workable compromise that takes into account both sides of the scale. The extended IT2FVIKOR procedure is explained step by step as follows.

Suppose that there is a finite set X of alternatives, where $X = \{x_1, x_2, \ldots, x_n\}$, and suppose that there is a finite set of F attributes, where $F = \{f_1, f_2, \ldots, f_m\}$. Suppose that there are k DMs $DM_1, DM_2, \ldots,$ and DM_k. The proposed method is now displayed as follows:

Step 1. Construct a hierarchical diagram of IT2FMCDM problem.

Structure the decision matrix Y_p of the pth DM and create the average decision matrix $\overline{Y}$, respectively, presented as follows:

$$\begin{aligned} Y_p &= \left[\tilde{\tilde{f}}_{ij}^{p}\right]_{m \times n} \\ &= x_1, x_2 \cdots x_n \\ &\begin{array}{c} f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \begin{bmatrix} \left(\left(f_{11}^{\approx p-} f_{11}^{\approx p+}\right)\left(Z_{11}^{\approx p-}; Z_{11}^{\approx p+}\right)\right) & \left(\left(f_{12}^{\approx p-} f_{12}^{\approx p+}\right)\left(Z_{12}^{\approx p-}; Z_{12}^{\approx p+}\right)\right) & \cdots & \left(\left(f_{1n}^{\approx p-} f_{1n}^{\approx p+}\right)\left(Z_{1n}^{\approx p-}; Z_{1n}^{\approx p+}\right)\right) \\ \left(\left(f_{21}^{\approx p-} f_{21}^{\approx p+}\right); \left(Z_{21}^{\approx p-}; Z_{21}^{\approx p+}\right)\right) & \left(\left(f_{22}^{\approx p-} f_{22}^{\approx p+}\right)\left(Z_{22}^{\approx p-}; Z_{22}^{\approx p+}\right)\right) & \cdots & \left(\left(f_{2n}^{\approx p-} f_{2n}^{\approx p+}\right)\left(Z_{2n}^{\approx p-}; Z_{2n}^{\approx p+}\right)\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\left(f_{m1}^{\approx p-} f_{m1}^{\approx p+}\right); \left(Z_{m1}^{\approx p-}; Z_{m1}^{\approx p+}\right)\right) & \left(\left(f_{m2}^{\approx p-} f_{m2}^{\approx p+}\right)\left(Z_{m2}^{\approx p-}; Z_{m2}^{\approx p+}\right)\right) & \cdots & \left(\left(f_{mn}^{\approx p-} f_{mn}^{\approx p+}\right)\left(Z_{mn}^{\approx p-}; Z_{mn}^{\approx p+}\right)\right) \end{bmatrix}, \\ \overline{Y} &= \left[\tilde{\tilde{f}}_{ij}\right]_{m \times n}, \end{aligned} \tag{14}$$

where $f_{ij} = (\tilde{\tilde{f}}^{1}_{ij} \oplus \tilde{\tilde{f}}^{2}_{ij} \oplus \cdots \oplus \tilde{\tilde{f}}^{k}_{ij}/k), (\tilde{\tilde{f}}^{p+}_{ij}, \tilde{\tilde{f}}^{p+}_{ij}))$ is the linguistic scale of restriction of the extended IT2FVIKOR using equitable linguistic scales and Z-Numbers and $(\tilde{\tilde{z}}^{p-}_{ij}, \tilde{\tilde{z}}^{p+}_{ij})$ is the linguistic scale of reliability of the extended IT2FVIKOR using equitable linguistic scales and Z-Numbers, $1 \le i \le m, 1 \le j \le n, 1 \le p \le k$, and k represents the number of DMs.

Step 2. Construct the weighted DMs' matrix.

The proposed entropy weight method based on IT2FSs is defined as follows.

Assign $\tilde{\tilde{f}}_{ij} = ((\tilde{\tilde{f}}^{L}_{ij}, \tilde{\tilde{f}}^{U}_{ij}); (\tilde{\tilde{z}}^{L}_{ij}, \tilde{\tilde{z}}^{U}_{ij}))$.

Thus,

$$\begin{aligned}
E\left(\tilde{\tilde{B}}^{L}_{ij}\right) &= \frac{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{\text{near}} - \left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{L}\right]}}{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{\text{far}} - \left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{L}\right]}},\\
E\left(\tilde{\tilde{B}}^{U}_{ij}\right) &= \frac{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{\text{near}} - \left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{U}\right]}}{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{\text{far}} - \left(\tilde{\tilde{f}}^{-}_{ij}, \tilde{\tilde{f}}^{+}_{ij}\right)^{U}\right]}},\\
zE\left(z\tilde{\tilde{B}}^{L}_{ij}\right) &= \frac{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{\text{near}} - \left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{L}\right]}}{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{\text{far}} - \left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{L}\right]}},\\
zE\left(\tilde{\tilde{B}}^{U}_{ij}\right) &= \frac{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{\text{near}} - \left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{U}\right]}}{\sqrt{\sum_{i=1}^{n}\left[\left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{\text{far}} - \left(\tilde{\tilde{z}}^{-}_{ij}, \tilde{\tilde{z}}^{+}_{ij}\right)^{U}\right]}},
\end{aligned} \tag{15}$$

where $E(\tilde{\tilde{B}}_{ij}) = [E(\tilde{\tilde{B}}^{L}_{ij}), E(\tilde{\tilde{B}}^{U}_{ij})], [zE(z\tilde{\tilde{B}}^{L}_{ij}), zE(\tilde{\tilde{B}}^{U}_{ij})]$.

Step 3. Divide by the maximal entropy value.

Then, all entropy values are divided by the maximal entropy value and the value of *ij*, *h* is employed to characterize the outcomes of the maximal entropy value. Therefore, it can be described as follows:

$$\begin{aligned}
\tilde{\tilde{h}}_{ij} &= \frac{1}{\mathrm{FOU}\left(\tilde{\tilde{h}}_{ij}\right)} = \left[\left(\tilde{\tilde{h}}_{ij}^{L}, \tilde{\tilde{h}}_{ij}^{U}\right); \left(\tilde{\tilde{zh}}_{ij}^{L}, \tilde{\tilde{zh}}_{ij}^{U}\right)\right],\\
\tilde{\tilde{h}}_{i1} &= \left[\left(\frac{E\left(\tilde{\tilde{B}}_{i1}^{L}\right)}{\max\left(E\left(\tilde{\tilde{B}}_{i1}^{L}\right)\right)}, \frac{E\left(\tilde{\tilde{B}}_{i1}^{U}\right)}{\max\left(E\left(\tilde{\tilde{B}}_{i1}^{U}\right)\right)}\right); \left(\frac{zE\left(z\tilde{\tilde{B}}_{i1}^{L}\right)}{\max\left(zE\left(z\tilde{\tilde{B}}_{i1}^{L}\right)\right)}, \frac{zE\left(z\tilde{\tilde{B}}_{i1}^{U}\right)}{\max\left(zE\left(z\tilde{\tilde{B}}_{i1}^{U}\right)\right)}\right)\right],\\
\tilde{\tilde{h}}_{i2} &= \left[\left(\frac{E\left(\tilde{\tilde{B}}_{i2}^{L}\right)}{\max\left(E\left(\tilde{\tilde{B}}_{i2}^{L}\right)\right)}, \frac{E\left(\tilde{\tilde{B}}_{i2}^{U}\right)}{\max\left(E\left(\tilde{\tilde{B}}_{i2}^{U}\right)\right)}\right); \left(\frac{zE\left(z\tilde{\tilde{B}}_{i2}^{L}\right)}{\max\left(zE\left(z\tilde{\tilde{B}}_{i2}^{L}\right)\right)}, \frac{zE\left(z\tilde{\tilde{B}}_{i2}^{U}\right)}{\max\left(zE\left(z\tilde{\tilde{B}}_{i2}^{U}\right)\right)}\right)\right], \ldots\\
\tilde{\tilde{h}}_{\mathrm{in}} &= \left[\left(\frac{E\left(\tilde{\tilde{B}}_{in}^{L}\right)}{\max\left(E\left(\tilde{\tilde{B}}_{in}^{L}\right)\right)}, \frac{E\left(\tilde{\tilde{B}}_{in}^{U}\right)}{\max\left(E\left(\tilde{\tilde{B}}_{in}^{U}\right)\right)}\right); \left(\frac{zE\left(z\tilde{\tilde{B}}_{in}^{L}\right)}{\max\left(zE\left(z\tilde{\tilde{B}}_{in}^{L}\right)\right)}, \frac{zE\left(z\tilde{\tilde{B}}_{in}^{U}\right)}{\max\left(zE\left(z\tilde{\tilde{B}}_{in}^{U}\right)\right)}\right)\right].
\end{aligned} \tag{16}$$

Then, the decision matrix D can be conveyed as follows:

$$D = \begin{bmatrix} \tilde{\tilde{h}}_{11} & \tilde{\tilde{h}}_{12} & \cdots & \tilde{\tilde{h}}_{1n} \\ \tilde{\tilde{h}}_{21} & \tilde{\tilde{h}}_{22} & \cdots & \tilde{\tilde{h}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{h}}_{m1} & \tilde{\tilde{h}}_{m2} & \cdots & \tilde{\tilde{h}}_{mn} \end{bmatrix}. \tag{17}$$

Step 4. Weight of criteria.

Compute the weight of criteria by using the fuzzy entropy weight-based IT2FS formula. W_j is used to present the outcome of the weight value of criteria j. Thus, it can be described as

$$\begin{aligned}
\tilde{\tilde{w}}_j &= \frac{1}{F\left(\tilde{\tilde{w}}_j\right)}\\
&= \left[\left(\tilde{\tilde{w}}_j^{L}, \tilde{\tilde{w}}_j^{U}\right); \left(\tilde{\tilde{zw}}_j^{L}, \tilde{\tilde{zw}}_j^{U}\right)\right],
\end{aligned} \tag{18}$$

$$\left[\left(\tilde{\tilde{w}}_j^{L}, \tilde{\tilde{w}}_j^{U}\right); \left(\tilde{\tilde{zw}}_j^{L}, \tilde{\tilde{zw}}_j^{U}\right)\right] = \left[\left(\frac{1-\tilde{\tilde{a}}_j^{L}}{\tilde{\tilde{T}}^{L}}, \frac{1-\tilde{\tilde{a}}_j^{U}}{\tilde{\tilde{T}}^{U}}\right), \left(\frac{1-z\tilde{\tilde{a}}_j^{L}}{z\tilde{\tilde{T}}^{L}}, \frac{1-\tilde{\tilde{za}}_j^{U}}{z\tilde{\tilde{T}}^{U}}\right)\right], \tag{19}$$

where $(\tilde{\tilde{b}}_j, \tilde{\tilde{zb}}_j) = (\tilde{\tilde{h}}_{i1} + \tilde{\tilde{h}}_{i2} + \cdots + \tilde{\tilde{h}}_{\mathrm{in}}/n; \tilde{\tilde{zh}}_{i1} + \tilde{\tilde{zh}}_{i2} + \cdots + \tilde{\tilde{zh}}_{\mathrm{in}}/n)$.

$$(\tilde{\tilde{T}}, \widetilde{zT}) = \left(\tilde{\tilde{h}}_{i1} + \tilde{\tilde{h}}_{i2} + \cdots + \tilde{\tilde{h}}_{in}; \tilde{\tilde{zh}}_{i1} + \tilde{\tilde{zh}}_{i2} + \cdots + \tilde{\tilde{zh}}_{in}\right), \ 1 \le j \le n. \tag{20}$$

Step 5. Construct the weighted value of decision matrix.

Construct the weighted decision matrix

$$x_1, x_2 \cdots x_n$$

$$\overline{Y}_w = \left[\tilde{\tilde{v}}_{ij}\right]_{m\times n}$$

$$= \begin{array}{c} f_1 \\ f_2 \\ \vdots \\ f_1 \end{array} \begin{bmatrix} \left(\tilde{\tilde{v}}_{11}; \tilde{\tilde{z}}v_{11}\right) & \left(\tilde{\tilde{v}}_{12}; \tilde{\tilde{z}}v_{12}\right) & \cdots & \left(\tilde{\tilde{v}}_{1n}; \tilde{\tilde{z}}v_{1n}\right) \\ \left(\tilde{\tilde{v}}_{21}; \tilde{\tilde{z}}v_{21}\right) & \left(\tilde{\tilde{v}}_{22}; \tilde{\tilde{z}}v_{22}\right) & \cdots & \left(\tilde{\tilde{v}}_{2n}; \tilde{\tilde{z}}v_{2n}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{\tilde{v}}_{m1}; \tilde{\tilde{z}}v_{m1}\right) & \left(\tilde{\tilde{v}}_{m2}; \tilde{\tilde{z}}v_{m2}\right) & \cdots & \left(\tilde{\tilde{v}}_{mn}; \tilde{\tilde{z}}v_{mn}\right) \end{bmatrix}, \tag{21}$$

where $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_j \oplus \tilde{\tilde{f}}_{ij}$, $1 \le i \le m$, and $1 \le j \le n$.

Step 6. Construct the fuzzy best value (FBV) and fuzzy worst value (FWV).

Choose the FBV and FWV values using the following equations:

$$\begin{aligned} \tilde{f}_j^* &= \max_i \tilde{\tilde{v}}_{ij}, \\ \tilde{f}_j^- &= \min_i \tilde{\tilde{v}}_{ij}, \ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n, \end{aligned} \tag{22}$$

where $\tilde{f}_j^*$ refers to FBV and $\tilde{f}_j^-$ refers to FWV. Then, defuzzify the value.

Step 7. Calculate the separation measures and defuzzification.

Compute the following values:

$$\begin{aligned} &\frac{W_i\left(\tilde{f}_i^* - \tilde{\tilde{v}}_{ij}\right)}{\tilde{f}_i^* - \tilde{f}_i^-}, \\ \tilde{M}_i &= \sum_{i=1}^{k} \frac{\tilde{w}_i\left(\tilde{f}_i^* - \tilde{\tilde{v}}_{ij}\right)}{\tilde{f}_i^* - \tilde{f}_i^-}, \\ \tilde{N}_i &= \max_j \left[\frac{\tilde{w}_i\left(\tilde{f}_i^* - \tilde{\tilde{v}}_{ij}\right)}{\tilde{f}_i^* - \tilde{f}_i^-}\right], \end{aligned} \tag{23}$$

where $\tilde{M}$ indicates the utility measure and $\tilde{N}$ indicates the regret measure, respectively. $\tilde{M}_i$ and $\tilde{N}_i$ can be computed using the sum of the FBV distance for all criteria.

Step 8. Defuzzify the utility measure value $(\tilde{M}_i)$ and regret measure value $(\tilde{N}_i)$.

Defuzzify the utility measure value $(\tilde{M}_i)$ and regret measure value $(\tilde{N}_i)$ using the following formula.

$$\text{Defuzzified} = \frac{\left[\left[\left(u_U - l_U\right) + \left(\beta_U . m_{1U} - l_U\right) + \left(\propto_U . m_{2U} - l_U\right)/4\right] + l_U\right] + \left[\left[\left(u_L - l_L\right) + \left(\beta_L . m_{1L} - l_L\right) + \left(\propto_L . m_{2L} - l_L\right)/4\right] + l_L\right]}{2}. \tag{24}$$

Step 9. Rank the alternatives.

Compute the subsequent values:

$$\begin{aligned} \tilde{M}^* &= \min_i \tilde{M}, \\ \tilde{M}^- &= \max_i \tilde{M}, \end{aligned} \tag{25}$$

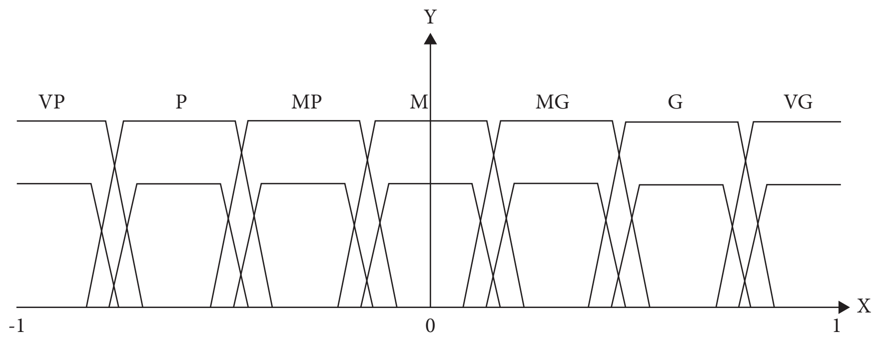

Figure 2: The positive and negative restrictions for the extended IT2FVIKOR.

Table 2: Linguistic scale of reliability for the extended IT2FVIKOR.

Linguistic terms	Linguistic scale
Strongly Unlikely (SU)	((−1.0, −0.9, −0.8, −0.7; 0.8, 0.8),(−1.0, −1.0, −0.8, −0.6; 1, 1))
Unlikely (U)	((−0.8, −0.7, −0.5, −0.4; 0.8, 0.8), (−0.9, −0.7, −0.5, −0.3; 1, 1))
Somewhat Unlikely (SWU)	((−0.5, −0.4, −0.2, −0.1; 0.8, 0.8), (−0.6, −0.4, −0.2, 0; 1, 1))
Neutral (N)	((−0.2, −0.1, 0.1, 0.2; 0.8, 0.8), (−0.3, −0.2, 0.2, 0.3; 1, 1))
Somewhat Likely (SWL)	((0.1, 0.2, 0.4, 0.5; 0.8, 0.8), (0, 0.2, 0.4, 0.6; 1, 1))
Likely (L)	((0.4, 0.5, 0.7, 0.8; 0.8, 0.8), (0.3, 0.5, 0.7, 0.9; 1, 1))
Strongly Likely (SLL)	((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1))

where $\widetilde{M}^*$ is the value for the maximum group of utility and $\widetilde{M}^-$ is the minimum value for the maximum group of utility.

$$\text{Then } \widetilde{N}^* = \min_i \widetilde{N}, \qquad \widetilde{N}^- = \max_i \widetilde{N}, \tag{26}$$

where $\widetilde{N}^*$ is the value of minimum individual regret of the opponent and $\widetilde{N}^-$ is the minimum value.

$$\text{Next } \widetilde{P}_i = v\frac{\left(\widetilde{M}_j - \widetilde{M}^*\right)}{\left(\widetilde{M}^- - \widetilde{M}^*\right)} + (1-v)\frac{\left(\widetilde{N}_j - \widetilde{N}^*\right)}{\left(\widetilde{N}^- - \widetilde{N}^*\right)}, \tag{27}$$

where $\widetilde{P}_i$ is the index for both $\widetilde{M}^*$ and $\widetilde{N}^*$, whereas v is the weight of the strategy to be used in the maximum group of utility, $v > 0.5$ refers to the maximum majority of rule, and $v \leq 0.5$ refers to the individual regret of the opponent. The normal value is when v is 0.5.

Lastly, $\widetilde{M}^*$, $\widetilde{N}^*$, and $\widetilde{P}_i$ are arranged and rated in decreasing order. Decreasing order improves to decrease the gaps in the criteria, and the most excellent one is chosen based on the lowest rank value.

Figure 4 shows the entire fundamental process involved in the proposed method. Generally, the conceptual framework can be split into three different phases. Phase 1 (Ranking Phase) focuses on achieving the preference scale. Then, Phase 2 (Weighting Phase) identifies weight for all the criteria. Phase 3 (Ranking Phase) ranks all the alternatives.

5. Application of Water Security in Malaysia

A real application on searching for the best strategies to enhance water supply security in Malaysia is utilized to demonstrate the processes and feasibility of the extended IT2FVIKOR. Six alternatives $(AL_1, AL_2, AL_3, AL_4, AL_5, AL_6)$ such as strengthening the protection of water source areas (AL_1), improving infrastructure to safeguard urban and rural water security (AL_2), developing water-saving system (AL_3), fully implementing the river chief system (AL_4), reinforcing groundwater monitoring and protection (AL_5), and strengthening the policy on water security (AL_6) are proposed as feasible alternatives to be assessed by five decision makers $(DM_1, DM_2, DM_3, DM_4, DM_5)$ corresponding to five criteria such as household water security (CR_1), economic water security (CR_2), urban water security (CR_3), environmental water security (CR_4), and resilience to water-related disasters (CR_5) to select the most suitable alternative for enhancing water security.

Step 1. Construct a hierarchical structure of weight factors associated with the strategies to enhance water supply security.

The hierarchical structure of evaluating the best strategies to enhance water supply security in Malaysia is given in Figure 5.

The comparison results involve six strategies as alternatives and five criteria based on water supply security. The rating of each alternative for each criterion is presented in Table 3.

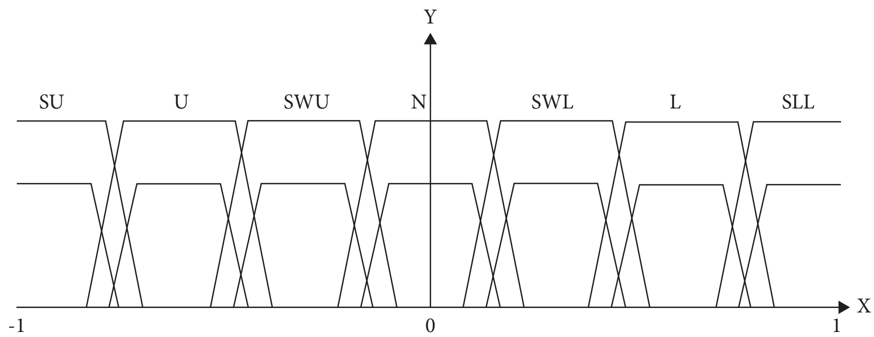

FIGURE 3: The positive and negative reliability for the extended IT2FVIKOR.

The data that reflect negative and positive sides with restriction and reliability and its transition (Tables 2 and 3) are described to create a matrix of attributes. Therefore, let us take the example of calculating $\tilde{\tilde{f}}_{11}$. The average for $\tilde{\tilde{f}}_{11}$:

$$\begin{aligned}(VG, L) &= ((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1))\\ &\quad((0.4, 0.5, 0.7, 0.8; 0.8, 0.8), (0.3, 0.5, 0.7, 0.9; 1, 1)),\\ (G, \mathrm{SWL}) &= ((0.4, 0.5, 0.7, 0.8; 0.8, 0.8), (0.3, 0.5, 0.7, 0.9; 1, 1))\\ &\quad((0.1, 0.2, 0.4, 0.5; 0.8, 0.8), (0, 0.2, 0.4, 0.6; 1, 1)),\\ (VG, \mathrm{SLL}) &= ((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1))\\ &\quad((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1)),\\ (VG, \mathrm{SLL}) &= ((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1))\\ &\quad((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1)),\\ (VG, \mathrm{SLL}) &= ((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1))\\ &\quad((0.7, 0.8, 1.0, 1.0; 0.8, 0.8), (0.6, 0.8, 1.0, 1.0; 1, 1)).\end{aligned} \tag{28}$$

Then, the average for (VG, L), (G, SWL), (VG, SLL), (VG, SLL), and (VG, SLL) is

$$\begin{aligned}&((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)\,(0.54, 0.74, 0.94, 0.98; 1, 1))\\ &((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)\,(0.42, 0.62, 0.82, 0.9; 1, 1)).\end{aligned} \tag{29}$$

Apply the same calculation as $\tilde{\tilde{f}}_{11}$. Thus, the whole results for the matrix of alternatives are summarized in Table 4.

Step 2. Construct the weighted DMs' matrix.

Use the fuzzy entropy with IT2FS formula (15) to compute each entropy value in the decision matrix. Therefore, the entropy value for $E(\tilde{\tilde{B}}_{11})$ is characterized as follows:

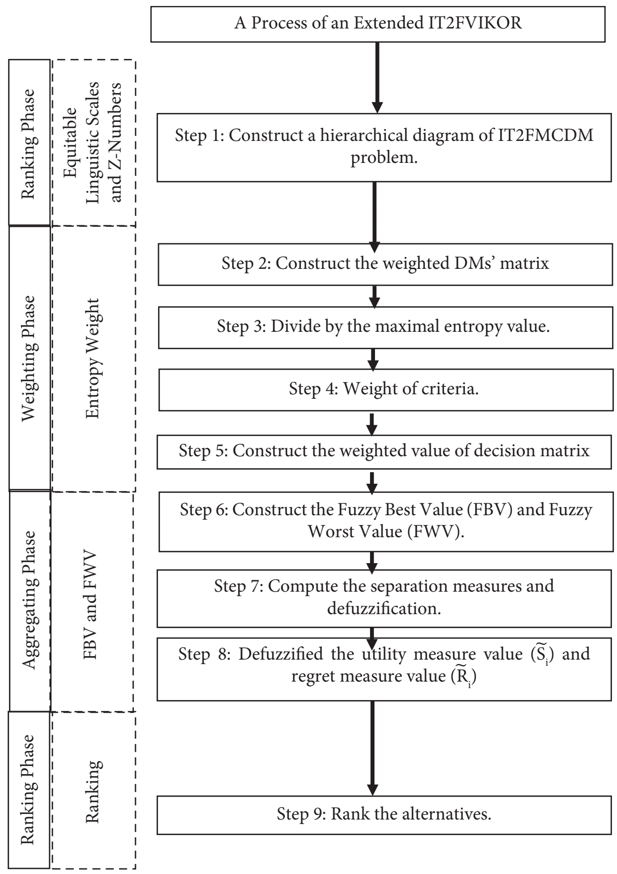

Figure 4: The positive and negative reliability for the extended IT2FVIKOR.

$$\begin{aligned}
E\left(\tilde{\tilde{B}}_{11}^{L}\right) &= \frac{\sqrt{(1-0.64)^2+(1-0.74)^2+(1-0.94)^2+(1-0.96)^2}}{\sqrt{(0-0.64)^2+(0-0.74)^2+(0-0.94)^2+(0-0.96)^2}} \\
&= 0.27, \\
E\left(\tilde{\tilde{B}}_{11}^{U}\right) &= \frac{\sqrt{(1-0.54)^2+(1-0.74)^2+(1-0.94)^2+(1-0.98)^2}}{\sqrt{(0-0.54)^2+(0-0.74)^2+(0-0.94)^2+(0-0.98)^2}} \\
&= 0.32, \\
zE\left(\widetilde{\widetilde{zB}}_{11}^{L}\right) &= \frac{\sqrt{(1-0.52)^2+(1-0.62)^2+(1-0.82)^2+(1-0.86)^2}}{\sqrt{(0-0.52)^2+(0-0.62)^2+(0-0.82)^2+(0-0.86)^2}} \\
&= 0.45, \\
zE\left(\widetilde{\widetilde{zB}}_{11}^{U}\right) &= \frac{\sqrt{(0-0.42)^2+(1-0.62)^2+(1-0.82)^2+(1-0.9)^2}}{\sqrt{(1-0.42)^2+(0-0.62)^2+(0-0.82)^2+(0-0.9)^2}} \\
&= 0.41, \\
E\left(\tilde{\tilde{B}}_{11}\right) &= ((0.27;0.8,0.8)(0.32;1,1)),((0.45;0.8,0.8)(0.41;1,1)).
\end{aligned} \tag{30}$$

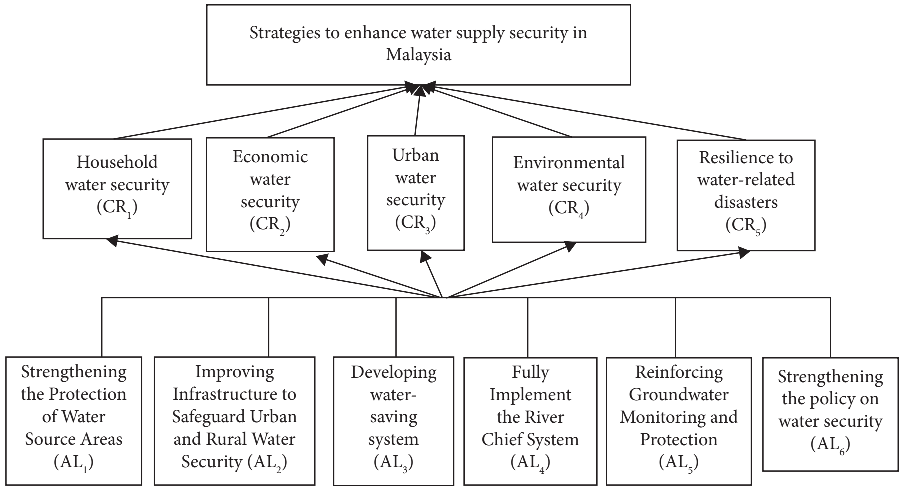

Figure 5: Hierarchy structure of the most suitable alternative for water security.

Table 3: The rating of each alternative under each criterion.

Decision makers	Criteria	Alternatives					
		AL_1	AL_2	AL_3	AL_4	AL_5	AL_6
DM_1	CR_1	(VG, L)	(G, SLL)	(VG, SLL)	(G, L)	(VG, SLL)	(VG, SLL)
	CR_2	(VG, L)	(VG, L)	(MG, L)	(VG, L)	(VG, SLL)	(VG, SLL)
	CR_3	(VG, L)	(VG, L)	(MG, L)	(VG, L)	(VG, SLL)	(VG, SLL)
	CR_4	(VG, L)	(VG, L)	(VG, L)	(VG, L)	(G, L)	(VG, L)
	CR_5	(VG, L)	(VG, L)	(VG, L)	(G, L)	(G, L)	(VG, L)
DM_2	CR_1	(G, SWL)	(MG, SWU)	(VG, L)	(G, SWU)	(VG, SLL)	(VG, SLL)
	CR_2	(VG, SLL)	(VG, SLL)	(VG, SLL)	(G, L)	(VG, SLL)	(VG, SLL)
	CR_3	(VG, SLL)	(G, SWL)	(VG, SLL)	(MG, N)	(VG, SLL)	(VG, SLL)
	CR_4	(VG, L)	(G, N)	(G, SWL)	(VG, L)	(VG, SLL)	(VG, SLL)
	CR_5	(MG,SWL)	(G, L)	(MG, N)	(G, SWL)	(VG, SLL)	(VG, SLL)
DM_3	CR_1	(VG, SLL)	(G, L)	(MG,SWL)	(G, SWL)	(G, L)	(MG,SWL)
	CR_2	(VG, SLL)	(G, L)	(VG, SLL)	(G, L)	(G, L)	(G, L)
	CR_3	(VG, SLL)	(VG, SLL)	(VG, SLL)	(MG,SWL)	(MG,SWL)	(MG,SWL)
	CR_4	(VG, SLL)	(MG,SWL)	(MG,SWL)	(VG, SLL)	(VG, SLL)	(G, SWL)
	CR_5	(MG,SWL)	(G, L)	(F, N)	(MG,SWL)	(G, L)	(VG, SLL)
DM_4	CR_1	(VG, SLL)	(G, SLL)	(VG, SLL)	(F, SWU)	(VG, SLL)	(VG, SLL)
	CR_2	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VP, SLL)	(VG, SLL)	(VG, SLL)
	CR_3	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VP, SLL)	(VG, SLL)	(VG, SLL)
	CR_4	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)
	CR_5	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)
DM_5	CR_1	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)
	CR_2	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)
	CR_3	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)
	CR_4	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)
	CR_5	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)	(VG, SLL)

Table 4: The judgment matrix.

	CR_1	CR_2	CR_3	CR_4	CR_5
AL_1	((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1)), ((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)(0.42, 0.62, 0.82, 0.90; 1, 1))	((0.70, 0.80, 1.00, 1.00; 0.8, 0.8)(0.60, 0.80, 1.00, 1.00; 1, 1)), ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))	((0.70, 0.80, 1.00, 1.00; 0.8, 0.8)(0.60, 0.80, 1.00, 1.00; 1, 1)), ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))	((0.70, 0.80, 1.00, 1.00; 0.8, 0.8)(0.60, 0.80, 1.00, 1.00; 1, 1)), ((0.58, 0.68, 0.88, 0.92; 0.8, 0.8)(0.48, 0.68, 0.88, 0.96; 1, 1))	((0.46, 0.56, 0.76, 0.80; 0.8, 0.8)(0.36, 0.56, 0.76, 0.84; 1, 1)), ((0.40, 0.50, 0.70, 0.76; 0.8, 0.8)(0.30, 0.50, 0.70, 0.82; 1, 1))
AL_2	((0.40, 0.50, 0.70, 0.78; 0.8, 0.8)(0.30, 0.50, 0.70, 0.86; 1, 1)), ((0.40, 0.50, 0.70, 0.74; 0.8, 0.8)(0.30, 0.50, 0.70, 0.78; 1, 1))	((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1)), ((0.58, 0.68, 0.88, 0.92; 0.8, 0.8)(0.48, 0.68, 0.88, 0.96; 1, 1))	((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1)), ((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)(0.42, 0.62, 0.82, 0.90; 1, 1))	((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)(0.42, 0.62, 0.82, 0.90; 1, 1)), ((0.34, 0.44, 0.64, 0.70; 0.8, 0.8)(0.24, 0.42, 0.66, 0.76; 1, 1))	((0.58, 0.68, 0.88, 0.92; 0.8, 0.8)(0.48, 0.68, 0.88, 0.96; 1, 1)), ((0.52, 0.62, 0.82, 0.88; 0.8, 0.8)(0.42, 0.62, 0.82, 0.94; 1, 1))
AL_3	((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1)), ((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)(0.42, 0.62, 0.82, 0.90; 1, 1))	((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1)), ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))	((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1)), ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))	((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)(0.42, 0.62, 0.82, 0.90; 1, 1)), ((0.40, 0.50, 0.70, 0.76; 0.8, 0.8)(0.30, 0.50, 0.70, 0.82; 1, 1))	((0.40, 0.50, 0.70, 0.74; 0.8, 0.8)(0.30, 0.48, 0.72, 0.78; 1, 1)), ((0.28, 0.38, 0.58, 0.64; 0.8, 0.8)(0.18, 0.34, 0.62, 0.70; 1, 1))
AL_4	((0.34, 0.44, 0.64, 0.72; 0.8, 0.8)(0.24, 0.42, 0.66, 0.8; 1, 1)), ((0.04, 0.14, 0.34, 0.42; 0.8, 0.8)(−0.06, 0.14, 0.34, 0.50; 1, 1))	((0.24, 0.34, 0.52, 0.58; 0.8, 0.8)(0.16, 0.32, 0.52, 0.64; 1, 1)), ((0.52, 0.62, 0.82, 0.88; 0.8, 0.8)(0.42, 0.62, 0.82, 0.94; 1, 1))	((0.12, 0.22, 0.40, 0.46; 0.8, 0.8)(0.04, 0.20, 0.40, 0.52; 1, 1)), ((0.34, 0.44, 0.64, 0.70; 0.8, 0.8)(0.24, 0.42, 0.66, 0.76; 1, 1))	((0.70, 0.80, 1.00, 1.00; 0.8, 0.8)(0.60, 0.80, 1.00, 1.00; 1, 1)), ((0.58, 0.68, 0.88, 0.92; 0.8, 0.8)(0.48, 0.68, 0.88, 0.96; 1, 1))	((0.46, 0.56, 0.76, 0.82; 0.8, 0.8)(0.36, 0.56, 0.76, 0.88; 1, 1)), ((0.40, 0.50, 0.70, 0.76; 0.8, 0.8)(0.30, 0.50, 0.70, 0.82; 1, 1))
AL_5	((0.64, 0.72, 0.88, 0.88; 0.8, 0.8)(0.56, 0.72, 0.88, 0.88; 1, 1)), ((0.64, 0.72, 0.88, 0.88; 0.8, 0.8)(0.56, 0.72, 0.88, 0.88; 1, 1))	((0.64, 0.72, 0.88, 0.88; 0.8, 0.8)(0.56, 0.72, 0.88, 0.88; 1, 1)), ((0.64, 0.72, 0.88, 0.88; 0.8, 0.8)(0.56, 0.72, 0.88, 0.88; 1, 1))	((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1)), ((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1))	((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1)), ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))	((0.58, 0.68, 0.88, 0.92; 0.8, 0.8)(0.48, 0.68, 0.88, 0.96; 1, 1)), ((0.58, 0.68, 0.88, 0.92; 0.8, 0.8)(0.48, 0.68, 0.88, 0.96; 1, 1))
AL_6	((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1)), ((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1))	((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1)), ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))	((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1)), ((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1))	((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1)), ((0.58, 0.68, 0.88, 0.90; 0.8, 0.8)(0.48, 0.68, 0.88, 0.92; 1, 1))	((0.70, 0.80, 1.00, 1.00; 0.8, 0.8)(0.60, 0.80, 1.00, 1.00; 1, 1)), ((0.70, 0.80, 1.00, 1.00; 0.8, 0.8)(0.60, 0.80, 1.00, 1.00; 1, 1))

Using a similar calculation, the entropy value for other attributes is calculated and recorded in Table 5.

Step 3. Divide by maximal entropy value.

All entropy values are divided by using the maximal entropy value following equation (16). Let us take $E(\tilde{\tilde{B}}_{1j})$ as an example for calculating the maximal entropy value as stated in Table 6.

The maximal value for $E(\tilde{\tilde{B}}_{1j})$ is

$$\max\left(E\left(\tilde{\tilde{B}}_{15}\right)\right) = ((0.53; 0.8, 0.8)(0.45; 1, 1)), ((0.58; 0.8, 0.8)(0.49; 1, 1)). \tag{31}$$

Thus,

$$\begin{aligned}
\tilde{\tilde{h}}_{11} &= \left[\left(\frac{\tilde{\tilde{E}}\left(\tilde{\tilde{B}}_{11}^{L}\right)}{\max\left(\tilde{\tilde{E}}\left(\tilde{\tilde{B}}_{15}^{L}\right)\right)}\right),\left(\frac{\tilde{\tilde{E}}\left(\tilde{\tilde{B}}_{11}^{U}\right)}{\max\left(\tilde{\tilde{E}}\left(\tilde{\tilde{B}}_{15}^{U}\right)\right)}\right)\right]\left[\left(\frac{\tilde{\tilde{E}}\left(\tilde{\tilde{Z}}_{11}^{L}\right)}{\max\left(\tilde{\tilde{E}}\left(\tilde{\tilde{Z}}_{15}^{L}\right)\right)}\right),\left(\frac{\tilde{\tilde{E}}\left(\tilde{\tilde{Z}}_{11}^{U}\right)}{\max\left(\tilde{\tilde{E}}\left(\tilde{\tilde{Z}}_{15}^{U}\right)\right)}\right)\right],\\
\tilde{\tilde{h}}_{11} &= \left[\left(\frac{0.27; 0.8, 0.8}{0.53; 0.8, 0.8}\right),\left(\frac{0.32; 1, 1}{0.45; 1, 1}\right)\right],\left[\left(\frac{0.45; 0.8, 0.8}{0.58; 0.8, 0.8}\right),\left(\frac{0.41; 1, 1}{0.49; 1, 1}\right)\right],\\
\tilde{\tilde{h}}_{11} &= ((0.52; 0.8, 0.8)(0.72; 1, 1)), ((0.79; 0.8, 0.8)(0.82; 1, 1)).
\end{aligned} \tag{32}$$

Table 5: The IT2 fuzzy entropy.

	CR_1	CR_2	CR_3	CR_4	CR_5
AL_1	((0.27; 0.8, 0.8) (0.32; 1, 1)), ((0.45; 0.8, 0.8) (0.41; 1, 1))	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.35; 0.8, 0.8) (0.38; 1, 1))	((0.53; 0.8, 0.8) (0.45; 1, 1)), ((0.58; 0.8, 0.8) (0.49; 1, 1))
AL_2	((0.57; 0.8, 0.8) (0.48; 1, 1)), ((0.59; 0.8, 0.8) (0.51; 1, 1))	((0.27; 0.8, 0.8) (0.32; 1, 1)), ((0.35; 0.8, 0.8) (0.38; 1, 1))	((0.27; 0.8, 0.8) (0.32; 1, 1)), ((0.45; 0.8, 0.8) (0.41; 1, 1))	((0.45; 0.8, 0.8) (0.41; 1, 1)), ((0.58; 0.8, 0.8) (0.49; 1, 1))	((0.59; 0.8, 0.8) (0.49; 1, 1)), ((0.57; 0.8, 0.8) (0.44; 1, 1))
AL_3	((0.36; 0.8, 0.8) (0.39; 1, 1)), ((0.45; 0.8, 0.8) (0.41; 1, 1))	((0.36; 0.8, 0.8) (0.39; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	((0.36; 0.8, 0.8) (0.39; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	((0.45; 0.8, 0.8) (0.41; 1, 1)), ((0.58; 0.8, 0.8) (0.49; 1, 1))	((0.59; 0.8, 0.8) (0.49; 1, 1)), ((0.57; 0.8, 0.8) (0.44; 1, 1))
AL_4	((0.56; 0.8, 0.8) (0.44; 1, 1)), ((0.36; 0.8, 0.8) (0.39; 1, 1))	((0.60; 0.8, 0.8) (0.51; 1, 1)), ((0.45; 0.8, 0.8) (0.40; 1, 1))	((0.46; 0.8, 0.8) (0.44; 1, 1)), ((0.57; 0.8, 0.8) (0.46; 1, 1))	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.35; 0.8, 0.8) (0.38; 1, 1))	((0.52; 0.8, 0.8) (0.44; 1, 1)), ((0.58; 0.8, 0.8) (0.49; 1, 1))
AL_5	((0.31; 0.8, 0.8) (0.36; 1, 1)), ((0.31; 0.8, 0.8) (0.36; 1, 1))	((0.31; 0.8, 0.8) (0.36; 1, 1)), ((0.31; 0.8, 0.8) (0.36; 1, 1))	((0.36; 0.8, 0.8) (0.39; 1, 1)), ((0.36; 0.8, 0.8) (0.39; 1, 1))	((0.27; 0.8, 0.8) (0.32; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	((0.35; 0.8, 0.8) (0.38; 1, 1)), ((0.35; 0.8, 0.8) (0.38; 1, 1))
AL_6	((0.36; 0.8, 0.8) (0.39; 1, 1)), ((0.36; 0.8, 0.8) (0.39; 1, 1))	((0.27; 0.8, 0.8) (0.32; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	((0.36; 0.8, 0.8) (0.39; 1, 1)), ((0.36; 0.8, 0.8) (0.39; 1, 1))	((0.27; 0.8, 0.8) (0.32, 1, 1)), ((0.36; 0.8, 0.8) (0.39; 1, 1))	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.20; 0.8, 0.8) (0.26; 1, 1))

Table 6: Choosing the maximal entropy value for $E(\tilde{\tilde{B}}_{1j})$.

	$E\ (\tilde{\tilde{B}}_{1_j})$	
CR_1	((0.27; 0.8, 0.8) (0.32; 1, 1)), ((0.45; 0.8, 0.8) (0.41; 1, 1))	$E\ (\tilde{\tilde{B}}_{11})$
CR_2	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	$E\ (\tilde{\tilde{B}}_{12})$
CR_3	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.27; 0.8, 0.8) (0.32; 1, 1))	$E\ (\tilde{\tilde{B}}_{13})$
CR_4	((0.20; 0.8, 0.8) (0.26; 1, 1)), ((0.35; 0.8, 0.8) (0.38; 1, 1))	$E\ (\tilde{\tilde{B}}_{14})$
CR_5	((0.53; 0.8, 0.8) (0.45; 1, 1)), ((0.58; 0.8, 0.8) (0.49; 1, 1))	$E\ (\tilde{\tilde{B}}_{15})$

Using the same calculation, the maximal entropy value is shown in Table 7.

Step 4. Weight of criteria.

Next, evaluate the weight of criteria by utilizing the weight formula from equations (18) and (19). The whole entropy-based weight is shown in Table 8.

Step 5. Construct the weighted value of decision matrices.

Then, construct the weighted value of DM's matrix with respect to aggregated matrix comparison using equation (26).

For example, let the value of the weighted matrix for $\tilde{\tilde{v}}_{11}$ be

$$\begin{aligned}
\tilde{\tilde{v}}_{11} &= \tilde{\tilde{f}}_{11} \times \tilde{\tilde{w}}_1,\\
\tilde{\tilde{v}}_{11} &= ((0.64, 0.74, 0.94, 0.96; 0.8, 0.8)(0.54, 0.74, 0.94, 0.98; 1, 1))\\
&\quad ((0.52, 0.62, 0.82, 0.86; 0.8, 0.8)(0.42, 0.62, 0.82, 0.9; 1, 1))\\
&\quad \times ((0.04; 0.8, 0.8)(0.02; 1, 1)), ((0.03; 0.8, 0.8)(0.02; 1, 1))\\
&= ((0.02, 0.03, 0.03, 0.04; 0.8, 0.8)(0.01, 0.02, 0.02, 0.02; 1; 1))\\
&\quad ((0.02, 0.02, 0.02, 0.03; 0.8, 0.8)(0.08, 0.01, 0.02, 0.02; 1, 1)).
\end{aligned} \tag{33}$$

Thus, the remaining values of the weighted DM's matrix are presented in Table 9.

Step 6. Construct the fuzzy best value (FBV) and fuzzy worst value (FWV).

Values for FBV and FWV are chosen using equation (22) and combined in Table 10.

Step 7. Compute the separation measures and defuzzification.

The utility measure $\tilde{M}_i$ and regret measure $\tilde{N}_i$ are calculated using the sum of the FBV distance with

Table 7: The maximal entropy value.

	CR_1	CR_2	CR_3	CR_4	CR_5
AL_1	((0.52; 0.8, 0.8) (0.72; 1, 1)), ((0.79; 0.8, 0.8) (0.82; 1, 1))	((0.39; 0.8, 0.8) (0.57; 1, 1)), ((0.47; 0.8, 0.8) (0.66; 1, 1))	((0.39; 0.8, 0.8) (0.57; 1, 1)), ((0.47; 0.8, 0.8) (0.66; 1, 1))	((0.39; 0.8, 0.8) (0.57; 1, 1)), ((0.61; 0.8, 0.8) (0.77; 1, 1))	((1.00; 0.8, 0.8) (1.00; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))
AL_2	((1.00; 0.8, 0.8) (1.00; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))	((0.48; 0.8, 0.8) (0.68; 1, 1)), ((0.60; 0.8, 0.8) (0.74; 1, 1))	((0.48; 0.8, 0.8) (0.68; 1, 1)), ((0.77; 0.8, 0.8) (0.80; 1, 1))	((0.80; 0.8, 0.8) (0.85; 1, 1)), ((0.96; 0.8, 0.8) (0.90; 1, 1))	((0.62; 0.8, 0.8) (0.79; 1, 1)), ((0.76; 0.8, 0.8) (0.78; 1, 1))
AL_3	((0.61; 0.8, 0.8) (0.80; 1, 1)), ((0.79; 0.8, 0.8) (0.82; 1, 1))	((0.61; 0.8, 0.8) (0.80; 1, 1)), ((0.47; 0.8, 0.8) (0.66; 1, 1))	((0.61; 0.8, 0.8) (0.80; 1, 1)), ((0.47; 0.8, 0.8) (0.66; 1, 1))	((0.77; 0.8, 0.8) (0.83; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))	((1.00; 0.8, 0.8) (1.00; 1, 1), ((0.98; 0.8, 0.8) (0.89; 1, 1))
AL_4	((0.93; 0.8, 0.8) (0.86; 1, 1)), ((0.62; 0.8, 0.8) (0.79; 1, 1))	((1.00; 0.8, 0.8) (1.00; 1, 1)), ((0.78; 0.8, 0.8) (0.80; 1, 1))	((0.77; 0.8, 0.8) (0.86; 1, 1)), ((0.98; 0.8, 0.8) (0.93; 1, 1))	((0.34; 0.8, 0.8) (0.50; 1, 1)), ((0.61; 0.8, 0.8) (0.77; 1, 1))	((0.86; 0.8, 0.8) (0.85; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))
AL_5	((0.87; 0.8, 0.8) (0.92; 1, 1)), ((0.87; 0.8, 0.8) (0.92; 1, 1))	((0.87; 0.8, 0.8) (0.92; 1, 1)), ((0.87; 0.8, 0.8) (0.92; 1, 1))	((1.00; 0.8, 0.8) (1.00; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))	((0.76; 0.8, 0.8) (0.84; 1, 1)), ((0.76; 0.8, 0.8) (0.84; 1, 1))	((0.99; 0.8, 0.8) (0.98; 1, 1)), ((0.99; 0.8, 0.8) (0.98; 1, 1))
AL_6	((1.00; 0.8, 0.8) (1.00; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))	((0.76; 0.8, 0.8) (0.84; 1, 1)), ((0.76; 0.8, 0.8) (0.84; 1, 1))	((1.00; 0.8, 0.8) (1.00; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))	((0.76; 0.8, 0.8) (0.84; 1, 1)), ((1.00; 0.8, 0.8) (1.00; 1, 1))	((0.57; 0.8, 0.8) (0.67; 1, 1)), ((0.57; 0.8, 0.8) (0.67; 1, 1))

Table 8: The entropy-based weights ($\tilde{\tilde{w}}_j$).

	$(\tilde{\tilde{W}}_j)$
CR_1	((0.04; 0.8, 0.8) (0.02; 1, 1)), ((0.03; 0.8, 0.8) (0.02; 1, 1))
CR_2	((0.09; 0.8, 0.8) (0.04; 1, 1)), ((0.08; 0.8, 0.8) (0.05; 1, 1))
CR_3	((0.07; 0.8, 0.8) (0.04; 1, 1)), ((0.05; 0.8, 0.8) (0.03; 1, 1))
CR_4	((0.09; 0.8, 0.8) (0.06; 1, 1)), ((0.04; 0.8, 0.8) (0.02; 1, 1))
CR_5	((0.03; 0.8, 0.8) (0.02; 1, 1)), ((0.02; 0.8, 0.8) (0.02; 1, 1))

regard to all criteria based on equation (26) till 26. The result of the calculation is listed in Table 11.

Step 8. Defuzzify the utility measure value (($\tilde{M}_i$) and regret measure value ($\tilde{N}_i$)).

This defuzzification step is needed to further the next calculation in Step 9. Therefore, each utility measure value ($\tilde{M}_i$) and regret measure value ($\tilde{N}_i$) from Table 11 are defuzzified using equation (31). Let us take an example of $\tilde{S}_1$ upper boundary:

$$\begin{aligned}\tilde{M}_1 &= \frac{(0.05-0.07)+(0.8\times(0.05-0.07))+(0.8\times(0.06-0.07))/4+0.07+[(0.04-0.05)+(1\times(0.03-0.05))+(1\times(0.04-0.05))/4+0.05]}{2},\\ \tilde{M}_1 &= 0.05.\end{aligned} \tag{34}$$

Thus, the rest of the defuzzification results for $\tilde{M}_1$ and $\tilde{N}_i$ are listed in Table 12.

Step 9. Rank the alternatives.

Table 13 lists all values for (1) maximum group of utility $\tilde{M}^*$, (2) minimum value for a maximum group of utility $\tilde{M}$, (3) minimum individual regret of the opponent $\tilde{N}^*$, and (4) minimum value $\tilde{N}$ using equation (33).

The final index for both $\tilde{M}^*$ and $\tilde{N}^*$ which is P_i is $AL_1 = 0.38$, $AL_2 = 0.88$, $AL_3 = 0.63$, $AL_4 = 0.70$, $AL_5 = 0.56$, and $AL_6 = 0$. That is, the best alternative is AL_6, and the ranking order of the alternatives is $AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$ which is listed in Table 14.

As a result, the five criteria, six alternatives, and the opinions of the five DMs are gathered to determine the best strategies to improve the security of the water supply in Malaysia. Among the first ten ranks, strengthening the policy on water security comes in as the first rank at 0, followed by strengthening the protection of water source areas, increasing groundwater monitoring and protection, developing water-saving systems, and fully implementing the river chief system. The last item on the list is enhancing infrastructure to safeguard urban and rural water security.

Table 9: Weight of decision matrix.

	CR_1	CR_2	CR_3	CR_4	CR_5
AL_1	((0.02, 0.03, 0.03, 0.04; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.05, 0.06, 0.08, 0.08; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.06, 0.06, 0.08, 0.08; 0.8, 0.8) (0.03, 0.04, 0.05, 0.05; 1, 1))	((0.05, 0.06, 0.07, 0.07; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.03, 0.03, 0.04, 0.04; 0.8, 0.8) (0.02, 0.02, 0.03, 0.03; 1, 1))	((0.07, 0.08, 0.09, 0.09; 0.8, 0.8) (0.04, 0.05, 0.06, 0.06; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))	((0.01, 0.02, 0.02, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1)), ((0.01, 0.01, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))
AL_2	((0.01, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1)), ((0.01, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.01, 0.02; 1, 1))	((0.05, 0.06, 0.07, 0.07; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.05, 0.06, 0.08, 0.08; 0.8, 0.8) (0.02, 0.03, 0.04, 0.05; 1, 1))	((0.04, 0.05, 0.06, 0.07; 0.8, 0.8) (0.02, 0.03, 0.03, 0.04; 1, 1)), ((0.02, 0.03, 0.04, 0.04; 0.8, 0.8) (0.01, 0.02, 0.03, 0.03; 1, 1))	((0.05, 0.06, 0.08, 0.08; 0.8, 0.8) (0.02, 0.04, 0.05, 0.05; 1, 1)), ((0.01, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.01, 0.01, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))
AL_3	((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.04, 0.05, 0.07, 0.07; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.06, 0.06, 0.08, 0.08; 0.8, 0.8) (0.03, 0.04, 0.05, 0.05; 1, 1))	((0.04, 0.05, 0.06, 0.07; 0.8, 0.8) (0.02, 0.03, 0.03, 0.03; 1, 1)), ((0.03, 0.03, 0.04, 0.04; 0.8, 0.8) (0.01, 0.02, 0.03, 0.03; 1, 1))	((0.05, 0.06, 0.08, 0.08; 0.8, 0.8) (0.02, 0.04, 0.05, 0.05; 1, 1)), ((0.01, 0.02, 0.02, 0.03; 0.8, 0.8) (0.01, 0.01, 0.01, 0.02; 1, 1))	((0.01, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.03; 1, 1)), ((0.01, 0.01, 0.01, 0.01; 0.8, 0.8) (0.00, 0.01, 0.01, 0.02; 1, 1))
AL_4	((0.01, 0.02, 0.02, 0.03; 0.8, 0.8) (0.01, 0.01, 0.01, 0.02; 1, 1)), ((0.00, 0.00, 0.01, 0.01; 0.8, 0.8) (0.00, 0.00, 0.01, 0.01; 1, 1))	((0.02, 0.03, 0.04, 0.04; 0.8, 0.8) (0.01, 0.01, 0.02, 0.03; 1, 1)), ((0.05, 0.05, 0.07, 0.08; 0, 8, 0.8) (0.02, 0.03, 0.04, 0.05; 1, 1))	((0.01, 0.02, 0.03, 0.03; 0.8, 0.8) (0.00, 0.01, 0.01, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.07, 0.08, 0.09, 0.09; 0.8, 0.8) (0.04, 0.05, 0.06, 0.06; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))	((0.01, 0.02, 0.02, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1)), ((0.01, 0.01, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))
AL_5	((0.02, 0.03, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.05, 0.06, 0.07, 0.07; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.06, 0.06, 0.08, 0.08; 0.8, 0.8) (0.03, 0.04, 0.04, 0.04; 1, 1))	((0.04, 0.05, 0.06, 0.06; 0.8, 0.8) (0.02, 0.03, 0.03, 0.03; 1, 1)), ((0.03, 0.03, 0.04, 0.04; 0.8, 0.8) (0.02, 0.02, 0.03, 0.03; 1, 1))	((0.06, 0.07, 0.09, 0.09; 0.8, 0.8) (0.03, 0.04, 0.06, 0.06; 1, 1)), ((0.02, 0.03, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))	((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.01, 0.01, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))
AL_6	((0.02, 0.02, 0.03, 0.03; 0, 8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.05, 0.06, 0.07, 0.07; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.06, 0.06, 0.08, 0.08, 8, 0.8) (0.03, 0.04, 0.05, 0.05; 1, 1))	((0.04, 0.05, 0.06, 0.06; 0.8, 0.8) (0.02, 0.03, 0.03, 0.03; 1, 1)), ((0.03, 0.03, 0.04, 0.04; 0.8, 0.8) (0.02, 0.02, 0.03, 0.03; 1, 1))	((0.06, 0.07, 0.09, 0.09; 0.8, 0.8) (0.03, 0.04, 0.06, 0.06; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))	((0.02, 0.03, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))

Table 10: Fuzzy best value and fuzzy worst value.

	Fuzzy best value	Fuzzy worst value
CR_1	((0.02, 0.03, 0.03, 0.04; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))	((0.01, 0.02, 0.02, 0.03; 0.8, 0.8) (0.01, 0.01, 0.01, 0.02; 1, 1)), ((0.00, 0.00, 0.01, 0.01; 0.8, 0.8) (−0.00, 0.00, 0.01, 0.01; 1, 1))
CR_2	((0.05, 0.06, 0.08, 0.08; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.06, 0.06, 0.08, 0.08; 0.8, 0.8), (0.03, 0.04, 0.05, 0.05; 1, 1))	((0.02, 0.03, 0.04, 0.04; 0.8, 0.8) (0.01, 0.01, 0.02, 0.03; 1, 1)), ((0.05, 0.05, 0.07, 0.08; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1))
CR_3	((0.05, 0.06, 0.07, 0.07; 0.8, 0.8) (0.02, 0.03, 0.04, 0.04; 1, 1)), ((0.03, 0.03, 0.04, 0.04; 0.8, 0.8) (0.02, 0.02, 0.03, 0.03; 1, 1))	((0.01, 0.02, 0.03, 0.03; 0.8, 0.8) (0.00, 0.01, 0.01, 0.02; 1, 1)), ((0.02, 0.02, 0.03, 0.03; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))
CR_4	((0.07, 0.08, 0.09, 0.09; 0.8, 0.8) (0.04, 0.05, 0.06, 0.06; 1, 1)), ((0.02, 0.03, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))	((0.05, 0.06, 0.08, 0.08; 0.8, 0.8) (0.02, 0.04, 0.05, 0.05; 1, 1)), ((0.01, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1))
CR_5	((0.02, 0.03, 0.03, 0.03; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.02, 0.02, 0.02; 1, 1))	((0.01, 0.02, 0.02, 0.02; 0.8, 0.8) (0.01, 0.01, 0.02, 0.02; 1, 1)), ((0.01, 0.01, 0.01, 0.01; 0.8, 0.8) (0.00, 0.01, 0.01, 0.02; 1, 1))

6. Analysis and Comparison

Prior to the comparisons, questionnaires were circulated to all DMs asking for their opinions and judgments about Malaysia's best strategies to enhance water supply security. Based on the nature of each method, different linguistic scales were used to set up the questionnaires. There were the same five DMs involved in the trials, and their opinions were constructed based on their experience. To visualize the agreement, we have used different methods: (1) extended IT2FVIKOR, (2) IT2FVIKOR with equitable linguistic scale only, (3) IT2FVIKOR with fuzzy entropy only, and (4) Interval Type-2 Trapezoidal Fuzzy VIKOR (IT2TrFVIKOR). Results for each comparison are listed in Table 15.

Considering the results from Table 15, it can be concluded that the output from the extended IT2FVIKOR is comparable with the output from the other methods. Results seem to be slightly different. However, the order of each alternative is still bearable. Thus, it shows that the extended IT2FVIKOR can become another alternative methods for handling uncertainty and unbalanced problems. They have a high chance of handling uncertainty due to the advantages of the proposed equitable linguistic scales, Z-Numbers, and fuzzy entropy. Furthermore, the proposed equitable

Table 11: $\widetilde{M}_i$ and $\widetilde{N}_i$.

	$\widetilde{M}_i$	$\widetilde{N}_i$
AL_1	((0.05, 0.05, 0.06, 0.08; 0.8, 0.8) (0.04, 0.03, 0.04, 0.05; 1, 1)), ((0.05, 0.05, 0.04, 0.04; 0.8, 0.8) (0.04, 0.03, 0.03, 0.02; 1, 1))	((0.04, 0.04, 0.04, 0.04; 0.8, 0.8) (0.02, 0.02, 0.02; 1, 1)), ((0.03, 0.03, 0.03, 0.03; 0.8, 0.8) (0.02, 0.02, 0.02, 0.02; 1, 1))
AL_2	((0.11, 0.12, 0.13, 0.13; 0.8, 0.8) (0.06, 0.06, 0.07, 0.07; 1, 1)), ((0.09, 0.10, 0.13, 0.17; 0.8, 0.8) (0.08, 0.06, 0.07, 0.07; 1, 1))	((0.08, 0.08, 0.08, 0.08; 0.8, 0.8) (0.04, 0.04, 0.04, 0.04; 1, 1)), ((0.09, 0.09, 0.09, 0.09; 0.8, 0.8) (0.05, 0.05, 0.05, 0.05; 1, 1))
AL_3	((0.11, 0.11, 0.11, 0.11; 0.8, 0.8) (0.06, 0.06, 0.06, 0.06; 1, 1)), ((0.06, 0.06, 0.06, 0.07; 0.8, 0.8) (0.04, 0.04, 0.05, 0.05; 1, 1))	((0.07, 0.07, 0.07, 0.07; 0.8, 0.8) (0.04, 0.04, 0.04, 0.04; 1, 1)), ((0.05, 0.05, 0.05, 0.05; 0.8, 0.8) (0.03, 0.03, 0.03, 0.03; 1, 1))
AL_4	((0.16, 0.16, 0.16, 0.15; 0.8, 0.8) (0.10, 0.10, 0.10, 0.08; 0.8, 0.8)), ((0.06, 0.06, 0.06, 0.05; 0.8, 0.8) (0.04, 0.03, 0.04, 0.03; 1, 1))	((0.09, 0.09, 0.09, 0.09; 0.8, 0.8) (0.06, 0.06, 0.06, 0.06; 1, 1)), ((0.03, 0.03, 0.03, 0.03; 0.8, 0.8) (0.02, 0.02, 0.02, 0.02; 1, 1))
AL_5	((0.12, 0.12, 0.12, 0.11; 0.8, 0.8) (0.09, 0.08, 0.09, 0.07; 1, 1)), ((0.08, 0.08, 0.08, 0.08; 0.8, 0.8) (0.07, 0.07, 0.08, 0.06; 1, 1))	((0.03, 0.03, 0.03, 0.03; 0.8, 0.8) (0.02, 0.02, 0.02, 0.02; 1, 1)), ((0.02, 0.02, 0.02, 0.03; 0.8, 0.8) (0.02, 0.02, 0.02, 0.02; 1, 1))
AL_6	((0, 0, 0, 0; 0.8, 0.8) (0, 0, 0, 0; 1, 1)), ((0, 0, 0, 0; 0.8, 0.8) (0, 0, 0, 0; 1, 1))	((0, 0, 0, 0; 0.8, 0.8) (0, 0, 0, 0; 1, 1)), ((0, 0, 0, 0; 0.8, 0.8) (0, 0, 0, 0; 1, 1))

Table 12: Defuzzification of $\widetilde{M}_1$ and $\widetilde{N}_i$.

	$\widetilde{M}_1$	$\widetilde{N}_i$
AL_1	(0.05) (0.04)	(0.03) (0.02)
AL_2	(0.09) (0.09)	(0.06) (0.06)
AL_3	(0.08) (0.05)	(0.05) (0.04)
AL_4	(0.12) (0.04)	(0.07) (0.02)
AL_5	(0.09) (0.07)	(0.03) (0.02)
AL_6	(0) (0)	(0) (0)

Table 13: $\widetilde{M}^*$, $\widetilde{M}^-$, $\widetilde{N}^*$, and $\widetilde{N}^-$.

$\widetilde{M}^*$	(0) (0)
$\widetilde{M}^-$	(0.12) (0.09)
$\widetilde{N}^*$	(0) (0)
$\widetilde{N}^-$	(0.07) (0.06)

Table 14: The rating of P_i and rank of each alternative.

m	$\widetilde{P}_i$	P_i	Rank
AL_1	(0.38) (0.38)	0.38	2
AL_2	(0.76) (1)	0.88	6
AL_3	(0.68) (0.57)	0.63	4
AL_4	(1) (0.40)	0.70	5
AL_5	(0.58) (0.54)	0.56	3
AL_6	(0) (0)	0	1

Table 15: Comparative analysis.

Methods	Results
The extended IT2FVIKOR	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
IT2FVIKOR with equitable linguistic scale only	$AL_5 > AL_6 > AL_1 > AL_3 > AL_2 > AL_4$
IT2FVIKOR with fuzzy entropy only	$AL_5 > AL_6 > AL_2 > AL_1 > AL_4 > AL_3$
IT2TrFVIKOR	$AL_5 > AL_6 > AL_2 > AL_1 > AL_3 > AL_4$

linguistic scales, Z-Numbers, and fuzzy entropy embedded inside the extended IT2FVIKOR are believed to be able to evaluate uncertainty, vagueness, and conflict in line with the previous method. This approach proved that it could handle the concept of rating and weighting when it comes to different shapes, negative type, and objective type. Besides, it is not compensative to each other due to the concept of the ranking phase. Besides, this proposed method can model the variation of decision-making problems demonstrated by various DMs' opinions. This led to providing a clearer agreement with human evaluations associated to Type-1 and Interval Type-2 Fuzzy Systems. It was found that the

Table 16: Ranking of alternatives in sensitivity runs when group utility (v) varies from 0.1 to 1.0.

v	Alternatives						Rank
	AL_1	AL_2	AL_3	AL_4	AL_5	AL_6	
0.1	0.3786	0.8762	0.6312	0.6960	0.3836	0	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
0.2	0.3796	0.8741	0.6310	0.7038	0.4281	0	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
0.3	0.3806	0.8721	0.6308	0.7116	0.4727	0	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
0.4	0.3816	0.8701	0.6305	0.7195	0.5173	0	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
0.5	0.3826	0.8681	0.6303	0.7273	0.5619	0	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
0.6	0.3836	0.8661	0.6301	0.7351	0.6064	0	$AL_6 > AL_1 > AL_5 > AL_3 > AL_4 > AL_2$
0.7	0.3846	0.8640	0.6298	0.7429	0.6510	0	$AL_6 > AL_1 > AL_3 > AL_5 > AL_4 > AL_2$
0.8	0.3856	0.8620	0.6296	0.7506	0.6956	0	$AL_6 > AL_1 > AL_3 > AL_5 > AL_4 > AL_2$
0.9	0.3866	0.8600	0.6294	0.7586	0.7401	0	$AL_6 > AL_1 > AL_3 > AL_5 > AL_4 > AL_2$
1.0	0.3876	0.8580	0.6292	0.7664	0.7847	0	$AL_6 > AL_1 > AL_3 > AL_4 > AL_5 > AL_2$

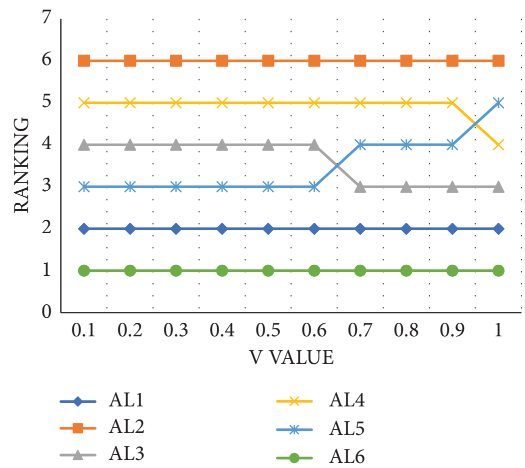

Figure 6: Results of sensitivity analysis.

subjective and qualitative factors in water supply security selection can merge with the proposed model to handle different opinions and factors of DMs. The positive IT2FE and negative IT2FE are good for narrow domains of conflict in DMs' decisions. Essentially, the positive IT2FE and negative IT2FE have been created by the intersection of the positive and negative sides to cope with the positive and negative aspects of water supply security selection. Besides, the fuzzy entropy weight-based IT2FS was proposed for the objective weights instead of subjective weights and considered all criteria. By using this approach, the subjective weighting process can be significantly simplified, and a more consistent weighting outcome can be achieved [54]. Therefore, we can say that the proposed approach gives a new dimension in the IT2MCDM area, which is particularly important for facilitating decision analysis in practical applications.

6.1. Sensitivity Analysis. The sensitivity analysis method is becoming progressively prevalent in a wide variety of engineering and science fields, including computational modeling, process simplification, and experimental data processing [55, 56]. A model's sensitivity can be described as how it depends on its input factors (whether numerical or otherwise) as described by Saltelli et al. [57]. It is a crucial step to perform sensitivity analysis for the FMCDM problems. Sensitivity analysis can help identify complicated systems' uncertainty [58]. An in-depth analysis of sensitivity and uncertainty in large-scale systems was made by Ionescu-Bujor and Cacuci [55].

Due to the imperfect and changeable input data of a FMCDM problem, DMs are eager to know how changes will affect their results and whether their decision results are balanced. Thus, sensitivity analysis is an effective tool for determining whether the results of a decision are stable. By using different weight assessment methods, Li et al. [59] computed the robustness of the water supply security selection towards the extended IT2FVIKOR by analyzing the sensitivity of the attribute weights. In order to determine how the valuation results would vary as the weights are altered, the sensitivity analysis is carried out on a water supply security study.

Table 16 indicates that the sort of water supply security selection did not vary until the unitary variation ratio reached 0.7. Figure 6 shows that the sort of the seven unitary variation ratios began to change at about $v = 0.7$, where the relative order for A_3 and A_5 was changed, and the rest were unchanged. It shows that the unitary variation ratio $v = 0.7$ did not cause much impact on the order of alternatives. However, when the unitary variation ratio reached 1.0, the order of water supply security selection significantly changed each other. This may be because it discriminates more among the alternatives. Besides, uncertainty may exist in the process of $v = 1.0$ and is more sensitive to the variation of input data.

Similarly, the results in Table 16 are illustrated in Figure 6.

In summary, these phenomena observed in Table 16 and Figure 6 reveal that small disturbances to all weights did not impact the assessment results. However, they started to change when the unitary variation ratio began to increase. At first, only two alternatives were slightly changed, and the rest remained unchanged. Until the unitary variation ratio begins to increase, three alternatives were changed, and other alternatives remained at the same rank. Therefore, an increase in the unitary variation ratio will alter the final assessment results. With the small unitary variation ratio, it can definitely be declared that the disruption to the weights of the parameter will not impact the valuation results.

The theoretical analysis and case study demonstrate that the extended IT2FVIKOR technique in water supply security selection is a reasonable and dependable method with regard to the sensitivity analysis to weights, and this research may be concluded accordingly. The final evaluation results can maintain a reasonable level of stability within the target range while maintaining a rather high sensitivity to weight variation. Furthermore, the model's sensitivity and the case study's validity are enhanced for credibility, showing that the approach is useful and is sufficient for accurately representing the situation for the purpose targeted.

7. Conclusion

In this research, we extend an IT2FVIKOR that thoroughly evaluates a group of DMs based on the synthesis of DM's preferences and opinions. When dealing with ambiguity and imprecision, the theory of the equitable linguistic scale and Z-Numbers are excellent tools for the IT2FVIKOR model. These equitable linguistic scales reflects exactly what the DM's mean, which focuses on the balance of the two sides. With the use of this, DM articulated his or her wants, needs, personality quirks, values, life experiences, and subjective assessments using the equitable linguistic scale. This equitable linguistic scale has demonstrated to be more consistent with people's tendency to make choices and act in accordance with their perceptions of reality. It avoids the single-sided judgment bias in the evaluation process because in real-life problems, positive and negative sides exist together, and these opposite sides can describe simultaneously a real condition of the alternatives [60]. Moreover, it allows to describe and assess the uncertainties among all DMs' opinions and judgments, resolving conflicts among various personal preferences with various alternatives and attributes. This work proposes Z-Numbers to incorporate the recommended equitable linguistic scale to create a concerted decision environment in IT2FVIKOR. The Z-Number theory was used considering the restriction and reliability decisions of the DMs. Using these equitable linguistic scales and Z-Numbers, the proposed method can produce the optimum decision based on the agreement of a group of DMs. Furthermore, it allows DMs to clearly convey their rational knowledge decisions using the language used to illustrate solutions. Moreover, this study also considers the objective weight in the weighting process. A relatively consistent weighting outcome can be achieved by using entropy weights to reduce decision-making burdens. Aside from being computationally simple, the entropy weight concept is also rational and understandable. It ensures that the outcome of the evaluation is not affected by interdependencies and inconsistencies in subjective weights [21]. In this way, we can escape the subjectivity of the DM's personal bias and verify the objectivity.

We have conducted experiments on solving the water security problems in Malaysia to evaluate six different strategies to find the best ways to enhance water supply security in Malaysia towards the five main criteria. Result shows that strengthening the protection of water source areas is ranked first, reinforcing groundwater monitoring and protections is ranked second, developing water-saving system is ranked third, and implementing the river chief system is ranked fourth. Improving infrastructure to safeguard urban and rural water security is ranked last. The proposed IT2FVIKOR seems to have better agreed with the DMs' decision than the existing IT2FVIKOR. It can be concluded that the output from the extended IT2FVIKOR is comparable with the output from the other methods. Results seem to be slightly different. However, the order of each alternative is still bearable. The extended IT2FVIKOR offers an alternative way of selecting the best water supply security selection. It allows the "water supply security selection" case study to have a better ranking via dealing with human and non-human factors. The new ranking values clearly show the best water supply security selection. The models successfully deal with vagueness and uncertainty of the data information provided by the DMs because it uses IT2FSs concept, equitable linguistic scales, Z-Numbers, and fuzzy entropy. Thus, it shows that the extended IT2FVIKOR can become another alternative method for handling uncertainty and unbalanced problems. The findings unequivocally demonstrate the theory behind the suggested model's ability to assess ambiguity, conflict, and uncertainty. We provided sensitivity analysis in addition to comparative analysis to evaluate the effectiveness of the extended IT2FVIKOR with regard to the problem of water supply security selection. It is a good platform to study how sensitive this model is towards the different weights' values. It offers a more confident model of the extended IT2FVIKOR for the water supply security selection after checking the model's efficiency via the sensitivity analysis. We intend to use universal type-2 membership functions in our upcoming work to increase the level of uncertainty in the decision-making process. The overall type-2 fuzzy application combines the different group DMs' viewpoints into a single endorsement that represents the uncertainty distribution (in the third dimension) related to the DMs. The use of general type-2 is anticipated to improve the levels of agreement between group DM choices and the IT2FTOPSIS system. The control decision system's ability to simulate a group of human decisions in the choice of water supply security increases with the agreement value.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interests.

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